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Theory of Double-Sided Flux Decorations

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Abstract

A novel two-sided Bitter decoration technique was recently employed by Yao et al. to study the structure of the magnetic vortex array in high-temperature superconductors. Here we discuss the analysis of such experiments. We show that two-sided decorations can be used to infer *quantitative* information about the bulk properties of flux arrays, and discuss how a least squares analysis of the local density differences can be used to bring the two sides into registry. Information about the tilt, compressional and shear moduli of bulk vortex configurations can be extracted from these measurements.

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I. INTRODUCTION

The nature of the ordering of the magnetic flux-line array in the mixed state of high-temperature superconductors is a topic of much current theoretical and experimental interest. Most direct measurements of the microscopic structure of the flux array have been obtained via Bitter decoration experiments at low fields [1]- [3]. The conventional Bitter technique employs small magnetic particles to decorate the tips of individual vortices as they emerge at one of the sample surfaces. Since in the high-T_c superconductors the vortices can wander considerably in the transverse direction as they traverse the sample [4] and in addition the intervortex interaction at the surface differs from that in bulk [5,6], it is difficult to unambigously infer bulk properties of flux arrays from conventional one-sided decorations that measure two-dimensional correlations of flux-line tips at the surface. Quantitative information on the three-dimensional structure of the vortex lines as they traverse the sample can be obtained via neutron scattering, but only very few such measurements have been carried out to-date due to their difficulty and cost. In addition neutron scattering is usually feasible only at much higher fields than probed by decorations. Very recently Yao et al. [7] used a novel two-sided decoration technique to study vortex structure in single crystals of BSCCO. These authors have simultaneously decorated both sides of the sample and analyzed how the two-dimensional translational and orientational order of the vortex array propagates across its thickness. Such "flux transmission spectroscopy" experiments are likely to become an important source of insight about vortex matter in the future. In this paper we discuss the analysis of these experiments and show how two-sided decorations can be used to infer *quantitative* information on bulk properties of the flux array. In principle all three bulk elastic constants of the flux array, the compressional, tilt and shear moduli, can be extracted from these measurements.

Much of the analysis described below was originally carried out for flux arrays in a liquid phase [4], [8]- [11]. This case may be relevant to the decoration experiments that are field-cooled below the irreversibility line. Because of long relaxation times, the observed flux patterns do not represent the equilibrium configuration of the vortices at the low temperature where the decoration takes place, but may be better approximated by the equilibrium configuration at a higher temperature T_f where the flux flux array falls out of equilibrium. The value of T_f is not known, but it is estimated to be very close to the experimentally observed irreversibility line. Depending on the field strength, the flux array may be in a crystalline or liquid-like state when it drops out of equilibrium. We summarize here results for liquid, hexatic and crystalline vortex arrays. We also discuss the extent to which configurations on opposite sides of a sample may be brought into registry by a least square fit of the difference in the local vortex density.

We discuss quenched random disorder here only for point pinning and weak surface disorder in the flux liquid. The effects of bulk point pinning at the elevated temperatures of the low fields irreversibility line are then weak because the impurity potential in thick samples is screened out by thermally induced vortex collisions [9]. It should be straightforward, however, to extend much of the analysis summarized here to other types of bulk and surface pinning in crystalline, hexatic and liquid phases. Strong surface disorder could certainly obscure the interpretation of double-sided decorations. If surface pinning is not a factor, it woud be especially interesting to consider the effect of *correlated* disorder, in the form of columnar pins, either parallel [12] or splayed [13], which pass completely through the sample. The key experimental question in this case is whether vortices always track a single column as they traverse the sample, or if they hop from column to column. This question plays a particularly important role in theories of vortex transport in the presence of splayed defects [13].

II. TRANSMISSION OF DENSITY FLUCTUATIONS

Density fluctuations of flux lines in three dimensions are described by the correlation function of a coarse-grained areal density field,

$$n(\mathbf{r}_\perp, z) = \sum_{i=1}^{N} \delta(\mathbf{r}_\perp - \mathbf{r}_i(z)), \tag{1}$$

where $\mathbf{r}_i(z)$ is the position of the i-th vortex in the (x, y) plane as it wanders along the $\hat{\mathbf{z}}$ ($\hat{\mathbf{z}} \parallel \mathbf{H}$) axis. In a sample of thickness L in the field direction and cross-sectional area A, translational correlations between the two opposite surfaces of the sample are described by

$$n_0 S(q_\perp, L) = \frac{1}{A}\Big[\overline{< \delta n(\mathbf{q}_\perp, L)\ \delta n(-\mathbf{q}_\perp, 0) >} - \overline{< \delta n(\mathbf{q}_\perp, L) >< \delta n(-\mathbf{q}_\perp, 0) >}\Big], \tag{2}$$

where $\delta n(\mathbf{q}_\perp, z) = n(\mathbf{q}_\perp, z) - n_0 A \delta_{\mathbf{q}_\perp, \mathbf{0}}$ denotes the fluctuation of the in-plane Fourier transform of the coarse-grained flux-line density from its equilibrium value $n_0 = B/\phi_0$. A factor of n_0 has been extracted in the definition of the structure factor so that $S(q_\perp, L) \to 1$ as $q_\perp \to \infty$. The angular brackets denote a thermal average and the overbar the average over quenched impurity disorder. The subtracted term on the right hand side of Eq. 2 vanishes in the absence of quenched disorder.

Almost thirty years ago Pearl [5] showed that the interaction between the tips of straight flux lines at a superconductor-vacuum interface decays as $1/r_\perp$ at large distances, with $r_\perp$ the distance between flux tips along the interface. In contrast, the interaction between flux-line elements in bulk decays exponentially at large distances. For this reason Huse argued that at low fields, where the intervortex separation is large compared to the penetration length, surface effects may play the dominant role in determining the magnetic flux patterns seen at the surface [6]. The question of the interplay between bulk and surface forces in determining the vortex structure at the surface was addressed by us [11] with a hydrodynamic model that incorporates the boundary condition on the flux-lines at the superconductor-vacuum interface - which is responsible for the $1/r_\perp$ interaction - as a surface contribution to the free energy of the flux array, coupled to the usual bulk free energy. This model neglects all spatial inhomogeneities in the z direction other than the presence of the sample boundaries. The thermal contribution to the structure factor defined in Eq. 2 was found to be

$$S_T(q_\perp, L) = S_{2T}(q_\perp) R(q_\perp, L), \tag{3}$$

where $S_{2T}(q_\perp)$ is the two-dimensional structure factor at one of the two surfaces and $R(q_\perp, L)$ measures the "transfer" of information about density fluctuations across the thickness of the sample. For the hydrodynamic model considered in Ref. [11] this transfer function is given by

$$R(q_\perp, L) = \frac{c_{11}(q_\perp)\xi_\parallel}{B_2(q_\perp)\sinh(L/\xi_\parallel) + c_{11}(q_\perp)\xi_\parallel \cosh(L/\xi_\parallel)}, \tag{4}$$

where $\xi_\parallel(q_\perp) = \sqrt{c_{44}(q_\perp)/c_{11}(q_\perp)}/q_\perp$ is the correlation length describing the decay of in-plane translational order in the z direction and $c_{11}(q_\perp)$ and $c_{44}(q_\perp)$ are the non-local compressional and tilt moduli of the bulk flux array, respectively. The nonlocality of the elastic constants in the z direction is negligible at low fields compared to the in-plane variation. Finally, $B_2(q_\perp)$ is the long wavelength compressional modulus of a $2d$ liquid of point vortices interacting via a $1/r_\perp$ potential at large distances, so that $B_2(q_\perp) \approx B^2/4\pi q_\perp$ as $q_\perp \to 0$. As discussed in [11], translational correlations at the surface are controlled by the $1/r_\perp$ surface interaction only for small wavevectors, such that $B_2(q_\perp) > c_{11}L$, or $q_\perp < {q_\perp}^s = q_\perp B_2/(Lc_{11})$. For $q_\perp > {q_\perp}^s$ the $2d$ surface structure factor is representative of that of a $2d$ cross-section of bulk and in the hydrodynamic model it is given by,

$$S_{2T}(q_\perp) \approx \frac{n_0 k_B T {q_\perp}^2}{c_{44}(q_\perp)\xi_\parallel^{-1}(q_\perp)} = \frac{n_0 k_B T}{\sqrt{c_{44}(q_\perp)c_{11}(q_\perp)}} q_\perp. \tag{5}$$

The transfer function $R(q_\perp, L)$ reduces then to

$$R(q_\perp, L) \approx [\cosh(L/\xi_\parallel)]^{-1} \approx 2e^{-L/\xi_\parallel(q_\perp)}, \tag{6}$$

where the second approximate equality holds provided $L >> \xi_\parallel$, i.e., if $q_\perp >> {q_\perp}^* = \sqrt{(c_{44}/c_{11})}/L$. If the elastic constants are calculated from Ginzburg-Landau theory [15], one finds ${q_\perp}^s \approx {q_\perp}^* \approx 1/L$. On the other hand, there is evidence for a strong downward renormalization of the compressional modulus c_{11} from entropic effects at low fields [4,9], as discussed below. In contrast the tilt modulus is expected to be accurately given by the Ginzburg-Landau theory. As a result, ${q_\perp}^* >> 1/L$. If the surface bulk modulus B_2 is *not* renormalized, then ${q_\perp}^s = B^2/(4\pi L c_{11}) \sim {q_\perp}^* >> 1/L$ and surface effects control the surface translational order up to wavevectors of order ${q_\perp}^* \sim 10^2/L \sim 5\mu m^{-1}$, where we have used the parameters of the Yao et al. experiment with $L = 20\mu m$ [7]. On the other hand, we argue below that B_2 may also be renormalized downward by entropic effects due to coupling of the surface tips to line wander in the bulk. In this case we expect $B_2 \sim c_{11}^R/q_\perp$ and ${q_\perp}^s \approx 1/L << {q_\perp}^*$, so that the long-range surface interaction only controls surface translational correlations for wavevectors much smaller than those probed by the decoration experiments.

The hydrodynamic model is very useful for describing the long-wavelength properties of the vortex array, and it allows us to incorporate the nonlocal effects of the intervortex interaction which are known to be important in flux crystals over much of the temperature-field phase diagram. An alternative more microscopic description can be obtained via the mapping of flux lines onto the world lines of two-dimensional bosons [4]. In the boson language the correlation length $\xi_\parallel(q_\perp)$ is determined by the Bogoliubov excitation spectrum $\epsilon(q_\perp)$ of a weakly interacting superfluid, according to [4],

$$\xi_\parallel^{-1}(q_\perp) \rightarrow \frac{\epsilon(q_\perp)}{k_BT} = \sqrt{\frac{n_0V_0}{\tilde{\epsilon}_1}{q_\perp}^2 + \Big(\frac{k_BT{q_\perp}^2}{2\tilde{\epsilon}_1}\Big)^2}, \tag{7}$$

where $V_0 = \phi_0^2/4\pi = 4\pi\lambda^2\epsilon_0$ is the energy scale of the bare intervortex interaction and $\tilde{\epsilon}_1$ is the single vortex tilt energy per unit length, with where λ is the penetration length in the ab plane. Although $\tilde{\epsilon}_1 \approx (M_\perp/M_z)\epsilon_0 \ln\kappa << \epsilon_0$ for fields $B >> \phi/\lambda^2$ ($M_\perp/M_z$ is the effective mass ratio), for $B \simeq \phi/\lambda^2$ (the regime relevant to decoration experiments) we have $\tilde{\epsilon}_1 \approx \epsilon_0$ because of magnetic couplings between the CuO_2 planes [14]. The result obtained from the analysis for the boson liquid agrees with the hydrodynamic result at small $q_\perp$, provided we make the identification $c_{44} = n_0\tilde{\epsilon}_1$ and $c_{11} = n_0^2V_0$.

Translational order can be quite sensitive to point disorder, which is present in all experimental samples. The question of whether the flux patterns seen in decoration experiments are controlled by quenched disorder or by thermal fluctuations is at present open. Weak point disorder both in the bulk and at the surface of the sample can be incorporated in the hydrodynamic model as a random potential with short-range correlations coupled to the vortex density [9]. Bulk point disorder yields an additive Lorentzian squared correction to the thermal three-dimensional structure factor. Neglecting surface effects, the disorder contribution to the structure factor defined in Eq. 2 is given by [11]

$$S_D(q_\perp, L) = S_{2D}(q_\perp)(1 + L/\xi_\parallel)e^{-L/\xi_\parallel}, \tag{8}$$

where $S_{2D}(q_\perp)$ is the quenched-disorder contribution to the two-dimensional structure factor at one of the surfaces,

$$S_{2D}(q_\perp) = n_0 \Delta_B \xi_\parallel \Big(\frac{{q_\perp}^2 \xi_\parallel}{2\tilde{\epsilon}_1}\Big)^2 \approx \Delta_B \frac{n_0}{2c_{44}} \Big(\frac{c_{44}}{c_{11}}\Big)^{3/2} q_\perp, \tag{9}$$

and Δ_B is the correlator of the bulk random impurity potential. The total structure function is $S(q_\perp, L) = S_T(q_\perp, L) + S_D(q_\perp, L)$. The transmittance of translational order across the sample is governed by the same length scale $\xi_\parallel(q_\perp)$ as in the thermal case independent of the strength of the quenched disorder. Weak surface disorder yields a contribution $S_{2SD}(q_\perp)$ to the two-dimensional surface structure factor that vanishes as ${q_\perp}^2$ at small wavevectors and can therefore be distinguished from the other contribuutions [11].

Double-sided decoration experiments of the type carried out by Yao et al. [7] can measure both the two-dimensional structure factor $S_2(q_\perp)$ at one of the two surfaces, as well the correlations across the thickness of the sample described by $S(q_\perp, L)$. The transfer function $R(q_\perp, L)$ and then the "excitation spectrum" $\epsilon(q_\perp)/k_B T$ are obtained from the ratio of these two correlation functions. Since the sample thickness is known, the slope of the Bogoliubov spectrum at small $q_\perp$ yields a measurement of the ratio $\sqrt{c_{11}/c_{44}}$. Yao et al. find $c_{11}/c_{44} \approx 1.5 \times 10^{-4}$ in BSCCO single crystals at $12G$, a value about four orders of magnitude smaller than predicted from the Ginzburg-Landau mean field theory [15]. If one assumes that the main contribution to $S_2(q_\perp)$ is the thermal one given by Eq. 5 and that the flux array falls out of equilibrium near the irreversibilty temperature T_{irr}, i.e., $T_f \approx T_{irr}$, one can also extract the geometric mean of the two elastic constants from the linear slope of $S_2(q_\perp)$ [11]. This gives $c_{44} \approx 27G^2$ and $c_{11} \approx 6 \times 10^{-3} G^2$ [7]. The value of c_{44} is essentially equal to $B^2/4\pi$ and is consistent with $c_{44} \approx n_0 \epsilon_0$, provided one uses the value of λ at the irreversibility line [17]. The expression for the nonlocal tilt modulus obtained from the Ginzburg-Landau theory can be found in the Appendix of Ref. [15]. If we use the expression for c_{44} that applies at low fields ($a_0 \leq \lambda$, but $q_\perp >> \sqrt{M_\perp/M_z}/\lambda$ so that nonlocal effects in the tilt modulus can be neglected), we obtain $c_{44} \approx n_0\epsilon_0/2$, consistent with the experimental measurement. The experimental value for c_{11} is about four order of magnitudes smaller than expected on the basis of Ginzburg-Landau theory, which neglects fluctuation effects.

To obtain an approximate understanding of the strong downward renormalization of the compressional modulus (or equivalently of the strength of the intervortex interaction), we recall that the Bogoliubov results can be made quantitatively accurate for dilute superfluids, provided the bare interaction potential is replaced by an effective interaction or "t-matrix" defined as the sum of an infinite series of ladder diagrams [16]. In a two-dimensional superfluid gas the renormalization of the $q_\perp = 0$ part of the intervortex interaction corresponds to the summation of a series in $1/\ln(1/n_0\lambda^2)$, where n_0 is the boson density and λ the range of the interaction, and leads to the replacement [4,9]

$$V_0 \rightarrow V_R = \frac{V_0}{1 + [V_0\tilde{\epsilon}_1/(k_BT)^2]\ln(1/n_0\lambda^2)/4\pi} \approx \frac{4\pi(k_BT)^2}{\tilde{\epsilon}_1 \ln(1/n_0\lambda^2)}. \tag{10}$$

The renormalized compressional modulus is then estimated as $c_{11}^R \approx n_0^2 V_R$. Substituting the material parameters appropriate to the experiments of Ref. [7], we find $c_{11}^R \sim 2 \times 10^{-4} G^2$. The experiments may not be in the limit of extreme dilution ($n_0\lambda^2 << 1$) required for the second line of Eq. 10, so it is not surprising that this result is even *lower* than the experimental value.

When $n\lambda^2 \geq \mathcal{O}(1)$ one can qualitatively expect an analogous downward renormalization to arise from entropic contributions to the free energy from vortex-line braiding [18]. In this dense limit each vortex line spends a certain "time", i.e., length along the z axis, in the tube or "cage" of radius a_0 provided by the repulsive interaction with its six neighbors. Flux lines wander within this cage until they escape to one of the approximately six neighboring cages. Collisions reduce the entropy of the interacting flux array relative to that of the noninteracting system. Escape events, which yield flux-line braiding, *increase*, however. the entropy, similar to the discussion of interstitial wandering in Ref. [18]. In the flux-line liquid, where escapes are frequent, the reduction in entropy due to collisions, or unsuccessful escapes, is a small correction. Each escape increases the entropy per vortex by $k_B \ln q$, with q an effective coordination number describing the different directions in which a vortex can hop. The average distance l_z between hops among lattice sites is given by $D_0 l_z \sim a_0^2$, with $D_0 = k_B T/\tilde{\epsilon}_1$ the vortex "diffusion constant" along $\hat{\mathbf{z}}$, or $l_z \approx \tilde{\epsilon}_1 a_0^2/k_B T$ [4]. In a sample of thickness L the total number of jumps is of order L/l_z and the corresponding entropic contribution to the Gibbs free energy per unit volume of the vortex array is $g_{ent} \approx -\frac{N}{AL} k_B T \frac{L}{l_z} \ln q = -\frac{(k_B T)^2}{\tilde{\epsilon}_1} n^2 \ln q$. The total Gibbs free energy per unit volume can be written as $g(n) \approx -n\frac{\phi_0}{4\pi}(H - H_{c1}) + g_{int}(n) + g_{ent}(n)$, where $g_{int} \approx \epsilon_0 n^2 \lambda^2$ is the contribution from intervortex interactions. Upon expanding about the minimum density to otain $c_{11} = (d^2 g/dn^2)|_{n=n_0}$, we see that the entropic contribution partially cancels the large contribution from interactions, consistent with experimental observations. The entropic and energetic contributions to c_{11} are comparable when $T \approx \sqrt{\epsilon_0 \tilde{\epsilon}_1}\lambda$, which is comparable to the melting temperature of the Abrikosov flux lattice in this field regime [18].

An analogous mechanism could lead to a strong downward renormalization of the surface interaction. This is because the flux tips at the sample surface are not true point vortices, but are connected to the flux lines in the bulk. Braiding effects of the type described above within a surface layer of thickness $\xi_z(q_\perp) \sim \sqrt{c_{44}/c_{11}}/q_\perp$ will increase the surface entropy of the flux-tips, yielding a contribution $g_{ent}^s \approx -k_B T n(\xi_z/l_z) \ln q \approx -\frac{(k_B T)^2}{\sqrt{\tilde{\epsilon}_1 V_0} q_\perp} n^{3/2} \ln q$ to the free energy per unit area, where we have used $c_{44} \sim n\tilde{\epsilon}_1$ and $c_{11} \sim n^2 V_0$. The corresponding free energy from surface interaction among the flux tips is $g_{int} \sim n^2 \frac{\phi_0^2}{4\pi q_\perp}$. Provided $\tilde{\epsilon}_1 \sim \epsilon_0$, as is appropriate for this low field regime, the two contributions to the energy (and hence to $B_2^R(q_\perp)$) are again comparable near the melting temperature.

The finite range of the intervortex interaction can be incorporated in the derivation of the Bogoliubov spectrum, which is then given by Eq. 7, with $V_0 \to V(q_\perp) = V_0/(1+{q_\perp}^2\lambda^2)$ [9,10]. The second term in Eq. 7 is unchanged since it represents the "kinetic energy" contribution to the spectrum, which is unrenormalized due to Galileian invariance of the equivalent boson problem. The full wavevector-dependent interaction $V(q_\perp)$ is again renormalized by resumming an infinite series of ladder diagrams, which leads to an integral equation for the effective t-matrix at finite wavevector [19]. Upon neglecting corrections logarithmic in the wavevector, we find that the screening length λ is not renormalized and one obtains $V_R(q_\perp) = V_R/(1 + {q_\perp}^2\lambda^2)$. The Bogoliubov spectrum can then be rewritten in a suggestive form that interpolates between the boson result and the hydrodynamic description as

$$\xi_\parallel^{-1}(q_\perp) \to \frac{\epsilon(q_\perp)}{k_BT} = \sqrt{\frac{c_{11}^R(q_\perp)}{c_{44}}{q_\perp}^2 + \Big(\frac{k_BT{q_\perp}^2}{2\tilde{\epsilon}_1}\Big)^2}, \tag{11}$$

where we have identified the renormalized local compressional modulus as $c_{11}^R(q_\perp) = n_0^2 V_R(q_\perp)$. The nonlocality of the tilt modulus is not important for the low fields of interest here [15], as confirmed by the experimental finding that $c_{44} \approx n_0\tilde{\epsilon}_1$. The renormalized Bogoliubov spectrum given in Eq. 11 is shown in Fig. 1.

The Bogoliubov spectrum is not expected to be quantitatively accurate for dense superfluids. In this regime the theory can be improved following Feynman and approximating this spectrum by

$$\frac{\epsilon(q_\perp)}{k_BT} = \frac{k_BT{q_\perp}^2}{2n_0\tilde{\epsilon}_1 S_2(q_\perp)}, \tag{12}$$

where $S_2(q_\perp)$ is the structure factor of a two-dimensional cross section of a dense vortex liquid. Its thermal contribution in hydrodynamic theory is given in Eq. 5. For more realistic functions $S_2(q_\perp)$, this formula leads to a "roton" minimum in the excitation spectrum at $q_\perp \approx k_{BZ} = \sqrt{4\pi n_0}$, at approximately the position of the first maximum in $S_2(q_\perp)$. This "roton" minimum has been observed in the experiments [7].

III. TRANSLATIONAL AND ROTATIONAL REGISTRY

A source of uncertainty arises in the experiments from the difficulty in matching the (x, y) locations of vortices being imaged on the two sides of the sample. This positional uncertainty can be decreased when the sample contains localized defects that run all the way across the sample, such as grain boundaries, since these can provide a common reference frame on the two sides [7]. Correlations on different length scales are in general affected differently by this mismatch. To quantify this effect for a given sample we define the *rms* density fluctuations arising from a translational mismatch d and an orientational mismatch ϕ averaged over the area A of the sample, as

$$\Delta(d,\phi,L) = \int \frac{d^2\mathbf{r}_\perp}{A} < [\delta n(\mathcal{R}_\phi \cdot \mathbf{r}_\perp + \mathbf{d}, L) - \delta n(\mathbf{r}_\perp, 0)]^2 >, \tag{13}$$

where $\mathcal{R}_\phi$ is a two-dimensional rotation matrix and we neglect effects due to quenched random disorder. We expect $\Delta(d,\phi,L)$ to be a minimum when the patterns on the two sides are defined relative to (x,y) coordinate systems with a common origin and orientation. Minimizing Δ with respect to d and ϕ using experimental data could be used to bring these coordinate systems into registry even in the absence of identifying features such as grain boundaries which penetrate across the entire crystal. Upon introducing the Fourier components of the density, Eq. 13 can be rewritten using the single-pole approximation in q-space for the structure factor,

$$\hat{S}(\mathbf{q}_\perp, q_z) =< |\delta\hat{n}(\mathbf{q}_\perp, q_z)|^2 >= \frac{n_0^2 k_B T q_\perp^2/c_{44}}{q_z^2 + [\epsilon(q_\perp)/k_B T]^2}, \tag{14}$$

as,

$$\Delta(d,\phi,L) = \frac{n_0^2 k_B T}{2\pi c_{44}} \int_0^{k_{BZ}} dq_\perp {q_\perp}^3 \frac{k_B T}{\epsilon_R(q_\perp)} \times \Big[1 - J_0(q_\perp d) \frac{J_1(2Rk_{BZ}\sin(\phi/2))}{Rk_{BZ}\sin(\phi/2)} e^{-L\epsilon_R(q_\perp)/k_B T}\Big], \tag{15}$$

where $J_0(x)$ and $J_1(x)$ are Bessel functions and R denotes the linear dimensions of the sample in the ab plane, with $A = \pi R^2$. The integral on the right hand side of Eq. 15 diverges and is cutoff by a circular Brillouin zone, $k_{BZ} = \sqrt{4\pi n_0}$. We have evaluated the dimensionless quantity $\tilde{\Delta}(d,\phi,L) = [\Delta(d,\phi,L) - \Delta(0,0,L)]/\Delta(0,0,L)$ using the hydrodynamic approximation $\epsilon_R(q_\perp)/k_B T = q_\perp\sqrt{c_{11}^R(q_\perp)/c_{44}}$. The function $\tilde{\Delta}(d,\phi,L)$ is shown in Figs.

2 as a function of both the angle ϕ and the translation d for a few values of the sample thickness. The function $\Delta(d,\phi,L)$ has a parabolic minimum at $d=0$, $\phi=0$, according to

$$\Delta(d,\phi,L)\approx\Delta(0,0,L)\Big\{1+\frac{1}{2}\alpha(L)\Big[d^2k_{BZ}^2/2+R^2k_{BZ}^2\sin^2(\phi/2)\Big]\Big\}, \tag{16}$$

where the dimensionless curvature $\alpha(L)$ is given by

$$\alpha(L)=\frac{\int_0^1 dx x^2\sqrt{1+x^2\lambda^2k_{BZ}^2}\,e^{-L^*k_{BZ}\frac{x}{\sqrt{1+x^2\lambda^2k_{BZ}^2}}}}{\int_0^1 dx x^2\sqrt{1+x^2\lambda^2k_{BZ}^2}\Big[1-e^{-L^*k_{BZ}\frac{x}{\sqrt{1+x^2\lambda^2k_{BZ}^2}}}\Big]}, \tag{17}$$

and $L^*=L\sqrt{c_{11}(q_\perp=0)/c_{44}}$. At low density, corresponding to $\lambda k_{BZ}<<1$, the behavior of the curvature is controlled by L^*k_{BZ}, with $\alpha\sim 1/(L^*k_{BZ})$ for $L^*k_{BZ}<<1$ and $\alpha\sim 1/(L^*k_{BZ})^5$ for $L^*k_{BZ}>>1$. At high density, corresponding to $\lambda k_{BZ}>>1$, the relevant length scale is $L^/\lambda$ and $\alpha\sim\lambda/L^*$ for $L^*<<\lambda$ and $\alpha\sim 1/(L^*k_{BZ})^5$ for $L^*>>\lambda$.

IV. TRANSMISSION OF ORIENTATIONAL ORDER

From the analysis of double-sided decorations one can also study the propagation of orientational order across the sample. Orientational order is much less sensitive to point pinning [20]. It is measured by correlations in the bond-orientational order parameter $\psi_6(\mathbf{r})=e^{6i\theta(\mathbf{r})}$, where $\theta(\mathbf{r})$ is the bond-angle field. The corresponding angular correlation across the sample thickness is

$$\begin{aligned}G_H(\mathbf{r}_\perp,L)=&< e^{6i[\theta(\mathbf{r}_\perp,L)-\theta(0,0)]}>\\ &\approx\exp[-18<[\theta(\mathbf{r}_\perp,L)-\theta(0,0)]^2>].\end{aligned} \tag{18}$$

The decay of bond-orientational order in a hexatic flux liquid was discussed in Ref. [8] in the hydrodynamic limit. In a bulk sample, ignoring boundary conditions and surface effects, the in-plane Fourier transform of the thermal part of the correlation function of the bond-orientational order parameter was found to be given by

$$G_H(q_\perp,L)=<|\psi_6|^2>\Big[A\delta_{\mathbf{q}_\perp,\mathbf{0}}+G_{H2}(q_\perp)e^{-L/\xi_H(q_\perp)}\Big], \tag{19}$$

with

$$G_{H2}(q_\perp) = \frac{9k_BT}{K_A^z}\xi_H(q_\perp) = \frac{9k_BT}{\sqrt{K_A^z K_A^\perp}}\frac{1}{q_\perp}. \tag{20}$$

Here $\xi_H(q_\perp) = \sqrt{K_A^z/K_A^\perp}/q_\perp$ is the correlation length governing the transmittance of hexatic order across an hexatic flux-line liquid and K_A^z and$K_A^\perp$ are the hexatic stiffnesses.

In a superconducting slab of finite thickness L we use free boundary conditions on the bond-angle field at the surface to find that Eq. 19 is replaced by,

$$G_H(q_\perp, L) =< |\psi_6|^2 > \Big[A\delta_{\mathbf{q}_\perp,\mathbf{0}} + G_{H2}(q_\perp)R_H(q_\perp, L)\Big], \tag{21}$$

with $G_{H2}(q_\perp) = \frac{k_BT}{K_A^z}\xi_H(q_\perp)\coth(L/\xi_H)$ and $R_H(q_\perp, L) = [\cosh(L/\xi_H)]^{-1}$.

In a flux lattice with long-range crystalline order the hexatic order parameter is not independent, but is simply related to the curl of the elastic diplacement field, $\theta = \frac{1}{2}\hat{\mathbf{z}}\cdot(\vec{\nabla}\times\vec{u})$. The correlation function of the bond-angle field in an infinite sample then follows immediately, with the result,

$$G_H^L(q_\perp, L) =< |\psi_6|^2 > \Big[A\delta_{\mathbf{q}_\perp,\mathbf{0}} + G_{H2}^L(q_\perp)e^{-L/\xi_H^L(q_\perp)}\Big], \tag{22}$$

where $\xi_H^L(q_\perp) = \sqrt{\frac{c_{44}}{c_{66}}}/q_\perp$ is the correlation length governing transmittance of hexatic order across a flux lattice, and

$$G_{H2}^L(q_\perp) = \frac{9k_BT}{4c_{44}\xi_H^L(q_\perp)} = \frac{9k_BT}{4\sqrt{c_{66}c_{44}}}q_\perp. \tag{23}$$

The corresponding expressions in a finite-thickness sample with free boundary condition on the bond-angle field are modified with the same finite-size functions of $L/\xi_H^L(q_\perp)$ as in the case of the hexatic flux liquid. This result shows that by measuring the correlation of bond order across the sample, as well as $G_{H2}^L(q_\perp)$ at one of the surfaces, one can infer the value of the tilt and shear moduli. In addition, Eqs. 20 and 23 show that surface bond-orientational order decays as $1/q_\perp$ in a hexatic liquid, but grows as $q_\perp$ in a lattice, providing a further mean to distinguish hexatic and crystalline order in the vortex array.

Bond order decays exponentially in flux liquids which are isotropic in a plane perpendicular to the field direction. The results in this case are similar to Eq. 19, except that the delta-function term is absent and $\lim_{q_\perp \to 0} \xi_H(q_\perp)$ is finite and equal to the hexatic correlation length along the z direction.

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Figure Captions

Fig. 1. The renormalized Bogoliubov spectrum given in Eq. (11) as a function of wavevector for $n_0\lambda^2 = 0.2, 0.5, 1$.

Fig. 2. The spatially averaged mismatch function $\tilde{\Delta}(d, \phi, L)$ is shown (a) at $\phi = 0$ as a function of d and (b) at $d = 0$ as a function of ϕ, for three values of L. Note that the sample thickness only enters in the dimensionless combination $Lk_{BZ}\sqrt{c_{11}(q_\perp = 0)/c_{44}}$. We have used $B = 12G$, $c_{11}(q_\perp = 0)/c_{44} = 1.5 \times 10^{-4}$ and $R = 0.2mm$.