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Glassy Motion of Elastic Manifolds

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We discuss the low-temperature dynamics of an elastic manifold driven through a random medium. For driving forces well below the $T = 0$ depinning force, the medium advances via thermally activated hops over the energy barriers separating favorable metastable states. We show that the distribution of waiting times for these hopping processes scales as a power-law. This power-law distribution naturally yields a nonlinear glassy response for the driven medium, $v \sim \exp(-\text{const} \times F^{-\mu})$.

The dynamics of elastic manifolds in random media has received an intense attention during the last decade and is in a state of rapid current development. The reasons for this interest are twofold. On one hand, elastic manifolds in a random environment are one of the simplest examples of a glassy system and yet exhibit a very rich static and dynamical behavior. Secondly, they can be used as models to study both nonlinear collective transport in driven disordered systems, such as charge density waves, polymers, driven interfaces, dislocations in solids, and magnetic flux lines in type II superconductors, as well as stochastic kinetic processes, such as stochastic growth and kinetic roughening (see [1,2] for a review).

The dynamics of driven elastic manifolds is the result of the interplay between quenched disorder and the interaction among the many elastic degrees of freedom that compose the manifold. A key physical quantity is the average velocity v of the driven manifold as a function of the applied force F. At zero temperature there is a depinning transition from a pinned state where $v = 0$ to a sliding state at a critical driving force F_c. A finite temperatures washes out the transition and the mean velocity is then finite for all driving forces. For low temperatures and driving forces well below the $T = 0$ depinning threshold, F_c, the dynamics of the driven manifold is controlled by thermally activated jumps of correlated regions over the pinning energy barriers separating different metastable states. In this region the mean velocity is highly nonlinear and has been evaluated via a scaling approach [3], with the result,

$$v \simeq \exp[-U(F)/T], \qquad (1)$$

where $U(F)$ is the optimal energy barrier for creep between favorable metastable states. Under the action of the external drive, sections of the manifold that are initially pinned move to a more favorable metastable state, determined by the condition that the energy gain due to the driving force equals the elastic deformation and pinning energy of the medium. For $F \ll F_c$ large sections of the manifold hop long distances to find the next optimal-energy state. This yields a large energy barrier

that diverges algebraically with vanishing driving force, $U(F) \sim (F_c/F)^\mu$, with μ a characteristic exponent. The form given in Eq. 1 was first proposed for the motion of dislocations in crystals [3] and is now widely used to describe the low temperature dynamics of vortex lines in type-II superconductors [1].

An important recent development has been the confirmation by numerical studies of an elastic string in $d = 2$ of the basic assumption of the scaling approach [3] that the barriers between the metastable states scale with the length of the fluctuating string segment in the same way as the fluctuations in the free energy (aside from logarithmic corrections) [4]. Despite a substantial analytical [5–7] and numerical [8–13] study of the dynamics of driven disordered elastic manifolds, many open questions remain.

In this Letter we focus on the nature of the mechanism by which the manifold selects the appropriate optimal segment controlling the dynamics. For simplicity we address this question for the case of an elastic string (magnetic flux line or interface) driven through two-dimensional point disorder. The paper is organized as follows. First we briefly review the results of the scaling theory of the thermally activated motion of elastic manifolds in disordered media following [3]. We then develop a description of the avalanche-like string motion.

The Hamiltonian of a D-dimensional elastic manifold driven through a d-dimensional disordered medium is

$$\mathcal{H} = \int d^d x \left[\frac{C}{2} \left(\frac{\partial \mathbf{u}}{\partial \mathbf{x}} \right)^2 + U(\mathbf{x}, \mathbf{u}) - \mathbf{F} \cdot \mathbf{u} \right], \qquad (2)$$

where C denotes an elastic stiffness constant (e.g., the linear tension of the string; for simplicity we consider the isotropic medium) and $\mathbf{u}(\mathbf{x}, t)$ is the n-dimensional transverse displacement field of the manifold. Eq. (2) describes for instance an elastic string ($D = 1$) in two ($d = 2$, $n = 1$) and three ($d = 3$, $n = 2$) dimensions, and two- or three-dimensional vortex lattices ($d = 2$, $D = 2$, $n = 2$ and $d = 3$, $D = 3$, $n = 2$, respectively). The disorder consists of uncorrelated point defects of density n_i. It is described by a random potential $U(\mathbf{x}, \mathbf{u})$ of

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range ξ and variance $\Delta = v_0 \sqrt{n_i \xi^d}$, with v_0 the maximum depth of the potential well of a single pinning center. In the absence of driving force the string adjusts to the random landscape and traverses the medium along rough optimal paths determined by balancing the elastic and pinning energies. The geometry of these optimal paths is characterized by the roughness of the manifold, defined as $w(L) = \langle [\mathbf{u}(\mathbf{x} + \mathbf{L}) - \mathbf{u}(\mathbf{x})]^2 \rangle^{1/2}$, where $\langle ... \rangle$ denotes the average over both thermal fluctuations and quenched disorder. At large distances the roughness scales as $w \approx \xi (L/L_c)^\zeta$, where $\zeta < 1$ is the roughness exponent and L_c is the pinning correlation length. We consider the case of weak disorder such that $L_c \gg \xi$. In this regime domains of linear size $L_c = \xi \left(C\xi^D/\Delta \right)^{2/(4-D)}$ are pinned coherently by disorder when $F = 0$. The pinning length L_c is the smallest scale on which barriers between metastable states exist at $F = 0$. The minimum average energy barrier between neighboring metastable positions of a pinned segment L_c is $U_c = C\xi^2/L_c^{(D-2)}$. Optimal metastable configurations of sizes $L > L_c$ are then separated by barriers $U(L) \simeq U_c(L/L_c)^{2\zeta+D-2}$. At $T = 0$ the string starts to slide when the applied force F can depin a region of linear size L_c, yielding a threshold force for sliding $F_c = C\xi/L_c^2$.

We now consider the dynamics at a finite but low temperature $T \ll U_c$, where the elementary pinning scales U_c and L_c are not renormalized significantly by thermal fluctuations [1]. Under the action of the driving force a domain of size L can be displaced to a new more favorable metastable state. For $F \ll F_c$ the dynamics can be described as the nucleation of an elementary excitation or nucleus of size L. The free energy cost for creating such a nucleus is given by $\mathcal{F}[L] = U_c (L/L_c)^{2\zeta+D-2} - FL_c^D \xi (L/L_c)^{\zeta+D}$. The size L_{opt} of the optimal excitation is obtained by minimizing this free energy cost, with the result $L_{opt} = L_c (F_c/F)^{1/(2-\zeta)}$. Nuclei with $L < L_{opt}$ collapse, while nuclei of size $L > L_{opt}$ expand. In other words excitations on scales larger than L_{opt} slide freely, while pinning barriers on scales $L < L_{opt}$ can be overcome only via thermally activated hops. The length scale L_{opt} determines the upper bound above which thermally activated processes are no longer relevant. The creep rate is determined by the corresponding optimal energy barrier, $U(F) = \mathcal{F}(L_{opt})$, with $U(F) = U_c (F_c/F)^\mu$, where $\mu = (2\zeta + D - 2)/(2 - \zeta)$. For an elastic string in two dimensions $\zeta = 2/3$, $D = 1$, and $\mu = 1/4$.

We now show that the form given in Eq. (1) for the mean motion arises naturally when the driven manifold sequentially nucleates all possible excitations on length scales $L < L_{opt}$ to overcome all the possible energy barriers separating metastable states on scales $E < U(F)$. For the sake of simplicity we consider an elastic string in $2d$. We demonstrate that the distribution of waiting times $\tau(E)$ for hops between metastable states separated by energy barriers $E < U(F)$ scales as a *power-law*,

$$\Psi(\tau) \simeq \tau^{-1-\alpha}, \tag{3}$$

with $\alpha < 1$. This microscopic mechanism of string dynamics can be viewed as an avalanche-like motion. The distribution of waiting times given in (3) is cutoff at the waiting time $\tau[U(F)]$ corresponding to the optimal barrier $U(F)$. For hopping times τ exceeding $\tau[U(F)]$ only very rare barriers with $E \gg U(F)$ can retard the motion (rare events) and the distribution function decays much faster than in Eq. (3). The average waiting time that controls the mean string velocity is therefore determined by the cutoff $\tau[U(F)]$.

We begin by considering a segment of finite length $L < L_{opt}$ initially pinned in a metastable configuration. We derive an expression for the probability $P_L(E)$ that this segment encounters energy barriers smaller than E when it samples all possible departures from its initial configuration under the action of the external force F. For any finite driving force the set of all configurations that the segment may sample during its activated motion form a connected cluster of *temporarily* pinned states. To describe this cluster, we fix the ends of the segment L and imagine dividing it in elementary units of longitudinal size L_c. An elementary move consists of the hop of the i-th unit L_c across a transverse distance ξ to a new metastable position by overcoming the elementary pinning energy barrier U_i. Here the $\{U_i\}$'s are random variables fluctuating around their average value, the minimal *average* pinning barrier, U_c. The segment L can be thought of as a singly-connected necklace of n (unit) beads, where the beads corresponds to the elementary sections L_c. To begin, we consider the simplest class of departures of L from its initial configuration and describe the advance of L via the hopping of individual beads over the fluctuating barriers U_i. The barrier associated with the motion of the segment L is then the largest of the barriers of the elementary hops L_c. To understand this we note that the hop of a section L of the string to a nearby metastable state can be thought of as the motion of the domain boundary between spin up and spin down regions in an Ising spin system with random couplings $\{J_i\}$. Each elementary unit L_c corresponds to a single Ising spin. The hop of the i-th segment L_c over the corresponding energy barrier U_i corresponds to a single spin flip. The flip of spin i_0 along the domain boundary will cost an energy of order $J_{i_0-1} + J_{i_0+1}$, where J_{i_0-1} and J_{i_0+1} are the couplings to the neighboring spins. The flipping of this particular spin will facilitate the flipping of the neighboring ones, since now overturning spin $i_0 - 1$ will only cost an energy J_{i_0-2}, which may be larger or smaller than $J_{i_0-1} + J_{i_0+1}$. In either case the process of overturning the entire domain boundary will be controlled by the largest energy cost for flipping a single spin. If L is composed of n elementary segments L_c with energy barriers U_i randomly fluctuating about U_c, then the energy barrier for the segment L is $U = \max U_i$. It

is important to stress here that we are not trying to find the optimal hop for the segment L and the associated optimal energy barrier, but we are simply discussing the distribution of all possible hops of the segment L from the initial to all available final metastable states. In order to construct all possible configurations of the advancing segment, we now imagine redefining the network of elementary hops by choosing a new unit $L_1 > L_c$ so that the section L is now composed of, say, n_1 sub-blocks of size L_1 and fluctuating energy barriers U_{1i}. The set of all possible configurations of the excitation L as it visits all available metastable states forms a new cluster of elementary excitations or subclusters L_1. Notice that the subclusters of length L_1 may in general be multiply connected, but the argument given above will still apply upon redefinition of the elementary unit. The global barrier for the motion of the new cluster will again be the maximum of the energy barriers of each subcluster. The linear structure of the string ensures that the cluster of all possible configurations of L remains singly-connected at all levels of rescaling. It is precisely this linear topology that enables us to carry out this rescaling procedure. Repeated application of this procedure will allow us to list all possible configurations of the segment in questions and yield the limiting distribution that we seek. Such a distribution, if it exists, must be stable under the max operation and therefore belongs to the class of so-called *extreme distributions* [15]. To briefly summarize the theory of extreme distributions, we consider a set of identically distributed independent random variables X_i, with $1 \leq i \leq n$ - in the present problem these are all the energy barriers of the elementary spins and/or block of spins composing the segment L. Let $M_n = \max\{X_1, ..., X_n\}$. If $F(x)$ is the probability of the event $X_i < x$ and $\mathcal{P}_n(x)$ the probability that $M_n < x$, it can be shown that in the limit of large n the probability distribution of the maxima can be approximated by an asymptotic form $\mathcal{P}(x)$, given by the solution of the functional equation $\lim_{n\to\infty} \mathcal{P}_n(x+a_n) = \lim_{n\to\infty} F^n(x+a_n) = \mathcal{P}(x)$, where $\mathcal{P}(x)$ is defined for $-\infty < x < \infty$. The extreme distribution we seek has the form $\mathcal{P}(x) = \exp[-\exp(-x)]$ [15]. The functional form of the distribution must remain invariant when the elementary unit of the cluster of all possible hops is redefined according to the procedure described earlier. Since $[\mathcal{P}(x)]^n = \exp[-n\exp(-x)]$, this requires $a_n = \ln n$. It thus follows that for the problem of interest here the appropriate variable x is $E - \ln L$, where we measure E in units of U_c and L in units of L_c. The probability distribution of energy barriers for a given segment L is then $\mathcal{P}_L(E) = \exp[-L\exp(-E)]$. The corresponding distribution density $p_L(E) = d\mathcal{P}_L(E)/dE$ is

$$p_L(E) = Le^{-E}\exp[-L\exp(-E)]. \tag{4}$$

The typical barrier of a segment of length L scales then as $E \sim U_c \ln L$. The global distribution density $W(E)$

of energy barriers for the string is obtained by integrating $p_L(E)$ over L with the proper weight n_L describing the density of the segments of length L. At the critical point $F = 0$, the cluster-sizes distribution is $n_L \sim L^{-\nu}$. At finite forces F the same distribution will describe the cluster network up to $L = L_{opt}$. For the usual percolation clusters $\nu = 1 + d/d_f$, with d the dimensionality of the medium and $D < d_f < d$ the fractal dimensionality of the clusters network [14]. For anisotropic directed percolation describing the depinning transition the exponent may be different and will be discussed elsewhere, but the relation $\nu > 2$ holds. By evaluating the integral we find

$$W(E) \sim e^{-(\nu-1)E/U_c}. \tag{5}$$

The corresponding probability density of finding a waiting time $\tau = \tau_\circ \exp(E/T)$, with $\tau_\circ$ a microscopic time scale, is given by

$$\Psi(\tau) \sim T(\tau_\circ/\tau)^{1+\alpha}, \quad \alpha = (\nu - 1)T/U_c. \tag{6}$$

The distribution of energy barriers given in Eq. 5 must be cut off at the energy $U(F)$ corresponding to the optimal segment $L_{opt}(F)$ since the segments of larger scales are, on average, sliding freely and do not participate in the activated dynamics. The notion of "waiting time" makes no sense for such excitations. The average waiting time controlling the motion is therefore

$$\langle \tau \rangle \sim \int^{\tau_{max}} d\tau \Psi(\tau)\tau \sim \exp[(1-\alpha)U(F)/T], \tag{7}$$

where $\tau_{max} = \tau_\circ \exp[U(F)/T]$. Since the motion is controlled by the largest barrier, one arrives at the average velocity given by $v \simeq u_{opt}/\langle\tau\rangle \simeq \exp[-(1-\alpha)U(F)/T]$, recovering the result of the scaling theory [3].

We now discuss some observable consequences of the result given in Eq. 6. The vortex lattice in the mixed state of high temperature superconductors is probably the best system for the experimental study of different aspects of glassy dynamics, since by tuning parameters such as the applied field one can probe different regimes. For a driven vortex lattice the avalanche-like low temperature dynamics just described will manifest itself in the spectral density of the density-density correlation function, $S(\mathbf{k} = 0, t) = \langle \delta n_v(\mathbf{r}, t)\delta n_v(\mathbf{r}, 0)\rangle$, which is accessible by Hall-probe measurements. The density-density correlation function can be expressed in terms of the correlation function of the displacement field $u(\mathbf{r}, t)$ as $S(\mathbf{k}, t) = k^2\langle u(\mathbf{k}, t)u(-\mathbf{k}, 0)\rangle$. On large distances displacements are uncorrelated in a glassy system and $\langle u(\mathbf{k}, t)u(-\mathbf{k}, 0)\rangle \propto (1/k^2)f_u(\mathbf{k}, t)$. The corresponding spectral density $f_u(\omega)$ can be expressed in terms of the distribution function of energy barriers $W(E)$,

$$f_u(\omega) \propto \int dE W(E)\frac{\tau(E)}{1 + [\tau(E)\omega]^2}, \tag{8}$$

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Using $W(E)$ from (5) we obtain a power law density noise spectrum,

$$S(\omega) \propto \frac{1}{\omega^{1-\alpha}}. \tag{9}$$

We see that at low temperatures, $T < U_c$, the exponent $\alpha \ll 1$, and the noise spectrum of vortex density fluctuations is nearly the $1/f$ spectrum.

We now turn to discuss the high temperature region, where thermal fluctuations renormalize the pinning energy barriers. The characteristic temperature T_{dp} separating the high temperature and low temperature regions is defined by the self-consistent equation $T_{dp} \simeq U_c(T_{dp})$ [1]. For elastic manifolds with $D \geq 2$, $U_c(T)$ grows much faster than T [1] for $T > T_{dp}$, and $\alpha < 1$. The results obtained earlier are therefore still relevant, with suitably renormalized parameters.

The situation is more subtle for the case of an elastic string, corresponding to $D = 1$. Above the string depinning temperature, T_{dp}^s, the minimal pinning energy of the string becomes of order T, i.e., $U_c(T) \simeq T$ [3,1]. The definition of the depinning temperature used here follows from a scaling theory based on the postulate that the creep barriers scale in the same way as the fluctuations in the free energy, i.e., the string statistical mechanics is controlled by a unique energy scale [3,4]. Such a scaling theory leaves numerical constants undetermined. To preserve the idea that a single energy scale controls the dynamics we define T_{dp}^s as the temperature where first $\alpha(T) = 1$. This definition avoids the introduction of a new unphysical energy scale depending on the fractal dimensionality of the cluster of the pinned states. For $T > T_{dp}^s$ the motion of the string on scales $L < L_{opt}$ is still governed by the waiting times distribution function of Eq. 6 with $\alpha = 1$, i.e., $\Psi(\tau) \sim (1/\tau^2)$. The average waiting time in this high temperature regime is then

$$\bar{\tau} = \tau_\circ \ln(\tau_{max}/\tau_\circ) \sim F^{-\mu}. \tag{10}$$

The characteristic energy and length of the segment hopping with this waiting time are $\bar{E} \simeq T \ln(\bar{\tau}/\tau_\circ) \sim \ln F$ and $\bar{L} = L_c(\bar{\tau}/\tau_\circ)$, respectively. The typical transverse displacement $\bar{u}$ of a length $\bar{L}$ of string is determined by $C\bar{u}^2/\bar{L} \simeq F\bar{u}\bar{L}$. Defining the string velocity as $v = \bar{u}/\bar{\tau}$ and recalling that the activated velocity contains a prefactor $\sim F$ ensuring that flux flow is recovered for $F > F_c$, we obtain

$$v \sim (F/F_c)^{2-\mu}, \;\; T > T_{dp}^s. \tag{11}$$

In the high temperature region, $T > T_{dp}^s$, the string velocity vanishes as a power law. This represents a kind of marginal glassy dynamics which may be relevant to the high temperature, low magnetic field region of type II superconductors. We conjecture that this marginal dynamics corresponds to a new glassy state in the regime of single vortex pinning which can be referred to as *critical glass*.

In conclusion, we have developed a microscopic model for the glassy dynamics of elastic manifolds. In our model the low-temperature creep is governed by a power law distribution of waiting times. The dynamics at low temperature, $T < T_{dp}$, and long times is dominated by the optimal barrier $U(F)$, corresponding to the maximally pinned configuration. As a result, the mean velocity of the manifold vanishes exponentially with vanishing driving force according to $v \simeq \exp[-U(F)/T]$, as obtained earlier by a scaling approach [1]. This form for the macroscopic response arises naturally as a result of the cutoff of the algebraic time distribution at the maximal waiting time. We also conjecture a qualitatively different marginal glassy response, with $v \propto [(F/F_c(T)]^{1-\mu}$, for a *single* elastic string and $T > T_{dp}^s$. Finally, we discuss how the power-law distribution of waiting times controls the noise spectrum of the vortex density and can therefore be probed experimentally.

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