Search and Offshoring in the Presence of 'Animal Spirits'

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ABSTRACT

Search and Offshoring in the Presence of “Animal Spirits”

In this paper, we introduce two sources of unemployment in a two-factor general equilibrium model: search frictions and fairness considerations. We find that a binding fair-wage constraint increases the unskilled unemployment rate and can at the same time lead to a higher unemployment rate for skilled workers, as compared to an equilibrium where fairness considerations are absent or non-binding. Starting from a constrained equilibrium, an increase in the fairness parameter leads to increases in both skilled and unskilled unemployment. The wage of unskilled workers increases but the wage of skilled workers decreases. Next we allow for offshoring of unskilled jobs in our model, and we find that, as a result, it becomes more likely that the fair-wage constraint binds. Offshoring of unskilled jobs always leads to an increase in skilled wage, a decrease in skilled unemployment and an increase in unskilled unemployment. The presence of fairness considerations increases the adverse impact of offshoring on unskilled unemployment. The unskilled wage can increase or decrease as a result of offshoring.

JEL Classification: E24, F16, F41

Keywords: fair wages, unemployment, strategic effect, offshoring

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1 Introduction

Akerlof and Shiller (2009) in their recent book explain the importance of paying attention to “animal spirits” in understanding how economies behave. By “animal spirits,” they mean “the thought patterns that animate people’s ideas and feelings.” Akerlof and Shiller look at various aspects of animal spirits that impact economic decision making. According to them, fairness is one such aspect (other aspects are confidence, corruption and antisocial behavior, money illusion etc.) and is the aspect of “animal spirits” that can lead to unemployment. They discuss in detail how our sense of fairness results, up to a limit, in a positive relationship between the effort we, as workers, put in and the wage we receive relative to what we believe is “fair,” and how in turn, that can result in the employer setting a wage above what clears the market. Obviously, this results in unemployment. There is now a whole strand in the unemployment literature, following Akerlof and Yellen (1990), that focuses on such fairness or fair-wage considerations. It is, however, important to point out that in addition to fair-wage models, macroeconomists use a wide variety of other models to explain the existence of unemployment. Prominent among them are minimum-wage models, insider-outsider labor union models, models based on implicit contracts, search-unemployment models and efficiency wages.¹

In a fair-wage model with skilled and unskilled workers, the fairness constraint usually binds for unskilled workers and is never binding for skilled workers (as long as a skilled worker makes more than an unskilled worker). Thus, the model generates zero unemployment for skilled workers and a positive rate of unemployment for unskilled workers. While surveys of managers and workers, sociological studies of work environments, firm-level studies of pay structures, experiments, personnel management textbooks etc. provide a wealth of evidence supporting the assumption or idea of a fair-wage (see for instance Akerlof and Yellen (1990), Bewley (2005) and Howitt (2002) for a survey of the evidence), the prediction of a “fair-wage” model, with skilled and unskilled labor, that skilled unemployment is zero is not very realistic. According to the OECD Employment Outlook (2007), while the unemployment rate in the US in 2006 for people with less than secondary education was roughly 9 percent, it was 5 percent and 2.6 percent for people with upper secondary and tertiary education, respectively. The EU unemployment rates for the same year for the same three cate-

¹In fact, the fair-wage model can be viewed as a kind of efficiency wage model.
gories were 13, 7 and 4 percent respectively. Thus, in addition to unskilled unemployment, skilled unemployment can be quite substantial.

In order to generate positive unemployment rates for both categories of workers and at the same time take the strong evidence in favor of the fair-wage hypothesis seriously, we incorporate this hypothesis into a search model of unemployment.\(^2\) Another reason for combining the two different strands of the unemployment literature is Solow’s (1980) insight that unemployment in the real world is caused by different sources. Moreover, we show that the two alternative sources of unemployment we focus on, namely search frictions and the fair-wage constraint do not lead to additively separable effects but actually interact with each other in important and interesting ways to produce outcomes when the economy is hit by different shocks. To illustrate this point, we perform comparative statics with respect to the fairness parameter (that measures the society’s preference for fairness) and the economy’s stock of skilled labor, and look at the impact of offshoring.

By combining search frictions with fair-wage concerns, we obtain several new results. One of the results, which we find quite interesting, is that introducing fairness considerations in a search model leads not only to an increase in the unemployment of unskilled workers, a group for whom the fair-wage constraint is binding, but can also possibly lead to an increase in the unemployment of skilled workers, a group for whom the fair-wage constraint is not binding. The intuition here can be explained as follows. Introducing fairness considerations (or increasing the preference for fairness) resulting in a binding fair-wage constraint makes it more expensive to hire unskilled workers, and therefore, less jobs are created for unskilled workers leading to increased unemployment of unskilled workers. Since the number of unskilled workers employed is now lower, given the complementarity between the two types of workers in production, we get a reduction in the marginal product of skilled workers. This reduces the demand for skilled workers as well, unless it is offset by the “strategic effect” (explained below) which causes increased hiring of skilled workers. Hence, their market tightness and wage rate may go down and we then also get an increase in their unemployment.

\(^2\)There is a very well-developed literature in macroeconomics on the search theory of unemployment. See Pissarides (2000) for an excellent and comprehensive treatment. In addition to the vast macroeconomic literature on search unemployment in a closed economy, it is important to note that there also exists a literature on search induced unemployment in an open economy (e.g. Davidson, Martin and Matusz (1999), Davidson and Matusz (2004), Moore and Ranjan (2005), Helpman and Itschkoki (2007)).
Introducing a fairness (“fair-wage”) constraint results in a net “strategic effect” in the wage and employment decision of firms, identified by Stole and Zweibel (1996), even if it is absent in net terms in the unconstrained model. The “strategic effect” in our set up can be explained as follows. In search models, wage is positively related to the market tightness, which also implies that the labor cost (wage plus recruitment cost) is positively related to market tightness. Therefore, the fairness constraint is likely to bind when the ratio of market tightness for skilled to unskilled labor is high. A firm correctly anticipating a binding fair-wage constraint has an incentive to increase the relative employment of skilled workers to reduce their marginal product to, in turn, reduce the wage obtained through wage bargaining. Therefore, this “strategic effect” causes the employment of skilled workers (relative to unskilled) to be higher than it would be in its absence.

In the unconstrained case (where there is no fairness constraint or the constraint does not bind), the "strategic effects" of the relative employment of skilled labor on skilled and unskilled wage rates are in opposite directions and cancel each other out in the determination of the wage bill. In the constrained case, there is always a non-zero net "strategic effect" unless the production function is such that the marginal product of skilled workers is independent of the number of skilled workers employed (as in the case of Leontief production function).

When the “strategic effect” is weak or absent, a binding fairness constraint leads to an increase in skilled unemployment, along with the increase in unskilled unemployment. For a marginal increase in the degree of fairness (when the “fair-wage” constraint is binding), the “strategic effect” is of the second order, and hence, there is always an increase in the skilled unemployment as well. Since the unemployment rate and wage of skilled workers move in opposite directions, the skilled wage decreases, however, the unskilled wage increases due to increased concern for fairness. What this implies is that when we compare two countries with different degrees of concern for fairness, we expect the country with a greater concern for fairness not only to have a higher unemployment for unskilled workers, but also to have a higher unemployment for skilled workers.

The unemployment figures we provided earlier show that unemployment rates are higher for Europe relative to the US for all educational attainment categories of workers. This is consistent with our model’s prediction that an increase in the fairness parameter leads to higher unemployment for both types of workers since, based on survey evidence of attitudes to poverty and income, one can argue that social norms of fairness are stronger in Europe than in the US. According to calculations
by Alesina and Glaeser (2005), based on data from the World Values Survey for the years 1983-97, while only 29 percent of the responders from the US believe that the poor are trapped in poverty, about 60 percent of the European responders believe this to be the case. Furthermore, only 30 percent in the US believe that luck determines income, while 54 percent in the EU believe in luck being a determinant. Alesina and Glaeser also find that 60 percent of the Americans surveyed believe that the poor are lazy, while only 26 percent of the EU nationals surveyed believe so.\(^3\) Finally, using the International Social Survey Program (ISSP) surveys of public opinion, Osberg and Smeeding (2006) find in the case of the US “less concern for leveling up at the bottom of the distribution than in other nations.” That is, there is less concern for raising the income of the poor relative to the mean income in the US than in other countries.

Next, we find that introducing fairness considerations in a search model of unemployment also leads to the possibility of multiple equilibria. As discussed earlier, the fair-wage constraint binds if the market for skilled labor is tight relative to the market for unskilled labor. If firms expect the ratio of market tightness of skilled to unskilled to be high and therefore the fair-wage constraint to bind, they will end up hiring more skilled workers relative to unskilled workers due to the “strategic effect” discussed earlier. This, in turn, will make the relative market tightness for skilled workers higher creating a “self-fulfilling prophecy.” We find that the possibility of multiple equilibria exists for intermediate levels skilled-to-unskilled labor endowment ratios. Thus, the implication here is that, in this intermediate range of skill abundance, countries with identical preference for fairness and with identical relative factor endowments can have different wages and unemployment rates.

Looking at the impact of a reduction in skill endowment on unemployment, we find that the presence of a fair-wage constraint (in addition to search) amplifies the adverse effect on unskilled unemployment. That is, the increase in unskilled unemployment is larger as a consequence of a decrease in skill endowment when the fair-wage constraint is binding. Therefore, the fair-wage constraint makes the impact of a decrease in skill endowment on unskilled unemployment worse.

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\(^3\) Alesina and Angeletos (2005) point to the fact that while the pre-tax inequality is much higher in the US than in Europe (Gini coefficient of 38.5 as opposed to 29.1), the redistributive policies are much more extensive and the tax structure much more progressive in the latter. They argue that "the difference in political support for redistribution appears, rather, to reflect a difference in social perceptions regarding the fairness of market outcomes and the underlying sources of income inequality."
Finally, we look at the impact of offshoring on unemployment and wages in the presence of fairness considerations and search frictions. Both in the presence and absence of fairness considerations (with search frictions present in both cases), offshoring of unskilled jobs increases skilled wage, reduces skilled unemployment and increases unskilled unemployment. The reason is that the input produced by foreign labor in the model is a substitute for domestic unskilled labor, and through this competitive effect, offshoring reduces the tightness in the unskilled labor market. Thus, the cost of hiring domestic unskilled labor also goes down. Due to the complementarity between skilled labor and the production input of domestic unskilled labor (or that produced by foreign labor), we get an increase in the demand for skilled labor and increase in skilled-labor market tightness.

The presence of fairness considerations makes the impact of this kind of offshoring on unskilled unemployment worse because in addition to the reduced market tightness in the unskilled labor market coming from the direct competitive pressure from the substitutable input produced by foreign labor, there is an indirect upward pressure on unskilled wages coming from the fact that offshoring can convert a non-binding fairness constraint into a binding one. This can make the impact of offshoring on the unskilled wage ambiguous. In fact, if the fair-wage constraint is binding before and after offshoring, contrary to general perceptions, we will get an increase in the unskilled wage as a result of offshoring (as skilled and unskilled wage will be moving in the same direction).

Finally, the fair-wage constraint also affects the amount of offshoring in equilibrium. We get more offshoring in the presence of fair-wage considerations. Also, the extent of offshoring increases as the society’s preference for fairness increases.

Looking at related literature, Kreickemeier and Nelson (2006) extend the Akerlof and Yellen (1990) model to a two-sector setting to study the impact of international trade and technology shocks on unemployment and relative wages. They show that trade between US and Europe, with the preference for fairness being greater in the latter, leads to reduction in wage inequality and increase in unemployment in the US and increase in wage inequality and reduction in unemployment in Europe. They also analyze how trade with the newly industrializing countries (NICs) affects wage inequality and unemployment differently in these two regions.

Cahuc and Zylberberg (2004) (in subsection 2.6 of Chapter 10 of their book) compare the Anglo-Saxon labor market with the European labor market. While search frictions with skilled and

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4 See also Agell and Lundborg (1995).
unskilled workers are incorporated in both these types of markets, the European labor market also has an endogenous minimum wage for unskilled workers that is proportional to the skilled wage, i.e., notions of fairness and equality are incorporated into the determination of the minimum wage. This is very similar to the concept of the fair wage that we use in this paper. However, the focus of Cahuc and Zylberberg (2004) is limited to a comparison of the effects of skill-biased technological change on the two types of labor in the two labor markets. They assume that each type of labor specifically produces exclusively a particular intermediate good in an industry devoted to such production, and then there is a final good industry where the two intermediate goods are combined to produce the final good. By separating the two types of workers to work in different firms in different sectors, Cahuc and Zylberberg abstract from what we call in our paper “strategic effect” and as a result do not have the "multiple-equilibria" case that we find. As well, the vast empirical literature on the fair-wage hypothesis suggests that while the notion of "fair wage" can be based on wage comparisons across occupations and skill types, it is hardly ever based on comparisons across sectors. To keep the comparison between skilled and unskilled wages in the same industry to arrive at the “fair wage”, both types of workers in our model are employed in the same firm. This feature gives rise to the cross-factor “strategic effect” (in addition to the own “strategic effect”) in our model, which means that not only can the employment level of one type of workers affect the negotiated wage of that worker type, it can also affect the negotiated wage of the other kind of workers. This is taken into account in both the firm’s maximization problem and the wage negotiation process. In addition, the employment of both types of workers by the firms allows us to study the implications of offshoring of the job done by one type of workers. Cahuc and Zylberberg do not look at offshoring to which we devote a substantial part of our paper. Finally, we perform several different types of comparative static exercises, including analysis of shocks that move us from a situation of nonbinding to binding "fair-wage" constraint.

Another recent related paper is Grossman and Helpman (2008). In that paper, the utility derived by a worker is increasing in her own wage but decreasing in the average wage of the firm. This utility has to be above a threshold for the participation constraint of the worker to be satisfied. Thus, under certain conditions, Grossman and Helpman are able to get an equilibrium where in some firms, a large number of unskilled workers (relative to the number of skilled workers) work at a very low wage (firm-level average wage is low) and in others, a small number of unskilled workers
work at very high wages (firm-level average wage is high). Since the effect of own wage and average firm wage are in opposite directions in the worker’s utility function, the two types of firms result in the same level of worker utility. There is no unemployment in the Grossman-Helpman model since workers can be employed at very low wages as long as the average wage in the firm is low. In order to get rid of inefficiencies caused by jealousies or fairness considerations, firms offshore the work of unskilled workers in their model. The assumption driving this result is that for a multinational firm, only the average firm wage for operations carried out within the domestic boundaries of a country enters the domestic worker’s utility function. Again, there is no unemployment of any kind in this case. A result in the Grossman-Helpman paper that is similar to ours is the direct positive relationship between the strength of the preference for fair wages and the extent of offshoring, even though the motivation for offshoring in our model is very different from theirs.

Before ending the introduction, we would like to reiterate and substantiate our earlier claim that the effects of shocks on wages and unemployment rates in our model are not additively separable into effects that one would see in a pure fair-wage model and a pure search model. By incorporating search frictions in a fair-wage model of unemployment, not only do we trivially generate skilled unemployment, but more substantively, identify another margin of adjustment to shocks in the form skilled unemployment or market tightness for skilled workers. In a pure fair wage model, the margins of adjustment are skilled wage and unskilled unemployment, therefore, the impact of shocks on unskilled unemployment may be exaggerated and the impact on skilled unemployment understated. For example, starting from an equilibrium where the fairness constraint binds, an increase in the preference for fairness has no impact on the skilled unemployment in the pure fair-wage model, but skilled unemployment increases unambiguously in the hybrid (fair wage plus search frictions) model. Looking at the offshoring of jobs done by unskilled labor, we find that offshoring can lead to complete unemployment of unskilled workers in a pure fair-wage model because skilled wage rises and consequently fair wage rises to a level that makes it unprofitable for firms to employ unskilled workers. However, in our hybrid model, skilled wage does not rise as much because a part of adjustment takes place through a tightening of skilled labor market, resulting in lower skilled unemployment. Thus, the increase in skilled wage, and hence, in the fair wage is less, which

\footnote{While the pure fair-wage model is worked out in the appendix, the unconstrained case in the main text is nothing but a pure search model.}
alleviates the impact of offshoring on unskilled unemployment.

2 The Model

2.1 The Goods Market

The economy comprises three types of agents: $S$ skilled workers, $L$ unskilled workers, and a large number of entrepreneurs. Entrepreneurs have access to technology to produce a final good, $Z$, using skilled and unskilled labor. The production function for the final good $Z$, which is constant returns to scale, is given by

$$Z = F(\varepsilon_s s, \varepsilon_l l)$$

(1)

where $s$ and $l$, respectively, are the numbers of skilled and unskilled workers employed, and $\varepsilon_s$ and $\varepsilon_l$ are the efforts undertaken by the two types of workers which depend on the fairness considerations as discussed below. We also assume that the final good, $Z$, is the numeraire.

2.2 The Labor Market

Our description of labor market is a combination of a static version of Pissarides (2000) (along the lines of Helpman and Itskhoki (2007)) and Akerlof and Yellen (1990). Entrepreneurs must post vacancies to hire skilled and unskilled workers to undertake production. Once a vacancy is matched with a worker, she is hired to work for that firm (entrepreneur). Each worker has one unit of labor to devote to market activities, however, workers choose their effort two work, $\varepsilon$, where $\varepsilon \in (0,1]$. Following Kreickemeier and Nelson (2006) we postulate the following instantaneous utility function for a worker of type-$i$:

$$G(C_i, \varepsilon_i) = g(C_i) + \Delta \varepsilon_i$$

(2)

where $C_i$ is the consumption of the final good by the worker of type-$i$ and $\Delta \varepsilon_i \equiv -|\varepsilon_i - \varepsilon_i^n|$ is the degree of norm violation for a worker of type-$i$. The effort norm of worker of type-$i$ is determined by

$$\varepsilon_i^n = \min\left(\frac{w_i^*}{w_i}, 1\right)$$

(3)

where $w_i^*$ denotes the fair wage for a worker of type-$i$ and $w_i$ is the actual wage paid. From the utility function in (2) it is obvious that once a worker’s wage is set, she always chooses $\varepsilon_i = \varepsilon_i^n$
to maximize utility. Therefore, (3) above implies that workers provide the normal level of effort, which is set to 1, if they receive at least their fair wage.

The labor markets for both skilled and unskilled are characterized by a matching technology that depends on the number of searchers (size of the labor force) and the number of job vacancies. Pissarides (2000) describes the empirical support for a constant returns to scale matching function, which is what we use in this paper.

Let \( u_i \) denote the unemployment rate of factor \( i \), \( \theta_i \) the vacancy rate (i.e., the number of vacancies divided by the labor force), \( S \) the economy’s endowment of skilled labor, and \( L \) the endowment of unskilled labor. Since the model is static where all workers search for a job, and a fraction \( 1 - u_i \) of workers of type-\( i \) is matched, \( \theta_i \) is also the measure of market tightness. Then, we write the number of matches for each factor as constant-returns-to-scale functions as follows:

\[
M(\theta_s S, S) = M(\theta_s, 1) S \quad (4)
\]

\[
M(\theta_l L, L) = M(\theta_l, 1) L \quad (5)
\]

Define \( m_s = \frac{M(\theta_s S, S)}{S} = M(\theta_s, 1) \) and \( m_l = \frac{M(\theta_l L, L)}{L} = M(\theta_l, 1) \) as the matching rates for the two factors, where \( m'(\theta_i) > 0 \). Define \( q(\theta_i) = \frac{m_i}{\theta_i} \). The constant returns to matching implies \( q'(\theta_i) < 0 \). With this notation, the probability of finding a job for a searcher of type-\( i \) is \( \theta_i q(\theta_i) \), and the probability of filling up a vacant job is \( q(\theta_i) \). The former is an increasing function of market tightness, and the latter is a decreasing function of market tightness. The number of vacancies that a firm needs to create for it to expect to create one job at the end of the matching process is \( \frac{1}{q(\theta_i)} \). For a large firm, by the law of large numbers the actual ratio of vacancies to jobs (matches) created will be \( \frac{1}{q(\theta_i)} \).

We will restrict attention to the case where skilled workers would never prefer to search for an unskilled job, even if that were possible.\(^6\) Note once again that the model is static (one-period). The unemployment rate for each factor is given by:

\[
u_i = 1 - m_i = 1 - \theta_i q(\theta_i) \quad (6)\]

An entrepreneur posting vacancies must pay a recruitment cost of \( c_i (i = s, l) \) units of the final good per vacancy posted. Since a firm needs to post \( \frac{1}{q(\theta_i)} \) vacancies to create one job, the vacancy

\(^6\)This can be done by imposing reasonable restrictions on the parameters of production and matching functions and on the relative factor endowments of skilled and unskilled labor.
cost per worker employed equals \( \frac{c_i}{q(\theta_i)} \). Once a job is filled, the entrepreneur receives the value of the marginal product of that factor less the factorial wage, \( w_i \), where the wage is denoted in units of the final good.

We solve the entrepreneur’s problem in two stages. In the first stage, employment and the number of vacancies are chosen, anticipating the wages as functions of skilled and unskilled employment (determined through bargaining in the second stage) correctly. Then given the employment levels chosen in the first stage, the wage rate is determined by a process of bargaining between the entrepreneur and the worker, along the lines of Stole and Zweibel (1996). A worker and her employer bargain with each other taking into account the impact of the worker’s possible exit on wages of other employees. In other words, we allow the possibility of renegotiation of the employer with other employees if bargaining fails with any employee, and this feature is completely factored into the bargaining process.\(^7\)

The discussion above implies that our overall equilibrium concept is one of subgame perfect equilibrium which is solved using backward induction. That is, taking as given the employment chosen in the first stage, in the second stage the wages are determined through a process of simultaneous Stole-Zweibel bargaining between the firm and the workers. Anticipating the second stage wage as a function of employment, the firm optimally chooses employment in the first stage.

Next, along the lines of Akerlof and Yellen (1990), we show in the appendix that a firm never pays a wage less than the fair wage, therefore, workers always work at full effort, and there is no loss of generality in assuming the production function given in (1) to be

\[
Z = F(s,l) \tag{7}
\]

With the fair-wage constraint never binding for skilled workers (as shown later), the entrepreneur solves the following problem in the first stage correctly anticipating the wages paid in the second stage of skilled and unskilled employment.

\[
Max_{s,l} F(s,l) - w_s(s,l)s - Max\{w_l(s,l), w_l^*\}l - \frac{c_s}{q(\theta_s)}s - \frac{c_l}{q(\theta_l)}l \tag{8}
\]

Below we describe the wage determination in two cases: when the fairness constraint does not bind and when it does bind.

\(^7\) A special case of this, where the worker and employer have equal bargaining weights, exactly boils down to the Shapley value solution to a cooperative, multilateral bargaining problem.
2.3 Wage Determination

2.3.1 Unconstrained case

The first-order conditions for the optimal choices of $s$ and $l$ are given by

$$
\begin{align*}
[F_1(s,l)] - w_s - s \frac{\partial w_s}{\partial s} - l \frac{\partial w_l}{\partial s} &= \frac{c_s}{q(\theta_s)} \quad (9) \\
[F_2(s,l)] - w_l - s \frac{\partial w_s}{\partial l} - l \frac{\partial w_l}{\partial l} &= \frac{c_l}{q(\theta_l)} \quad (10)
\end{align*}
$$

where subscripts “1” and “2” denote partial derivatives of the production function with respect to the first and second arguments, respectively. Denote the expressions on the l.h.s in the above two equations by $J_i$, $i = s, l$, where $J_i$ is the surplus of the firm from hiring the marginal worker of type-$i$. Assuming unemployment benefit to be zero, the bargaining weight of a worker to be $\beta$, the bargained wage for a worker of type-$i$ is obtained as follows.

$$
w_i^b = \arg \max_{w_i} w_i^\beta J_i^{1-\beta} \quad (11)
$$

As mentioned earlier, a worker and her employer bargain with each other taking into account the impact of the worker’s possible exit on wages of other employees. In other words, we allow the possibility of renegotiation of the employer with other employees if bargaining fails with any employee, and this feature is completely factored into the bargaining process. Using (9), (10), the first-order conditions of the above maximization problem yields the following expressions for wages for the two types of workers.

$$
\begin{align*}
w_s &= \beta [F_1(s,l) - s \frac{\partial w_s}{\partial s} - l \frac{\partial w_l}{\partial s}] \quad (12) \\
w_l &= \beta [F_2(s,l) - s \frac{\partial w_s}{\partial l} - l \frac{\partial w_l}{\partial l}] \quad (13)
\end{align*}
$$

The above is a system of differential equations, where each worker’s bargained wage is a fraction of the surplus she creates in the form of her marginal product plus the reduction in the wage bill (or minus the increase in the wage bill) of the existing workers through her employment (relative to the situation where she exits and wages with other workers are renegotiated). As seen from the above differential equations, there are own as well as cross effects of skilled and unskilled employment on wages.
It is important to note here that if \( w_s \) and \( w_l \) are homogeneous of degree zero in \( s \) and \( l \), the two first-order conditions above result in the zero-profit condition for the firm being satisfied.\(^8\) Taking this as an important hint in finding the solution to the above set of differential equations, we write \( w_s \) and \( w_l \) as functions of \( s/l \). Denote \( s/l \) by \( t \). Given that \( F(s, l) \) is CRS, we can write \( F(s, l) = \ell F(t; 1) \). Denote \( F(t; 1) \) by \( f(t) \) and it follows that \( F_1(s, l) = f'(t) \) and \( F_2(s, l) = f(t) - tf'(t) \).

In the appendix we show that the solutions to (12) and (13), namely bargained wage rates in the second (bargaining) stage for any relative skilled employment, \( t \) set in the first (prior) stage- are given by

\[
\text{Lemma 1: } w_s(t) = \beta f'(t); w_l(t) = \beta (f(t) - tf'(t))
\]

That is, the wages are simply a fraction \( \beta \) of the respective marginal products of labor. This is despite the presence of the “strategic effect” mentioned in the introduction which are captured by terms \( i \frac{\partial w_i}{\partial j} \), for \( i, j = s, l \) in the first-order conditions (9) and (10). For example, \( s \frac{\partial w_s}{\partial s} \) captures the effect of hiring an additional skilled worker on skilled wage. If hiring an additional skilled worker lowers their marginal product it will reduce the Nash bargained wage that firms have to pay to skilled workers. Therefore, the value of a skilled job to the firm would exceed the marginal product of skilled labor due to this effect. However, an additional skilled worker also increases the marginal product of unskilled workers which would lead to an increase in the unskilled wage (\( \frac{\partial w_l}{\partial s} > 0 \)), leading to a reduced value of a skilled job for the firm. For a constant returns to scale production function, these two “strategic effects” cancel out in the determination of the wage bill. Thus, the surplus that is shared between the worker and the firm is the worker’s marginal product and it is shared according to their bargaining weights.

\[2.3.2\] Constrained Case

Suppose the fair wage is \( w_t^* \), and the firm expects it to be the unskilled wage in the second stage. Assuming that \( w_t^* \), determined in general equilibrium, is taken by the firm parametrically (as given), the first-order conditions for the employment choice in the first stage become

\[8\] Multiplying (9) by \( s \) and (10) by \( l \) and adding we get \( F(s, l) = (w_s + \frac{q_s}{q_t} w_t^*) s + (w_l + \frac{q_l}{q_t} w_t^*) l + s \frac{\partial w_s}{\partial s} + l \frac{\partial w_l}{\partial l} \). Therefore, the zero profit condition holds if \( s \frac{\partial w_s}{\partial s} + l \frac{\partial w_l}{\partial l} = l \frac{\partial w_l}{\partial l} + s \frac{\partial w_s}{\partial s} = 0 \), which always holds if if \( w_s \) and \( w_l \) are homogeneous of degree zero in \( s \) and \( l \).
The value of an extra skilled worker for a firm is given by the l.h.s of (14). As in the unconstrained case, if $w_s$ is homogeneous of degree zero in $s$ and $l$, the two first-order conditions above result in the zero-profit condition for the firm being satisfied. We thus write $w_s$ as a function of $t = s/l$. Using Nash bargaining, as in the unconstrained case, the wage of a skilled worker is given by the following differential equation

$$ w_s(t) = \beta[f'(t) - tw'_s(t)] $$

The solution to the above differential equation is given by

$$ w_s(t) = t^{-\frac{1}{\gamma}} \int_0^t x^{-\frac{1}{\gamma}} f'(x)dx $$

In the case of Cobb-Douglas production function given by

$$ F = \frac{s^\gamma l^{1-\gamma}}{\gamma^\gamma (1-\gamma)^{1-\gamma}} $$

the skilled wage given in (17) takes the following simple form

$$ w_s(t) = \phi \beta f'(t) $$

where $\phi \equiv \frac{1}{1-(1-\gamma)\beta}$. Since $\phi > 1$, it implies that the skilled wage is a greater fraction of the marginal product of skilled workers in the constrained case than in the unconstrained case. This happens because of the “strategic effect” mentioned earlier. Since the fair wage of unskilled workers is determined outside the firm, an additional skilled worker does not increase the wage of unskilled workers giving rise only to the positive “strategic effect” of reducing the wage for all employed skilled workers in the firm. Therefore, the value of a skilled job exceeds the marginal product of skilled labor, which is reflected in the skilled wage being a higher fraction of marginal product of skilled labor in the case of Cobb-Douglas production function. For analytical tractability and to obtain closed form solutions, in rest of the paper we make the following assumption.

**Assumption 1:** The production function for the final good is of the Cobb-Douglas form given in (18) and hence $f(t) = \frac{t^\gamma}{\gamma^\gamma (1-\gamma)^{1-\gamma}}$. 

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Having determined wages in the constrained and unconstrained cases, we next solve for the equilibria in the two cases.

3 Autarky Equilibrium

3.1 Unconstrained case

It is shown in the appendix that the first-order conditions (9), (10), and lemma 1 imply

\[ \text{Lemma 2: } w_s(t) = \beta f'(t) = \frac{\beta}{1-\beta} \frac{c_s q}{q(\theta_s)}; \quad w_l(t) = \beta (f(t) - tf'(t)) = \frac{\beta}{1-\beta} \frac{c_l}{q(\theta_l)}. \]

The above relationships are determined by the fact that the surplus from a job-i in equilibrium equals the hiring cost of \( \frac{c_i}{q(\theta_i)} \). Since the firms take \( \theta_i \) as given, lemma 2 provides us with two expressions for the relative demand \( t^d \) for a firm, both of which must be true in equilibrium. Dividing one expression by the other we get the following expression for the relative demand \( t^d \) as a function of \( \theta_s \) and \( \theta_l \).

\[ \frac{f'(t^d)}{f(t^d) - t^d f'(t^d)} = \frac{c_s q(\theta_l)}{c_l q(\theta_s)} \]

Using the functional form for \( f(t) \) given in Assumption 1, the above can be written as

\[ t^d = \frac{\gamma}{1-\gamma} \left( \frac{c_s q(\theta_s)}{c_l q(\theta_l)} \right) \]

To obtain closed form solutions, we assume that the matching function is also of Cobb-Douglas form given as follows.

**Assumption 2**: \( m(\theta_i) = k(\theta_i)^{\delta} \) and hence \( q(\theta_i) = k(\theta_i)^{\delta-1} \).

Under the above assumption, the relative demand is decreasing in \( \frac{\theta_s}{\theta_l} \). This is shown using a downward sloping curve denoted by \( RLD^u \) in Figure 1 in \((\frac{S}{L}, \frac{\theta_s}{\theta_l})\) space. This is intuitive as the relative cost of employing skilled labor (relative to unskilled) is increasing in its relative market tightness.

Having obtained an expression for relative demand, next we derive an expression for the relative supply of two types of labor. Denoting the economy’s endowments of skilled and unskilled labor by \( S \) and \( L \) respectively, the relative supply (available for employment) is given by \( \frac{S(1-u_s)}{L(1-u_l)} \), which using (6) becomes

\[ t^s = \frac{S\theta_s q(\theta_s)}{L\theta_l q(\theta_l)} = \frac{S}{L} \left( \frac{\theta_s}{\theta_l} \right)^\delta \]

\[ (21) \]

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where the last equality follows from the functional form for the matching function given in Assumption 2. The above is clearly increasing in \( \frac{\theta_s}{\theta_l} \). In other words, because the relative employment rate is increasing in the relative market tightness of skilled labor (through the Beveridge curve relationship between the unemployment rate and labor market tightness), the relative supply of skilled labor available for employment is also increasing in its relative market tightness.

The intersection of the downward sloping relative demand with the upward sloping relative supply determines the autarky equilibrium in the unconstrained case as shown in Figure 1. The unconstrained equilibrium \( \frac{\theta_s}{\theta_l} \) and \( t \) are given by

\[
\frac{\theta_s}{\theta_l} = \frac{c_l \gamma L}{c_s (1 - \gamma) S}; t = \left( \frac{\gamma c_l}{(1 - \gamma) c_s} \right)^{\delta} \left( \frac{L}{S} \right)^{\delta^{-1}} \tag{22}
\]

The corresponding equilibrium values of \( w_s, w_l, \theta_s, \) and \( \theta_l \) are obtained as follows. As mentioned earlier, when both (9) and (10) are satisfied, firms make zero profits. From the two expressions in lemma 2, which are derived from the two first-order conditions (9) and (10), we get the following zero profit condition

\[
\frac{1}{1 - \beta} \left( \frac{c_s}{q(\theta_s)} \right)^{\gamma} \left( \frac{c_l}{q(\theta_l)} \right)^{1 - \gamma} = 1 \tag{23}
\]

The above simply states that to maintain zero profits in equilibrium, an increase in \( \theta_s \) must be associated with a decrease in \( \theta_l \), and vice-versa. This negative relationship between \( \theta_s \), and \( \theta_l \) is plotted in Figure 2 and denoted by \( \text{ZPC}^u \). Once we know the equilibrium \( \frac{\theta_s}{\theta_l} \) from (22), we can determine the equilibrium values of \( \theta_s \), and \( \theta_l \) from \( \text{ZPC}^u \), and the corresponding \( w_i \) from lemma 2. Denote the unconstrained equilibrium wage and market tightness variables by \( w_i^u, w_s^u, \theta_i^u, \theta_s^u \).

### 3.2 Constrained case

The fair-wage constraint in our model arises from social norms about the maximum permissible wage inequality. Akerlof and Yellen (1990) model fair wage for type-\( i \) as a linear combination of the wage of the other type and the market clearing wage for type-\( i \). Given the search friction and wage bargaining, there is no market clearing wage in our framework. In principle, we could use the unconstrained bargained wage in place of the market clearing wage, that is we could use a fair-wage specification as follows.

\[
w'_i = \tau w_s + (1 - \tau) w_i^u \tag{24}
\]
where $w_l^u$ is the unconstrained equilibrium fair wage. Without loss of generality, and to simplify the exposition considerably, we assume that our fair wage, $w_l^*$, takes the following simple form

**Assumption 3:** $w_l^* = \tau w_s$

Now, there are two possibilities: either the fair-wage constraint does not bind ($w_l^u > \tau w_s^u$) or it does bind. Therefore, the expression for the unskilled wage is given by

$$w_l = \text{Max}\{w_l^u, \tau w_s^c\}$$

where $w_s^c$ is the equilibrium skilled wage in the case where the wage of unskilled workers is constrained to be equal to its fair wage.$^9$

When the fair-wage constraint binds, the constrained relative demand as a function of $\theta_s$ and $\theta_l$ is derived as follows. Re-write (14) and (15) using (16) and (19) as

$$w_s(t) = \phi \beta f'(t) = \frac{\beta}{1 - \beta} \frac{c_s}{q(\theta_s)}$$

$$f(t) - tf'(t) + t^2 w_s'(t) = f(t) - \phi tf'(t) = w_l^* + \frac{c_l}{q(\theta_l)}$$  \hspace{1cm} (26)

Dividing (25) by (26) we get

$$\frac{\phi \beta f'(t)}{f(t) - \phi tf'(t)} = \frac{\beta}{1 - \beta} \frac{c_s}{q(\theta_s)} \frac{w_l^* + \frac{c_l}{q(\theta_l)}}{w_l^* + \frac{c_l}{q(\theta_l)}}$$  \hspace{1cm} (27)

which upon using the functional form of $f(t)$ yields

$$t_d = \frac{\gamma (w_l^* + \frac{c_l}{q(\theta_l)})}{(1 - \gamma) \frac{c_s}{q(\theta_s)}}$$  \hspace{1cm} (28)

The above is the relative demand for a firm taking $w_l^*$, $\theta_l$, and $\theta_s$ as given. Comparing the above relative demand with the one in the unconstrained case given in (20) note that for any given $\theta_l$ and $\theta_s$, the constrained relative demand for skilled labor is higher. This is due to the “strategic effect” mentioned earlier. Hiring an extra skilled worker lowers the wages of all skilled workers, and therefore, firms have an incentive to hire more skilled workers than in the unconstrained case. Also, hiring an additional unskilled worker increases the skilled wage, therefore, firms want to reduce the hiring of unskilled workers. Both these effects tend to increase the relative demand for skilled workers in the constrained case.

$^9$We can also have exactly the same fair-wage constraint for skilled workers in terms of the unskilled wage. However, as long as the skilled wage is greater than unskilled wage (which is assumed to be true throughout by choice of parameters), it is obvious that the constraint is never binding for skilled workers.
If the fair wage \( w^* \) were exogenous, say a policy determined minimum wage, then we would use (21) and (28) to determine the equilibrium values of \( \theta_l \) and \( \theta_s \). Using the fair wage specified in assumption 3, the equation for the relative demand becomes

\[
d^d = \frac{\gamma (\tau w_s(t) + \frac{c_l}{q(\theta_l)})}{(1 - \gamma) \frac{c_s}{q(\theta_s)}}
\]  

(29)

To obtain a simplified expression for relative demand, we use the following lemma which is proved in the appendix.

**Lemma 3:** \( w_s(t) = \frac{\beta}{1 - \beta} \frac{c_s}{q(\theta_s)} \) holds even in the constrained case.

The reason that the expression for the skilled wage as function of \( \theta_s \) is unchanged from the expression in lemma 2 for the unconstrained case is the following. Nash bargaining implies that the skilled wage, \( w_s \) equals \( \frac{\beta}{1 - \beta} J_s \), where \( J_s \) is the surplus of the firm from hiring the marginal skilled worker. Note from (9) and (14) that, for any given market tightness that the firms and workers take as given, the interaction between a firm and its workers it decides to hire (through its employment decision and wage bargaining) always results in an outcome in which \( J_s \) must equal the recruitment cost of hiring an additional worker, \( \frac{c_s}{q(\theta_s)} \).

Using lemma 3 to substitute out \( w_s(t) \) in (29) we get

\[
d^d = \frac{\gamma}{(1 - \gamma)} \left( \frac{\tau \beta}{1 - \beta} + \frac{c_s q(\theta_s)}{c_s q(\theta_l)} \right)
\]  

(30)

The relative demand above is decreasing in \( \frac{\theta_s}{\theta_l} \). Comparing (20) and (30) it is easy to see that the constrained relative demand \( RLD^c \) in Figure 2 lies to the right of the unconstrained demand \( RLD^u \) when the constraint is binding. The relative supply curve, which is an increasing function of \( \frac{\theta_s}{\theta_l} \), remains unchanged. Therefore, the equilibrium values of \( \frac{\theta_s}{\theta_l} \) and \( t \) are higher in the constrained case (with the constraint binding) compared to the unconstrained case.

To obtain the equilibrium values of \( \theta_l \), \( \theta_s \), and \( w_s \) in the constrained case, we again make use of the two first-order conditions (25) and (26) to eliminate \( t \) to get our zero profit condition in terms of \( \theta_l \) and \( \theta_s \) given by

\[
\frac{1}{(1 - \beta)\phi} \left( \frac{c_s}{q(\theta_s)} \right)^\gamma \left( \left( \frac{\tau \beta}{1 - \beta} \right) \left( \frac{c_s}{q(\theta_s)} \right) + \frac{c_l}{q(\theta_l)} \right)^{1-\gamma} = 1
\]  

(31)

The above is again a negative relationship between \( \theta_l \), \( \theta_s \), which yields equilibrium values of \( \theta_l \), and \( \theta_s \) for an equilibrium \( \frac{\theta_s}{\theta_l} \). It is shown in the appendix that, in the range of \( \frac{\theta_s}{\theta_l} \) for which
the constraint is binding, the curve representing (31) lies to the left of (23) in Figure 2. Intuitively, for any $\theta_l$ and $\theta_s$ satisfying (23), the introduction of a binding fairness constraint increases the cost of hiring unskilled labor without affecting the cost of hiring skilled labor. Therefore, firms start making losses. To restore zero profits, $\theta_l$ must be lower for each $\theta_s$ and vice-versa.

Below we derive the range of $\frac{\theta_s}{\theta_l}$ in which the constraint is binding.

### 3.3 When does the constraint bind?

The constraint binds when $w_u^u < \tau w_s^u$. From lemma 2 the constraint binds if

$$\frac{c_l}{q(\theta_u^l)} < \tau \frac{c_s}{q(\theta_u^s)}$$

which upon using $q(\theta) = k\theta^{\delta-1}$ from assumption 2 becomes

$$\left(\frac{\theta_u^s}{\theta_u^l}\right) > \left(\frac{c_l}{\tau c_s}\right)^{\frac{1}{\delta}}$$

Thus, the relative demand curve with the possibility of the fair-wage constraint becomes the one denoted by $RLD^c$ in Figure 1 which has two segments, one for values of $\frac{\theta_s}{\theta_l}$ greater than $\left(\frac{c_l}{\tau c_s}\right)^{\frac{1}{\delta}}$, and the other for values below it which corresponds to the unconstrained relative demand curve.

Given the shape of the relative demand curve with fair-wage constraint, it is possible to get multiple equilibria. To see this, suppose the relative supply curve is one denoted by $RLS'$ in Figure 1. Now, if the society did not have any concern for fairness, then the equilibrium would be at $e_3$. However, fairness concerns cause a stepward shift in the relative demand curve making $e_4$ a candidate for equilibrium as well. That is, both $e_3$ and $e_4$ are possible equilibria when fairness considerations are present.

The intuition for “multiple equilibria” here is the following. Since the relative cost of labor (wage plus recruitment cost) is positively related to the relative market tightness, the fair-wage constraint binds if $\frac{\theta_s}{\theta_l}$ is high. If firms expect $\frac{\theta_s}{\theta_l}$ to be high and therefore the fair-wage constraint to bind, they will end up hiring more skilled workers relative to unskilled workers due to the “strategic effect” discussed earlier. This, in turn, will make the relative market tightness for skilled workers higher creating a “self-fulfilling prophecy.” Similarly, if $\frac{\theta_s}{\theta_l}$ is expected to be low, the fair-wage constraint is then not expected to bind, the relative demand for skilled labor is lower, which in turn leads to a low $\frac{\theta_s}{\theta_l}$ and thus, an effectively unconstrained outcome.
As seen from Figure 1, we have multiple possible expectations regarding $\frac{\theta_s}{\theta_l}$, when relative supply is in the intermediate range, which happens only when the relative endowment of skilled labor is in the intermediate range. In other words, if the relative endowment of skilled labor is very high (low), which the firms know, they will expect the relative market tightness of skilled labor to be always low (high).

Using (22) which gives the value of unconstrained equilibrium $\frac{\theta_s}{\theta_l}$, the condition (32) can be written as

$$\tau > \left( \frac{c_l}{c_s} \right)^\delta \left( \frac{(1 - \gamma)S}{\gamma L} \right)^{1-\delta}$$

Therefore, if the condition above is satisfied, the constraint binds, and the equilibrium must be obtained using the constrained relative demand curve. For example, in the case where $c_l = c_s$, $L = 2S$, $\gamma = \delta = .5$, $\tau$ must be greater than .7 for the constraint to bind.

In Figure 2, if $\frac{\theta_s}{\theta_l}$ is greater than a threshold value, the fair-wage constraint binds and for such values the zero profit curve under the binding constraint, $ZPC^c$ is to the left of the unconstrained one, $ZPC^u$.

### 3.4 Comparing constrained and unconstrained equilibria

Let us compare the autarky equilibrium in the absence of fairness considerations given by point $e_1$ in Figure 1 with the constrained autarky equilibrium given by point $e_2$. Since the equilibrium $\frac{\theta_s}{\theta_l}$ is higher in the constrained case, Figure 2 implies a lower $\theta_l$ and consequently a higher unemployment rate of unskilled in the constrained case. Since the impact on $\theta_s$ is ambiguous, the impact on skilled unemployment and skilled wage is ambiguous. Since the impact on skilled wage is ambiguous, the impact on unskilled wage is ambiguous as well.

It is shown in the appendix (section 6.6) that in the absence of the “strategic effect”, there is an increase in skilled unemployment as well. In the absence of the “strategic effect”, a binding fairness constraint has two effects on the demand for skilled labor. Firstly, since unskilled labor becomes more expensive, firms substitute skilled labor for unskilled labor. Secondly, given the complementarity between skilled and unskilled labor, lower employment of unskilled labor reduces the marginal product of skilled labor and hence reduces the demand for skilled labor. It is shown that, for an individual firm there is an increase in the relative demand for skilled labor at each $\frac{\theta_s}{\theta_l}$ compared to the unconstrained case. However, at the aggregate level, the increased ratio of skilled
to unskilled employment results in a lower marginal product of skilled labor and consequently a lower market tightness for skilled labor. Lower market tightness for skilled labor implies lower skilled wage and higher skilled unemployment. Given that the the ratio of skilled to unskilled employment is higher, the marginal product of unskilled labor is higher. With a lower recruitment cost per unskilled worker due to a lower market tightness in the constrained equilibrium, this higher marginal product implies a higher unskilled wage. The result is summarized below.

**Proposition 1:** In a constrained equilibrium where \( w_l^* \geq w_u^* \), unskilled unemployment is higher and skilled unemployment (and skilled wage) may be higher or lower compared to an unconstrained equilibrium. If the “strategic effect” in employment choice is absent (as is the case with a Leontief production function), then skilled unemployment is higher as well and while skilled wage is lower, the unskilled wage is higher.

The proposition above highlights the importance of the “strategic effect” in determining the impact of fair-wage considerations on skilled unemployment. When the constraint is binding, the possibility of paying a lower wage to skilled workers in the second stage (first stage employment choice determines marginal product, and hence wage through wage bargaining in the second stage) induces firms to hire more skilled workers (and fewer unskilled workers) in the first stage which reduces skilled unemployment (and increases unskilled unemployment). The strength of the “strategic effect” depends on the elasticity of substitution between the two factors of production. In the extreme case of zero elasticity of substitution (Leontief production function) the “strategic effect” is zero as well (See appendix).

### 3.5 Comparative statics with respect to \( \tau \)

Let us assume that the economy is at a constrained equilibrium, i.e., the relative supply curve in Figure 1 intersects the fair-wage relative demand curve in its upper right-hand downward sloping part (that lies above the cut-off value of \( \frac{\theta_s}{\theta_l} \) at which the fairness constraint binds). An increase in \( \tau \) implies a rightward shift in the upper right-hand segment of the relative demand for skilled labor from (30). Also, from (32), it implies a reduction in the cut-off value of \( \frac{\theta_s}{\theta_l} \) at which the fairness constraint binds. Since the relative supply remains unchanged, an increase in \( \tau \) increases the equilibrium values of \( \frac{\theta_s}{\theta_l} \) and \( t \). Since the corresponding part of the zero-profit curve shifts to the left in Figure 2 (and the ray through the origin indicating the threshold value of \( \frac{\theta_s}{\theta_l} \) rotates...
clockwise), there is an unambiguous decrease in $\theta_l$. An increase in $t$ implies from (25) a decrease in $\theta_s$ as well. Decreases in $w_s$, $\theta_s$, and $\theta_l$ imply from (31) (and from our definition of the fair unskilled wage) that the unskilled wage must rise. Recall that decreases in $\theta_s$ and $\theta_l$ will reduce the cost of recruiting both types of workers, and since the skilled wage declines, the unskilled wage must increase to restore the zero profit condition. Therefore, we get the following result.

**Proposition 2:** Starting from a constrained equilibrium, an increase in the fairness parameter $\tau$ leads to increases in both skilled and unskilled unemployment. Skilled wage falls and unskilled wage rises.

While a comparison of unconstrained and constrained equilibria yields ambiguous results on the skilled unemployment as mentioned in proposition 1, proposition 2 shows that any increase in $\tau$, starting from a constrained equilibrium, leads to an unambiguous increase in skilled unemployment. As discussed in proposition 1, in the absence of the “strategic effect”, a move from unconstrained to constrained equilibrium leads to an increase in skilled unemployment. However, the “strategic effect” confounds this by providing increased incentive to hire skilled workers. Starting from a constrained equilibrium, for an incremental increase in $\tau$ the “strategic effect” is of the second order (the effect is present in the initial and final equilibrium, both of which are constrained and so at least partially get canceled out when looking at the difference between the two situations), and hence skilled unemployment increases.

### 3.6 Change in the relative endowment of skilled labor

It is easy to see from the discussion of autarky equilibrium in the constrained and unconstrained cases that an increase in the relative endowment of skilled labor rotates the relative supply curve to the right in Figure 1. This leads to a decrease in the equilibrium $\frac{\theta_s}{\theta_l}$. From Figure 2, a decrease in $\frac{\theta_s}{\theta_l}$ implies a decrease in $\theta_s$ and an increase in $\theta_l$. Therefore, skilled unemployment increases and unskilled unemployment decreases in both cases. In the unconstrained case it also leads to a decrease in wage inequality. Since wage inequality is fixed at $\frac{1}{\tau}$ in the constrained case, there is no change in wage inequality as a result of increased relative endowment of skilled labor.

Next, note from (33) that an increase in $\frac{S}{L}$ makes it less likely that the constraint binds. Therefore, it is possible that an increase in $\frac{S}{L}$ makes a binding constraint non-binding. This is shown in Figure 1 when the there is a rightward shift in the relative supply curve from RLS to
In this case, there is a decrease in wage inequality because the wage inequality decreases from $\frac{1}{\tau}$ to the inverse of the expression on the r.h.s of (33).

To compare the effects of relative endowment changes in the presence and absence of fairness considerations and to keep the initial equilibrium the same under both situations for a fair comparison, let us consider the reverse case of a reduction in $\frac{S}{L}$. In the presence of fairness considerations, let us assume that the fair-wage constraint does not bind initially, which gives us the same equilibrium as when there are no fairness considerations at all. The impact on unemployment can be analyzed using Figures 1 and 2. In Figure 1, the initial relative supply curve is $RLS''$ and therefore, the intersection with the relative demand curve is in the unconstrained part at $e_5$, irrespective of whether we have fairness considerations or not. After the reduction in $\frac{S}{L}$, the relative supply curve moves from $RLS''$ to $RLS$. Now, there is a greater increase in the equilibrium $\theta$ under fairness considerations since this reduction now makes the fair-wage condition bind. In Figure 2, the zero profit curve now is the one corresponding to the constrained case whose relevant portion now lies to the left of the unconstrained one. Therefore, under fairness considerations, there is a larger reduction in $\theta_l$, but the impact on $\theta_s$ becomes ambiguous. In other words, the increase in unskilled unemployment is magnified by the fact that this decrease in the skilled labor force makes the fair-wage constraint binding.

4 Impact of offshoring

For offshoring to be possible, firms have to be able to fragment their production in such a way that semi-finished output, whose production only requires the application of unskilled labor, can be produced in another country (South) and then imported back to be combined with skilled labor at home to produce the final product. To keep things simple, we assume that one unit of this semi-finished good is a perfect substitute for a unit of domestic unskilled labor. We assume the price of imported input inclusive of the search and trade costs as fixed, which is equivalent to a small country assumption. (This assumption is reasonable if we believe that the South has large quantities of unskilled labor and that fixed labor productivity in a large subsistence, numeraire sector there fixes their unskilled wage.) Suppose the unit cost (faced at home) of offshored input is $p_m$.
Let the amount of this input imported be denoted by \( m \). As far as the timing of offshoring is concerned, we assume that the quantity of offshored input is chosen in the first stage along with skilled labor. This assumption may be closer in spirit to the offshoring of services, while the assumption of freely adjustable input will be closer to the case of imported intermediate good. Later we will note the implication of allowing this input to be imported freely at any time.

A firm that decides to offshore and not hire unskilled workers domestically, solves the following problem:

\[
\max_{s,m} F(s,m) - w_s(s,m)s - p_m m - \frac{c_s}{q(\theta_s)}s
\]

The first order conditions for the above maximization are given by

\[
F_1(s,m) - w_s - s \frac{\partial w_s}{\partial s} = \frac{c_s}{q(\theta_s)}
\]

\[
F_2(s,m) - s \frac{\partial w_s}{\partial m} - p_m = 0
\]

(34)

(35)

Using Nash bargaining, the wage of skilled is given by the following differential equation

\[
w_s(t) = \beta [f'(t) - tw'_s(t)]
\]

(36)

where \( t \) equals \( s/m \) in the offshoring case. Note from above that the problem of offshoring firms is similar to that of firms in autarky when the fairness constraint binds. Therefore, the solution to (36), when \( f(t) \) is one given in assumption 1, is \( w_s(t) = \phi \beta f'(t) \) where \( \phi \equiv \frac{1}{1-(1-\gamma)\beta} \). As well, since lemma 3 still holds for offshoring firms: \( w_s(t) = \phi \beta f'(t) = \frac{\beta c_s}{1-\beta q(\theta_s)} \). For non-offshoring firms, either lemma 2 or lemma 3 will hold depending on whether the fairness constraint binds or not. In either case the equilibrium wage of skilled workers is going to equal \( \frac{\beta c_s}{1-\beta q(\theta_s)} \). That is, the offshoring and non-offshoring firms will end up paying the same wage to the skilled workers.

Ruling out the case of fair-wage constraint binding before but not after offshoring (as shown in the appendix), we discuss three possible cases of offshoring: 1) the fairness constraint does not bind before and after offshoring; 2) it does not bind before but binds after offshoring; 3) it binds before and after offshoring. Note that in the post-offshoring equilibrium the issue of fairness constraint binding or not binding is applicable only for firms that are not offshoring or sourcing their production input domestically.
To analyze the impact of offshoring on wage and unemployment we make use of the zero profit conditions in (23) and (31) which in turn are based on the firms’ optimization exercises in the unconstrained and constrained cases, respectively. Analogous to the derivation of (31), using the two first-order conditions (34) and (35), we derive the following zero profit condition for the offshoring firm

$$\frac{1}{(1 - \beta) \phi} \left( \frac{c_s}{q(\theta_s)} \right)^\gamma (p_m)^{1-\gamma} = 1 \quad (37)$$

We analyze here the different possible cases as follows:

### 4.1 Case 1: Fairness constraint non-binding before and after offshoring

This case also isolates the impact of offshoring on unemployment when unemployment is caused only by search frictions and there are no fairness considerations, and hence serves as a benchmark for considering the issues arising out of fairness considerations.

Denote the autarky values of variables with a superscript $A$ and the post-offshoring values with a superscript $o$. Recall that the autarky zero profit condition (23) can be written using superscript $A$ as

$$\frac{1}{1 - \beta} \left( \frac{c_s}{q(\theta_s^A)} \right)^\gamma \left( \frac{c_l}{q(\theta_l^A)} \right)^{1-\gamma} = 1 \quad (38)$$

At the autarky values of $\theta_s^A$ and $\theta_l^A$, in order for a firm to be induced to offshore, it should make a positive profit from offshoring. Using (37) we can write this condition as

$$\frac{1}{(1 - \beta) \phi} \left( \frac{c_s}{q(\theta_s^A)} \right)^\gamma (p_m)^{1-\gamma} < 1 \quad (39)$$

Therefore, starting from autarky, in order for any firm to want to offshore, it must be the case that

$$\frac{p_m^{1-\gamma}}{\phi} < \left( \frac{c_l}{q(\theta_l^A)} \right)^{1-\gamma} \quad (40)$$

That is, for a given $p_m$, the higher the $\theta_l^A$, the greater the likelihood of offshoring.

In the post-offshoring equilibrium, the zero profit condition of an offshoring firm derived in (37) can be written using subscript $o$ as

$$\frac{1}{(1 - \beta) \phi} \left( \frac{c_s}{q(\theta_s^o)} \right)^\gamma (p_m)^{1-\gamma} = 1 \quad (41)$$

while the zero profit condition for a non-offshoring firm is given by

$$\frac{1}{1 - \beta} \left( \frac{c_s}{q(\theta_s^o)} \right)^\gamma \left( \frac{c_l}{q(\theta_l^o)} \right)^{1-\gamma} = 1 \quad (42)$$
Since firms must be indifferent between offshoring and non-offshoring, (41) and (42) imply
\[
\frac{p_m^{1-\gamma}}{\phi} = \left( \frac{c_l}{q(\theta_l^f)} \right)^{1-\gamma}
\]  
Conditions (40) and (43) together imply that \( \theta_l^o < \theta_l^A \). This implies from lemma 2 that the unskilled wage falls and the unskilled unemployment rises due to offshoring. By comparing the zero profit conditions of non-offshoring firms in the pre and post offshoring equilibriums ((38) and (42)) note that \( \theta_l^o < \theta_l^A \) implies \( \theta_l^o > \theta_s^A \), thus, the skilled wage rises, and skilled unemployment falls. There is a rise in wage inequality as well since the skilled wage rises and the unskilled wage falls.

Note that the cost of hiring a unit of unskilled labor for non-offshoring firms is \( w_o^o + \frac{c_l}{q(\theta_l^f)} \), which from lemma 2 equals \( \frac{1}{1-\beta} \frac{e_l}{q(\theta_l^f)} \). Therefore, (43) can be re-written as
\[
p_m^{1-\gamma} = \phi(1 - \beta)^{1-\gamma} \left( w_o^o + \frac{c_l}{q(\theta_l^f)} \right)^{1-\gamma}
\]
Since \( \phi(1 - \beta)^{1-\gamma} < 1 \), the above implies that \( p_m < w_o^o + \frac{c_l}{q(\theta_l^f)} \) in an offshoring equilibrium. That is, there is a wedge between the cost of domestic unskilled labor and the price at which the input is offshored. Since firms are wage setters in the model, they value the marginal worker not only for her marginal product but also for her impact on the wage bill. An extra unskilled worker lowers the marginal product of an unskilled worker and therefore, when firms can control the unskilled wage through employment, the wage of existing unskilled workers goes down. This monopsony power is valued by firms and to keep this monopsony power, they are willing to bear a higher cost of domestic unskilled labor compared to the cost of offshoring the production input. Algebraically, note from (35) that \( F_2(s,m) = p_m - t^2 w_s^o(t) \), and since \( w_s^o(t) < 0 \), the marginal expenditure by a firm on a unit of offshored input exceeds \( p_m \) because offshored input increases the marginal product of domestic skilled labor and hence their wage through Nash bargaining.

4.2 Case 2: Constraint non-binding before and binding after offshoring

Since the fairness constraint is not binding in autarky, starting from autarky, in order for a firm to offshore, condition (40) must be satisfied. The zero profit condition of offshoring firms is again given by (41). Since the fairness constraint is binding in the post-offshoring equilibrium, the zero profit condition of non-offshoring firms, obtained in (31) can be written using superscript \( o \) as
\[
\frac{1}{(1-\beta)\phi} \left( \frac{c_s}{q(\theta_s^o)} \right)^\gamma \left( \frac{\tau\beta}{1-\beta} \left( \frac{c_s}{q(\theta_s^o)} \right) + \frac{c_l}{q(\theta_l^f)} \right)^{1-\gamma} = 1
\]  
25
Now, (41) and (44) imply $p_m = w^o + \frac{c_l}{q(\theta_l)} = \left(\frac{\tau}{\beta} \right) \left(\frac{c_s}{q(\theta_s)}\right) + \frac{c_l}{q(\theta_l)}$. This along with (40) implies $w_1^A + \frac{c_l}{q(\theta_l)} > w_0^o + \frac{c_l}{q(\theta_l)}$. That is, the cost of hiring unskilled labor is lower in the post-offshoring equilibrium.

Next, (38), (40), and (41) imply that $\theta^o_s > \theta^A_s$, which in turn implies that the post-offshoring skilled wage is higher and unemployment lower than in autarky: $w_s^o > w_s^A, w_0^o < w_0^A$.

Since the fair-wage constraint did not bind in autarky, it must be the case that $w_1^A > \tau w_s^A$ in the autarky unconstrained equilibrium. Since $w_0^o > w_s^A$, the unskilled fair wage in the post-offshoring equilibrium, $\tau w_s^o$, is higher than $\tau w_s^A$. Therefore, it is possible for post-offshoring unskilled wage to be higher than the autarky unskilled wage. However, the post-offshoring unskilled unemployment must be higher than the autarky unemployment as is shown below.

**Lemma 4:** When offshoring converts a nonbinding fair-wage constraint into a binding one, it must be the case that $\theta^o_i < \theta^A_i$ (Proof in appendix).

To understand the intuition behind this lemma, note first that since the fairness constraint is binding in the post-offshoring equilibrium, there is no wedge between the cost of domestic unskilled labor and the cost of offshoring the input: $p_m = w^o + \frac{c_l}{q(\theta_l)}$. A binding fairness constraint implies the absence of monopsony power vis-a-vis unskilled workers, and hence there cannot be a wedge. Secondly, note that the cost of hiring (wage plus vacancy cost) a unit of unskilled labor under autarky has to be higher than the cost of offshoring the input for offshoring to actually take place (This is a necessary condition). Given that in the offshoring equilibrium domestic unskilled labor and offshored imported input are perfect substitutes, the latter exerts downward competitive pressure on the domestic unskilled labor market. This downward pressure on unskilled labor market tightness is further accentuated by the fact that the fair-wage constraint is binding and a higher wage will have to be paid in comparison to what unskilled workers would have been paid in the absence of such a constraint.

One of the things that clearly emerges from this analysis is that offshoring makes it more likely that the fairness constraint will bind. In other words, the cutoff value of $\tau$ above which the fairness constraint binds is lower under offshoring.

Cases 1 and 2 also allow us to compare the effects of offshoring in the absence and presence of the fair-wage constraint. The autarky equilibria under the two regimes are the same when, under fairness considerations, the fair-wage constraint is not binding in autarky. Essentially, we are
comparing the post-offshoring outcomes in cases 1 and 2 above. Note that the zero profit condition of offshoring firms is given by (41) in both cases, while for non-offshoring firms it is given by (42) if the constraint does not bind, and (44) if the constraint does bind. Using superscripts \( u \) and \( c \) to denote the variables in the unconstrained and constrained cases, respectively, it can be easily verified from (41), (42), and (44) that \((\theta^o_\text{os})^u = (\theta^o_\text{os})^c\) and \((\theta^o_\text{ol})^u > (\theta^o_\text{ol})^c\). The former implies that skilled wage and unemployment remain unchanged in the offshoring equilibrium if the constraint binds. However, the unskilled unemployment is higher in the latter case. Finally, the unskilled wage, given by \(\tau w^o_u\), is higher in the constrained case.

4.3 Case 3: Constraint binding before and after offshoring

The results for this case are formally derived in the appendix, but are intuitively explained here as follows. Since there is a net strategic effect (but no strategic effect to be exercised on unskilled wage or cost of the imported input) for firms wanting to offshore as well as those wanting to produce the input domestically (employ domestic unskilled labor), there is no “wedge” between domestic labor costs (wage plus vacancy costs) and the cost of offshoring the production input. Offshoring here pushes down the cost of hiring domestic unskilled labor (wage plus vacancy costs). The reason is that domestic unskilled labor comes in direct competition with the imported input that is a perfect substitute for it in the offshoring equilibrium. In the offshoring equilibrium, we know that a firm will be indifferent between offshoring the production input and producing it domestically using domestic unskilled labor. Given the zero-profit condition of the final good \( Z \), the lower domestic cost of an unskilled worker allows for a higher labor cost and therefore a higher market tightness for skilled workers. Thus skilled unemployment is lower and skilled wages are higher. Since the unskilled wage is proportional to skilled wage by the fair-wage condition, unskilled wages are also higher. Given that the cost of hiring (wage plus vacancy costs) an unskilled worker has gone down, this means that the vacancy cost per worker hired has gone down (unskilled labor market tightness has gone down). Therefore, unskilled unemployment has gone up.

We summarize the result on offshoring in the proposition below.

**Proposition 3:** Irrespective of the fairness constraint, offshoring leads to an increase in skilled wage and a decrease in skilled unemployment. As well, there is an unambiguous increase in unskilled unemployment. If the constraint does not bind before or after offshoring, then the unskilled wage
decreases unambiguously. If the constraint binds both before and after offshoring, then the unskilled wage increases unambiguously. Even if the constraint is not binding in autarky, it may become binding upon offshoring. In this case, the impact on unskilled wage is ambiguous.

Another thing to note from the above analysis is that final unemployment rates and wages of skilled and unskilled workers in an offshoring equilibrium are independent of the country’s relative factor endowments and depend totally on the price of the offshored input, $p_m$. On the other hand, we have shown that autarky unemployment and wages depend on endowments.

4.4 Impact of an increase in $\tau$ on offshoring

In this sub-section, we want to study the impact of an increase in $\tau$ on the likelihood of offshoring, and starting from an offshoring equilibrium, the impact of an increase in $\tau$ on the extent of offshoring. From proposition 2 we know that in autarky: $\frac{d\theta_A^s}{d\tau} < 0$, $\frac{d\theta_A^l}{d\tau} < 0$. Let us use the following notational simplification in this section.

**Notation:** Denote $\omega(\tau) \equiv \left( \frac{\tau^\beta}{1-\beta} \right) \left( \frac{c_s}{q(\theta_A^s)} \right) + \frac{c_l}{q(\theta_A^l)}$

Using the zero profit condition in autarky when the constraint is binding given by (31) and proposition 2, we get $\omega'(\tau) > 0$. Starting from a constrained autarky equilibrium, in order for firms to offshore it must be the case that $\omega(\tau) > p_m$. Therefore, an increase in $\tau$ makes offshoring more likely.

Starting from an offshoring equilibrium where the fairness constraint is binding, an increase in $\tau$ has no effect on $\theta_A^s$ as is easily verified from the zero profit condition of offshoring firms given in (41). Therefore, there is no effect on skilled wage and unemployment. However, unskilled wage increases because $w^o_s = \tau w^o_s$. Next, the zero profit condition of the non-offshoring firms given in (44) implies that $\frac{d\theta_l^o}{d\tau} < 0$. That is, the effect of an increase in $\tau$, starting from an offshoring equilibrium is to increase unskilled wage and unskilled unemployment. As well, (41) and (44) together imply that $\left( \frac{\tau^\beta}{1-\beta} \right) \left( \frac{c_s}{q(\theta_A^s)} \right) + \frac{c_l}{q(\theta_A^l)} = p_m$, and therefore, the relative demand for skilled labor by non-offshoring firms given in (29) is unchanged as is the relative demand $s/m$ for offshoring firms. Since the effective relative supply of domestic skilled labor has risen due to increased unemployment of unskilled workers, more firms must be offshoring now. Therefore, an increase in $\tau$ leads to an increase in offshoring.

**Proposition 4:** An increase in fairness parameter, $\tau$, increases the likelihood of offshoring.
Starting from an offshoring equilibrium where the fairness constraint is binding, an increase in \( \tau \) leads to an increase in offshoring, an increase in unskilled wage, and an increase in unskilled unemployment, while the unemployment rate and wage of skilled workers remain unchanged.

Before ending the discussion of offshoring we note the implications of allowing the adjustment of the imported input any time, in particular after the skilled employment has been chosen and at the same time as (or even after) they bargain over wages with skilled workers. In this case the optimal choice of imported input must always satisfy \( F_2(s, m) = p_m \), and the equilibrium \( t \) will be pinned down by this condition. Therefore, the strategic effect will disappear \( (\frac{\partial w_s}{\partial s} = 0) \), and the first-order condition for the choice of skilled labor will become \( f'(t) - w_s(t) = \frac{\sigma}{\phi(\theta)} \), and the Nash bargained wage will simply be \( w_s(t) = \beta f'(t) \). In this case, the cost of employing a unit of offshored input equals \( p_m \). The results regarding the increase in unskilled unemployment and decline in skilled unemployment obtain even in this case. One difference from the case when the imported input is chosen along with the skilled wage in the first stage is that now the wedge between the price of imported input and the cost of domestic unskilled labor obtains when the fairness constraint binds in the post-offshoring equilibrium, while earlier it obtained when the fairness constraint did not bind in the offshoring equilibrium.

5 Discussions and Concluding Remarks

In this paper, we have introduced fairness considerations in a model of unemployment with search frictions. The purpose is to generate positive unemployment rates for both categories of workers as found in the macroeconomic data across countries and at the same time take the strong evidence in favor of the fair-wage hypothesis seriously. In our model the fair-wage constraint operated at the level of the industry. That is, the firm took it as given while deciding on its employment decision in the first period. In models with competitive labor market such as Akerlof and Yellen (1990) or Kreickemeier and Nelson (2006), when looking at an integrated equilibrium where both types of labor are employed by all firms, one cannot make a distinction between the cases when the fair-wage constraint operates at the firm level or the industry level. However, in our framework where firms have monopsonistic power in the labor market, it is possible to make a distinction between the two cases. If the fair wage is determined at the firm level, then a firm takes this into account while
choosing its employment in the first stage. To be more precise, the objective function of the firm given in (8) now becomes

$$\max_{s,l} \left\{ F(s,l) - w_s(s,l)s - \max \{ w_l(s,l), \tau w_s(s,l) \} \left[ l - \frac{c_s}{q(\theta_s)} s - \frac{c_l}{q(\theta_l)} l \right] \right\}$$

(45)

where $w^*_s$ has now been replaced by $\tau w_s(s,l)$ since it is determined within the firm. As a result, the first order condition for the optimal choice of $s$ in the case where the constraint is expected to bind is given by

$$f'(t) - w_s - s \frac{\partial w_s}{\partial s} - l\tau \frac{\partial w_s}{\partial s} = \frac{c_s}{q(\theta_s)}$$

(46)

Note the extra term $l\tau \frac{\partial w_s}{\partial s}$ which captures the fact that any change in skilled wage affects the fair wage as well, which is taken into account by the firm. Using the Nash bargaining as before, the wage of the skilled workers is given by the following differential equation

$$w_s(t) = \beta \left[ f'(t) - (t + \tau) w_s'(t) \right]$$

(47)

It can be verified that the solution to the above differential equation is given by

$$w_s(t) = (t + \tau)^{-\frac{1}{\beta}} \int_0^t (x + \tau)^{1-\frac{1}{\beta}} f'(x) dx$$

(48)

Intuitively, since hiring an extra skilled worker reduces the fair wage by lowering the skilled wage, the value of a skilled worker for a firm is higher than in the case when the fair wage is exogenous from the firm’s point of view. A higher value of skilled worker implies a higher wage for skilled workers as well. Therefore, the ‘strategic effect’ in the hiring of skilled workers becomes stronger than in the case discussed earlier. Note that when the firm takes the fair wage as given, then both in autarky in the case of offshoring, the only ‘strategic effect’ is that of skilled employment affecting skilled wage. When the fair wage is determined inside the firm, there is an additional strategic effect in autarky because skilled employment affects fair wage, however, it vanishes in the case of offshoring because the firm takes the price of offshored input as given. A full fledged analysis of the case when the fair wage is determined within the firm is left for future research.

Let us conclude here by summarizing our key findings in this paper. Firstly, we find that the effect of search frictions and the fair-wage constraint in this encompassing model on the response of wages, wage inequality and unemployment rates to various shocks is not additively separable. There are important interaction effects that can be missed unless these two characteristics of the
labor market are combined in the same model. Specifically, we find that a binding fair-wage constraint increases the unskilled unemployment rate and can also lead to an increase in the skilled unemployment rate as compared to an equilibrium where fairness considerations are absent or non-binding. This is consistent with the experiences of Europe and the US.

Secondly, allowing for offshoring of unskilled jobs in our model, we find that it becomes more likely that the fair-wage constraint binds. Offshoring always leads to an increase in skilled wage, a decrease in skilled unemployment and an increase in unskilled unemployment. The presence of fairness considerations increases the adverse impact of offshoring on unskilled unemployment. The unskilled wage can increase or decrease as a result of offshoring. If the fair-wage constraint binds both before and after offshoring, then the unskilled wage increases unambiguously.

Finally, the difference between our combined model and that of a model purely based on fairness considerations is also important to note. It is shown in the appendix (section 6.11) that in a pure fair-wage model there is no skilled unemployment. As a result, skilled unemployment is not affected by shocks such as changes in the degree of fairness, changes in the relative factor endowments, or offshoring. More importantly, while in the hybrid (search plus fair wage) model presented in the text the adjustment to any shock takes place through changes in both skilled and unskilled unemployment, in the fair-wage model only the unskilled unemployment adjusts which can result in extremely high levels of unskilled unemployment. For example, we show in the appendix that in the fair-wage model offshoring leads to complete unemployment of unskilled workers.

References


### 6 Appendix

#### 6.1 Firms never pay a wage less than the fair wage

Given the production function in (1) a firm maximizes

\[
\max F(\hat{s}, \hat{l}) - \hat{w}_s \hat{s} - \hat{w}_l \hat{l} - \frac{\hat{c}_s}{q(\theta_s)} \hat{s} - \frac{\hat{c}_l}{q(\theta_l)} \hat{l}
\]

where \( \hat{i} = \epsilon_i i, \hat{w}_i = \frac{w_i}{\epsilon_i}, i = s, l \). Here \( \frac{\hat{c}_i}{q(\theta_i)} \) is the recruitment cost per efficiency unit of labor , and \( \theta_i \) is the market tightness in terms of efficiency units of labor. \( \hat{c}_i \) is clearly decreasing in the number of efficiency units per worker if the recruitment cost per worker (irrespective of the efficiency units provided) is constant (for given \( \theta_s, \theta_l \)).

We show below that paying an unskilled worker a wage equal to \( w^*_l \) leaves the profits gross of recruitment costs unchanged relative to what is obtained under a lower wage, for given total number of efficiency units of labor, \( \hat{l} = \varepsilon_l l \), where \( l \) is the number of workers. If \( w_l \leq w^*_l \), then \( \varepsilon_l = \frac{w_l}{w^*_l} \leq 1 \). Therefore, the wage per efficiency unit of labor, \( \hat{w}_l = \frac{w_l}{\varepsilon_l} = w^*_l \), which is taken as given by the firm. Thus, as long as the firm pays a wage of \( w^*_l \) per efficiency unit of labor and hires a fixed total of \( \hat{l} \) efficiency units of labor, its profit gross of recruitment costs remains unchanged.

The profits net of recruitment costs will be higher at per worker wage \( w^*_l \) than any lower wage for the following reason. Paying a per-worker wage of \( w^*_l \) would result in each worker working at full
efficiency. Therefore, fewer workers will be needed to provide a given number of efficiency units, \( \hat{\ell} \). Given that recruitment cost per worker is constant for given \( \theta_s, \theta_l \), profits net of recruitment costs will be higher.

Even if the recruitment cost per efficiency unit is not declining in the efficiency units per worker, the profits will not decline and will remain unchanged. As in Akerlof and Yellen (1990), we assume that firms have a preference for fairness if the fair wage can be provided at no cost to their profits.

Thus, firms never pay less than the fair wage.

6.2 Proof of Lemma 1

Re-write (12) and (13) as

\[
\begin{align*}
w_s(t) &= \beta [f'(t) - tw'_s(t) - w'_l(t)] \\
w_l(t) &= \beta [f(t) - tf'(t) + t^2 w'_s(t) + tw'_l(t)]
\end{align*}
\]

The above is a system of differential equations in the argument \( t \). We guess the following solution:

\[
\begin{align*}
w_s(t) &= \beta f'(t); w_l(t) = \beta (f(t) - tf'(t)), \text{ and verify that it satisfies (49)-(50).}
\end{align*}
\]

The guessed solution implies

\[
w'_s(t) = \beta f''(t); w'_l(t) = \beta [-tf''(t)]
\]

Therefore, \( tw'_s(t) + w'_l(t) = 0 \), and hence (49)-(50) imply \( w_s(t) = \beta f'(t); w_l(t) = \beta (f(t) - tf'(t)). QED \)

6.3 Proof of Lemma 2

The first-order conditions in (9) and (10) can be written as

\[
\begin{align*}
f'(t) - w_s(t) - tw'_s(t) - w'_l(t) &= \frac{c_s}{q(\theta_s)} \\
f(t) - tf'(t) - w_l(t) + t^2 w'_s(t) + tw'_l(t) &= \frac{c_l}{q(\theta_l)}
\end{align*}
\]

Using \( tw'_s(t) + w'_l(t) = 0 \), and \( w_s(t) = \beta f'(t) \) from lemma 1 in (52) we get

\[
w_s(t) = \frac{\beta c_s}{1 - \beta q(\theta_s)}
\]

Similarly, \( w_l(t) = \frac{\beta c_l}{1 - \beta q(\theta_l)} QED. \)
6.4 Proof of Lemma 3

Proof. Note that \( w_s = \phi \beta f'(t) \). The first-order condition with respect to \( s \) given in (14) can be re-written as

\[
f'(t) - w_s(t) - tw'_s(t) = \frac{c_s}{q(\theta_s)}
\]

The wage determination equation is

\[
w_s(t) = \beta[f'(t) - tw'_s(t)]
\]

The above two imply that

\[
w_s(t) = \frac{\beta}{1 - \beta} \frac{c_s}{q(\theta_s)}
\]

QED.

6.5 Constrained \( ZPC^c \) lies to the left of the unconstrained \( ZPC^u \) in \( (\theta_l, \theta_s) \) space

Proof: Recall from the text that the two zero profit conditions are

\[
\frac{1}{1 - \beta} \left( \frac{c_s}{q(\theta_s)} \right)^\gamma \left( \frac{c_l}{q(\theta_l)} \right)^{1-\gamma} = 1
\]

(54)

\[
\frac{1}{(1 - \beta)\phi} \left( \frac{c_s}{q(\theta_s)} \right)^\gamma \left( \gamma \left( \frac{\beta}{1 - \beta} \right) \left( \frac{c_s}{q(\theta_s)} \right) + \frac{c_l}{q(\theta_l)} \right)^{1-\gamma} = 1
\]

(55)

Now, for the unconstrained case, obtain \( \theta_s \) and \( \theta_l \) corresponding to any \( \frac{\theta_s}{\theta_l} \) from (54). Next, verify that at this \( \theta_s \) and \( \theta_l \) the l.h.s. of (55) is greater than 1. To see this divide (55) by (54) to obtain

\[
\phi^{-1} \left( \tau \left( \frac{\beta}{1 - \beta} \right) \left( \frac{c_s q(\theta_l)}{c_l q(\theta_s)} \right) + 1 \right)^{1-\gamma}
\]

(56)

Now, if the constraint binds, then \( \tau \left( \frac{c_s}{q(\theta_s)} \right) > \left( \frac{c_l}{q(\theta_l)} \right) \), and therefore,

\[
\phi^{-1} \left( \tau \left( \frac{\beta}{1 - \beta} \right) \left( \frac{c_s q(\theta_l)}{c_l q(\theta_s)} \right) + 1 \right)^{1-\gamma} > \phi^{-1} \left( \left( \frac{\beta}{1 - \beta} \right) + 1 \right)^{1-\gamma} = \phi^{-1} (1 - \beta)^{\gamma - 1} > 1
\]

where the last inequality is true for any \( 0 < \beta < 1, 0 < \gamma < 1 \).
6.6 Constrained skilled unemployment higher in the absence of strategic effect

Proof: Note that the absence of strategic effect in wage setting in the constrained case implies \( \frac{\partial w_s}{\partial s} = 0 \). Therefore, the solution to (16) is simply \( w_s(t) = \beta f'(t) \). The two first-order condition become

\[
\begin{align*}
f'(t) &= \frac{c_s}{(1 - \beta)q(\theta_s)} \\
f(t) - tf'(t) &= w^*_t + \frac{c_l}{q(\theta_l)}
\end{align*}
\]

Therefore, the relative demand for skilled labor is given by

\[
\frac{f'(t)}{f(t) - tf'(t)} = \frac{1 - \beta}{w^*_t + \frac{c_l}{q(\theta_l)}}
\]

The above can be re-written as

\[
t^d = \frac{\gamma(1 - \beta)(w^*_t + \frac{c_l}{q(\theta_l)})}{(1 - \gamma)\frac{c_s}{q(\theta_s)}} \tag{57}
\]

Using \( w^*_t = \tau w_s(t) = \frac{\tau c_s}{(1 - \beta)q(\theta_s)} \), the above becomes

\[
t^d = \frac{\gamma}{(1 - \gamma)} \left( \tau + \frac{(1 - \beta)q(\theta_s)}{c_s q(\theta_l)} \right)
\]

In the constrained case, when (33) is satisfied, the constrained relative demand above lies to the right of the unconstrained one, given by \( t^d = \frac{\gamma}{(1 - \gamma)} \left( \frac{c_l q(\theta_l)}{c_s q(\theta_s)} \right) \), for values of \( \frac{\theta_s}{\theta_l} > \left( \frac{1}{\tau c_s} \right)^{\frac{1}{1 - \gamma}} \) where the fair-wage constraint binds. This is simply because unskilled labor is more expensive due to its higher wage, and hence firms economize on unskilled labor. Since the relative supply curve is unchanged, the constrained equilibrium \( \frac{\theta_s}{\theta_l} \) (when the fair-wage constraint binds) is higher than the unconstrained one even in the absence of the strategic effect.

To find the equilibrium values of \( \theta_s \) and \( \theta_l \) corresponding to the equilibrium \( \frac{\theta_s}{\theta_l} \), eliminate \( t \) from the two first-order conditions to obtain

\[
\left( \frac{1}{1 - \beta} \right)^\gamma \left( \frac{c_s}{q(\theta_s)} \right)^\gamma \left( \frac{\tau \beta}{1 - \beta} \right) \left( \frac{c_s}{q(\theta_l)} \right) + \frac{c_l}{q(\theta_l)} \right)^{1 - \gamma} = 1 \tag{58}
\]

Again, it can be verified that the curve representing (58) above lies to the left of the unconstrained one in (54) in \((\theta_l, \theta_s)\) space for values of \((\theta_l, \theta_s)\) for which the fairness constraint binds. Therefore, the constrained equilibrium \( \theta_l \) is still lower than the unconstrained one. Note that the skilled wage
is given by \( w_s(t) = \beta f'(t) \) in both the unconstrained and constrained cases. We have already established that the equilibrium \( t \) is higher in the constrained case. This immediately implies that \( w_s(t) \) is lower. Since \( w_s(t) \) is increasing in \( \theta_s \), \( (w_s(t) = \frac{\beta C_s}{(1-\beta)q(\theta_s)} \) \), \( \theta_s \) must decrease as well. Also, since \( f(t) - tf'(t) = w_l^* + \frac{c_t}{q(\theta_t)} \) in the constrained case and \( f(t) - tf'(t) = w_l + \frac{c_t}{q(\theta_t)} \) in the unconstrained case, and given that we have shown that \( \theta_l \) is lower in the constrained case, with a higher \( t \) in the constrained case (that leads to a higher \( f(t) - tf'(t) \)), unskilled wage is higher in the constrained case. \( Q.E.D \)

6.7 Leontief Case

The production function is Leontief, but matching functions remain Cobb-Douglas as above.

*Wage determination*

\( F(s,l) = \min(\varphi s, l) \). In this case when \( \varphi s > l \), \( MPL = \infty \) and \( MPS = 0 \), while if \( \varphi s < l \), we have \( MPL = 0 \) and \( MPS = \infty \). To have a well-defined bargaining problem in the second stage, our bargaining set should be bounded and therefore, we assume that there is an upper bound (constant) on skilled and unskilled wages, given by \( \bar{w}_s \) and \( \bar{w}_l \) that can be paid. It is easy to see that the following will be a bargaining solution that satisfies the Stole-Zweibel intrafirm bargaining conditions:

**Unconstrained case:** When \( \varphi s > l \), \( w_l = \bar{w}_l \) and \( w_s = 0 \), while if \( \varphi s < l \), we have \( w_l = 0 \) and \( w_s = \bar{w}_s \).

**Constrained case:** \( w_l = w_l^* \). When \( \varphi s > l \), \( w_s = 0 \), while if \( \varphi s < l \), we have \( w_s = \bar{w}_s \).

**Equilibrium**

When faced with these wage functions, the firm will always set \( \varphi s = l \) in the first stage. In other words, \( \varphi \) is always set to \( \frac{1}{\varphi} \).

Thus, the strategic effect is completely absent in the Leontief case.

6.8 Ruling out the case of fair-wage constraint binding before but not after offshoring

From the analysis done in Case 1 in the text, it is clear that in the absence of any fair-wage considerations (equivalent to the case where the fairness constraint never binds), the relative wage goes up after offshoring: \( \left( \frac{w_s^o}{w_l} \right)^u > \left( \frac{w_s^A}{w_l} \right)^u \) where superscript \( u \) denotes the unconstrained case. Now
if the fairness constraint is binding in autarky, then \((\frac{w_s^A}{w_l^A})^u > \frac{1}{\tau_s}\), and therefore, \((\frac{w_l^o}{w_l^A})^u > \frac{1}{\tau_l}\), that is, the constraint must bind in the post-offshoring equilibrium. Therefore, this case can be ruled out as a possibility.

### 6.9 Proof of Lemma 4.

Proof: Suppose not. Suppose \(\theta_l^o > \theta_l^A\). This implies that the unconstrained unskilled wage corresponding to \(\theta_l^o\), denoted by \(\tilde{w}_l^o\) is greater than \(w_l^A\). Since the constrained unskilled wage \(w_l^o > \tilde{w}_l^o\) by definition, \(w_l^o > w_l^A\), and hence, \(w_l^o + \frac{c_l}{q(\theta_l^o)} < w_l^o + \frac{c_l}{q(\theta_l^A)}\), which is a contradiction. \(Q.E.D.\)

### 6.10 Derivation of results for case 3 under offshoring: Fairness constraint binding before and after offshoring

If the fairness constraint binds in autarky, the zero profit condition is

\[
\frac{1}{(1 - \beta)\phi} \left( \frac{c_s}{q(\theta_s^A)} \right)^\gamma \left( \left( \frac{\tau \beta}{1 - \beta} \right) \left( \frac{c_s}{q(\theta_s^A)} \right) + \frac{c_l}{q(\theta_l^A)} \right)^{1-\gamma} = 1
\]

(59)

Given the zero profit condition of offshoring firms in (41), starting from autarky equilibrium, in order for offshoring to be profitable, it must be the case that

\[
\left( \frac{\tau \beta}{1 - \beta} \right) \left( \frac{c_s}{q(\theta_s^A)} \right) + \frac{c_l}{q(\theta_l^A)} > p_m
\]

(60)

In the post-offshoring equilibrium the zero profit condition of the offshoring firms is given by (41), while that of non-offshoring firms is given by (44). The two together imply that

\[
p_m = w_l^o + \frac{c_l}{q(\theta_l^o)} = \left( \frac{\tau \beta}{1 - \beta} \right) \left( \frac{c_s}{q(\theta_s^o)} \right) + \frac{c_l}{q(\theta_l^o)}
\]

(61)

This immediately implies from (60) that

\[
\left( \frac{\tau \beta}{1 - \beta} \right) \left( \frac{c_s}{q(\theta_s^o)} \right) + \frac{c_l}{q(\theta_l^o)} < \left( \frac{\tau \beta}{1 - \beta} \right) \left( \frac{c_s}{q(\theta_s^A)} \right) + \frac{c_l}{q(\theta_l^A)}
\]

(62)

Next, (44) and (59) imply that \(\theta_s^o > \theta_s^A\), and consequently, \(w_s^o > w_s^A\). Therefore, skilled wage increases and skilled unemployment decreases compared to autarky. Since the fairness constraint binds, unskilled wage, \(w_l^o = \tau w_s^o > w_l^A = \tau w_s^A\). Since the unskilled wage increases while the unskilled labor cost decreases as shown in (62), \(\theta_l\) must decrease. Therefore, both unskilled wage and unskilled unemployment increase unambiguously.

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6.11 Outcomes in the standard fair-wage model

The firm’s maximization problem can be written as:

\[
\max_{s,l} \frac{s^\gamma l^{1-\gamma}}{\gamma^\gamma(1-\gamma)^{1-\gamma}} - w_s s - \max\{w_l, w_l^*\} l - c_s s - c_l l
\]

(63)

where \(c_i\) is a constant recruitment cost per worker of type \(i = s, l\). Since there is no search for workers, recruitment cost does not depend on market tightness in this case (Every vacancy posted is filled with probability, \(q = 1\)).

6.11.1 Fair-wage constraint does not bind (same as the unconstrained case) under autarky

Assuming that firms decide on employment given market wages, the first-order conditions of the firm’s maximization problem are:

\[
\left[\gamma^\gamma (1-\gamma)^{1-\gamma}\right]^{-1} \gamma s^\gamma - l^{1-\gamma} = w_s + c_s; \quad \left[\gamma^\gamma (1-\gamma)^{1-\gamma}\right]^{-1} (1-\gamma)s^\gamma l^{-\gamma} = w_l + c_l
\]

(64)

Dividing one first-order condition by the other we can solve for relative labor demand as follows:

\[
\frac{s}{l} = \frac{\gamma}{1-\gamma} \left[ \frac{w_l + c_l}{w_s + c_s} \right]
\]

(65)

The two first order conditions, with some manipulation, give us the zero profit condition as follows:

\[(w_s + c_s)^\gamma (w_l + c_l)^{1-\gamma} = 1\]

(66)

The relative factor demand equals relative factor supply, which is written as

\[
\frac{s}{l} = \frac{S}{L} \Rightarrow \frac{\gamma}{1-\gamma} \left[ \frac{w_l + c_l}{w_s + c_s} \right] = \frac{S}{L} \Rightarrow \left[ \frac{w_l + c_l}{w_s + c_s} \right] = \frac{1-\gamma S}{\gamma L}.
\]

(67)

Plugging this into our zero profit condition, we can get our unconstrained solutions as \(w^*_s = \left(\frac{1-\gamma S}{\gamma L}\right)^{\gamma-1} - c_s\) and \(w^*_l = \left(\frac{1-\gamma S}{\gamma L}\right)^{\gamma} - c_l\). Thus, \(w^*_s\) is decreasing in \(\frac{S}{L}\) and \(w^*_l\) is increasing in it. Wages of skilled and unskilled labor are inversely related to their respective recruitment costs. There is no unemployment.
6.11.2 Fair-wage constraint is binding (constrained equilibrium), under autarky

We will have similar first-order conditions as in unconstrained (or non-binding) case. Only $w_i$ gets replaced by the fair-wage $w_i^*$. Dividing one first-order condition by the other and rearranging terms and plugging our original fair-wage relation $w_i^* = \tau w_s$, we can get the labor demand as

$$\frac{s}{l} = \frac{\gamma}{1 - \gamma} \left[ \frac{\tau w_s + c_l}{w_s + c_s} \right]$$

The zero-profit condition can be written as

$$(w_s + c_s)^\gamma (\tau w_s + c_l)^{1-\gamma} = 1$$

For given $\tau$, the zero-profit condition above solves for the constrained equilibrium skilled wage $w_s^c$. Since the left-hand side of the zero-profit condition is increasing in both $w_s$ and $\tau$, it is clear that $w_s^c$ is decreasing in $\tau$, and if that is so the constrained unskilled wage $\tau w_s^c$ is increasing in $\tau$ (to keep the left-hand side equal to unity). Thus, relative labor demand, $\frac{s}{l} = \frac{\gamma}{1 - \gamma} \left[ \frac{\tau w_s^c + c_l}{w_s + c_s} \right]$ is increasing in $\tau$ since the numerator is increasing and the denominator is decreasing in $\tau$. Skilled labor here is fully employed since their wages are fully flexible. Therefore, we have $\frac{s}{l} = \frac{\gamma}{1 - \gamma} \left[ \frac{\tau w_s^c + c_l}{w_s + c_s} \right] = \frac{S}{L(1-u_l)}$

where $u_l$ is the unemployment rate of unskilled labor. Since the equilibrium relative labor demand, $\frac{\gamma}{1 - \gamma} \left[ \frac{\tau w_s^c + c_l}{w_s^c + c_s} \right]$ goes up as a result of an increase in $\tau$, $(1 - u_l)$ has to go down, i.e., $u_l$ has to go up. Skilled unemployment will always be zero. Note that equilibrium skilled wage and hence equilibrium relative demand do not depend on the economy’s labor endowments. Thus, an increase in $S$ or a reduction in $L$ will require a reduction in $u_l$ for $\frac{s}{l} = \frac{S}{L(1-u_l)}$ to hold (as skilled labor will always be fully employed).

**When does the fair-wage constraint bind?** From the above, we can also see that the fair-wage constraint will bind when $\tau w_s^c > w_u^u$. This, from the expressions for unconstrained wages, can be written as

$$\tau \left[ \left( \frac{1 - \gamma S}{\gamma L} \right)^{\gamma - 1} - c_s \right] > \left( \frac{1 - \gamma S}{\gamma L} \right)^{\gamma} - c_l$$

The right-hand side of this inequality is increasing in $\frac{S}{L}$ and the left-hand side is decreasing in it but increasing in $\tau$. Thus, the likelihood that the constraint binds decreases with $\frac{S}{L}$ and increases with $\tau$. 

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As we move from an unconstrained to a constrained equilibrium due to an increase in $\tau$, skilled unemployment remains at zero, unskilled unemployment increases from zero to a positive value, skilled wages fall and the unskilled wage rises because $\frac{s}{l}$ increases.

6.11.3 Offshoring in the pure fair-wage model

Finally, we can look at the case of offshoring. In an offshoring equilibrium, the following equation (zero-profit condition) will hold for firms that offshore:

$$(w_s^o + c_s)^\gamma p_m^{1-\gamma} = 1 \quad (71)$$

where superscript $o$ stands for offshoring. This fixes $w_s$ in the offshoring equilibrium. Offshoring will take place only if $w_A^d + c_l > p_m$, where superscript $A$ stands for autarky. Clearly in that case $w_s$ will go up after offshoring. Starting from a constrained autarky equilibrium, can we have an offshoring equilibrium where some firms offshore and others hire domestic unskilled workers? For the latter firms, the zero profit condition will be

$$(w_s^o + c_s)^\gamma (w_s^o + c_l)^{1-\gamma} = 1 \quad (72)$$

Therefore, $p_m = w_l^o + c_l$. If the fair-wage constraint is binding in autarky, then the unskilled wage is $\tau w_s^A$, and hence $\tau w_s^A + c_l > p_m = w_l^o + c_l$.

That is, $\tau w_s^A > w_l^o$. We also know that $w_s^o > w_s^A$.

Therefore, $w_l^o < \tau w_s^o$.

Thus, unskilled workers can be hired in an offshoring equilibrium only if their wage, $w_l^o$, is less than the fair wage, $\tau w_s^o$. However, at a wage less than the fair wage, unskilled do not put in full effort, and therefore, firms do not hire them. In other words, the unskilled unemployment rate will be 100%. Offshoring leads to a huge increase in unemployment.
Figure 1: Determination of equilibrium
Figure 2: Zero profit conditions