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**JOINT LM TEST FOR HOMOSKEDASTICITY
IN A ONE-WAY
ERROR COMPONENT MODEL**

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and Alain Pirotte**

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Joint LM test for homoskedasticity in a one-way error component model

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Abstract

This paper considers a general heteroskedastic error component model using panel data, and derives a joint LM test for homoskedasticity against the alternative of heteroskedasticity in both error components. It contrasts this joint LM test with marginal LM tests that ignore the heteroskedasticity in one of the error components. Monte Carlo results show that misleading inference can occur when using marginal rather than joint tests when heteroskedasticity is present in both components.

JEL classification: C23

Keywords: Panel data; Heteroskedasticity; Lagrange multiplier tests; Error components; Monte Carlo simulations

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1 Introduction

Mazodier and Trognon (1978) seem to be the first to deal with the problem of heteroskedasticity in panel data. Since then, several papers have followed, see Rao, Kaplan and Cochran (1981), Magnus (1982), Baltagi (1988), Baltagi and Griffin (1988), Randolph (1988), Wansbeek (1989), Li and Stengos (1994), Lejeune (1996), Holly and Gardiol (2000), Roy (2002) and Phillips (2003). These papers usually consider a regression model with one-way error component disturbances: $u_{it} = \mu_i + v_{it}$, $i = 1, \dots, N$, $t = 1, \dots, T$, where the index i refers to the N individuals and the index t to the T time series observations. Both Mazodier and Trognon (1978) and Baltagi and Griffin (1988) were concerned with the estimation of a model allowing for heteroskedasticity on the individual-specific error term, *i.e.*, assuming that $\mu_i \sim (0, \sigma_{\mu_i}^2)$ while $v_{it} \sim IID(0, \sigma_v^2)$. In contrast, Rao, Kaplan and Cochran (1981), Magnus (1982), Baltagi (1988) and Wansbeek (1989) adopted a symmetrically opposite specification allowing for heteroskedasticity on the remainder error term, *i.e.*, assuming that $\mu_i \sim IID(0, \sigma_{\mu}^2)$ while $v_{it} \sim (0, \sigma_{v_i}^2)$. Randolph (1988) allowed for a more general heteroskedastic error component model assuming that *both* the individual and remainder error terms were heteroskedastic, *i.e.*, $\mu_i \sim (0, \sigma_{\mu_i}^2)$ and $v_{it} \sim (0, \sigma_{v_{it}}^2)$, with the latter varying with every observation over time and individuals. As Lejeune (1996) pointed out, if heteroskedasticity is say due to differences in size across individuals, firms or countries, then *both* error components are expected to be heteroskedastic and it may be difficult to argue that only one component of the error term is heteroskedastic but not the other. Early on, Verbon (1980) derived a Lagrange multiplier test where the null hypothesis is that of a standard normally distributed homoskedastic model against the heteroskedastic alternative $\mu_i \sim (0, \sigma_{\mu_i}^2)$ and $v_{it} \sim (0, \sigma_{v_i}^2)$. In Verbon's model, however, $\sigma_{\mu_i}^2$ and $\sigma_{v_i}^2$ are, up to a multiplicative constant, *identical* parametric functions of a vector of time invariant exogenous variables z'_i , *i.e.*, $\sigma_{\mu_i}^2 = \sigma_{\mu}^2 f(z'_i \theta_2)$ and $\sigma_{v_i}^2 = \sigma_v^2 f(z'_i \theta_1)$. Lejeune (1996) on the other hand, dealt with maximum likelihood estimation and Lagrange multiplier testing of a general heteroskedastic one-way error components regression model assuming that $\mu_i \sim (0, \sigma_{\mu_i}^2)$ and $v_{it} \sim (0, \sigma_{v_{it}}^2)$ where $\sigma_{\mu_i}^2$ and $\sigma_{v_{it}}^2$ are *distinct* parametric functions of exogenous variables z'_{it} and f'_i , *i.e.*, $\sigma_{v_{it}}^2 = \sigma_v^2 h_v(z'_{it} \theta_1)$ and $\sigma_{\mu_i}^2 = \sigma_{\mu}^2 h_{\mu}(f'_i \theta_2)$. In the context of incomplete panels, Lejeune (1996) derived two joint LM tests for no individual effects and homoskedasticity in the remainder error term. The first LM test considers a random effects one-way error component model with $\mu_i \sim IIN(0, \sigma_{\mu}^2)$ and a remainder error term that is heteroskedastic $v_{it} \sim N(0, \sigma_{v_{it}}^2)$ with $\sigma_{v_{it}}^2 = \sigma_v^2 h_v(z'_{it} \theta_1)$. The joint hypothesis

$\theta_1 = \sigma_\mu^2 = 0$ renders OLS the restricted MLE. Lejeune argued that there is no need to consider a variance function for μ_i since one is testing σ_μ^2 equal to zero. While the computation of the LM test statistic is simplified under this assumption, *i.e.*, $\mu_i \sim IIN(0, \sigma_\mu^2)$, this is not in the original spirit of Lejeune's ML estimation where both μ_i and v_{it} have general variance functions. Lejeune's second LM test considers a fixed effects one-way error component model where μ_i is a fixed parameter to be estimated and the remainder error term is heteroskedastic with $v_{it} \sim N(0, \sigma_{v_{it}}^2)$ and $\sigma_{v_{it}}^2 = \sigma_v^2 h_v(z'_{it} \theta_1)$. The joint hypothesis is $\mu_i = \theta_1 = 0$ for all $i = 1, 2, \dots, N$. This renders OLS the restricted MLE.

With regards to estimation, Li and Stengos (1994) suggested an adaptive estimation procedure for an error component model allowing for heteroskedasticity of unknown form on the remainder error term, *i.e.*, assuming that $\mu_i \sim IID(0, \sigma_\mu^2)$ while $v_{it} \sim (0, \sigma_{v_{it}}^2)$, where $\sigma_{v_{it}}^2$ is a nonparametric function $f(z'_{it})$ of a vector of exogenous variables z'_{it} . Li and Stengos (1994) also suggested a robust version of the Breusch and Pagan (1980) LM test for no random individual effects $\sigma_\mu^2 = 0$ by allowing for adaptive heteroskedasticity of unknown form on the remainder error term. Holly and Gardiol (2000) proposed a Rao score test for homoskedasticity assuming the existence of individual effects. The unrestricted model assumes that $\mu_i \sim N(0, \sigma_{\mu_i}^2)$ and $v_{it} \sim IIN(0, \sigma_v^2)$ where $\sigma_{\mu_i}^2$ is a parametric function of exogenous variables f'_i , *i.e.*, $\sigma_{\mu_i}^2 = \sigma_\mu^2 h_\mu(f'_i \theta_2)$. Under the null hypothesis $\theta_2 = 0$, with $h_\mu(0) = 1$ and the restricted model reverts to the homoskedastic one-way error component model. Roy (2002) dealt with adaptive estimation of an error component model assuming heteroskedasticity of unknown form for the individual-specific error term, *i.e.*, assuming that $\mu_i \sim (0, \sigma_{\mu_i}^2)$ while $v_{it} \sim IID(0, \sigma_v^2)$, where $\sigma_{\mu_i}^2$ is a nonparametric function $f(\bar{z}'_i)$ of a vector of individual means of exogenous variables z'_{it} with $\bar{z}'_i = \sum_{t=1}^T z'_{it}/T$. More recently, Phillips (2003) followed Mazodier and Trognon (1978) in considering a one-way stratified error component model. As unobserved heterogeneity occurs through individual-specific variances changing across strata, Phillips provided an EM algorithm for estimating this model and suggested a bootstrap test for identifying the number of strata.

In the spirit of the general heteroskedastic model of Randolph (1988) and Lejeune (1996), this paper derives a *joint* Lagrange multiplier test for homoskedasticity, *i.e.*, $\theta_1 = \theta_2 = 0$. Under the null hypothesis, the model is a homoskedastic one-way error component regression model and is estimated by restricted MLE. Note that this is different from Lejeune (1996), where under the null, $\sigma_\mu^2 = 0$, so that the restricted MLE is OLS and not MLE on a one-way homoskedastic error component model. Allowing for $\sigma_\mu^2 > 0$ is more

likely to be the case in panel data where heterogeneity across the individuals is likely to be present even if heteroskedasticity is not. The model under the null is exactly that of Holly and Gardiol (2000) but it is more general under the alternative since it does not assume a homoskedastic remainder error term. We also derive an LM test for the null hypothesis of homoskedasticity of the individual random effects assuming homoskedasticity of the remainder error term, *i.e.*, $\theta_2 = 0 \mid \theta_1 = 0$. Not surprisingly, we get the Holly and Gardiol (2000) LM test. In addition, we derive an LM test for the null hypothesis of homoskedasticity of the remainder error term assuming homoskedasticity of the individual effects, *i.e.*, $\theta_1 = 0 \mid \theta_2 = 0$. The rest of the paper is organized as follows: Section 2 reviews the general heteroskedastic one-way error component model. Section 3 derives the marginal and joint Lagrange multiplier tests described above. Section 4 performs Monte Carlo simulations comparing the size and power of these LM tests. Section 5 concludes.

2 The general heteroskedastic one-way error component model

We consider the following regression model:

$$y_{it} = x'_{it}\beta + u_{it}, \quad u_{it} = \mu_i + v_{it}, \quad i = 1, \dots, N, \quad t = 1, \dots, T \quad (1)$$

where y_{it} , u_{it} , μ_i and v_{it} are scalars, x'_{it} is a $(1 \times k_\beta)$ vector of strictly exogenous regressors (the first element being a constant) and β is a $(k_\beta \times 1)$ vector of parameters. The index i refers to the N individuals and the index t to the T observations of each individual i . The total number of observations is NT . The error terms μ_i and v_{it} are assumed mutually independent and normally distributed according to:

$$\begin{aligned} v_{it} &\sim N\left(0, \sigma_{v_{it}}^2\right), & \sigma_{v_{it}}^2 &= \sigma_v^2 h_v(z'_{it}\theta_1), & i &= 1, \dots, N, \quad t = 1, \dots, T \\ \mu_i &\sim N\left(0, \sigma_{\mu_i}^2\right), & \sigma_{\mu_i}^2 &= \sigma_\mu^2 h_\mu(f'_i\theta_2), & i &= 1, \dots, N \end{aligned} \quad (2)$$

where $h_v(\cdot)$ and $h_\mu(\cdot)$ are arbitrary non-indexed (strictly) positive twice continuously differentiable functions¹ satisfying $h_v(\cdot) > 0$, $h_\mu(\dots) > 0$, $h_v(0) = 1$, $h_\mu(0) = 1$ and $h_v^{(1)}(0) \neq 0$, $h_\mu^{(1)}(0) \neq 0$ where $h_\mu^{(1)}(x)$ denotes the first derivative of $h_\mu(x)$ with respect to x . z'_{it} and f'_i are respectively $(1 \times k_{\theta_1})$ and $(1 \times k_{\theta_2})$ vectors of strictly exogenous regressors while θ_1 and θ_2 are respectively $(k_{\theta_1} \times 1)$ and $(k_{\theta_2} \times 1)$ vectors of parameters². We will denote by

¹Different choices are possible for the variance functions $h_v(\dots)$ and $h_\mu(\cdot)$. Among them, the additive and the multiplicative heteroskedastic forms appear to be attractive. See Breusch and Pagan (1979).

²If both $\theta_1 = 0$ and $\theta_2 = 0$, system (1)-(2) reduces to the classical homoskedastic one-way error component model.

$\varphi = (\sigma_v^2, \sigma_\mu^2, \theta'_1, \theta'_2)'$ ($\equiv \{\varphi_p\}'$, $p = 1, \dots, 4$) the vector of variance-specific parameters.

Stacking the T observations of each individual i , (1) may be written as:

$$y_i = X_i\beta + u_i, u_i = \iota_T\mu_i + v_i, i = 1, \dots, N \quad (3)$$

where ι_T is a $(T \times 1)$ vector of ones, y_i , u_i and v_i are $(T \times 1)$ vectors and X_i a $(T \times k_\beta)$ matrix of regressors. From (2), the $(T \times T)$ covariance matrix Ω_i of u_i may be written as:

$$\Omega_i = \sigma_v^2 \text{diag}(h_v(Z_i\theta_1)) + \sigma_\mu^2 J_T h_\mu(f'_i\theta_2), i = 1, \dots, N \quad (4)$$

where $J_T = \iota_T \iota_T'$ and Z_i is a $(T \times k_{\theta_1})$ matrix of regressors with a typical row being z'_{it} . Here, $\text{diag}(h_v(Z_i\theta_1))$ denotes a diagonal $(T \times T)$ matrix with its t -th diagonal element being the t -th element of the $(T \times 1)$ vector $h_v(Z_i\theta_1)$. Finally, stacking again the above vectors and matrices, we obtain the general matrix form of the model:

$$y = X\beta + u, u = Z_\mu\mu + v, Z_\mu = I_N \otimes \iota_T \quad (5)$$

$$\Omega(\theta) = \sigma_v^2 \text{diag}(h_v(Z\theta_1)) + \sigma_\mu^2 Z_\mu \text{diag}(h_\mu(F\theta_2)) Z_\mu' \quad (6)$$

y , u and v are $(NT \times 1)$ vectors, μ is the $(N \times 1)$ vector of individual effects X , Z and F are respectively $(NT \times k_\beta)$, $(NT \times k_{\theta_1})$ and $(N \times k_{\theta_2})$ matrix of regressors and Ω is the $(NT \times NT)$ block-diagonal covariance matrix of u . Here, $\text{diag}(h_v(Z\theta_1))$ denotes a diagonal $(NT \times NT)$ matrix with its it -th diagonal element being the it -th element of the $(NT \times 1)$ vector $h_v(Z\theta_1)$. Similarly, $\text{diag}(h_\mu(F\theta_2))$ denotes a diagonal $(N \times N)$ matrix with its i -th diagonal element being the i -th element of the $(N \times 1)$ vector $h_\mu(F\theta_2)$.

3 The marginal and joint Lagrange Multiplier tests

Based on equations (3)-(4), the log-likelihood function L may be written as:

$$L(y|X, Z, F; \beta, \sigma_v^2, \sigma_\mu^2, \theta_1, \theta_2) = -\frac{NT}{2} \ln(2\pi) - \frac{1}{2} \sum_{i=1}^N \ln |\Omega_i| - \frac{1}{2} \sum_{i=1}^N u_i' \Omega_i^{-1} u_i \quad (7)$$

where $u_i = y_i - X_i\beta$. Following Magnus (1978) and Lejeune (1996), the information matrix is given by $\Psi^{NT} = -E_0[H|X, Z, F]$ with $E_0[\cdot]$ denoting expectation taken with respect to the true distribution. H is the hessian matrix with elements $\left(H_{\beta\beta} = \frac{\partial^2 L}{\partial\beta\partial\beta'}\right)$, $\left(H_{\beta\varphi_p} = \frac{\partial^2 L}{\partial\beta\partial\varphi_p'}\right)$ and $\left(H_{\varphi_p\varphi_q} = \frac{\partial^2 L}{\partial\varphi_p\partial\varphi_q'}\right)$.

The information matrix Ψ^{NT} is block diagonal between the β and φ parameters. Therefore, the Lagrange Multiplier (LM) statistic for testing, $H_0 : \varphi = 0$, may be written as:

$$LM_{\varphi=0} = \tilde{g}'_{\varphi} \left(\tilde{\Psi}_{\varphi\varphi}^{NT} \right)^{-1} \tilde{g}_{\varphi} \quad (8)$$

(see Breusch and Pagan (1980) p. 245) where g_{φ} is the gradient of the log-likelihood with respect to φ , $\Psi_{\varphi\varphi}^{NT}$ is the φ block of the information matrix and $\tilde{\cdot}$ denotes quantities evaluated under the null. Under the null, this statistic is asymptotically distributed as a χ^2 with k_{φ} degrees of freedom, k_{φ} being the number of parameters in the vector φ .

3.1 Marginal LM test for $\theta_2 = 0$ assuming $\theta_1 = 0$

For the Mazodier and Trognon (1978), Baltagi and Griffin (1988) and Roy (2002) papers, the remainder error term is assumed to be homoskedastic, *i.e.*, θ_1 is assumed to be zero and the covariance matrix Ω_i in (4) becomes:

$$\Omega_i = \sigma_v^2 I_T + \sigma_{\mu}^2 J_T h_{\mu} (f'_i \theta_2)$$

Testing for homoskedasticity in this model amounts to testing:

$$H_0^a : \theta_2 = 0 \mid \sigma_{\mu}^2 > 0, \sigma_v^2 > 0, \theta_1 = 0$$

the LM statistic for H_0^a is given by³:

$$LM_{\theta_2=0} = \frac{1}{2\tilde{\sigma}_1^4} S' \underline{F} (\underline{F}' \underline{F})^{-1} \underline{F}' S \quad (9)$$

with $\tilde{\sigma}_1^2 = (T\tilde{\sigma}_{\mu}^2 + \tilde{\sigma}_v^2)$ where $\tilde{\sigma}_{\mu}^2$ and $\tilde{\sigma}_v^2$ are the restricted ML estimates of σ_{μ}^2 and σ_v^2 , $\underline{F} = (I_N - \bar{J}_N) F$ with $\bar{J}_N = J_N/N$ and S is a $(N \times 1)$ vector with typical element $S_i = \tilde{u}'_i \bar{J}_T \tilde{u}_i$. The \tilde{u}_i 's are vectors of restricted ML residuals obtained from a one-way error component model with no heteroskedasticity. These can be obtained from standard regression packages, for e.g., using xtreg in Stata with the mle option. F is the matrix of regressors $(N \times k_{\theta_2})$ with typical row f'_i . Under the null H_0^a , this statistic is asymptotically distributed as χ^2 with (k_{θ_2}) degrees of freedom. This statistic is exactly identical to the Rao score statistic proposed by Holly and Gardiol (2000). The LM statistic (9) is simply one half the explained sum of squares (ESS) from the artificial least squares regression of $\left(\frac{1}{\tilde{\sigma}_1^2} S - \iota_N \right)$ on \underline{F} .

³Technical appendices giving the derivation of all LM tests considered in this paper are not reproduced here to save space and can be obtained upon request from the authors.

3.2 Marginal LM test for $\theta_1 = 0$ assuming $\theta_2 = 0$

For the Rao, Kaplan and Cochran (1981), Magnus (1982), Baltagi (1988), Wansbeek (1989) and Li and Stengos (1994) papers, the individual error component is assumed to be homoskedastic, *i.e.*, θ_2 is assumed to be zero and the covariance matrix Ω_i in (4) becomes:

$$\Omega_i = \sigma_v^2 \text{diag}(h_v(Z_i\theta_1)) + \sigma_\mu^2 J_T, \quad i = 1, \dots, N$$

Testing for homoskedasticity amounts to testing

$$H_0^b : \theta_1 = 0 \mid \sigma_\mu^2 > 0, \sigma_v^2 > 0, \theta_2 = 0$$

the LM statistic for H_0^b is given by:

$$LM_{\theta_1=0} = \frac{1}{2} GZ (Z'MZ)^{-1} Z'G' \quad (10)$$

where

$$G = \left[\left(\tilde{u}'\tilde{\Omega}^{-1} \right) \odot \left(\tilde{u}'\tilde{\Omega}^{-1} \right) \right] - \frac{(\tilde{\sigma}_1^2 - \tilde{\sigma}_\mu^2)}{\tilde{\sigma}_1^2 \tilde{\sigma}_v^2} \iota'_{NT}$$

and

$$M = \left[\tilde{\Omega}^{-1} \odot \tilde{\Omega}^{-1} \right] - \left[\frac{\tilde{\sigma}_v^4 + \tilde{\sigma}_1^4(T-1)}{T\tilde{\sigma}_1^4\tilde{\sigma}_v^4} \right] \bar{J}_{NT}$$

where \odot stands for the Hadamard product, *i.e.*, an element-by-element multiplication. Z is the matrix of $(NT \times k_{\theta_1})$ regressors. Under the null, the covariance matrix is: $\tilde{\Omega} = \tilde{\sigma}_1^2 P + \tilde{\sigma}_v^2 Q$ where $P = I_N \otimes \bar{J}_T$ and $Q = I_N \otimes E_T$, see Baltagi (2005). P and Q are the Between and Within transformations and $E_T = I_T - \bar{J}_T$ while $\bar{J}_T = J_T/T$. Under the null H_0^b , this statistic is asymptotically distributed as χ^2 with (k_{θ_1}) degrees of freedom. The LM statistic (10) is approximately equal⁴ to one half the explained sum of squares (ESS) from the artificial least squares regression of $\left(\left[\left(\tilde{\Omega}^{-1} \tilde{u} \right) \odot \left(\tilde{\Omega}^{-1} \tilde{u} \right) \right] / \left(\frac{\tilde{\sigma}_1^2 - \tilde{\sigma}_\mu^2}{\tilde{\sigma}_1^2 \tilde{\sigma}_v^2} \right) - \iota_{NT} \right)$ on Z .

As a special case of this heteroskedasticity specification, one may only want the variance of v_{it} to vary with $i = 1, \dots, N$, so that $\mu_i \sim IID(0, \sigma_\mu^2)$ while $v_{it} \sim (0, \sigma_{v_i}^2)$. In this particular case, $\theta_2 \equiv 0$ and $\text{diag}(h_v(Z_i\theta_1))$ is replaced by $h_v(h'_i\theta_1) I_T$ where h'_i is a $(1 \times k_{\theta_1})$ vector of strictly exogenous regressors. So, the covariance matrix Ω_i becomes:

$$\Omega_i = \sigma_v^2 h_v(h'_i\theta_1) I_T + \sigma_\mu^2 J_T, \quad i = 1, \dots, N$$

⁴This approximation of the LM statistic by one half of the ESS from the artificial regression seems to perform well even when T is small and the difference between the exact formula and the approximation becomes negligible when T increases. Our simulations show that this approximation does not affect the size and power of the test significantly.

Testing for homoskedasticity amounts to testing H_0^b , but the alternative is now different. For this reason we call the null H_0^b to distinguish this special case from the more general alternative in H_0^b . The corresponding LM statistic is given by:

$$LM_{\theta_1=0} = \frac{1}{2a} \underline{S}' \underline{H} (\underline{H}' \underline{H})^{-1} \underline{H}' \underline{S} \quad (11)$$

where

$$a = \frac{\tilde{\sigma}_v^4 + \tilde{\sigma}_1^4 (T-1)}{\tilde{\sigma}_1^4 \tilde{\sigma}_v^4}, \quad \underline{H} = (I_N - \bar{J}_N) H \quad \text{and} \quad \underline{S} = \left(\frac{1}{\tilde{\sigma}_1^4} S + \frac{1}{\tilde{\sigma}_v^4} S^* \right)$$

with S denoting an $(N \times 1)$ vector with typical element $S_i = \tilde{u}_i' \bar{J}_T \tilde{u}_i$ and S^* denoting an $(N \times 1)$ vector with typical element $S_i^* = \tilde{u}_i' E_T \tilde{u}_i$. The \tilde{u}_i 's are vectors of restricted ML residuals obtained from a one-way error component model with no heteroskedasticity. H is the matrix of regressors $(N \times k_{\theta_2})$ with typical row h_i' . Under the null, this statistic is asymptotically distributed as χ^2 with (k_{θ_1}) degrees of freedom. The LM statistic (11) is simply one half the explained sum of squares (ESS) from the artificial least squares regression of $\left(\frac{1}{\sqrt{a}} \underline{S} - \iota_N \right)$ on \underline{H} .

3.3 Joint LM test

For the general heteroskedastic one-way error component model described by equations (3)-(4), testing for homoskedasticity in this model amounts to jointly testing:

$$H_0^c : \theta_1 = 0 \text{ and } \theta_2 = 0 \mid \sigma_\mu^2 > 0, \sigma_v^2 > 0$$

Under the null H_0^c , the variance-covariance matrix of u_i is given by:

$$\Omega_i = \sigma_v^2 I_T + \sigma_\mu^2 J_T = \sigma_1^2 \bar{J}_T + \sigma_v^2 E_T$$

see Baltagi (2005), where $\sigma_1^2 = (T\sigma_\mu^2 + \sigma_v^2)$, $E_T = I_T - \bar{J}_T$ and $\bar{J}_T = J_T/T$. The corresponding LM statistic is given by:

$$\begin{aligned} LM_{\theta_1=0, \theta_2=0} &= \frac{1}{2} GZ (Z' MZ)^{-1} Z' G' \quad (12) \\ &+ \frac{1}{2\tilde{\sigma}_1^4} GZ (Z' MZ)^{-1} \underline{Z}' (\underline{F} \otimes \iota_T) \left[\tilde{\Gamma} \right]^{-1} (\underline{F}' \otimes \iota_T') \underline{Z} (Z' MZ)^{-1} Z' G' \\ &- \frac{T}{\tilde{\sigma}_1^4} GZ (Z' MZ)^{-1} \underline{Z}' (\underline{F} \otimes \iota_T) \left[\tilde{\Gamma} \right]^{-1} \underline{F}' S \\ &+ \frac{T^2}{2\tilde{\sigma}_1^4} S' \underline{F} \left[\tilde{\Gamma} \right]^{-1} \underline{F}' S \end{aligned}$$

where G , Z , M , \underline{F} and S were defined above. Also,

$$\underline{Z} = (I_{NT} - \bar{J}_{NT})Z$$

and

$$\left[\tilde{\Gamma}\right]^{-1} = \left[T^2 (\underline{F}'\underline{F}) - \frac{1}{\tilde{\sigma}_1^4} (\underline{F}' \otimes \iota'_T) \underline{Z} (Z' M Z)^{-1} \underline{Z}' (\underline{F} \otimes \iota_T)\right]^{-1}$$

Under the null H_0^c , this statistic is asymptotically distributed as χ^2 with $(k_{\theta_1} + k_{\theta_2})$ degrees of freedom.

The expression (12) may be computationally cumbersome and the associated artificial regression is not easy to obtain⁵. But, if one ignores the last term in the expression for $\tilde{\Gamma}$, one gets

$$\left[\tilde{\Gamma}\right]^{-1} \simeq [T^2 (\underline{F}'\underline{F})]^{-1}$$

and, as the two middle terms of (12) tend to zero, as T gets large, the joint LM test can be approximated by:

$$\begin{aligned} LM_{\theta_1=0, \theta_2=0} &\simeq \frac{1}{2} GZ (Z' M Z)^{-1} Z' G' & (13) \\ &+ \frac{1}{2\tilde{\sigma}_1^4} S' \underline{F} (\underline{F}'\underline{F})^{-1} \underline{F}' S \end{aligned}$$

i.e., the joint LM test can be approximated by the sum of the two marginal LM tests $LM_{\theta_1=0}$ and $LM_{\theta_2=0}$.

So, the joint LM test (12) is approximatively⁶ the sum of one half the explained sum of squares from the artificial least squares regressions of $\left(\frac{1}{\tilde{\sigma}_1^2} S - \iota_N\right)$ on \underline{F} and one half the explained sum of squares from the artificial least squares regression of $\left(\left[\left(\tilde{\Omega}^{-1}\tilde{u}\right) \odot \left(\tilde{\Omega}^{-1}\tilde{u}\right)\right] / \left(\frac{(\tilde{\sigma}_1^2 - \tilde{\sigma}_\mu^2)}{\tilde{\sigma}_1^2 \tilde{\sigma}_v^2}\right) - \iota_{NT}\right)$ on Z .

⁵We cannot derive artificial regressions a la Davidson and MacKinnon (1990, 2003) since we cannot derive the jacobian of the variance-covariance matrix Ω_i when $\sigma_{v_{it}}^2$ depends on t . In that case, and as already shown by Mazodier and Trognon (1978, p. 456), both characteristic roots and characteristic vectors of Ω_i do depend on the variances (the σ_μ^2 's and the $\sigma_{v_{it}}^2$'s) which are unknown. Moreover, this dependance is generally intricate, so that it would not help much in that respect if we knew σ_μ^2 and $\sigma_{v_{it}}^2$.

⁶This approximation performs well in our simulations even when T is small and improves as T gets large. It does not change the size and power of the test significantly.

4 Monte Carlo results

The design of our Monte Carlo experiments follows closely that of Li and Stengos (1994) and Roy (2002) for panel data, which in turn adapted it from Rilstone (1991) and Delgado (1992) for cross-section data. Consider the following simple regression model:

$$y_{it} = \beta_0 + \beta_1 x_{it} + \mu_i + v_{it}, \quad i = 1, \dots, N, \quad t = 1, \dots, T \quad (14)$$

where

$$x_{it} = w_{i,t} + 0.5w_{i,t-1} \quad (15)$$

We generate $w_{i,t}$ as *i.i.d* $U(0, 2)$. The parameters β_0 and β_1 are assigned values 5 and 0.5 respectively. We choose $N = 50$ and $N = 200$ and $T = 5$ and 10. For each x_i , we generate $T + 10$ observations and drop the first ten observations in order to reduce the dependency on initial values.

- **Case 1:** For the Roy (2002) set up, we generate v_{it} as *i.i.d* $N(0, \sigma_v^2)$ and $\mu_i \sim N(0, \sigma_{\mu_i}^2)$ where

$$\begin{cases} \sigma_{\mu_i}^2 = \sigma_{\mu_i}^2(\bar{x}_i) = \sigma_{\mu}^2(1 + \lambda_{\mu}\bar{x}_i)^2 \\ \text{or} \\ \sigma_{\mu_i}^2 = \sigma_{\mu_i}^2(\bar{x}_i) = \sigma_{\mu}^2 \exp(\lambda_{\mu}\bar{x}_i) \end{cases} \quad (16)$$

where \bar{x}_i is the individual mean of x_{it} . Denoting the expected variance of μ_i by $\overline{\sigma_{\mu_i}^2}$ and following Roy (2002), we fix the expected total variance $\overline{\sigma^2} = \overline{\sigma_{\mu_i}^2} + \sigma_v^2 = 8$ to make it comparable across the different data generating processes. We let σ_v^2 take the values 2 and 6. For each fixed value of σ_v^2 , λ_{μ} is assigned values 0, 1, 2 and 3 with $\lambda_{\mu} = 0$ denoting the homoskedastic individual specific error. For a fixed value of σ_v^2 , we obtain a value of $\overline{\sigma_{\mu_i}^2} = (8 - \sigma_v^2)$ and using a specific value of λ_{μ} , we get the corresponding value for σ_{μ}^2 from (16). Of course, we can choose a quadratic or an exponential heteroskedastic specification for $\sigma_{\mu_i}^2 = \sigma_{\mu}^2 h_{\mu}(f_i' \theta_2)$ with $h_{\mu}(f_i' \theta_2) = (1 + \lambda_{\mu}\bar{x}_i)^2$ or $h_{\mu}(f_i' \theta_2) = \exp(\lambda_{\mu}\bar{x}_i)$. For case 1, the appropriate hypothesis to test is $H_0^a : \theta_2 = 0 \mid \sigma_{\mu}^2 > 0, \sigma_v^2 > 0, \theta_1 = 0$.

- **Case 2:** For the Li and Stengos (1994) set up, we generate μ_i as *i.i.d* $N(0, \sigma_{\mu}^2)$ and $v_{it} \sim N(0, \sigma_{v_{it}}^2)$ where

$$\begin{cases} \sigma_{v_{it}}^2 = \sigma_{v_{it}}^2(x_{it}) = \sigma_v^2(1 + \lambda_v x_{it})^2 \\ \text{or} \\ \sigma_{v_{it}}^2 = \sigma_{v_{it}}^2(x_{it}) = \sigma_v^2 \exp(\lambda_v x_{it}) \end{cases} \quad (17)$$

Denoting the expected variance of v_{it} by $\overline{\sigma_{v_{it}}^2}$ and following Li and Stengos (1994), we set the expected total variance $\overline{\sigma^2} = \sigma_\mu^2 + \overline{\sigma_{v_{it}}^2} = 8$ to make it comparable across the different data generating processes. We let σ_μ^2 take the values 2 and 6. For each fixed value of σ_μ^2 , λ_v is assigned values 0, 1, 2 and 3 with $\lambda_v = 0$ denoting the homoskedastic remainder error term. For a fixed value of σ_μ^2 , we obtain a value of $\overline{\sigma_{v_{it}}^2} = (8 - \sigma_\mu^2)$ and using a specific value of λ_v , we get the corresponding value for σ_v^2 from (17). Again, we can choose a quadratic or an exponential heteroskedastic specification for $\sigma_{v_{it}}^2 = \sigma_v^2 h_v(z'_{it}\theta_1)$ with $h_v(z'_{it}\theta_1) = (1 + \lambda_v x_{it})^2$ or $h_v(z'_{it}\theta_1) = \exp(\lambda_v x_{it})$. For case 2, the appropriate hypothesis to test is $H_0^b : \theta_1 = 0 \mid \sigma_\mu^2 > 0, \sigma_v^2 > 0, \theta_2 = 0$.

- **Case 3:** For the special case where the variance of v_{it} varies only with $i = 1, 2, \dots, N$, we generate μ_i as *i.i.d* $N(0, \sigma_\mu^2)$ and $v_{it} \sim N(0, \sigma_{v_i}^2)$ where

$$\begin{cases} \sigma_{v_i}^2 = \sigma_{v_i}^2(\bar{x}_i) = \sigma_v^2 (1 + \lambda_v \bar{x}_i)^2 \\ \text{or} \\ \sigma_{v_i}^2 = \sigma_{v_i}^2(\bar{x}_i) = \sigma_v^2 \exp(\lambda_v \bar{x}_i) \end{cases} \quad (18)$$

We let σ_μ^2 take the values 2 and 6 and for each fixed value of σ_μ^2 , λ_v is assigned values 0, 1, 2 and 3 with $\lambda_v = 0$ denoting the homoskedastic remainder error term. For a fixed value of σ_μ^2 , we obtain a value of $\overline{\sigma_{v_i}^2} = (8 - \sigma_\mu^2)$ and using a specific value of λ_v , we get the corresponding value for σ_v^2 from (18). For case 3, the appropriate hypothesis to test is $H_0^b : \theta_1 = 0 \mid \sigma_\mu^2 > 0, \sigma_v^2 > 0, \theta_2 = 0$.

- **Case 4:** We generate $\mu_i \sim N(0, \sigma_{\mu_i}^2)$ and $v_{it} \sim N(0, \sigma_{v_{it}}^2)$ where

$$\begin{cases} \sigma_{\mu_i}^2 = \sigma_{\mu_i}^2(\bar{x}_i) = \sigma_\mu^2 (1 + \lambda_\mu \bar{x}_i)^2 \\ \sigma_{v_{it}}^2 = \sigma_{v_{it}}^2(x_{it}) = \sigma_v^2 (1 + \lambda_v x_{it})^2 \\ \text{or} \\ \sigma_{\mu_i}^2 = \sigma_{\mu_i}^2(\bar{x}_i) = \sigma_\mu^2 \exp(\lambda_\mu \bar{x}_i) \\ \sigma_{v_{it}}^2 = \sigma_{v_{it}}^2(x_{it}) = \sigma_v^2 \exp(\lambda_v x_{it}) \end{cases} \quad (19)$$

and we set the expected total variance $\overline{\sigma^2} = \overline{\sigma_{\mu_i}^2} + \overline{\sigma_{v_{it}}^2} = 8$ to make it comparable across the different data generating processes. We let $\overline{\sigma_{\mu_i}^2}$ take the values 2 and 6. For each fixed value of $\overline{\sigma_{\mu_i}^2}$, λ_μ is assigned values 0, 1, 2 and 3 with $\lambda_\mu = 0$ denoting the homoskedastic individual specific error term. For a fixed value of $\overline{\sigma_{\mu_i}^2}$, we obtain a value of $\overline{\sigma_{v_{it}}^2} = (8 - \overline{\sigma_{\mu_i}^2})$ and using a specific value of λ_μ , we get the corresponding value for σ_μ^2 from (19). Second, for each fixed value of $\overline{\sigma_{\mu_i}^2}$, we obtain the corresponding value for $\overline{\sigma_{v_{it}}^2}$. λ_v is assigned values 0, 1, 2 and 3, with

$\lambda_v = 0$ denoting the homoskedastic remainder error term. For a specific value of λ_v , we get the corresponding value for σ_v^2 from (19). For case 4, the appropriate hypothesis to test is $H_0^c : \theta_1 = 0$ and $\theta_2 = 0 \mid \sigma_\mu^2 > 0, \sigma_v^2 > 0$.

For each replication, we compute the restricted one-way error component MLE. Using the \tilde{u} 's, *i.e.*, the vectors of restricted ML residuals and $\tilde{\sigma}_\mu^2$ and $\tilde{\sigma}_v^2$ which are the corresponding restricted ML estimates of σ_μ^2 and σ_v^2 , we compute the marginal and joint LM tests derived above for H_0^a , H_0^b , $H_0^{b'}$ and H_0^c . For each experiment, 5000 replications are performed and we obtain the empirical size for each test at the 5% level.

[Put Table 1]

Table 1 reports the percentage of rejections of the null hypothesis based on nominal critical values in 5000 replications at the 5% significance level. We first look at case 1, for $(N, T) = (50, 5)$ and $(E[\sigma_{\mu_i}^2] = \overline{\sigma_{\mu_i}^2} = 6, E[\sigma_{v_{it}}^2] = \sigma_v^2 = 2)$. This is the set up of Roy (2002) where homoskedasticity is assumed on the remainder error term, *i.e.*, $\theta_1 = 0$. The parameter (λ_μ) determines the degree of heteroskedasticity on the random individual specific effect μ_i . When we test the null hypothesis $H_0^a : \theta_2 = 0 \mid \sigma_\mu^2 > 0, \sigma_v^2 > 0, \theta_1 = 0$, we use the the marginal LM test defined in (9) and also derived by Holly and Gardiol (2000). When there is no heteroskedasticity, *i.e.*, $\lambda_\mu = 0$, we get $\theta_2 = 0$, and the null H_0^a is true. Table 1 shows that the empirical size of this test is not significantly different from 5%. However, the power of this test is poor especially for small λ_μ and the quadratic form of heteroskedasticity. For example, for a low degree of heteroskedasticity ($\lambda_\mu = 1$), the power is 24.1% for the quadratic type of heteroskedasticity and 33.6% for the exponential type of heteroskedasticity. This power increases with the degree of heteroskedasticity and for $\lambda_\mu = 3$, it yields 41% rejection of the null of homoskedasticity for the quadratic form and 98.4% for the exponential form of heteroskedasticity. The power for the exponential form of heteroskedasticity is better than that of the quadratic case. Doubling the size of T from 5 to 10, does not help in improving the power of this test⁷. However, increasing N from 50 to 200 improves the power drastically, see Fig. 1. For case 1, H_0^a is the right test to perform, while testing for H_0^b or $H_0^{b'}$ is checking for the wrong form of heteroskedasticity on the wrong error component. As expected, the marginal tests derived in (10) and (11) yield a rejection rate for the null between 4.6% and 5.6% for H_0^b and for $H_0^{b'}$. In other words, the

⁷Table 2 showing results for $T = 10$ and $N = 50, 200$ can be obtained upon request from the authors.

power of these tests is close to the nominal size of 5%. If we interpret these results as not rejecting a homoskedastic error component model, we reach the wrong conclusion whenever λ_μ is different from zero. If we perform the joint LM test defined in (12) for $H_0^c : \theta_1 = 0$ and $\theta_2 = 0 \mid \sigma_\mu^2 > 0, \sigma_v^2 > 0$ under case 1, the size is not significantly different from 5% and the power is a little lower than that of the right marginal test for H_0^a defined in (9). Note that, with the joint LM test, we are overtesting in case 1 and still performing a close second to the preferred marginal LM test. For example, when $\lambda_\mu = 3$, the joint LM test yield a rejection of the null in 31.8% for the quadratic form of heteroskedasticity and 96% for the exponential form of heteroskedasticity. These are below the 41% and 98.4% rejection rates reported above for the marginal test defined in (9).

For case 2, for $(N, T) = (50, 5)$ and $\left(E \left[\sigma_{\mu_i}^2 \right] = \sigma_\mu^2 = 6, E \left[\sigma_{vit}^2 \right] = \overline{\sigma_{vit}^2} = 2 \right)$, this is the set up of Li and Stengos (1994) where homoskedasticity is assumed on the individual error term, *i.e.*, $\theta_2 = 0$. The parameter (λ_v) determines the degree of heteroskedasticity on the remainder error term v_{it} . When we test the null hypothesis $H_0^b : \theta_1 = 0 \mid \sigma_\mu^2 > 0, \sigma_v^2 > 0, \theta_2 = 0$, we use the the marginal LM test defined in (10). When there is no heteroskedasticity, *i.e.*, $\lambda_v = 0$, we get $\theta_1 = 0$, and the null H_0^b is true. Table 1 shows that the empirical size of this test is not significantly different from 5%. Also, the power of this test is excellent, yielding 99.4 to 100% rejection rates for $\lambda_v \neq 0$ for the quadratic form as well as the exponential form of heteroskedasticity. For case 2, testing H_0^b is the right hypothesis test to perform, while testing for H_0^a is checking for the wrong form of heteroskedasticity on the wrong error component. As expected, the marginal test derived in (9) yields a rejection rate for the null between 4.1% and 4.7% for H_0^a under the quadratic form of heteroskedasticity and between 4.3% and 6.1% under the exponential form of heteroskedasticity. In other words, the power of this test is close to the nominal size of 5%, except when $\sigma_\mu^2 = 2$ and λ_v is larger than 2 for the exponential form of heteroskedasticity. If we interpret these results as not rejecting a homoskedastic error component model, we reach the wrong conclusion whenever λ_v is different from zero. If we perform the joint LM test for $H_0^c : \theta_1 = 0$ and $\theta_2 = 0 \mid \sigma_\mu^2 > 0, \sigma_v^2 > 0$ under case 2, the size is not significantly different from 5% and the power is above 98.7%. Note that, with the joint LM test, we are overtesting in case 2 and still performing about the same as the preferred test (10). For case 2, the Monte Carlo design is more in favor of the test in (10) for H_0^b than the test in (11) for $H_0^{b'}$. The latter test is still testing for heteroskedasticity in the remainder error form but misspecifies that it does not vary over time but only over individuals. This test has empirical size that is not significantly different from 5%. Its power is good exceeding 78.8% for the quadratic form of heteroskedasticity

and 90.3% for the exponential form of heteroskedasticity.

For case 3, for $(N, T) = (50, 5)$ and $\left(E \left[\sigma_{\mu_i}^2\right] = \sigma_{\mu}^2 = 6, E \left[\sigma_{v_{it}}^2\right] = \overline{\sigma_{v_i}^2} = 2\right)$. This is the set up in line with the heteroskedastic form of Baltagi (1988) and Wansbeek (1989) where homoskedasticity is assumed on the individual error term, *i.e.*, $\theta_2 = 0$ just like case 2. However, unlike case 2, the parameter (λ_v) determines the degree of heteroskedasticity on the remainder error term v_{it} which now varies only over i and not over t . In this case, $v_{it} \sim N(0, \sigma_{v_i}^2)$. The design is in favor of testing the null hypothesis H_0^b with the LM test defined in (11). Table 1 shows that the empirical size of this test is 5.6% for $T = 5$ and 5.2% for $T = 10$, so it is close to its nominal size (5%). However, its power is good exceeding 83% for the quadratic form of heteroskedasticity and 94% for the exponential form of heteroskedasticity. For case 3, testing for H_0^a is checking for the wrong form of heteroskedasticity on the wrong error component. As expected, the marginal test derived in (9) yields a rejection rate for the null between 4.3% and 5.4% for H_0^a under the quadratic form of heteroskedasticity and between 4.3% and 7% under the exponential form of heteroskedasticity. In other words, the power of this test is close to the nominal size of 5%, except when $\sigma_{\mu}^2 = 2$ and λ_v is larger than 1. If we interpret these results as not rejecting a homoskedastic error component model, we reach the wrong conclusion whenever λ_v is different from zero. If we perform the joint LM test for $H_0^c : \theta_1 = 0$ and $\theta_2 = 0 \mid \sigma_{\mu}^2 > 0, \sigma_v^2 > 0$ under case 3, the size is not significantly different from 5%. However, the power is poor for the quadratic form of heteroskedasticity varying between 15% and 57%. The power is much higher for the exponential form of heteroskedasticity varying between 20% and 99.9%. Note that, with the joint LM test, we are overtesting in case 3 and not performing as well as the preferred marginal test (11). For case 3, the Monte Carlo design is more in favor of the test in (11) for H_0^b than the test in (10) for H_0^b . The latter test is still testing for heteroskedasticity in the remainder error form, but misspecifies its form, assuming that it varies over time and individuals when it only varies over individuals. This test has empirical size not significantly different from the 5% level. Its power is low for the quadratic form of heteroskedasticity varying between 19.8% and 68.4%, while it is better for the exponential form of heteroskedasticity varying between 26.4% and 99.9%.

Table 1 also reports the percentage of rejections of the null hypothesis in 5000 replications at the 5% significance level for case 4. This is the general heteroskedastic set up of Lejeune (1996). The design is in favor of testing the null hypothesis $H_0^c : \theta_1 = 0$ and $\theta_2 = 0 \mid \sigma_{\mu}^2 > 0, \sigma_v^2 > 0$ with the joint LM test defined by (12). For $(N, T) = (50, 5)$ and $\left(E \left[\sigma_{\mu_i}^2\right] = \overline{\sigma_{\mu_i}^2} = 6, E \left[\sigma_{v_{it}}^2\right] = \overline{\sigma_{v_{it}}^2} = 2\right)$, the empirical size of this test is between 4.3% and 5.1%,

so it is not significantly different from the 5% level. Its power is excellent as long as λ_v is different from zero. In fact, the power for $\lambda_v > 0$ always exceeds 98.7%. Low power occurs for $\lambda_v = 0$ and $\lambda_\mu \neq 0$. This power can be as low as 18% for $\lambda_v = 0$ and $\lambda_\mu = 1$ for the quadratic form of heteroskedasticity and 25.9% for the exponential form of heteroskedasticity. In general, the test has more power under the exponential form of heteroskedasticity rather than the quadratic form. The marginal LM tests are designed to test for heteroskedasticity in one error component assuming homoskedasticity on the other error component. Obviously, the design for case 4 is not in the marginal LM tests favor if the other error component is heteroskedastic and this shows in Table 1. When $\lambda_\mu = 0$, but $\lambda_v \neq 0$, the marginal test for H_0^a given by (9) yield power between 4.1% and 5.1% for the quadratic form of heteroskedasticity and between 4.3% and 6.1% for the exponential form of heteroskedasticity. When $\lambda_v = 0$, but $\lambda_\mu \neq 0$, the marginal test for H_0^b given by (10) and $H_0^{b'}$ given by (11) yield power between 4.5% and 5.1% for the quadratic form of heteroskedasticity and for the exponential form of heteroskedasticity.

[Put Fig. 1]

Fig. 1 replicates Table 1 for $T = 5$ and $N = 200$. Larger N is more likely encountered in micro-panels. As clear from Fig. 1, the power of these tests improves drastically as we increase N . However, it is still the case that performing the wrong test for heteroskedasticity yields misleading results.

5 Conclusion

For the random error component model popular in panel data applications, the researcher does not know whether heteroskedasticity is absent from both error components or whether it is in one or both components. This paper derived a joint as well as marginal LM tests for homoskedasticity against heteroskedasticity in one or both error components. Monte Carlo experiments show that this joint LM test performs well when both error components are heteroskedastic, and it performs second best when one of the components is homoskedastic while the other is not. In contrast, the marginal LM tests perform best when heteroskedasticity is present in the right error component. They yield misleading results if heteroskedasticity is present in the wrong error component.

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Table 1 – Size and power of joint and marginal LM tests - $T=5$, 5000 replications

	Quadratic heteroskedasticity								Exponential heteroskedasticity							
	H_0^a		H_0^b		$H_0^{b'}$		H_0^c		H_0^a		H_0^b		$H_0^{b'}$		H_0^c	
	$N=50$	$N=200$	$N=50$	$N=200$	$N=50$	$N=200$	$N=50$	$N=200$	$N=50$	$N=200$	$N=50$	$N=200$	$N=50$	$N=200$	$N=50$	$N=200$
Case 1 λ_μ	$E[\sigma_{\mu_i}^2] = \overline{\sigma_{\mu_i}^2} = 6, E[\sigma_{v_i}^2] = \overline{\sigma_{v_i}^2} = 2$								$E[\sigma_{\mu_i}^2] = \overline{\sigma_{\mu_i}^2} = 6, E[\sigma_{v_i}^2] = \overline{\sigma_{v_i}^2} = 2$							
0	4.3	4.4	5.3	4.6	5.6	5.3	5.1	4.5	4.3	4.4	5.3	4.6	5.6	5.3	5.1	4.5
1	24.1	77.5	4.9	4.9	4.8	4.6	18.0	66.7	33.6	91.4	4.8	4.5	4.7	5.2	26.2	85.6
2	35.6	91.1	5.3	4.5	5.2	5.3	27.5	85.1	82.7	100.0	5.3	5.0	5.1	5.6	74.5	100.0
3	41.0	94.8	4.5	4.3	5.5	5.5	31.8	90.3	98.4	100.0	4.4	5.5	5.5	5.9	96.0	100.0
	$E[\sigma_{\mu_i}^2] = \overline{\sigma_{\mu_i}^2} = 2, E[\sigma_{v_i}^2] = \overline{\sigma_{v_i}^2} = 6$								$E[\sigma_{\mu_i}^2] = \overline{\sigma_{\mu_i}^2} = 2, E[\sigma_{v_i}^2] = \overline{\sigma_{v_i}^2} = 6$							
0	4.6	4.8	5.2	4.9	5.5	5.1	4.6	5.1	4.6	4.8	5.2	4.9	5.5	5.1	4.6	5.1
1	12.8	43.1	4.9	5.4	4.8	5.8	10.2	34.1	17.5	61.5	5.1	5.8	5.0	6.5	13.8	51.5
2	18.0	59.4	5.7	5.2	5.0	6.9	14.4	48.2	50.0	98.2	5.8	7.3	5.9	12.4	41.4	96.6
3	20.6	67.8	4.7	5.1	5.5	7.6	15.9	57.7	73.9	99.9	5.7	10.0	8.3	21.5	65.0	99.9
Case 2 λ_v	$E[\sigma_{\mu_i}^2] = \overline{\sigma_{\mu_i}^2} = 6, E[\sigma_{v_i}^2] = \overline{\sigma_{v_i}^2} = 2$								$E[\sigma_{\mu_i}^2] = \overline{\sigma_{\mu_i}^2} = 6, E[\sigma_{v_i}^2] = \overline{\sigma_{v_i}^2} = 2$							
0	4.3	4.5	5.3	4.6	5.6	5.3	5.1	4.5	4.3	4.4	5.3	4.6	5.6	5.3	5.1	4.5
1	4.1	5.1	99.4	100.0	78.8	100.0	98.7	100.0	4.3	5.8	99.9	100.0	90.3	100.0	99.8	100.0
2	4.7	5.6	100.0	100.0	91.5	100.0	99.9	100.0	5.3	7.4	100.0	100.0	99.8	100.0	100.0	100.0
3	4.7	5.9	100.0	100.0	94.4	100.0	100.0	100.0	6.1	9.4	100.0	100.0	99.9	100.0	100.0	100.0
	$E[\sigma_{\mu_i}^2] = \overline{\sigma_{\mu_i}^2} = 2, E[\sigma_{v_i}^2] = \overline{\sigma_{v_i}^2} = 6$								$E[\sigma_{\mu_i}^2] = \overline{\sigma_{\mu_i}^2} = 2, E[\sigma_{v_i}^2] = \overline{\sigma_{v_i}^2} = 6$							
0	4.6	4.8	5.2	4.9	5.5	5.1	4.6	5.1	4.6	4.8	5.2	4.9	5.5	5.1	4.6	5.1
1	7.7	17.9	99.7	100.0	79.1	100.0	99.2	100.0	9.3	25.9	100.0	100.0	90.7	100.0	99.9	100.0
2	9.0	23.6	100.0	100.0	92.1	100.0	99.9	100.0	18.3	63.7	100.0	100.0	99.8	100.0	100.0	100.0
3	9.4	27.7	100.0	100.0	94.9	100.0	100.0	100.0	27.4	83.2	100.0	100.0	99.9	100.0	100.0	100.0
Case 3 λ_v	$E[\sigma_{\mu_i}^2] = \overline{\sigma_{\mu_i}^2} = 6, E[\sigma_{v_i}^2] = \overline{\sigma_{v_i}^2} = 2$								$E[\sigma_{\mu_i}^2] = \overline{\sigma_{\mu_i}^2} = 6, E[\sigma_{v_i}^2] = \overline{\sigma_{v_i}^2} = 2$							
0	4.3	4.4	5.3	4.6	5.6	5.3	5.1	4.5	4.3	4.4	5.3	4.6	5.6	5.3	5.1	4.5
1	4.3	4.8	44.9	95.0	82.4	100.0	34.6	90.6	4.4	5.8	59.7	99.5	93.7	100.0	50.3	98.7
2	4.6	5.7	60.6	99.4	95.5	100.0	51.4	98.6	5.5	8.1	97.8	100.0	100.0	100.0	96.0	100.0
3	4.7	6.0	68.4	99.7	97.6	100.0	57.4	99.3	7.0	11.4	99.9	100.0	100.0	100.0	99.9	100.0
	$E[\sigma_{\mu_i}^2] = \overline{\sigma_{\mu_i}^2} = 2, E[\sigma_{v_i}^2] = \overline{\sigma_{v_i}^2} = 6$								$E[\sigma_{\mu_i}^2] = \overline{\sigma_{\mu_i}^2} = 2, E[\sigma_{v_i}^2] = \overline{\sigma_{v_i}^2} = 6$							
0	4.6	4.8	5.2	4.9	5.5	5.1	4.6	5.1	4.6	4.8	5.2	4.9	5.5	5.1	4.6	5.1
1	7.7	18.9	42.5	93.9	83.3	100.0	34.2	89.7	9.5	27.1	56.5	99.2	94.1	100.0	49.0	98.5
2	9.8	26.4	58.5	99.0	95.7	100.0	49.6	98.3	23.6	72.3	97.2	100.0	100.0	100.0	95.5	100.0
3	10.1	30.8	65.6	99.6	98.0	100.0	55.8	99.1	40.4	93.4	99.9	100.0	100.0	100.0	99.9	100.0

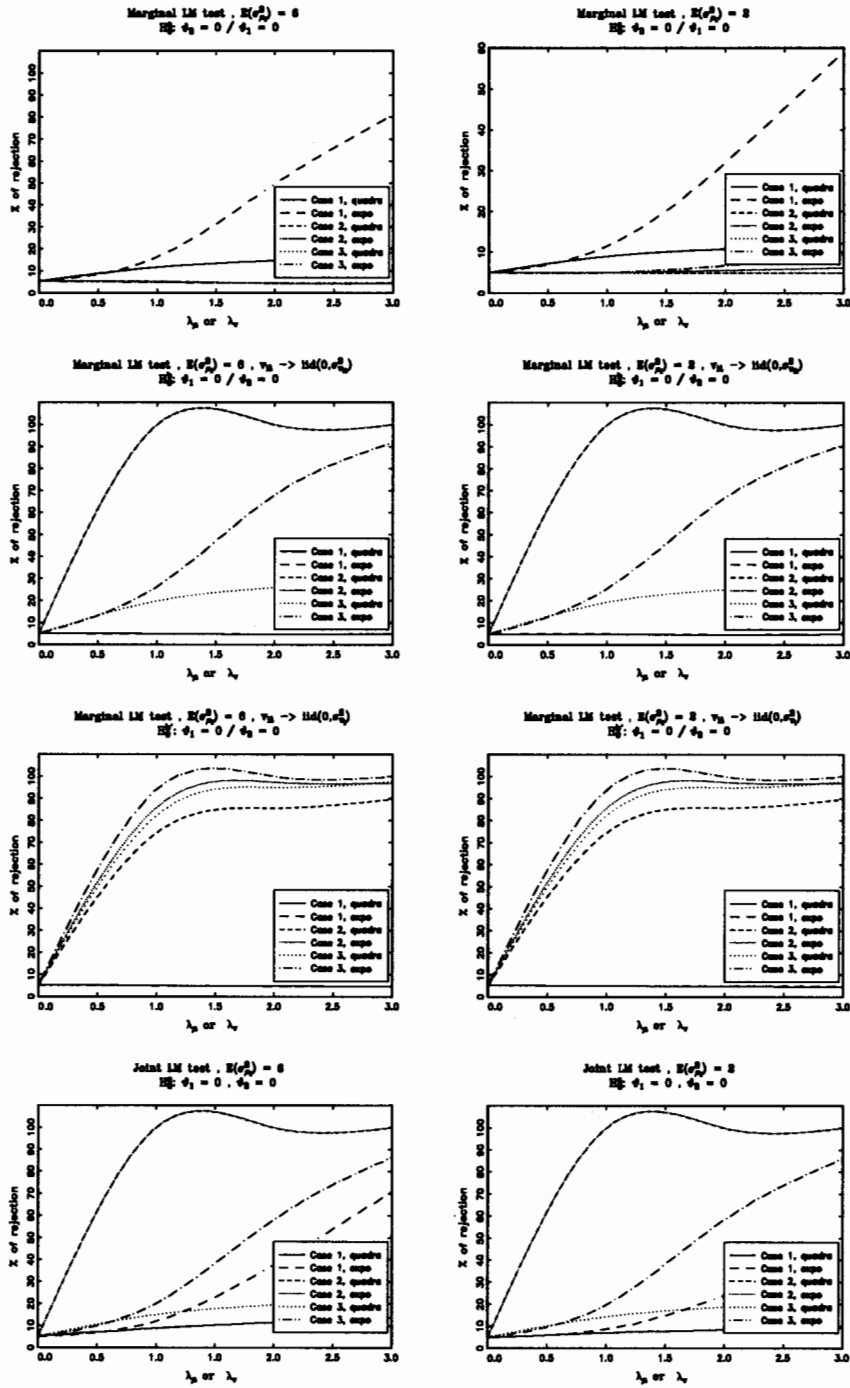
Table 2 – Size and power of joint and marginal LM tests - $N=50, T=10, 5000$ replications

	Quadratic heteroskedasticity				Exponential heteroskedasticity			
	H_0^a	H_0^b	$H_0^{b'}$	H_0^c	H_0^a	H_0^b	$H_0^{b'}$	H_0^c
Case 1 λ_μ	$E[\sigma_{\mu_t}^2] = \overline{\sigma_{\mu_t}^2} = 6, E[\sigma_{v_{it}}^2] = \sigma_v^2 = 2$				$E[\sigma_{\mu_t}^2] = \overline{\sigma_{\mu_t}^2} = 6, E[\sigma_{v_{it}}^2] = \sigma_v^2 = 2$			
0	5.4	5.1	5.2	5.2	5.4	5.1	5.2	5.1
1	11.7	5.0	5.0	8.9	16.3	4.9	4.9	12.0
2	14.8	4.6	4.7	11.6	49.6	4.7	4.6	37.7
3	18.3	4.9	4.6	13.8	81.1	4.8	4.6	70.8
	$E[\sigma_{\mu_t}^2] = \overline{\sigma_{\mu_t}^2} = 2, E[\sigma_{v_{it}}^2] = \sigma_v^2 = 6$				$E[\sigma_{\mu_t}^2] = \overline{\sigma_{\mu_t}^2} = 2, E[\sigma_{v_{it}}^2] = \sigma_v^2 = 6$			
0	5.0	4.8	5.1	4.8	5.0	4.8	5.1	4.9
1	9.0	4.8	5.0	7.0	11.5	4.9	5.0	8.7
2	10.8	4.4	4.7	8.4	32.0	4.5	4.8	24.1
3	12.2	4.9	4.6	9.5	59.2	4.9	4.9	48.1
Case 2 λ_v	$E[\sigma_{\mu_t}^2] = \sigma_{\mu_t}^2 = 6, E[\sigma_{v_{it}}^2] = \overline{\sigma_{v_{it}}^2} = 2$				$E[\sigma_{\mu_t}^2] = \sigma_{\mu_t}^2 = 6, E[\sigma_{v_{it}}^2] = \overline{\sigma_{v_{it}}^2} = 2$			
0	5.4	5.1	5.2	5.1	5.4	5.1	5.2	5.1
1	5.1	100.0	74.4	100.0	5.2	100.0	85.9	100.0
2	4.3	100.0	85.6	100.0	4.3	100.0	97.3	100.0
3	4.4	100.0	89.7	100.0	4.3	100.0	97.0	100.0
	$E[\sigma_{\mu_t}^2] = \sigma_{\mu_t}^2 = 2, E[\sigma_{v_{it}}^2] = \overline{\sigma_{v_{it}}^2} = 6$				$E[\sigma_{\mu_t}^2] = \sigma_{\mu_t}^2 = 2, E[\sigma_{v_{it}}^2] = \overline{\sigma_{v_{it}}^2} = 6$			
0	5.0	4.8	5.1	4.9	5.0	4.8	5.1	4.9
1	5.0	100.0	74.6	100.0	5.0	100.0	85.8	100.0
2	4.8	100.0	85.7	100.0	5.5	100.0	97.4	100.0
3	4.8	100.0	89.7	100.0	6.1	100.0	97.0	100.0
Case 3 λ_v	$E[\sigma_{\mu_t}^2] = \sigma_{\mu_t}^2 = 6, E[\sigma_{v_{it}}^2] = \overline{\sigma_{v_{it}}^2} = 2$				$E[\sigma_{\mu_t}^2] = \sigma_{\mu_t}^2 = 6, E[\sigma_{v_{it}}^2] = \overline{\sigma_{v_{it}}^2} = 2$			
0	5.4	5.1	5.2	5.1	5.4	5.1	5.2	5.1
1	5.0	19.8	82.3	15.0	4.9	26.4	94.1	20.1
2	4.4	25.9	94.9	19.5	4.4	68.0	100.0	58.2
3	4.4	29.9	97.6	23.1	4.4	91.9	100.0	86.5
	$E[\sigma_{\mu_t}^2] = \sigma_{\mu_t}^2 = 2, E[\sigma_{v_{it}}^2] = \overline{\sigma_{v_{it}}^2} = 6$				$E[\sigma_{\mu_t}^2] = \sigma_{\mu_t}^2 = 2, E[\sigma_{v_{it}}^2] = \overline{\sigma_{v_{it}}^2} = 6$			
0	5.0	4.8	5.1	4.9	5.0	4.8	5.1	4.8
1	4.9	19.3	82.3	14.3	5.0	25.6	94.4	19.7
2	4.9	25.1	94.9	18.8	6.7	66.9	100.0	58.4
3	4.9	28.9	97.6	22.3	9.3	90.9	100.0	86.1

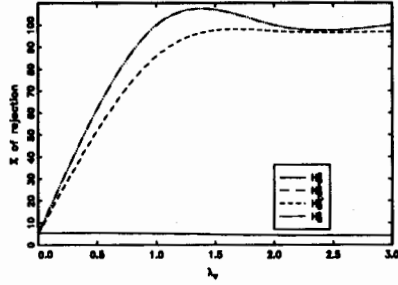
Table 2 (continued)

		Quadratic heteroskedasticity				Exponential heteroskedasticity			
		H_0^a	H_0^b	$H_0^{b'}$	H_0^c	H_0^a	H_0^b	$H_0^{b'}$	H_0^c
Case 4									
λ_μ	λ_ν	$E[\sigma_{\mu_t}^2] = \overline{\sigma_{\mu_t}^2} = 6, E[\sigma_{\nu_t}^2] = \overline{\sigma_{\nu_t}^2} = 2$				$E[\sigma_{\mu_t}^2] = \overline{\sigma_{\mu_t}^2} = 6, E[\sigma_{\nu_t}^2] = \overline{\sigma_{\nu_t}^2} = 2$			
0	0	5.4	5.1	5.2	5.1	5.4	5.1	5.2	5.1
0	1	5.1	100.0	74.5	100.0	5.2	100.0	85.9	100.0
0	2	4.3	100.0	85.6	100.0	4.3	100.0	97.3	100.0
0	3	4.4	100.0	89.7	100.0	4.3	100.0	97.0	100.0
1	0	11.4	4.5	5.1	8.8	15.4	4.5	5.1	11.5
1	1	11.9	100.0	75.1	100.0	16.5	100.0	85.6	100.0
1	2	12.0	100.0	86.6	100.0	16.9	100.0	97.8	100.0
1	3	11.6	100.0	90.6	100.0	16.4	100.0	97.0	100.0
2	0	15.9	4.8	5.0	11.8	49.9	4.7	5.0	38.5
2	1	17.6	100.0	74.0	100.0	52.5	100.0	84.9	100.0
2	2	16.8	100.0	86.6	100.0	51.5	100.0	97.8	100.0
2	3	16.5	100.0	89.7	100.0	51.7	100.0	97.2	100.0
3	0	17.8	4.8	5.1	13.3	81.2	4.9	5.1	69.9
3	1	19.8	100.0	75.7	100.0	83.4	100.0	86.6	100.0
3	2	18.7	100.0	86.7	100.0	82.5	100.0	97.8	100.0
3	3	18.5	100.0	90.1	100.0	82.3	100.0	97.4	100.0
λ_μ	λ_ν	$E[\sigma_{\mu_t}^2] = \overline{\sigma_{\mu_t}^2} = 2, E[\sigma_{\nu_t}^2] = \overline{\sigma_{\nu_t}^2} = 6$				$E[\sigma_{\mu_t}^2] = \overline{\sigma_{\mu_t}^2} = 2, E[\sigma_{\nu_t}^2] = \overline{\sigma_{\nu_t}^2} = 6$			
0	0	5.0	4.8	5.1	4.8	5.0	4.8	5.1	4.8
0	1	5.0	100.0	74.6	100.0	5.0	100.0	85.8	100.0
0	2	4.8	100.0	85.7	100.0	5.5	100.0	97.9	100.0
0	3	4.8	100.0	89.7	100.0	6.1	100.0	97.0	100.0
1	0	8.1	4.5	5.2	7.0	10.2	4.5	5.3	8.6
1	1	11.2	100.0	76.9	100.0	15.6	100.0	86.9	100.0
1	2	12.7	100.0	87.6	100.0	19.8	100.0	98.0	100.0
1	3	12.1	100.0	91.1	100.0	21.9	100.0	97.2	100.0
2	0	11.4	5.2	5.2	9.2	32.7	5.2	5.4	25.2
2	1	15.6	100.0	76.1	100.0	43.5	100.0	86.9	100.0
2	2	17.0	100.0	87.6	100.0	47.7	100.0	98.0	100.0
2	3	15.9	100.0	90.6	100.0	50.4	100.0	97.4	100.0
3	0	12.1	4.8	5.2	9.5	59.2	4.8	5.5	47.4
3	1	18.1	100.0	77.8	100.0	70.0	100.0	89.7	100.0
3	2	17.4	100.0	88.0	100.0	73.2	100.0	98.3	100.0
3	3	17.8	100.0	90.9	100.0	75.7	100.0	97.7	100.0

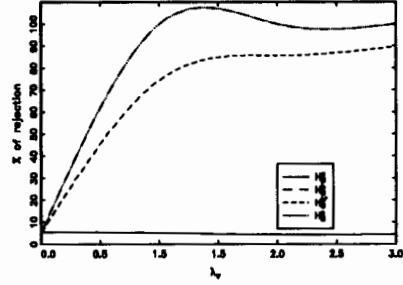
Figure 1 - Rejection proportions of LM tests ($N = 50, T = 10$).



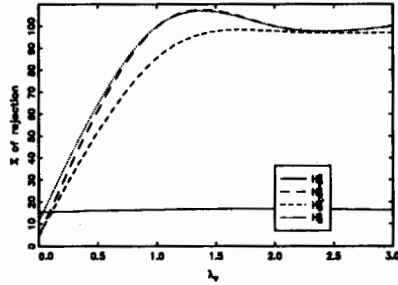
Marginal and Joint LM tests, $E(\sigma_{it}^2) = 6$
Case 4, Exponential heteroskedasticity, $\lambda_0 = 0$



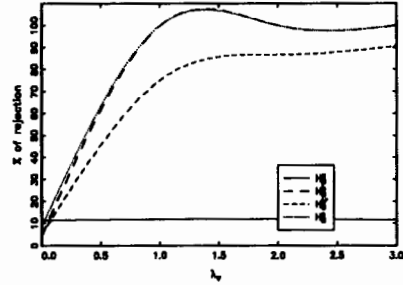
Marginal and Joint LM tests, $E(\sigma_{it}^2) = 6$
Case 4, Quadratic heteroskedasticity, $\lambda_0 = 0$



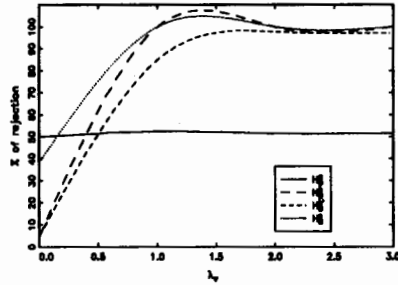
Marginal and Joint LM tests, $E(\sigma_{it}^2) = 6$
Case 4, Exponential heteroskedasticity, $\lambda_0 = 1$



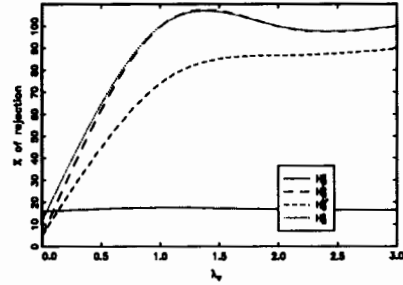
Marginal and Joint LM tests, $E(\sigma_{it}^2) = 6$
Case 4, Quadratic heteroskedasticity, $\lambda_0 = 1$



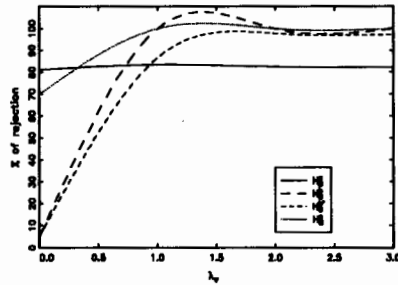
Marginal and Joint LM tests, $E(\sigma_{it}^2) = 6$
Case 4, Exponential heteroskedasticity, $\lambda_0 = 2$



Marginal and Joint LM tests, $E(\sigma_{it}^2) = 6$
Case 4, Quadratic heteroskedasticity, $\lambda_0 = 2$



Marginal and Joint LM tests, $E(\sigma_{it}^2) = 6$
Case 4, Exponential heteroskedasticity, $\lambda_0 = 3$



Marginal and Joint LM tests, $E(\sigma_{it}^2) = 6$
Case 4, Quadratic heteroskedasticity, $\lambda_0 = 3$

