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A survey of various frequency domain integral equations for the analysis of scattering from three-dimensional dielectric objects

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**A SURVEY OF VARIOUS FREQUENCY DOMAIN INTEGRAL EQUATIOUS FREQUENCY DOMAIN
INTEGRAL EQUATIONS FOR THE ANALYSIS OF
SCATTEBING FROM THREE DIMENSIONAL A SURVEY OF VARIOUS FREQUENCY DOMAIN
INTEGRAL EQUATIONS FOR THE ANALYSIS OF
SCATTERING FROM THREE-DIMENSIONAL
DIELECTRIC OBJECTS** INTEGRAL EQUATIONS F(
SCATTERING FROM THRI
DIELECTRIC OBJECTS

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Syracuse N. 13944 H Department of Electrical Engin
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Syracuse, NY 13244, USA **Abstract**—In this paper, we present four different formulations for

Abstract—In this paper, we present four different formulations for
the analysis of electromagnetic scattering from arbitrarily shaped
three dimensional $(2, D)$ homogeneous dialectric hody in the frequency Abstract—In this paper, we present four different formulations for
the analysis of electromagnetic scattering from arbitrarily shaped
three-dimensional (3-D) homogeneous dielectric body in the frequency
domain. The four in the analysis of electromagnetic scattering from arbitrarily shaped
three-dimensional $(3-D)$ homogeneous dielectric body in the frequency
domain. The four integral equations treated here are the electric field
integral equ three-dimensional $(3-D)$ homogeneous dielectric body in the frequency domain. The four integral equations treated here are the electric field integral equation (MFIE), the combined field integral equation (CFIE), and the domain. The four integral equations treated here are the electric field integral equation (EFIE), the magnetic field integral equation (MFIE), the combined field integral equation (CFIE), and the PMCHW (Poggio, Miller, Chang, Harrington, and Wu) formulation. For the CEIE assessment proposes si the combined field integral equation (CFIE), and the PMCHW
(Poggio, Miller, Chang, Harrington, and Wu) formulation. For the
CFIE case, we propose eight separate formulations with different
combinations of supersian and tes (Poggio, Miller, Chang, Harrington, and Wu) formulation. For the CFIE case, we propose eight separate formulations with different combinations of expansion and testing functions that result in sixteen different formulatio \overline{C} FIE case, we propose eight separate formulations with different combinations of expansion and testing functions that result in sixteen different formulations of CFIE. One of the objectives of this paper is to illu combinations of expansion and testing functions that result in sixteen
different formulations of CFIE. One of the objectives of this paper is
to illustrate that not all CFIE are valid methodologies in removing
defects, whi different formulations of CFIE. One of the objectives of this paper is
to illustrate that not all CFIE are valid methodologies in removing
defects, which occur at a frequency corresponding to an internal
recononce of the s to illustrate that not all CFIE are valid methodologies in removing
defects, which occur at a frequency corresponding to an internal
resonance of the structure. Numerical results involving the equivalent
electric and magne defects, which occur at a frequency corresponding to an internal resonance of the structure. Numerical results involving the equivalent electric and magnetic currents, far scattered fields, and radar cross resonance of the structure. Numerical results involving the equivalent
electric and magnetic currents, far scattered fields, and radar cross
section (RCS) are presented for three canonical dielectric scatterers,
wiz a sphe electric and magnetic currents, far scattered fields, and radar cross
section (RCS) are presented for three canonical dielectric scatterers,
viz. a sphere, a cube, and a finite circular cylinder, to illustrate which
formul section (RCS) are presented for three caviz. a sphere, a cube, and a finite circula
formulation works and which does not.

- **1 Introduction**
- **2 Integral Equations**

**2 Integral Equations
3 Numerical Implementation** Integral Equations
Numerical Implementat
3.1 EFIE Formulation Numerical Implementati
3.1 EFIE Formulation
3.2 MFIE Formulation

-
-
- 3.1 EFIE Formulation
3.2 MFIE Formulation
3.3 PMCHW Formulation 3.2 MFIE Formulation
3.3 PMCHW Formulation
3.4 CFIE Formulation
-

4 Study of the Various Formulations 5 Numerical Examples

- **6 Conclusion**
-

Appendix A.

- A.1 Integrals (26), (73), and (74)
- **pendix A.**
A.1 Integrals (26), (73), and (74)
A.2 Integrals (29), (75), and (76)
- A.1 Integrals (26) , (73) , and (74)
A.2 Integrals (29) , (75) , and (76)
A.3 Integrals (35) , (77) , and (78)
- A.3 Integrals (35), (77), and (78)
A.4 Integrals (37), (51), (79), and (80)
- A.5 Integrals (40), (46), (50), (54), (72), and (89)

References

1. INTRODUCTION

The analysis of electromagnetic scattering from arbitrarily shaped 3- The analysis of electromagnetic scattering from arbitrarily shaped 3-
D homogeneous dielectric body in the frequency domain has been
of considerable interest in recent wears. In the analysis of dielectric The analysis of electromagnetic scattering from arbitrarily shaped 3-
D homogeneous dielectric body in the frequency domain has been
of considerable interest in recent years. In the analysis of dielectric
bodies at frequen D homogeneous dielectric body in the frequency domain has been
of considerable interest in recent years. In the analysis of dielectric
bodies at frequencies, which correspond to an internal resonance of
the structure often of considerable interest in recent years. In the analysis of dielectric bodies at frequencies, which correspond to an internal resonance of the structure, often spurious solutions are obtained for the EFIE or MEIE One poss bodies at frequencies, which correspond to an internal resonance of
the structure, often spurious solutions are obtained for the EFIE or
MFIE. One possible way of obtaining a unique solution at an internal
resonant frequen the structure, often spurious solutions are obtained for the EFIE or MFIE. One possible way of obtaining a unique solution at an internal resonant frequency of the structure under analysis is to combine a maighted linear s MFIE. One possible way of obtaining a unique solution at an internal
resonant frequency of the structure under analysis is to combine a
weighted linear sum of the EFIE with MFIE and thereby eliminate the
rewricus solution resonant frequency of the structure under analysis is to combine a
weighted linear sum of the EFIE with MFIE and thereby eliminate the
spurious solutions [1]. This combination results in the CFIE. Although
an integral equa weighted linear sum of the EFIE with MFIE and thereby eliminate the
spurious solutions [1]. This combination results in the CFIE. Although
an integral equation formulation has been used for 3-D dielectric bodies
in the fr spurious solutions [1]. This combination results in the CFIE. Although
an integral equation formulation has been used for 3-D dielectric bodies
in the frequency domain, only a few researchers have applied it to the an integral equation formulation has been used for 3-D dielectric bodies
in the frequency domain, only a few researchers have applied it to the
analysis of scattering by arbitrarily shaped 3-D objects with triangular
patc in the frequency domain
analysis of scattering by
patch modeling $[2-6]$.
The integral equat ysis of scattering by arbitrarily shaped 3-D objects with triangular
h modeling $[2-6]$.
The integral equation used in $[2]$ and $[5]$ is the PMCHW formu-
n in which Bas Wilton Glisson (BWG) functions described in $[7]$

patch modeling $[2-6]$.
The integral equation used in $[2]$ and $[5]$ is the PMCHW formulation, in which Rao-Wilton-Glisson (RWG) functions described in $[7]$ The integral equation used in [2] and [5] is the PMCHW formulation, in which Rao-Wilton-Glisson (RWG) functions described in [7] has been used both as the basis and testing functions, to approximate both the electric and lation, in which Rao-Wilton-Glisson (RWG) functions described in [7]
has been used both as the basis and testing functions, to approximate
both the electric and the magnetic currents. For the EFIE formulation
[2] the elec has been used both as the basis and testing functions, to approximate both the electric and the magnetic currents. For the EFIE formulation [3], the electric current is expanded using the RWG functions, but the

magnetic current is expanded using another set of basis functions given \hat{n} is the unit permutation of the point-wise spatially orthogonal to the RWG extends in the unit permutation of the surface. In magnetic current is expanded using another set of basis functions given
by $\hat{n} \times RWG$ which are point-wise spatially orthogonal to the RWG
set. Here \hat{n} is the unit normal pointing outward from the surface. In
addition by $\hat{n} \times \text{RWG}$ which are point-wise spatially orthogonal to the RWG
set. Here \hat{n} is the unit normal pointing outward from the surface. In
addition RWG is also used as the testing functions. Rao and Wilton set. Here \hat{n} is the unit normal pointing outward from the surface. In addition RWG is also used as the testing functions. Rao and Wilton proposed the CFIE with EFIE and MFIE for the analysis of scattering by orbitrari addition RWG is also used as the testing functions. Rao and Wilton
proposed the CFIE with EFIE and MFIE for the analysis of scattering
by arbitrarily shaped 3-D dielectric bodies for the first time [4]. In their
work, BWG proposed the CFIE with EFIE and MFIE for the analysis of scattering
by arbitrarily shaped 3-D dielectric bodies for the first time [4]. In their
work, RWG functions is used to approximate the electric current, but by arbitrarily shaped 3-D dielectric bodies for the first time [4]. In their work, RWG functions is used to approximate the electric current, but
the magnetic current is approximated by $\hat{n} \times RWG$ as in [3], and a line
te work, RWG functions is used to approximate the electric current, but
the magnetic current is approximated by $\hat{n} \times RWG$ as in [3], and a line
testing is used. In a recent paper [6], Sheng et al. proposed a CFIE
formulatio the magnetic current is approximated by $\hat{n} \times \text{RWG}$ as in [3], and a line testing is used. In a recent paper [6], Sheng et al. proposed a CFIE formulation for this problem. In their work the RWG functions are used as b testing is used. In a recent paper [6], Sheng et al. proposed a CFIE formulation for this problem. In their work the RWG functions are used as basis functions to approximate both the electric and magnetic currents, and RW formulation for this problem. In their work the RWG functions are used as basis functions to approximate both the electric and magnetic functions. This yields a well-conditioned matrix. They also presented $\langle RWG, EFIE \rangle + \langle \hat{n} \times RWG, EFIE \rangle + \langle RWG, MFIE \rangle + \langle \hat{n} \times RWG, MFIE \rangle.$ In this work, we investigate various integral formulations and a set of four CFIE formulations by dropping one of the testing terms in

 $\langle RWG, EFIE \rangle + \langle \hat{n} \times RWG, EFIE \rangle + \langle RWG, MFIE \rangle + \langle \hat{n} \times RWG, MFIE \rangle$.
In this work, we investigate various integral formulations and
propose several combination of CFIE with different choices of testing
functions. The scal is to ill In this work, we investigate various integral formulations and
propose several combination of CFIE with different choices of testing
functions. The goal is to illustrate that not all CFIE formulations
are stable. This pape propose several combination of CFIE with different choices of testing
functions. The goal is to illustrate that not all CFIE formulations
are stable. This paper is organized as follows. In the next section,
we describe the functions. The goal is to illustrate that not all CFIE formulations are stable. This paper is organized as follows. In the next section, we describe the integral equation formulations such as EFIE, MFIE, are stable. This paper is organized as follows. In the next section,
we describe the integral equation formulations such as EFIE, MFIE,
CFIE, and PMCHW. In Section 3, the triangular patch basis functions
are described and we describe the integral equation formulations such as EFIE, MFIE,
CFIE, and PMCHW. In Section 3, the triangular patch basis functions
are described and the numerical implementation of EFIE, MFIE,
CEIE, and PMCHW is develo CFIE, and PMCHW. In Section 3, the triangular patch basis functions
are described and the numerical implementation of EFIE, MFIE,
CFIE, and PMCHW is developed in detail. Section 4 presents several
formulations of CEIE and are described and the numerical implementation of EFIE, MFIE, CFIE, and PMCHW is developed in detail. Section 4 presents several formulations of CFIE and some numerical solutions. In Section 5, CFIE, and PMCHW is developed in detail. Section 4 presents several
formulations of CFIE and some numerical solutions. In Section 5,
numerical results for a dielectric sphere, a cube, and a finite circular
guinder are prese formulations of CFIE and some numerical solutions. In Section 5, numerical results for a dielectric sphere, a cube, and a finite circular cylinder are presented and compared with other available solutions. numerical results for a dielectric sphere, a cube, a
cylinder are presented and compared with other
Finally, conclusions are presented in Section 6. Finally, conclusions are presented in Section 6.
2. INTEGRAL EQUATIONS

2. INTEGRAL EQUATIONS
In this section, we describe the detailed mathematical steps to obtain
a pair of coupled integral equations to applyze the electromographic In this section, we describe the detailed mathematical steps to obtain
a pair of coupled integral equations to analyze the electromagnetic
sections from enhitment shaped 3 D homogeneous dislecties hodies In this section, we describe the detailed mathematical steps to obtain
a pair of coupled integral equations to analyze the electromagnetic
scattering from arbitrary shaped 3-D homogeneous dielectric bodies.
From these set a pair of coupled integral equations to analyze the electromagnetic
scattering from arbitrary shaped 3-D homogeneous dielectric bodies.
From these set of integral equations, we develop EFIE, MFIE,
PMCHW and CEIE For the sa scattering from arbitrary shaped 3-D homogeneous dielectric bodies.
From these set of integral equations, we develop EFIE, MFIE,
PMCHW, and CFIE. For the sake of clarity, we present the formula-
tions for a single dielectr From these set of integral equations, we develop EFIE, MFIE, PMCHW, and CFIE. For the sake of clarity, we present the formulations for a single dielectric body. Extending the formulation to handle multiple dielectric bodie PMCHW, and CFIE. For the sake of clarity, we present the formulations for a single dielectric body. Extending the formulation to handle multiple dielectric bodies is quite straightforward.
Consider a homogeneous dielectri tions for a single dielectric body. Extending the formulation to handle

and a permeability μ_2 placed in an infinite homogeneous medium with Consider a homogeneous dielectric body with a permittivity ε_2
and a permeability μ_2 placed in an infinite homogeneous medium with
a permittivity ε_1 and a permeability μ_1 . A lossy material body can be
har and a permeability μ_2 placed in an infinite homogeneous medium with
a permittivity ε_1 and a permeability μ_1 . A lossy material body can be
handled by considering ε_2 , or μ_2 , to be complex. The structur a permittivity ε_1 and a permeability μ_1 . A lossy material body can be handled by considering ε_2 , or μ_2 , to be complex. The structure is now illuminated by an incident plane wave denoted by $(\underline{E}^i, \underline{H}$ handled by considering ε_2 , or μ_2 , to be complex. The structure is now illuminated by an incident plane wave denoted by $(\underline{E}^i, \underline{H}^i)$. It may be noted that the incident field is defined to be that which would

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space if the structure were not present. By invoking the equivalence principle if the structure were not present. By invoking the equivalence
principle [8], the following two problems are formulated, which are
unlid for the periods attempt and internal to the dielectric hedy in space if the structure were not present. By invoking the equivalence
principle [8], the following two problems are formulated, which are
valid for the regions external and internal to the dielectric body, in
tarms of the principle [8], the following two problems are formulated, which are valid for the regions external and internal to the dielectric body, in terms of the equivalent electric and magnetic current \overline{J} and \overline{M} on the valid for the regions external and internations of the equivalent electric and magnetized surface S of dielectric body, respectively. s of the equivalent electric and magnetic current \underline{J} and \underline{M} on the
ce S of dielectric body, respectively.
By enforcing the continuity of the tangential electric and magnetic
et S where S is selected to b

surface S of dielectric body, respectively.
By enforcing the continuity of the tangential electric and magnetic
field at $S_$, where $S_$ is selected to be slightly interior to S, the
following equations are obtained: By enforcing the continuity of t
field at S_{-} , where S_{-} is selected
following equations are obtained: tained:
 $s^s(S, M)$ _{tan} = $[\underline{E}^i]_{\text{tan}}$

$$
[-\underline{E}_1^s(\underline{J}, \underline{M})]_{\text{tan}} = [\underline{E}^i]_{\text{tan}}
$$
\n
$$
[-\underline{H}_1^s(\underline{J}, \underline{M})]_{\text{tan}} = [\underline{H}^i]_{\text{tan}}
$$
\n(1)\n(2)

$$
[-\underline{H}_1^s(\underline{J}, \underline{M})]_{\tan} = [\underline{H}^i]_{\tan} \tag{2}
$$

where the subscript '1' represents the medium in which the scattered where the subscript '1' represents the medium in which the scattered
field $(\underline{E}_1^s, \underline{H}_1^s)$ is computed. Because the field in the interior region
to the dielectric body is zero, the entire space is now filled with the where the subscript '1' represents the medium in which the scattered
field $(\underline{E}_1^s, \underline{H}_1^s)$ is computed. Because the field in the interior region
to the dielectric body is zero, the entire space is now filled with the field $(\underline{E}_1^s, \underline{H}_1^s)$ is computed. Because the field in the interior region
to the dielectric body is zero, the entire space is now filled with the
dielectric medium (ε_1, μ_1) , which originally was only external to the dielectric body is zero, the entire space is now filled with the dielectric medium (ε_1, μ_1) , which originally was only external to the dielectric body. By enforcing the continuity of the tangential electric a dielectric medium (ε_1, μ_1) , which originally was only external to the dielectric body. By enforcing the continuity of the tangential electric and magnetic field at S_+ , where S_+ is the surface slightly exterior dielectric body. By enforcing the contin
and magnetic field at S_+ , where S_+ is t
S, the following equations are derived: s derived:
 $s^s_2(\underline{J}, \underline{M})|_{\tan} = 0$ (3)

$$
[-\underline{E}_2^s(\underline{J}, \underline{M})]_{tan} = 0
$$
\n
$$
[-\underline{H}_2^s(\underline{J}, \underline{M})]_{tan} = 0
$$
\n(3)\n(4)

$$
[-\underline{H}_2^s(\underline{J}, \underline{M})]_{\text{tan}} = 0 \tag{4}
$$

 $[-\underline{H}_2^s(\underline{J}, \underline{M})]_{\text{tan}} = 0$ (4)
where the subscript '2' represents the medium in which the scattered where the subscript '2' represents the medium in which the scattered
field $(\underline{E}_2^s, \underline{H}_2^s)$ is evaluated. Again since the fields in the external region
to the material body is zero, the external region can be replaced where the subscript '2' represents the medium in which the scattered
field $(\underline{E}_2^s, \underline{H}_2^s)$ is evaluated. Again since the fields in the external region
to the material body is zero, the external region can be replaced field $(\underline{E}_2^s, \underline{H}_2^s)$ is evaluated. Again since the fields in the external region
to the material body is zero, the external region can be replaced by the
material (ε_2, μ_2) so that now the currents are located to the material body is zero, the external region can be replaced by the material (ε_2, μ_2) so that now the currents are located in a homogeneous medium with properties of the material which was internal to the mater material (ε_2, μ_2) so that now the currents are located in a homogeneous
medium with properties of the material which was internal to the
material body of the original problem. In (1)–(4), the subscript 'tan'
refers medium with properties of the material
material body of the original problem. In
refers to the tangential component only.
Now that both problems (internal as erial body of the original problem. In $(1)-(4)$, the subscript 'tan'
s to the tangential component only.
Now that both problems (internal and external) are formulated
rms of electric and magnetic currents redisting in a ho

refers to the tangential component only.
Now that both problems (internal and external) are formulated
in terms of electric and magnetic currents radiating in a homogeneous
medium the various fields can be expressed analyt Now that both problems (internal and external) are formulated
in terms of electric and magnetic currents radiating in a homogeneous
medium the various fields can be expressed analytically. The scattered
electric and magne in terms of electric and magnetic currents radiating in a homogeneous
medium the various fields can be expressed analytically. The scattered
electric and magnetic fields due to the electric current \underline{J} and magnetic
g medium the various fields can be expressed analytically. The scattered electric and magnetic fields due to the electric current \underline{J} and magnetic current M are given by

$$
\underline{E}_v^s(\underline{J}) = -j\omega \underline{A}_v - \nabla \phi_v \tag{5}
$$

$$
\underline{E}_v^s(\underline{J}) = -j\omega \underline{A}_v - \nabla \phi_v \tag{5}
$$
\n
$$
\underline{E}_v^s(\underline{M}) = -\frac{1}{\varepsilon_v} \nabla \times \underline{F}_v \tag{6}
$$
\n
$$
\underline{H}_v^s(I) = \frac{1}{\varepsilon_v} \nabla \times \underline{A} \tag{7}
$$

$$
\underline{E}_v^s(\underline{M}) = -\frac{1}{\varepsilon_v} \nabla \times \underline{F}_v \tag{6}
$$
\n
$$
\underline{H}_v^s(\underline{J}) = \frac{1}{\mu_v} \nabla \times \underline{A}_v \tag{7}
$$
\n
$$
\underline{H}_v^s(\underline{M}) = -j\omega \underline{F}_v - \nabla \psi_v \tag{8}
$$

$$
\underline{H}^s_v(\underline{M}) = -j\omega \underline{F}_v - \nabla \psi_v \tag{8}
$$

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where the magnetic and electric vector potentials \underline{A}_v and \underline{F}_v , and the
scalar potentials \underline{A}_v and \underline{w}_v for $v = 1, 2$ are given by where the magnetic and electric vector potentials \underline{A}_v and
scalar potentials ϕ_v and ψ_v , for $v = 1, 2$ are given by

where
$$
\omega
$$
 and ω are given by

\n
$$
\Delta_v(\mathbf{r}) = \frac{\mu_v}{4\pi} \int_S \underline{J(\mathbf{r'})} G_v(\mathbf{r}, \mathbf{r'}) dS'
$$
\n(9)

$$
\underline{A}_v(\underline{r}) = \frac{\mu_v}{4\pi} \int_S \underline{J}(\underline{r}') G_v(\underline{r}, \underline{r}') dS'
$$
(9)

$$
\underline{F}_v(\underline{r}) = \frac{\varepsilon_v}{4\pi} \int_S \underline{M}(\underline{r}') G_v(\underline{r}, \underline{r}') dS'
$$
(10)

$$
\underline{F}_v(\underline{r}) = \frac{\partial}{\partial \underline{\pi}} \int_S \underline{M}(\underline{r}') G_v(\underline{r}, \underline{r}') dS' \tag{10}
$$
\n
$$
\phi_v(\underline{r}) = \frac{j}{4\pi\omega\varepsilon_v} \int_S \nabla'_S \cdot \underline{J}(\underline{r}') G_v(\underline{r}, \underline{r}') dS' \tag{11}
$$

$$
\phi_v(\underline{r}) = \frac{J}{4\pi\omega\varepsilon_v} \int_S \nabla'_S \cdot \underline{J}(\underline{r}') G_v(\underline{r}, \underline{r}') dS' \tag{11}
$$
\n
$$
\psi_v(\underline{r}) = \frac{j}{4\pi\omega\mu_v} \int_S \nabla'_S \cdot \underline{M}(\underline{r}') G_v(\underline{r}, \underline{r}') dS' \tag{12}
$$

$$
\varphi_v(\underline{L}) = \frac{1}{4\pi\omega\mu_v} \int_S \nabla S \cdot \underline{M}(\underline{L}) G_v(\underline{L}, \underline{L}) dS \tag{12}
$$
\n
$$
G_v(\underline{r}, \underline{r}') = \frac{e^{-jk_vR}}{R}; \qquad R = |\underline{r} - \underline{r}'|.
$$
\n
$$
\text{In (9)-(12), a } e^{j\omega t} \text{ time dependence has been assumed and suppressed.}
$$
\n
$$
R \text{ represents the distance between the observation point } x \text{ and the}
$$

 $C_v(L, L) = \frac{R}{R}$, $R = |L - L|$. (13)
In (9)–(12), a $e^{j\omega t}$ time dependence has been assumed and suppressed.
R represents the distance between the observation point r and the In (9)–(12), a $e^{j\omega t}$ time dependence has been assumed and suppressed.

R represents the distance between the observation point <u>r</u> and the source point <u>r'</u> with respect to a global coordinate origin. $k_v = \omega \sqrt{\mu_v \varepsilon_v$ R represents the distance between the observation point $\frac{r}{\Delta}$ and the source point $\frac{r'}{L'}$ with respect to a global coordinate origin. $k_v = \omega \sqrt{\mu_v \varepsilon_v}$ is the wave number. ω is the angular frequency in rad/se source point <u>r'</u> with respect to a global coordinate origin. $k_v = \omega \sqrt{\mu_v \varepsilon_v}$ is the wave number. ω is the angular frequency in rad/sec. Note that v is either 1 or 2 depending on the medium in which the currents <u>J</u> is the wave number. ω is the angular frequency in rad/sec. Note that
 v is either 1 or 2 depending on the medium in which the currents <u>J</u>
and <u>M</u> are radiating. Equation (13) represents the Green's function
for a h v is either 1 or 2 depending c
and \underline{M} are radiating. Equation
for a homogeneous medium.
 I_{R} (1) (4) there are two <u>M</u> are radiating. Equation (13) represents the Green's function
homogeneous medium.
In (1)–(4), there are two unknowns <u>J</u> and <u>M</u>, and four equations
ing them. It is possible to develop verticus combinations for the

for a homogeneous medium.
In $(1)-(4)$, there are two unknowns <u>J</u> and <u>M</u>, and four equations relating them. It is possible to develop various combinations for the In (1)–(4), there are two unknowns <u>J</u> and <u>M</u>, and four equations
relating them. It is possible to develop various combinations for the
solution of these equations. If we take only two equations, (1) and
(2) we have the relating them. It is possible to develop various combinations for the solution of these equations. If we take only two equations, (1) and (3), we have the EFIE formulation. Dual to the EFIE formulation, we can obtain the solution of these equations. If we take only two equations, (1) and (3), we have the EFIE formulation. Dual to the EFIE formulation, we can obtain the MFIE formulation by choosing only (2) and (4) from the set (1) (4) How (3) , we have the EFIE formulation. Dual to the EFIE formulation, we can obtain the MFIE formulation by choosing only (2) and (4) from can obtain the MFIE formulation by choosing only (2) and (4) from
the set (1)–(4). However, both EFIE and MFIE formulations fail at
frequencies at which the surface S, when covered by a perfect electric
conductor and fill the set (1) – (4) . However, both EFIE and MFIE formulations fail at frequencies at which the surface S , when covered by a perfect electric conductor and filled with the materials of the exterior medium, forms a recove frequencies at which the surface S , when covered by a perfect electric conductor and filled with the materials of the exterior medium, forms a resonant cavity. An alternative way of combining the four equations is conductor and filled with the materials of the exterior medium, forms a
resonant cavity. An alternative way of combining the four equations is
the PMCHW formulation. In this formulation, the set of four equations
is reduc resonant cavity. An alternative way of combining the four equations is
the PMCHW formulation. In this formulation, the set of four equations
is reduced to two by adding (1) to (3) and (2) to (4). This gives a pair
of cous the PMCHW f
is reduced to tv
of equations adding (1) to (3) and (2) to (4). T
 $S_1^s(\underline{J}, \underline{M}) - \underline{E}_2^s(\underline{J}, \underline{M})|_{\tan} = [\underline{E}^i]_{\tan}$

$$
[-\underline{E}_1^s(\underline{J}, \underline{M}) - \underline{E}_2^s(\underline{J}, \underline{M})]_{\text{tan}} = [\underline{E}^i]_{\text{tan}}
$$
(14)

$$
[-\underline{H}_1^s(\underline{J}, \underline{M}) - \underline{H}_2^s(\underline{J}, \underline{M})]_{\text{tan}} = [\underline{H}^i]_{\text{tan}}
$$
(15)

$$
-\underline{H}_1^s(\underline{J}, \underline{M}) - \underline{H}_2^s(\underline{J}, \underline{M})|_{\tan} = [\underline{H}^i]_{\tan} \tag{15}
$$

For the CFIE formulation, a set of two integral equations are formed
 $[-\underline{H}_1^s(\underline{J}, \underline{M}) - \underline{H}_2^s(\underline{J}, \underline{M})]_{\text{tan}} = [\underline{H}^i]_{\text{tan}}$ (15)

For the CFIE formulation, a set of two integral equations are formed $[-\underline{H}_1^s(\underline{J}, \underline{M}) - \underline{H}_2^s(\underline{J}, \underline{M})]_{\text{tan}} = [\underline{H}]^s$
For the CFIE formulation, a set of two integral eq
from the set (1)–(4) using the following form from the set (1) – (4) using the following form

from the set (1)–(4) using the following form
\n
$$
[-\underline{E}_v^s(\underline{J}, \underline{M})]_{\tan} + \eta_1[-\underline{H}_v^s(\underline{J}, \underline{M})]_{\tan} = \begin{cases} [\underline{E}^i]_{\tan} + \eta_1[\underline{H}^i]_{\tan}, & v = 1\\ 0, & v = 2\\ (16) \end{cases}
$$

where η_1 is the wave impedance of region 1.

3. NUMERICAL IMPLEMENTATION
3. NUMERICAL IMPLEMENTATION

3. NUMERICAL IMPLEMENTATION
The structure to be analyzed is approximated by planar triangular
patches. The triangular patches have the shility to conform to any The structure to be analyzed is approximated by planar triangular patches. The triangular patches have the ability to conform to any reconstrued surface of boundary. As in reference $[7]$ we define the The structure to be analyzed is approximated by planar triangular patches. The triangular patches have the ability to conform to any geometrical surface of boundary. As in reference $[7]$, we define the patches. The triangular patches have the
geometrical surface of boundary. As in
vector basis function associated the nth equals to the ability
in reference
 $\frac{t}{t}$ reference

on associated the *n*th edge as
\n
$$
\underline{f}_n(\underline{r}) = \underline{f}_n^+(\underline{r}) + \underline{f}_n^-(\underline{r})
$$
\n(17a)

$$
\frac{f_n(\mathbf{r})}{f_n^{\pm}(\mathbf{r})} = \begin{cases}\n\frac{f_n(\mathbf{r}) + f_n(\mathbf{r})}{2A_n^{\pm} \rho_n^{\pm}}, & \mathbf{r} \in T_n^{\pm} \\
0, & \mathbf{r} \notin T_n^{\pm}\n\end{cases}
$$
\n(17a)\n(17a)\n(17b)

where l_n is the length of the n^{th} edge and A_n^{\pm} is the area of the triangle T_n^{\pm} . ρ_n^{\pm} is the position vector with respect to the free vertex of T_n^{\pm} . Fig. 1 illustrates the various variables. The position vector ρ_n^+ ι_n is the length of the n^{th} edge and A_n^{\pm} is the area of the ile T_n^{\pm} . $\underline{\rho}_n^{\pm}$ is the position vector with respect to the free vertex . Fig. 1 illustrates the various variables. The position vector triangle T_n^{\pm} . ρ_n^{\pm} is the position vector with respect to the free vertex
of T_n^{\pm} . Fig. 1 illustrates the various variables. The position vector ρ_n^+
is directed from the free vertex of T_n^+ toward poin of T_n^{\pm} . Fig. 1 illustrates the various variables. The position vector $\underline{\rho}_n^+$
is directed from the free vertex of T_n^+ toward points in T_n^+ . Similar
remarks apply to the position $\underline{\rho}_n^-$ except that it is is directed from the free vertex of T_n^+ toward points in T_n^+ . S
remarks apply to the position $\underline{\rho}_n^-$ except that it is directed towar
free vertex of T_n^- . The surface divergence of (17) is given by $n \cdot$

$$
\overline{r}_n^-, \text{ The surface divergence of (17) is given by}
$$

$$
\nabla_S \cdot \underline{f}_n(\underline{r}) = \nabla_S \cdot \underline{f}_n^+(\underline{r}) + \nabla_S \cdot \underline{f}_n^-(\underline{r})
$$
(18a)

$$
\nabla_S \cdot \underline{f}_n(\underline{r}) = \nabla_S \cdot \underline{f}_n^+(\underline{r}) + \nabla_S \cdot \underline{f}_n^-(\underline{r})
$$
(18a)

$$
\nabla_S \cdot \underline{f}_n^+(\underline{r}) = \begin{cases} \pm \frac{l_n}{A_n^{\pm}} \underline{\rho}_n^{\pm}, & \underline{r} \in T_n^{\pm} \\ 0, & \underline{r} \notin T_n^{\pm} \end{cases}
$$
(18b)
Another vector basis function is defined through [3]

$$
\begin{aligned}\n\text{ction is defined through [3]}\\ \n\underline{g}_n(\underline{r}) &= \hat{n} \times \underline{f}_n(\underline{r}).\n\end{aligned}\n\tag{19}
$$

 $\underline{g}_n(\underline{r}) = \hat{n} \times \underline{f}_n(\underline{r}).$ (19)
The functions \underline{f}_n and \underline{g}_n are point-wise orthogonal in the triangle
 \vdots \underline{F} . The set of \underline{F} . The MEIE MEIE $\frac{g_n(z)}{z_n(z)}$. The functions \underline{f}_n and \underline{g}_n are point-wise orthogonal in the triangle
pair. These functions are used in the four formulations, EFIE, MFIE,
PMCHW and CEIE, The various supersion and testing functi The functions \underline{f}_n and \underline{g}_n are point-wise orthogonal in the triangle
pair. These functions are used in the four formulations, EFIE, MFIE,
PMCHW, and CFIE. The various expansion and testing functions
for the var pair. These functions are used in the four formulations, EFIE, MFIE, PMCHW, and CFIE. The various expansion and testing functions for the various currents used in each formulation are summarized in Table 1 PMCHW, a
for the vario
Table 1. for the various currents used in each formulation are summarized in Table 1.
We consider the numerical procedure in detail. Although the EFIE

formulation is described in [3] and [4], we develop the implementation procedure in detail, from which other formulations are derived simply. The evaluation of integrals in the matrix elements is considered in the Appendix.

3.1. EFIE Formulation

3.1. EFIE Formulation
The electric current \underline{J} and the magnetic current \underline{M} on the structure
to be applyzed may be approximated in tarms of two yester basis **S.1. EFTE FORTHURGION**
The electric current \underline{J} and the magnetic current \underline{M} on the structure
to be analyzed may be approximated in terms of two vector basis

Figure 1. Two triangular patches associated with an edge.

Tigure 1. Two triangular patches associated with an edge.
Table 1. Expansion and testing functions used in four formulations.

			le 1. Expansion and testing functions used in four formulation	
Formulation	Testing		Current expansion function	
	function			
EFIE	m		$\hat{n} \times$	
MFIE	m	$\hat{n} \times$		
PMCHW				
CFIE				

functions (17) and (19) as

$$
\underline{J}(\underline{r}) = \sum_{n=1}^{N} I_n \underline{f}_n(\underline{r})
$$
\n(20)

$$
\underline{M}(\underline{r}) = \sum_{n=1}^{N} M_n \underline{g}_n(\underline{r})
$$
\n(21)

 $\frac{M(T)}{n=1} = \sum_{n=1}^{M_n} \frac{M_n g_n(T)}{n}$ (21)
where I_n and M_n are constants yet to be determined and N is the
number of edges on the dislecting surface for the triangulated model where I_n and M_n are constants yet to be determined and N is the
number of edges on the dielectric surface for the triangulated model
numerimative the surface of the dielectric hody. The next stap in the where I_n and M_n are constants yet to be determined and N is the number of edges on the dielectric surface for the triangulated model approximating the surface of the dielectric body. The next step in the application o number of edges on the dielectric surface for the triangulated model
approximating the surface of the dielectric body. The next step in the
application of the method of moments is to select a suitable testing approximating the surface of the dielectric body. The next step in the application of the method of moments is to select a suitable testing procedure. As testing functions, we choose the expansion functions in (17) . Sin procedure. As testing functions, we choose the expansion functions in

$$
\langle \underline{f} \cdot \underline{g} \rangle = \int_{S} \underline{f} \cdot \underline{g} dS. \tag{22}
$$

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We test (1) and (3) with \underline{f}_m , yielding

We test (1) and (3) with
$$
\underline{f}_m
$$
, yielding
\n
$$
\langle \underline{f}_m, -\underline{E}_v^s(\underline{J}) \rangle + \langle \underline{f}_m, -\underline{E}_v^s(\underline{M}) \rangle = \begin{cases} \langle \underline{f}_m, \underline{E}^i \rangle, & v = 1 \\ 0, & v = 2 \end{cases}
$$
\nfor $m = 1, 2, ..., N$. The first term in (23) with (5) is given by

...,*N*. The first term in (23) with (5) is given by

$$
\langle \underline{f}_m, -\underline{E}_v^s(\underline{J}) \rangle = \langle \underline{f}_m, j\omega \underline{A}_v \rangle + \langle \underline{f}_m, \nabla \phi_v \rangle.
$$
 (24)

Substituting (9) into the testing of the magnetic vector potential and using the current expansion (20), we have

$$
\langle \underline{f}_m, j\omega \underline{A}_v \rangle = \sum_{n=1}^N j\omega \mu_v A_{mn,v} I_n \tag{25}
$$

where

where
\n
$$
A_{mn,v} = \frac{1}{4\pi} \int_S \underline{f}_m(\underline{r}) \cdot \int_S \underline{f}_n(\underline{r}') G_v(\underline{r}, \underline{r}') dS' dS.
$$
\n(26)
\nNext, we consider the testing of the gradient of the electric scalar

potential in (24). Using the vector identity $\nabla \cdot \phi \underline{A} = \underline{A} \cdot \nabla \phi + \phi \nabla \cdot \underline{A}$ and the properties of the basis function [9], we have

$$
\langle \underline{f}_m, \nabla \phi_v \rangle = -\int_S \nabla S \cdot \underline{f}_m \phi_v dS. \tag{27}
$$

Substitution of (11) into (27) with current expansion (20) yields

$$
\langle \underline{f}_m, \nabla \phi_v \rangle = \sum_{n=1}^N \left(-\frac{j}{\omega \varepsilon} \right) B_{mn,v} I_n \tag{28}
$$

where

$$
B_{mn,v} = \frac{1}{4\pi} \int_S \nabla_S \cdot \underline{f}_m(\underline{r}) \int_S \nabla_S' \cdot \underline{f}_n(\underline{r}') G_v(\underline{r}, \underline{r}') dS' dS. \tag{29}
$$

Therefore, substitution of (25) and (28) into (24) yields

$$
\langle \underline{f}_m, -\underline{E}_v^s(\underline{J}) \rangle = \sum_{n=1}^N j k_v \eta_v \left(A_{mn,v} - \frac{B_{mn,v}}{k_v^2} \right) I_n \tag{30}
$$

where $k_v \eta_v = \omega \mu_v$ and η_v is the intrinsic impedance of the medium numbered v.

Now, consider the second term of (23). Extracting the Cauchy Now, consider the second term of (23) . Extracting the Cauchy principal value from the curl term on the planar surface in (6) with (10) we may write Now, consider th
principal value from t
(10), we may write

$$
\text{write}
$$
\n
$$
\frac{1}{\varepsilon_v} \nabla \times \underline{F}_v(\underline{r}) = \pm \frac{1}{2} \hat{n} \times \underline{M}(\underline{r}) + \frac{1}{\varepsilon_v} \nabla \times \underline{\tilde{F}}_v(\underline{r}) \tag{31}
$$

 $-\frac{1}{\varepsilon_v} \nabla \times \underline{F}_v(\underline{r}) = \pm \frac{1}{2} n \times \underline{M}(\underline{r}) + \frac{1}{\varepsilon_v} \nabla \times \underline{F}_v(\underline{r})$ (31)
where $\underline{\tilde{F}}_v$ is defined by (10) with $\underline{r} = \underline{r}'$, or $R = 0$, term removed from
the integration. In (31), the positive sign where $\underline{\tilde{F}}_v$ is defined by (10) with $\underline{r} = \underline{r}'$, or $R = 0$, term removed from
the integration. In (31), the positive sign is used when $v = 1$ and
negative sign otherwise. Heing (31), we can get where $\underline{\tilde{F}}_v$ is defined by (10) with $\underline{r} = \underline{r}'$, or $R = 0$,
the integration. In (31), the positive sign is use
negative sign otherwise. Using (31), we can get

$$
\langle \underline{f}_m, -\underline{E}_v^s(\underline{M}) \rangle = \langle \underline{f}_m, \pm \frac{1}{2} \hat{n} \times \underline{M} \rangle + \langle \underline{f}_m, \frac{1}{\varepsilon_v} \nabla \times \underline{\tilde{F}}_v \rangle. \tag{32}
$$

Consider the inner product integrals in (32). Substitution of the
mscneti current expansion defined in (21) in the first term located

Consider the inner product integrals in (32). Substitution of the magnetic current expansion defined in (21) in the first term located in the right hand side of (32) violds Consider the inner product integrals
magnetic current expansion defined in
in the right-hand side of (32) yields

$$
\langle \underline{f}_m, \pm \frac{1}{2} \hat{n} \times \underline{M} \rangle = \sum_{n=1}^{N} C_{mn,v} M_n \tag{33}
$$

where

$$
C_{mn,v} = \begin{cases} +C_{mn}, & v = 1\\ -C_{mn}, & v = 2 \end{cases}
$$
 (34)

$$
C_{mn} = \frac{1}{2} \int_{S} \underline{f}_m(\underline{r}) \cdot \hat{n} \times \underline{g}_n(\underline{r}) dS.
$$
 (35)

 $C_{mn} = \frac{1}{2} \int_S \underline{f}_m(\underline{r}) \cdot \hat{n} \times \underline{g}_n(\underline{r}) dS.$ (35)
Next, we consider the second term on the right-hand side of (32). Using
(10) in association with (21), we obtain Next, we consider the second term on the rig (10) in association with (21), we obtain

$$
\langle \underline{f}_m, \frac{1}{\varepsilon_v} \nabla \times \underline{\tilde{F}}_v \rangle = \sum_{n=1}^N D_{mn,v} M_n \tag{36}
$$

where

$$
D_{mn,v} = \frac{1}{4\pi} \int_S \underline{f}_m(\underline{r}) \cdot \int_S \underline{g}_n(\underline{r}') \nabla' G_v(\underline{r}, \underline{r}') dS' dS. \tag{37}
$$

Substitution of (33) and (36) into (32) yields

$$
\langle \underline{f}_m, -\underline{E}_v^s(\underline{M}) \rangle = \sum_{n=1}^N (C_{mn,v} + D_{mn,v}) M_n. \tag{38}
$$

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Finally, substituting (30) and (38) into (23), we obtain

$$
\sum_{n=1}^{N} j k_{v} \eta_{v} \left(A_{mn,v} - \frac{B_{mn,v}}{k_{v}^{2}} \right) I_{n} + \sum_{n=1}^{N} (C_{mn,v} + D_{mn,v}) M_{n} = V_{m,v}^{E}.
$$
 (39)

where

$$
V_{m,v}^{E} = \begin{cases} \int_{S} \underline{f}_{m}(\underline{r}) \cdot \underline{E}^{i}(\underline{r}) dS, & v = 1 \\ 0, & v = 2 \end{cases}
$$
 (40)

Equation (39) is associated with each edge, $m = 1, 2, ..., N$. Therefore
(39) may be written in a matrix form as Equation (39) is associated with each edge, $n(39)$ may be written in a matrix form as (39) may be written in a matrix form as

$$
\begin{bmatrix}\n[jk_1\eta_1(A_{mn,1}-B_{mn,1}/k_1^2)] & [C_{mn,1}+D_{mn,1}] \\
[jk_2\eta_2(A_{mn,2}-B_{mn,2}/k_2^2)] & [C_{mn,2}+D_{mn,2}] \n\end{bmatrix}\n\begin{bmatrix}\n[I_n] \\
[M_n]\n\end{bmatrix} =\n\begin{bmatrix}\n[V_{m,1}^E] \\
[V_{m,2}^E] \\
[N_{m,2}^E]\n\end{bmatrix}.
$$
\nEquation (41) is a 2*N* × 2*N* system of linear equations.

Equation (41) is a $2N \times 2N$ sys
3.2. MFIE Formulation

3.2. MFIE Formulation
In this paper, MFIE has been defined in a different form from that
of $\begin{bmatrix} 4 & 1 \\ 2 & 1 \end{bmatrix}$ is the dual of EEIF. For the MEIF formulation, the electric **S.2.** WIFIE FORMULATION
In this paper, MFIE has been defined in a different form from that
of [4]. It is the dual of EFIE. For the MFIE formulation, the electric
guyment Land the magnetic survent M may be approximated by In this paper, MFIE has been defined in a different form from that
of [4]. It is the dual of EFIE. For the MFIE formulation, the electr
current \underline{J} and the magnetic current \underline{M} may be approximated by current \underline{J} and the magnetic current \underline{M} may be approximated by

$$
\underline{J}(\underline{r}) = \sum_{n=1}^{N} I_n \underline{g}_n(\underline{r}) \tag{42}
$$

$$
\underline{M}(\underline{r}) = \sum_{n=1}^{N} M_n \underline{f}_n(\underline{r}). \tag{43}
$$

These current expansions are similar to (20) and (21) in the EFIE
formulation Heing (22) we test (2) and (4) with f widding These current expansions are similar to (20) and (21) if formulation. Using (22), we test (2) and (4) with \underline{f}_m , yiel 21) in the E , yielding

equation. Using (22), we test (2) and (4) with
$$
\underline{f}_m
$$
, yielding

\n
$$
\langle \underline{f}_m, -\underline{H}_v^s(\underline{J}) \rangle + \langle \underline{f}_m, -\underline{H}_v^s(\underline{M}) \rangle = \begin{cases} \langle \underline{f}_m, \underline{H}^i \rangle, & v = 1 \\ 0, & v = 2 \end{cases} \tag{44}
$$

Using a formulation as described for the EFIE formulation, we get

$$
-\sum_{n=1}^{N} (C_{mn,v} + D_{mn,v}) I_n + \sum_{n=1}^{N} j \frac{k_v}{\eta_v} \left(A_{mn,v} - \frac{B_{mn,v}}{k_v^2} \right) M_n = V_{mn,v}^H
$$
 (45)

where

$$
V_{m,v}^H = \begin{cases} \int_S \underline{f}_m(\underline{r}) \cdot \underline{H}^i(\underline{r}) dS, & v = 1 \\ 0, & v = 2 \end{cases}
$$
 (46)

 $v_{m,v} = \begin{pmatrix} v_{m,v} & v_{m,v} \\ v_{m,v} & 0 \end{pmatrix}$
We note that (45) can be obtained directly from (39) by using the
principle of duality [8]. Equation (45) is associated with each edge. We note that (45) can be obtained directly from (39) by using the
principle of duality [8]. Equation (45) is associated with each edge,
 $m = 1.2$ M. Therefore (45) may be written in a matrix form as We note that (45) can be obtained directly from (39) by using the principle of duality [8]. Equation (45) is associated with each edge, $m = 1, 2, ..., N$. Therefore, (45) may be written in a matrix form as $m=1,2,\ldots,N.$ Therefore, (45) may be written in a matrix form as

$$
\begin{bmatrix}\n[-(C_{mn,1}+D_{mn,1})] & [jk_1/\eta_1(A_{mn,1}-B_{mn,1}/k_1^2)] \\
[-(C_{mn,2}+D_{mn,2})] & [jk_2/\eta_2(A_{mn,2}-B_{mn,2}/k_2^2)]\n\end{bmatrix}\n\begin{bmatrix}\n[I_n] \\
[M_n]\n\end{bmatrix} =\n\begin{bmatrix}\n[V_{m,1}^H \\
[V_{m,2}^H\n\end{bmatrix}.\n\tag{47}
$$

Equation (47) is dual to (41) and the integrals $A_{mn,v}$, $B_{mn,v}$, $C_{mn,v}$ and $D_{mn,v}$ in the matrix elements are same as those of (41).
3.3. PMCHW Formulation

In the PMCHW integral equations (14) and (15), the electric current J and the magnetic current M may be approximated by (20) and (43), In the PMCHW integral equations (14) and (15), the electric current \underline{J} and the magnetic current \underline{M} may be approximated by (20) and (43), respectively. Also, \underline{f}_m is used as the testing function. Applying t \underline{J} and the magnetic current \underline{M} may
respectively. Also, \underline{f}_m is used as t
testing procedure to (14), we get testing procedure to (14) , we get

ure to (14), we get
\n
$$
\langle \underline{f}_m, -\underline{E}_1^s(\underline{J}, \underline{M}) - \underline{E}_2^s(\underline{J}, \underline{M}) \rangle = \langle \underline{f}_m, \underline{E}^i \rangle.
$$
\n(48)

 $\langle \underline{f}_m, -\underline{E}_1^s(\underline{J}, \underline{M}) - \underline{E}_2^s(\underline{J}, \underline{M}) \rangle = \langle \underline{f}_m, \underline{E}^i \rangle.$ (48)
Equation (48) is evaluated simply by using (39), which is derived from
(23). The result is given by Equation (48) is evaluated simp (23). The result is given by

$$
\sum_{n=1}^{N} \sum_{v=1}^{2} j k_v \eta_v \left(A_{mn,v} - \frac{B_{mn,v}}{k_v^2} \right) I_n + \sum_{n=1}^{N} \sum_{v=1}^{2} D_{mn,v} M_n = V_m^E \tag{49}
$$

where

$$
V_m^E = \int_S \underline{f}_m(\underline{r}) \cdot \underline{E}^i(\underline{r}) dS. \tag{50}
$$

 $V_m^E = \int_S \underline{f}_m(\underline{r}) \cdot \underline{E}^i(\underline{r}) dS.$ (50)
In (49), $A_{mn,v}$ and $B_{mn,v}$ are same as (26) and (29), respectively.
Because the expansion functions of the magnetic eurons are different $\int_S 2^{-m}$ $\left| \frac{1}{S} \right|$ = $\left| \frac{1}{S} \right|$
In (49), $A_{mn,v}$ and $B_{mn,v}$ are same as (26) and (29), respectively.
Because the expansion functions of the magnetic current are different, In (49), $A_{mn,v}$ and
Because the expansio
 $D_{mn,v}$ is given by

given by
\n
$$
D_{mn,v} = \frac{1}{4\pi} \int_S f_m(\mathbf{r}) \cdot \int_S f_n(\mathbf{r}') \nabla' G_v(\mathbf{r}, \mathbf{r}') dS' dS.
$$
\n(51)

Note that the first term located in the right-hand side of (31) is Note that the first term located in the right-hand side of (31) is
eliminated by adding \underline{E}_1^s to \underline{E}_2^s in (48). Therefore, the term $C_{mn,v}$
is not seen in (40) Note that the first
eliminated by addin
is not seen in (49).

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Next, we apply the testing procedure to (15), we get

Next, we apply the testing procedure to (15), we get
\n
$$
\langle \underline{f}_m, -\underline{H}_1^s(\underline{J}, \underline{M}) - \underline{H}_2^s(\underline{J}, \underline{M}) \rangle = \langle \underline{f}_m, \underline{H}^i \rangle.
$$
\n(S2)
\nSimilarly, by using the result of the MFIE formulation given by (45),
\n(52) can be expressed as

 $\lim_{m \to \infty}$ can be expressed as
(52) can be expressed as

$$
-\sum_{n=1}^{N} \sum_{v=1}^{2} D_{mn,v} I_n + \sum_{n=1}^{N} \sum_{v=1}^{2} j \frac{k_v}{\eta_v} \left(A_{mn,v} - \frac{B_{mn,v}}{k_v^2} \right) M_n = V_m^H \quad (53)
$$

where

$$
V_m^H = \int_S \underline{f}_m(\underline{r}) \cdot \underline{H}^i(\underline{r}) dS.
$$
 (54)

 $V_m^H = \int_S \underline{f}_m(\underline{r}) \cdot \underline{H}^i(\underline{r}) dS.$ (54)
The integral of elements $A_{mn,v}$, $B_{mn,v}$, and $D_{mn,v}$ are same as those
in (49) respectively. We note that (53) can be obtained directly from The integral of elements $A_{mn,v}$, $B_{mn,v}$, and $D_{mn,v}$ are same as those
in (49), respectively. We note that (53) can be obtained directly from The integral of elements $A_{mn,v}$, $B_{mn,v}$, and $D_{mn,v}$ are same as those
in (49), respectively. We note that (53) can be obtained directly from
(49) by using duality without additional any effort. Equation (49) and
(53) a in (49), respectively. We note that (53) can be obtained directly from (49) by using duality without additional any effort. Equation (49) and (53) are associated with each edge, $m = 1, 2, ..., N$. Therefore, we have a matrix e (49) by using duality without additional any effort. Equation (49) and

$$
\left[\begin{bmatrix} \sum_{v=1}^{2} j k_{v} \eta_{v} (A_{mn,v} - B_{mn,v}/k_{v}^{2}) \\ - \sum_{v=1}^{2} D_{mn,v} \end{bmatrix} \begin{bmatrix} \sum_{v=1}^{2} D_{mn,v} \\ \sum_{v=1}^{2} j k_{v} / \eta_{v} (A_{mn,2} - B_{mn,v}/k_{v}^{2}) \end{bmatrix}\right]
$$

$$
\cdot \begin{bmatrix} [I_{n}] \\ [M_{n}] \end{bmatrix} = \begin{bmatrix} [V_{m}^{E}] \\ [V_{m}^{H}] \end{bmatrix}.
$$
(55)

3.4. CFIE Formulation

In the CFIE formulation, the basis functions defined in (17) are used to expand both the electric current \underline{J} and the magnetic current \underline{M} as In the CFIE formulation, the basis functions defined in (17) are used
to expand both the electric current <u>J</u> and the magnetic current <u>M</u> as
in the PMCHW formulation and then we use $\underline{f}_m + \underline{g}_m$ as the testing to expand both the electric current J and the magnetic current M as
in the PMCHW formulation and then we use $\underline{f}_m + \underline{g}_m$ as the testing
functions to convert the CFIE into a matrix equation. Applying the
testing p in the PMCHW formulation and the functions to convert the CFIE into testing procedure to (16) , we get

$$
\langle \underline{f}_m + \underline{g}_m, -\underline{E}^s_v \rangle + \eta_1 \langle \underline{f}_m + \underline{g}_m, -\underline{H}^s_v \rangle
$$

=
$$
\begin{cases} \langle \underline{f}_m + \underline{g}_m, \underline{E}^i \rangle + \eta_1 \langle \underline{f}_m + \underline{g}_m, \underline{H}^i \rangle, & v = 1 \\ 0, & v = 2 \end{cases}
$$
 (56)

Another way to represent the set of four boundary integral equations in $(1)–(4)$ is the following [1]: $\hat{f}_1^s(\underline{J}, \underline{M}) = \hat{n} \times \underline{E}^i$

$$
-\hat{n} \times \underline{E}_1^s(\underline{J}, \underline{M}) = \hat{n} \times \underline{E}^i \tag{57}
$$

$$
-\hat{n} \times \underline{E}_1^s(\underline{J}, \underline{M}) = \hat{n} \times \underline{E}^i
$$
(57)
\n
$$
-\hat{n} \times \underline{H}_1^s(\underline{J}, \underline{M}) = \hat{n} \times \underline{H}^i
$$
(58)
\n
$$
-\hat{n} \times \underline{E}_2^s(\underline{J}, \underline{M}) = 0
$$
(59)
\n
$$
-\hat{n} \times \underline{H}_2^s(\underline{J}, \underline{M}) = 0.
$$
(60)

$$
-\hat{n} \times \underline{E_2^s(\underline{J}, M)} = 0 \tag{59}
$$

$$
-\hat{n} \times \underline{H}_2^s(\underline{J}, \underline{M}) = 0. \tag{60}
$$

 $-n \times \underline{E_2(\underline{J}, \underline{M})} = 0$ (39)
 $-\hat{n} \times \underline{H_2^s(\underline{J}, \underline{M})} = 0.$ (60)

Adding the set of equations (1)–(4) to the set of equations (57)–(60), respectively, we may obtain another CFIE similar to (16). By applying Adding the set of equations (1)–(4) to the set of equations (
respectively, we may obtain another CFIE similar to (16). By
the testing procedure with \underline{f}_m as testing functions, we get

$$
\langle \underline{f}_m, -\underline{E}^s_v - \hat{n} \times \underline{E}^s_v \rangle + \eta_1 \langle \underline{f}_m, -\underline{H}^s_v - \hat{n} \times \underline{H}^s_v \rangle
$$

=
$$
\begin{cases} \langle \underline{f}_m, \underline{E}^i + \hat{n} \times \underline{E}^i \rangle + \eta_1 \langle \underline{f}_m, \underline{H}^i + \hat{n} \times \underline{H}^i \rangle, & v = 1 \\ 0, & v = 2 \end{cases}
$$
(61)

By using the vector identity $\underline{A} \cdot \underline{B} \times \underline{C} = \underline{C} \cdot \underline{A} \times \underline{B}$, we obtain the following relationship. K
By using the vector iden
following relationship

following relationship
\n
$$
\langle \underline{f}_m, \hat{n} \times \underline{E} \rangle = \langle -\hat{n} \times \underline{f}_m, \underline{E} \rangle = \langle -\underline{g}_m, \underline{E} \rangle
$$
\n(62)
\nwhere E denotes the electric or the magnetic field. Using (62), we may

where \underline{E} denotes
write (61) as

$$
\langle \underline{f}_m - \underline{g}_m, -\underline{E}_v^s \rangle + \eta_1 \langle \underline{f}_m - \underline{g}_m, -\underline{H}_v^s \rangle
$$

=
$$
\begin{cases} \langle \underline{f}_m - \underline{g}_m, \underline{E}^i \rangle + \eta_1 \langle \underline{f}_m - \underline{g}_m, \underline{H}^i \rangle, & v = 1 \\ 0, & v = 2 \end{cases}
$$
 (63)

It is important to note that the testing function is $f_m + g_m$ in (56) and It is important to note that the testing function is $\underline{f}_m + \underline{g}_m$ in (56) and $\underline{f}_m - \underline{g}_m$ in (63) for the same CFIE. We also note that we can formulate It is important to note that the testing function is $\underline{f}_m + \underline{g}_m$ in (56) and
 $\underline{f}_m - \underline{g}_m$ in (63) for the same CFIE. We also note that we can formulate

by testing either EFIE or MFIE with $\underline{f}_m + \underline{g}_m$ or \u $\underline{f}_m - \underline{g}_m$ in (63) for the same CFIE. We also note that we can formulate
by testing either EFIE or MFIE with $\underline{f}_m + \underline{g}_m$ or $\underline{f}_m - \underline{g}_m$. This
different ways to obtain the CFIE formulation results in eight by testing either EFIE or MFIE with $\underline{f}_m + \underline{g}_m$ or $\underline{f}_m - \underline{g}_m$. This
different ways to obtain the CFIE formulation results in eight different
formulations by combining $\langle \underline{f}_m \pm \underline{g}_m, \underline{E} \rangle \pm \langle \underline{f}_m \pm \underline{g}_m$ by testing either EFIE or MFIE with $\underline{f}_m + \underline{g}_m$ or $\underline{f}_m - \underline{g}_m$. This
different ways to obtain the CFIE formulation results in eight different
formulations by combining $\langle \underline{f}_m \pm \underline{g}_m, \underline{E} \rangle \pm \langle \underline{f}_m \pm \underline{g}_m$ formulations by combining $\langle \underline{f}_m \pm \underline{g}_m, \underline{E} \rangle \pm$
we present a general expression for CFIE usi:
conjunction with the testing functions as

$$
\begin{aligned} &(1-\kappa)\langle f_E \underline{f}_m + g_E \underline{g}_m, -\underline{E}^s_v \rangle + \kappa \eta_1 \langle f_H \underline{f}_m + g_H \underline{g}_m, -\underline{H}^s_v \rangle \\ &= \begin{cases} &(1-\kappa)\langle f_E \underline{f}_m + g_E \underline{g}_m, \underline{E}^i \rangle + \kappa \eta_1 \langle f_H \underline{f}_m + g_H \underline{g}_m, \underline{H}^i \rangle, & v=1\\ & 0, & v=2 \end{cases} \end{aligned} \eqno{(64)}
$$

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where κ is the usual combination parameter which can have any value between 0 and 1. The testing coefficients f_E , g_E , f_H , and g_H may be where κ is the usual combination parameter which can have any value
between 0 and 1. The testing coefficients f_E , g_E , f_H , and g_H may be
+1 or −1. If $f_E = 1$, $g_E = 1$, $f_H = 1$, and $g_H = 1$, (64) is the same
as between 0 and 1. The testing coefficients f_E , g_E , f_H , and g_H may be +1 or -1. If $f_E = 1$, $g_E = 1$, $f_H = 1$, and $g_H = 1$, (64) is the same as (56). Equation (64) becomes (63) when $f_E = 1$, $g_E = -1$, $f_H = 1$, and g_H (64) becomes (63) when $f_E = 1$, $g_E = -1$, $f_H = 1$,
 $g_H = -1$.

To convert (64) into a matrix equation by using the above

meters to result in FFIF MEIF and CEIF formulations we

and $g_H = -1$.
To convert (64) into a matrix equation by using the above
parameters to result in EFIE, MFIE, and CFIE formulations, we To convert (64) into a matrix equation by using the above
parameters to result in EFIE, MFIE, and CFIE formulations, we
separate (64) into two categories, the electric field and the magnetic
field parts. First, we wri parameters to result in EFIE, MFIE, and CFIE formulations, we separate (64) into two categories, the electric field and the magnetic field parts. First, we write the equation related to the electric field only from (64) separate (64) into two
field parts. First, we
only from (64) as

nly from (64) as
\n
$$
\langle f_E \underline{f}_m + g_E \underline{g}_m, -\underline{E}_v^s(\underline{J}, \underline{M}) \rangle = \begin{cases} \langle f_E \underline{f}_m + g_E \underline{g}_m, \underline{E}^i \rangle, & v = 1 \\ 0, & v = 2 \end{cases} .
$$
\n(65)

This equation is termed as the TENE formulation in [6]. Equation (65)
is of the same form as (23) event the testing functions are different This equation is termed as the TENE formulation in $[6]$. Equation (65) is of the same form as (23) except the testing functions are different. This equation is termed as the TENE formulation in [6]. Eq
is of the same form as (23) except the testing functions ar
Thus, by using a similar procedure as in EFIE, we have

$$
\sum_{n=1}^{N} j k_{v} \eta_{v} \left(A_{mn,v}^{E} - \frac{B_{mn,v}^{E}}{k_{v}^{2}} \right) I_{n} + \sum_{n=1}^{N} (C_{mn,v}^{E} + D_{mn,v}^{E}) M_{n} = V_{m,v}^{E} \tag{66}
$$

where

$$
A_{mn,v}^E = f_E A_{mn,v}^f + g_E A_{mn,v}^g \tag{67}
$$

$$
B_{mn,v}^{E} = f_E B_{mn,v}^{f} + g_E B_{mn,v}^{g}
$$

\n
$$
C_{mn,v}^{E} = \begin{cases} +C_{mn}^{E}, & v = 1\\ \dots & \dots \end{cases}
$$
\n(68)

$$
C_{mn,v}^{E} = \begin{cases} +C_{mn}^{E}, & v = 1\\ -C_{mn}^{E}, & v = 2 \end{cases}
$$
(69)

$$
C_{mn}^E = f_E C_{mn}^f + g_E C_{mn}^g \tag{70}
$$

$$
D_{mn,v}^E = f_E D_{mn,v}^f + g_E D_{mn,v}^g \tag{71}
$$

$$
E_{mn,v}^{E} = f_E D_{mn,v}^{f} + g_E D_{mn,v}^{g}
$$

\n
$$
V_{m,v}^{E} = \begin{cases} \int_{S} (f_E f_m + g_E g_m) \cdot \underline{E}^i dS, & v = 1 \\ 0, & v = 2 \end{cases}
$$
 (71)

In (67)–(71), the elements having the superscript f are the inner In (67)–(71), the elements having the superscript 'f' are the inner
products with f_m and the elements having superscript 'g' are the inner In (67)–(71), the elements having the superscript
products with \underline{f}_m and the elements having superscript
products with \underline{g}_m . These elements are given by

$$
A_{mn,v}^f = \frac{1}{4\pi} \int_S \underline{f}_m(\underline{r}) \cdot \int_S \underline{f}_n(\underline{r}') G_v(\underline{r}, \underline{r}') dS' dS \tag{73}
$$

s of scattering from dielectric objects
\n
$$
A_{mn,v}^g = \frac{1}{4\pi} \int_S \underline{g}_m(\underline{r}) \cdot \int_S \underline{f}_n(\underline{r}') G_v(\underline{r}, \underline{r}') dS' dS
$$
\n(74)

$$
A_{mn,v}^s = \frac{1}{4\pi} \int_S \frac{g_m(\mathbf{r}) \cdot \int_S \mathbf{f}_n(\mathbf{r}) G_v(\mathbf{r}, \mathbf{r}) dS dS}{} \tag{74}
$$

$$
B_{mn,v}^f = \frac{1}{4\pi} \int_S \nabla_S \cdot \underline{f}_m(\mathbf{r}) \int_S \nabla_S' \cdot \underline{f}_n(\mathbf{r'}) G_v(\mathbf{r}, \mathbf{r'}) dS' dS \tag{75}
$$

$$
B_{mn,v}^g = \frac{1}{4\pi} \int_S \nabla S \cdot \underline{f}_m(\underline{r}) \int_S \nabla' S \cdot \underline{f}_n(\underline{r}) G_v(\underline{r}, \underline{r}) dS' dS \qquad (75)
$$

$$
B_{mn,v}^g = \frac{1}{4\pi} \int_S \underline{g}_m(\underline{r}) \cdot \int_S \nabla' S \cdot \underline{f}_n(\underline{r}') \nabla' G_v(\underline{r}, \underline{r}') dS' dS \qquad (76)
$$

$$
C_{mn}^f = \frac{1}{2} \int_S \underline{f}_m(\underline{r}) \cdot \hat{n} \times \underline{f}_n(\underline{r}) dS \tag{77}
$$

$$
C_{mn}^g = \frac{1}{2} \int_S \underline{g}_m(\underline{r}) \cdot \hat{n} \times \underline{f}_n(\underline{r}) dS \tag{78}
$$

$$
C_{mn}^{g} = \frac{1}{2} \int_{S} \underline{g}_{m}(\underline{r}) \cdot n \times \underline{f}_{n}(\underline{r}) dS \qquad (78)
$$

$$
D_{mn,v}^{f} = \frac{1}{4\pi} \int_{S} \underline{f}_{m} \cdot \int_{S} \underline{f}_{n} \nabla' G_{v}(\underline{r}, \underline{r}') dS' dS \qquad (79)
$$

$$
D_{mn,v}^g = \frac{1}{4\pi} \int_S \underline{f}_m \cdot \int_S \underline{f}_n \nabla G_v(\underline{r}, \underline{r}) dS dS \qquad (79)
$$

$$
D_{mn,v}^g = \frac{1}{4\pi} \int_S \underline{g}_m \cdot \int_S \underline{f}_n \nabla' G_v(\underline{r}, \underline{r}') dS' dS \qquad (80)
$$
Therefore, we can obtain a matrix equation for (66) as

Therefore, we can obtain a matrix equation for (66) as\n
$$
\begin{bmatrix}\n[jk_1\eta_1(A_{mn,1}^E - B_{mn,1}^E/k_1^2)] & [C_{mn,1}^E + D_{mn,1}^E] \\
[jk_2\eta_2(A_{mn,2}^E - B_{mn,2}^E/k_2^2)] & [C_{mn,2}^E + D_{mn,2}^E] \\
[N_k] & \end{bmatrix}\n\begin{bmatrix}\n[I_n] \\
[M_n]\n\end{bmatrix}\n=\n\begin{bmatrix}\n[V_{m,1}^E] \\
[V_{m,2}^E]\n\end{bmatrix}.
$$
\nNext, we write the equation corresponding to the magnetic field

Next, we write $\frac{1}{\pi}$ only from (64) as

only from (64) as
\n
$$
\langle f_H \underline{f}_m + g_H \underline{g}_m, -\underline{H}^s_v(\underline{J}, \underline{M}) \rangle = \begin{cases} \langle f_H \underline{f}_m + g_H \underline{g}_m, \underline{H}^i \rangle, & v = 1 \\ 0, & v = 2 \end{cases}
$$
 (82)

This equation is termed as the THNH formulation in [6]. Equation This equation is termed as the THNH formulation in [6]. Equation (82) is of the same form as (44) except for the testing functions. Thus, by using a similar precedure as in the MEIF, we have This equation is termed as the THNH formulation in $[(82)$ is of the same form as (44) except for the testing fundby using a similar procedure as in the MFIE, we have

$$
-\sum_{n=1}^{N} (C_{mn,v}^{H} + D_{mn,v}^{H}) I_n + \sum_{n=1}^{N} j \frac{k_v}{\eta_v} \left(A_{mn,v}^{H} - \frac{B_{mn,v}^{H}}{k_v^2} \right) M_n = V_{m,v}^{H} \tag{83}
$$

where

$$
A_{mn,v}^H = f_H A_{mn,v}^f + g_H A_{mn,v}^g \tag{84}
$$

$$
B_{mn,v}^H = f_H B_{mn,v}^f + g_H B_{mn,v}^g \tag{85}
$$

$$
B_{mn,v}^H = f_H B_{mn,v}^f + g_H B_{mn,v}^g
$$
\n
$$
C_{mn,v}^H = \begin{cases}\n+ C_{mn}^H, & v = 1 \\
- C_{mn}^H, & v = 2\n\end{cases}
$$
\n(85)

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$$
C_{mn}^H = f_H C_{mn}^f + g_H C_{mn}^g \tag{87}
$$

$$
D_{mn,v}^H = f_H D_{mn,v}^f + g_H D_{mn,v}^g \tag{88}
$$

$$
V_{mn,v}^{H} = f_H D_{mn,v}^{f} + g_H D_{mn,v}^{g}
$$
(88)

$$
V_{m,v}^{H} = \begin{cases} \int_{S} (f_H \underline{f}_m + g_H \underline{g}_m) \cdot \underline{H}^i dS, & v = 1 \\ 0, & v = 2 \end{cases}
$$
(89)

In (84)–(88), the elements with superscript superscript 'f' and 'g' are
same as those in (73)–(80) respectively. Note that we may obtain (83) In (84) – (88) , the elements with superscript superscript 'f' and 'g' are
same as those in (73) – (80) , respectively. Note that we may obtain (83)
directly from (66) by using duality. The matrix equation correspon In (84) – (88) , the elements with superscript superscript 'f' and 'g' are
same as those in (73) – (80) , respectively. Note that we may obtain (83)
directly from (66) by using duality. The matrix equation correspon same as those in (73) – (80) , respectively. Note that we may obtain (83) directly from (66) by using duality. The matrix equation corresponding to (83) is given by

to (83) is given by
\n
$$
\begin{bmatrix}\n[-(C_{mn,1}^H + D_{mn,1}^H)] \ [jk_1/\eta_1(A_{mn,1}^H - B_{mn,1}^H/k_1^2)] \\
[-(C_{mn,2}^H + D_{mn,2}^H)] \ [jk_2/\eta_2(A_{mn,2}^H - B_{mn,2}^H/k_2^2)]\n\end{bmatrix}\n\begin{bmatrix}\n[I_n] \\
[N_n]\n\end{bmatrix}\n=\n\begin{bmatrix}\n[V_{m,1}^H] \\
[V_{m,2}^H]\n\end{bmatrix}.
$$
\n(90)
\nWe may rewrite the matrix equations (81) and (90), respectively, as

ix equations (81) and (90), respectively, as
\n
$$
[Z_{mn}^E][C_n] = [V_m^E]
$$
\n
$$
[Z_{mn}^H][C_n] = [V_m^H]
$$
\n(92)

$$
[Z_{mn}^H][C_n] = [V_m^H] \tag{92}
$$

 $[Z_{mn}^H][C_n] = [V_m^H]$ (92)
where $C_n = I_n$ and $C_{(N+n)} = M_n$ for $n = 1, 2, ..., N$. Finally, by
combining TENE and THNH given in (91) and (92) respectively we where $C_n = I_n$ and $C_{(N+n)} = M_n$ for $n = 1, 2, ..., N$. Finally, by
combining TENE and THNH given in (91) and (92), respectively, we
have a metrix equation associated with (64) as where $C_n = I_n$ and $C_{(N+n)} = M_n$ for $n = 1, 2$,
combining TENE and THNH given in (91) and (9
have a matrix equation associated with (64) as sociated with (64) as
 $[Z_{mn}][C_n] = [V_m]$ (93)

$$
[Z_{mn}][C_n] = [V_m] \tag{93}
$$

 $[Z_{mn}][C_n] = [V_m] \label{eq:zmn}$ where the matrix elements are given by

$$
Z_{mn} = (1 - \kappa)Z_{mn}^{E} + \kappa \eta_1 Z_{mn}^{H}
$$
\n
$$
V_m = (1 - \kappa)V_m^{E} + \kappa \eta_1 V_m^{H}
$$
\n(94)\n
$$
(95)
$$

$$
V_m = (1 - \kappa)V_m^E + \kappa \eta_1 V_m^H \tag{95}
$$

for $m = 1, 2, \ldots, 2N$ and $n = 1, 2, \ldots, 2N$.

4. STUDY OF THE VARIOUS FORMULATIONS

4. STUDY OF THE VARIOUS FORMULATIONS
In this section, we study the general CFIE formulation described by
 (64) , which results in the matrix equation (03) . As discussed in In this section, we study the general CFIE formulation described by (64) , which results in the matrix equation (93) . As discussed in the above section we have sight different formulations for a CEIE In this section, we study the general CFIE formulation described by (64) , which results in the matrix equation (93) . As discussed in the above section, we have eight different formulations for a CFIE formulation with (64) , which results in the matrix equation (93) . As discussed in the above section, we have eight different formulations for a CFIE formulation with different testing coefficients. They are summarized along with the a the above section, we have eight different formulations for a CFIE formulation with different testing coefficients. They are summarized along with the appropriate testing coefficients in Table 2. To obtain numerical result formulation with different testing coefficients. They are summarized of 1m and a relative permittivity $\varepsilon_r = 2$, centered at the origin, as

Figure 2. Triangle surface patching of a dielectric sphere (radius 0.5 m). θ - and ϕ -directed arrows represent position and direction of sampled guypotte *L* and *M*, respectively. **Figure 2.** Triangle surface patching of 0.5 m). θ - and ϕ -directed arrows represents sampled currents <u>J</u> and <u>M</u>, respectively.

Table 2. Eight CFIE formulations with the different combination
of testing eoefficients and the averaged difference of monostatic RCS Table 2. Eight CFIE formulations with the different combination
of testing coefficients and the averaged difference of monostatic RCS
between Mie and CEIE solution for the dielectric sphere in Fig. 2 Table 2. Eight CFIE formulations with the different combination of testing coefficients and the averaged difference of monostatic RCS between Mie and CFIE solution for the dielectric sphere in Fig. 2. between Mie and CFIE solution for the dielectric sphere in Fig. 2.

Formulation			Testing coefficients	$\Delta\sigma$ (dBm ²)	
	ĴΕ	g_E	fн	g_H	
CFIE-1				1	1.02
$CFIE-2$				-1	4.61
CFIE-3			-1		0.40
$CFIE-4$	1		-1	-1	0.73
$CFIE-5$		-1			0.41
CFIE-6		-1			0.85
CFIE-7		-1	-1		1.10
CFIE-8					$4.61\,$

shown in Fig. 2. There are twelve and twenty-four divisions along θ shown in Fig. 2. There are twelve and twenty-four divisions along θ and ϕ directions, respectively. This results in a sphere discretized by 528 patches and 702 edges. In the numerical solution the sphere is shown in Fig. 2. There are twelve and twenty-four divisions along θ and ϕ directions, respectively. This results in a sphere discretized by 528 patches and 792 edges. In the numerical calculation, the sphere is illu and ϕ directions, respectively. This results in a sphere discretized by 528 patches and 792 edges. In the numerical calculation, the sphere is illuminated from the top by an incident x-polarized plane wave with 528 patches and 792 edges. In the numerical calculation, the sphere is illuminated from the top by an incident x-polarized plane wave with the propagation vector $\hat{k} = -\hat{z}$. The analysis is to be done over a illuminated from the top by an incident x-polarized plane wave with
the propagation vector $\hat{k} = -\hat{z}$. The analysis is to be done over a
frequency range of $0 < f \le 400$ MHz at an interval of 4 MHz with 100
samples. Nume the propagation vector $\hat{k} = -\hat{z}$. The analysis is to be done over a frequency range of $0 < f \le 400$ MHz at an interval of 4 MHz with 100 samples. Numerical results are compared with the Mie series solution

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Figure 3. Monostatic RCS of the dielectric
by eight CFIE formulations in Table 2. by eight CFIE formulations in Table 2.
between 100–400 MHz.

Fig. 3 represents the monostatic RCS of the sphere obtained for between 100–400 MHz.
Fig. 3 represents the monostatic RCS of the sphere obtained for
all the eight CFIE formulations described in Table 2. We used $\kappa = 0.5$.
As ovident from the figures, only two CEIE formulations, i.e., Fig. 3 represents the monostatic RCS of the sphere obtained for all the eight CFIE formulations described in Table 2. We used $\kappa = 0.5$.
As evident from the figures, only two CFIE formulations, i.e., CFIE-
2 and CEIE 5, all the eight CFIE formulations described in Table 2. We used $\kappa = 0.5$.
As evident from the figures, only two CFIE formulations, i.e., CFIE-
3 and CFIE-5, compare well with the Mie solution. Other solutions
broak down a As evident from the figures, only two CFIE formulations, i.e., CFIE-
3 and CFIE-5, compare well with the Mie solution. Other solutions
break down as the interior resonance problem manifests itself or they
interior discree 3 and CFIE-5, compare well with the Mie solution. Other solutions
break down as the interior resonance problem manifests itself or they
just disagree with the Mie solution. It is interesting to note that
CEIE 1 and CEIE 4 break down as the interior resonance problem manifests itself or they
just disagree with the Mie solution. It is interesting to note that
CFIE-1 and CFIE-4 in Fig. 3(a) and CFIE-6 and CFIE-7 in Fig. 3(b)
show a good agree just disagree with the Mie solution. It is interesting to note that
CFIE-1 and CFIE-4 in Fig. 3(a) and CFIE-6 and CFIE-7 in Fig. 3(b)
show a good agreement with the exact solution except near resonant
frequencies of 362 a CFIE-1 and CFIE-4 in Fig. 3(a) and CFIE-6 and CFIE-7 in Fig. 3(b)
show a good agreement with the exact solution except near resonant
frequencies of 262 and 369 MHz. Table 2 also shows the averaged
difference between numer show a good agreement with the exact solution except near resonant
frequencies of 262 and 369 MHz . Table 2 also shows the averaged
difference between numerical and Mie solution for the monostatic RCS
of the sphane. T frequencies of 262 and 369 MHz. Table 2 also shows the averaged
difference between numerical and Mie solution for the monostatic RCS
of the sphere. The averaged difference of monostatic RCS is computed
by using the definit difference between numeric
of the sphere. The average
by using the definition

$$
\Delta \sigma = \frac{\sum_{l}^{M} |\sigma_{\text{Mie}} - \sigma_{\text{nume}}|}{M}
$$
\n(96)

 $\Delta \sigma = \frac{\sum |\sigma_{\text{Mile}}|}{M}$ (96)
where σ denotes the RCS and *M* is the number of samples, which is
100 in this asse where σ denotes t
100 in this case.
There are four 100 in this case.
There are four terms when one combines the TENE and THNH to

form a CFIE, i.e., CFIE = E FIE + $\hat{n} \times E$ FIE + MFIE + $\hat{n} \times M$ FIE with There are four terms when one combines the TENE and THNH to
form a CFIE, i.e., CFIE = EFIE + $\hat{n} \times$ EFIE + MFIE + $\hat{n} \times$ MFIE with
 f_m as the testing functions or CFIE = EFIE + MFIE with $f_m + g_m$ as form a CFIE, i.e., CFIE = EFIE + $\hat{n} \times$ EFIE + MFIE + $\hat{n} \times$ MFIE with \underline{f}_m as the testing functions or CFIE = EFIE + MFIE with $\underline{f}_m + \underline{g}_m$ as the testing functions. It was suggested to drop one of these ter f_m as the testing functions or CFIE = EFIE + MFIE with $f_m + g_m$ as
the testing functions. It was suggested to drop one of these terms [6].
These formulations are named as TENE-TH, TENE-NH, TE-THNH,
and NE THNH depending the testing functions. It was suggested to drop one of these terms [6].
These formulations are named as TENE-TH, TENE-NH, TE-THNH,
and NE-THNH, depending on which term is neglected. Applying this

Analysis of scattering from dielectric objects and the state of the different combination of testing coefficients and the sygmered difference of normalized for field and Table 3. CFIE formulations with the different combination of testing coefficients and the averaged difference of normalized far field and monostatic BCS between Mig and CEIE solution for the dialectric Table 3. CFIE formulations with the different combination of testing
coefficients and the averaged difference of normalized far field and
monostatic RCS between Mie and CFIE solution for the dielectric
sphare in Fig. 2. coefficients and the averaged difference of normalized far field and monostatic RCS between Mie and CFIE solution for the dielectric sphere in Fig. 2.

ohere in Fig. 2.								
Formulation		Testing coefficients				Δe_{θ}	Δe_ϕ	$\Delta \sigma$
		f_E	g_E	fн	g_H	(mV)	(mV)	(dBm ²)
	$\left(1\right)$	$\mathbf{1}$	1	1	θ	12.0	16.7	0.79
TENE-TH	$\left(2\right)$	1	$\mathbf{1}$	-1	Ω	4.1	6.4	0.30
	$\left(3\right)$	$\mathbf{1}$	$^{-1}$	1	θ	4.7	6.0	0.34
	$\left(4\right)$	$\mathbf{1}$	-1	$^{-1}$	θ	4.5	24.0	0.51
	$\left(1\right)$	1	$\mathbf{1}$	$\overline{0}$	$\mathbf{1}$	3.5	12.9	0.28
TENE-NH	$\left(2\right)$	1	$\mathbf{1}$	$\overline{0}$	$^{-1}$	7.8	32.1	0.22
	$\left(3\right)$	$\mathbf{1}$	-1	$\overline{0}$	$\mathbf{1}$	2.4	20.3	0.37
	$\left(4\right)$	$\mathbf{1}$	-1	$\overline{0}$	-1	8.2	21.4	0.48
TE-THNH	$\left(1\right)$	$\mathbf{1}$	$\overline{0}$	$\mathbf{1}$	$\mathbf{1}$	9.4	5.5	0.61
	$\left(2\right)$	$\mathbf{1}$	$\overline{0}$	$\mathbf{1}$	-1	8.1	9.8	0.59
	$\left(3\right)$	$\mathbf{1}$	$\overline{0}$	-1	1	8.7	7.2	0.55
	(4)	1	$\overline{0}$	$^{-1}$	$^{-1}$	6.9	16.4	0.49
NE-THNH	$\left(1\right)$	$\overline{0}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	5.1	12.9	0.42
	$\left(2\right)$	θ	$\mathbf{1}$	$\mathbf{1}$	-1	5.7	16.4	0.48
	$\left(3\right)$	$\overline{0}$	$\mathbf{1}$	$^{-1}$	$\mathbf{1}$	5.7	21.5	0.52
	$\left(4\right)$	θ	$\mathbf{1}$	-1	-1	5.3	15.6	0.45

scheme to the eight different CFIE formulations of Table 2, we may
have sixteen pessible ages of CEIE with different testing sofficients scheme to the eight different CFIE formulations of Table 2, we may
have sixteen possible cases of CFIE with different testing coefficients,
which are summarized in Table 2. The four figures in Fig. 4 show the scheme to the eight different CFIE formulations of Table 2, we may
have sixteen possible cases of CFIE with different testing coefficients,
which are summarized in Table 3. The four figures in Fig. 4 show the
monostatic PC have sixteen possible cases of CFIE with different testing coefficients, which are summarized in Table 3. The four figures in Fig. 4 show the monostatic RCS of the sphere displayed in Fig. 2. For comparison, we also presen which are summarized in Table 3. The four figures in Fig. 4 show the monostatic RCS of the sphere displayed in Fig. 2. For comparison, we also present the Mie solution, which is represented by the solid gurup, and all the monostatic RCS of the sphere displayed in Fig. 2. For comparison, we also present the Mie solution, which is represented by the solid curve, and all the results compare well. We note that the resonance problem is not obse curve, and all the results compare well. We note that the resonance
problem is not observed in any one of the sixteen CFIE results. It curve, and all the results compare well. We note that the resonance
problem is not observed in any one of the sixteen CFIE results. It
should be noted that only two formulations of the CFIE in Table 2,
for which any of the problem is not observed in any one of the sixteen CFIE results. It
should be noted that only two formulations of the CFIE in Table 2,
for which any of the testing coefficient is not zero, give valid solutions,
but in all t should be noted that only two formulations of the CFIE in Table 2, for which any of the testing coefficient is not zero, give valid solutions, but in all the sixteen formulations one of the terms may be dropped as Table 3 for which any of the testing coefficient is not zero, give valid solutions,
but in all the sixteen formulations one of the terms may be dropped as
Table 3 without breaking down. There is a small difference between
the syn but in all the sixteen formulations one of the terms may be dropped as
Table 3 without breaking down. There is a small difference between
the exact solution in the high frequency region. We also present the
sygmetric diffe Table 3 without breaking down. There is a small difference between
the exact solution in the high frequency region. We also present the
averaged difference in the far field and in the monostatic RCS between

²¹² Jung, Sarkar, and Chung

Figure 4. Monostatic RCS of the dielectric sphere by the sixteen CFIE formulations of Table 3.

by the sixteen CFTE formulations of Table 3.
the numerical and the Mie solution, which are summarized in Table 3.
The systemed difference between the for field is also semputed using the numerical and the Mie solution, which are summarized in Table 3.
The averaged difference between the far field is also computed using
the definition in (06) . The smallest averaged difference in the PGS The averaged difference between the far field is also computed using
the definition in (96). The smallest averaged difference in the RCS The averaged difference between the far field is also computed using
the definition in (96). The smallest averaged difference in the RCS
is 0.22 dBm² for the TENE-NH (2) case. Comparing the differences
in the for field, the definition in (96). The smallest averaged difference in the RCS
is 0.22 dBm² for the TENE-NH (2) case. Comparing the differences
in the far field, the results of TENE-TH (2) are most accurate for
both θ and ϕ is 0.22 dBm² for the TENE-NH (2) case. Comparing the differences
in the far field, the results of TENE-TH (2) are most accurate for
both θ - and ϕ -component in the far field. Fig. 5 compares the far
field for TENE in the far field, the results of TENE-TH (2) are most accurate for both θ - and ϕ -component in the far field. Fig. 5 compares the far field for TENE-TH (2) and TENE-NH (2) with the Mie solution. It is clearly seen th field for TENE-TH (2) and TENE-NH (2) with the Mie solution. It results than the TENE-NH (2) in this case. Numerical results using CFIE in the next section are presented using the testing coefficients as

sphere in Fig. 2 computed by Mie and CFIE solution. (a) TENE-**Figure 5.** Comparison of the normalized far field for the dielectric sphere in Fig. 2 computed by Mie and CFIE solution. (a) TENE-TH (2) $(f_E = 1, g_E = 1, f_H = -1, g_H = 0)$. (b) TENE-NH (2) $(f_E = 1, g_E = 1, f_H = 0, g_H = -1)$ sphere in Fig. 2 computed by Mie an
TH (2) $(f_E = 1, g_E = 1, f_H = -1,$
 $(f_E = 1, g_E = 1, f_H = 0, g_H = -1).$

 $f_E = 1$, $g_E = 1$, $f_H = -1$, and $g_H = 1$ for TENE-THNH, which is
CFIE-3 in Table 2 and includes TENE-TH (2) $f_E = 1$, $g_E = 1$, $f_H = -1$, and $g_H = 1$ for TE.
CFIE-3 in Table 2 and includes TENE-TH (2). = 1, $g_E = 1$, $f_H = -1$, and $g_H = 1$ for TENE-THNH, which is
E-3 in Table 2 and includes TENE-TH (2).
Now, we investigate the effect of the combination parameter κ in
Eig 6 compares the solutions of TENE TH (2) with the

CFIE-3 in Table 2 and includes TENE-TH (2).
Now, we investigate the effect of the combination parameter κ in
(64). Fig. 6 compares the solutions of TENE-TH (2) with the Mie Now, we investigate the effect of the combination parameter κ in (64). Fig. 6 compares the solutions of TENE-TH (2) with the Mie solution as we vary κ from 0.3 to 0.7 at an interval of 0.1. All the (64). Fig. 6 compares the solutions of TENE-TH (2) with the Mie
solution as we vary κ from 0.3 to 0.7 at an interval of 0.1. All the
results agree well with the exact solution. Table 4 summarizes the
sygmetric differe solution as we vary κ from 0.3 to 0.7 at an interval of 0.1. All the results agree well with the exact solution. Table 4 summarizes the averaged difference in the far field and monostatic RCS. It is evident from Eig 6 results agree well with the exact solution. Table 4 summarizes the averaged difference in the far field and monostatic RCS. It is evident from Fig. 6 and Table 4 that the solution of CFIE is not so sensitive to μ . The averaged difference in the far field and monostatic RCS. It is evident
from Fig. 6 and Table 4 that the solution of CFIE is not so sensitive to
 κ . The choice of κ can be selected within a wide range. For the resul κ . The choice of κ can be selected within a wide range. For the results

5. NUMERICAL EXAMPLES

5. NUMERICAL EXAMPLES
In this section, we present and compare the numerical results obtained
from top different formulations. These are EEIE MEIE PMCHW, and In this section, we present and compare the numerical results obtained
from ten different formulations. These are EFIE, MFIE, PMCHW, and
sayon CEIE formulations that are described in Toble 5. In this work In this section, we present and compare the numerical results obtained
from ten different formulations. These are EFIE, MFIE, PMCHW, and
seven CFIE formulations that are described in Table 5. In this work,
given though TEN from ten different formulations. These are EFIE, MFIE, PMCHW, and
seven CFIE formulations that are described in Table 5. In this work,
even though TENE or THNH consists of only electric or magnetic field, seven CFIE formulations that are described in Table 5. In this work,
even though TENE or THNH consists of only electric or magnetic field,
respectively, we consider this as a special case of CFIE with $\kappa = 0$ or
 $\kappa = 1$ even though TENE or THNH consists of only electric or magnetic field,
respectively, we consider this as a special case of CFIE with $\kappa = 0$ or
 $\kappa = 1$ to differentiate from EFIE or MFIE described in Section 3. The
numer respectively, we consider this as a special case of CFIE with $\kappa = 0$ or $\kappa = 1$ to differentiate from EFIE or MFIE described in Section 3. The numerical results are obtained for representative 3-D scatterers with $\kappa = 1$ to differentiate from EFIE or MFIE described in Section 3. The numerical results are obtained for representative 3-D scatterers with a relative permittivity $\varepsilon_r = 2$, viz. a sphere, a cube, and a cylinder. numerical results are obtained for representative 3-D scatterers with
a relative permittivity $\varepsilon_r = 2$, viz. a sphere, a cube, and a cylinder.
In the numerical calculation, the scatterers are illuminated from the
top by a relative permittivity $\varepsilon_r = 2$, viz. a sphere, a cube, and a cylinder.
In the numerical calculation, the scatterers are illuminated from the
top by an incident x-polarized plane wave with a propagation vector $k = -\hat{z}$ as used in the above section. The frequency range over which

Table 4. Averaged difference of the normalized far field and monostatic RCS for the dielectric sphere in Fig. 2 computed using **Table 4.** Averaged difference of the normalized far field and monostatic RCS for the dielectric sphere in Fig. 2 computed using TENE-TH (2) ($f_E = 1$, $g_E = 1$, $f_H = -1$, $g_H = 0$) formulation as varying parameter κ . monostatic RCS for the TENE-TH (2) $(f_E = 1)$
varying parameter κ .

κ	Δe_{θ} (mV)	Δe_{ϕ} (mV)	$\Delta\sigma$ (dBm ²)
0.3	6.0	5.0	0.41
0.4	4.8	5.7	0.34
0.5	4.1	6.4	0.30
0.6	4.0	6.8	0.29
$0.7\,$	4.5	6.9	0.31

Figure 6. Monostatic RCS of the dielectric sphere in Fig. 2 computed by TENE-TH (2) formulation ($f_E = 1$, $g_E = 1$, $f_H = -1$, $g_H = 0$) as varying CFIE combining parameter κ . **Figure 6.** Monostatic RCS of the dieler by TENE-TH (2) formulation $(f_E = 1$ varying CFIE combining parameter κ .

Formulation	ŤΕ	ЯE	ŤН	ЯH
TENE				
THNH				
TENE-THNH		1		
TENE-TH		1		
TENE-NH				
TE-THNH				
NE-THNH		1		

Table 5. Seven CFIE formulations using different testing coefficients.

the results are calculated is $0 < f \leq 400$ MHz at an interval of 4 MHz. the results are calculated is $0 < f \le 400$ MHz at an interval of 4 MHz.
We compute the equivalent currents, the far field, and the monostatic
RCS, We shoose $f = 0.5$ when CEIF is used. We sempare all the the results are calculated is $0 < f \le 400$ MHz at an interval of 4 MHz.
We compute the equivalent currents, the far field, and the monostatic
RCS. We choose $\kappa = 0.5$ when CFIE is used. We compare all the
computed equival We compute the equivalent currents, the far field, and the monostatic RCS. We choose $\kappa = 0.5$ when CFIE is used. We compare all the computed equivalent currents with those obtained from the PMCHW formulation. Also, we c RCS. We choose $\kappa = 0.5$ when CFIE is used. We compare all the computed equivalent currents with those obtained from the PMCHW formulation. Also, we compare the computed far fields and RCS with computed equivalent currents with those obtained from the PMCHW
formulation. Also, we compare the computed far fields and RCS with
the Mie solution for a sphere and WIPL-D [10] solution for a cube
and a sulinder. Mie and W formulation. Also, we compare the computed far fields and RCS with
the Mie solution for a sphere and WIPL-D [10] solution for a cube
and a cylinder. Mie and WIPL-D solutions are obtained at the same
interval of $4\,\text{MHz}$ the Mie solution fo
and a cylinder. Mie
interval of 4 MHz . a cylinder. Mie and WIPL-D solutions are obtained at the same
val of 4 MHz .
As a first example, we consider the dielectric sphere of Fig. 2 used
a shows section. The θ directed electric surront and the ϕ directed

interval of 4 MHz.
As a first example, we consider the dielectric sphere of Fig. 2 used
in the above section. The θ-directed electric current and the φ-directed
magnetic current, as indicated by arrows in Fig. 2, are obs As a first example, we consider the dielectric sphere of Fig. 2 used
in the above section. The θ -directed electric current and the ϕ -directed
magnetic current, as indicated by arrows in Fig. 2, are observed. Fig. 7

rigure s. EFTE results for a dielectric sphere.
shows the results for the PMCHW. The far field and RCS agree well
with Mic solution over for a small difference in the high fracuator. shows the results for the PMCHW. The far field and RCS agree well
with Mie solution except for a small difference in the high frequency
region. The sumpris are used to sempore with any other numerical shows the results for the PMCHW. The far field and RCS agree well
with Mie solution except for a small difference in the high frequency
region. The currents are used to compare with any other numerical
results and not sho with Mie solution except for a small difference in the high frequency
region. The currents are used to compare with any other numerical
results and not shown here. Fig. 8 and Fig. 9 show the results of EFIE
and MEIE shtei region. The currents are used to compare with any other numerical
results and not shown here. Fig. 8 and Fig. 9 show the results of EFIE
and MFIE obtained by using (41) and (47), respectively. Because
the bosis function fo results and not shown here. Fig. 8 and Fig. 9 show the results of EFIE
and MFIE obtained by using (41) and (47), respectively. Because
the basis function for the magnetic current in the EFIE is different
from that in PMCH and MFIE obtained by using (41) and (47) , respectively. Because
the basis function for the magnetic current in the EFIE is different
from that in PMCHW, we cannot compare together them in Fig. 8(b).
Similarly, the bas the basis function for the magnetic current in the EFIE is different
from that in PMCHW, we cannot compare together them in Fig. 8(b).
Similarly, the basis function for the electric current in the MFIE is from that in PMCHW, we cannot compare together them in Fig. $8(b)$.
Similarly, the basis function for the electric current in the MFIE is
different from that in the PMCHW, hence we do not compare them in
Fig. 9(a). The ele Similarly, the basis function for the electric current in the MFIE is
different from that in the PMCHW, hence we do not compare them in
Fig. 9(a). The electric current in Fig. 8(a) and the magnetic current
in Fig. 9(b) ex different from that in the PMCHW, hence we do not compare them in Fig. 9(a). The electric current in Fig. 8(a) and the magnetic current in Fig. 9(b) agree well with those of PMCHW except at the resonant frequencies. It is in Fig. $9(b)$ agree well with those of PMCHW except at the resonant frequencies. It is clearly seen that there are peaks and discontinuities

rigure 9. MPTE results for a dielectric sphere.
near the resonant frequencies of 262 and 369 MHz in the far field and
RCS results near the reson
RCS results.
Figures 1 the resonant frequencies of 262 and 369 MHz in the far field and
results.
Figures 10 and 11 show the numerical results for TENE and
 H_{H} respectively. As discussed explicit in Fig. 8 and Fig. 10 the

RCS results.
Figures 10 and 11 show the numerical results for TENE and
THNH, respectively. As discussed earlier, in Fig. 8 and Fig. 10 the
results are presented using only the electric field and in Fig. 0 and Figures 10 and 11 show the numerical results for TENE and THNH, respectively. As discussed earlier, in Fig. 8 and Fig. 10 the results are presented using only the electric field and in Fig. 9 and Fig. 11 using only the meg THNH, respectively. As discussed earlier, in Fig. 8 and Fig. 10 the results are presented using only the electric field and in Fig. 9 and Fig. 11 using only the magnetic field. For this reason, peaks at the reconnect freq results are presented using only the electric field and in Fig. 9 and Fig. 11 using only the magnetic field. For this reason, peaks at the resonant frequencies are observed in the figures. It is interesting to note that th Fig. 11 using only the magnetic field. For this reason, peaks at the resonant frequencies are observed in the figures. It is interesting to note that the electric current in the EFIE formulation and the magnetic current i resonant frequencies are observed in the figures. It is interesting
to note that the electric current in the EFIE formulation and the
magnetic current in the MFIE formulation agree better with the
 $PMCHW$ solution than thes magnetic current in the MFIE formulation agree better with the PMCHW solution than those of the TENE and THNH, even though magnetic current in the MFIE formulation agree better with the
PMCHW solution than those of the TENE and THNH, even though
 g_m is used as expansion function for the magnetic current in EFIE PMCHW solution than those of the TENE and THNH, even though g_m is used as expansion function for the magnetic current in EFIE and for the electric current in MFIE, which violates the property of

Figure 10. TENE re
1, $f_H = 0$, $g_H = 0$).

1, $f_H = 0$, $g_H = 0$).
each current. Fig. 12 gives the results computed by TENE-THNH,
combining TENE and THNH formulation. In this CEIE, the reconnect each current. Fig. 12 gives the results computed by TENE-THNH,
combining TENE and THNH formulation. In this CFIE, the resonant each current. Fig. 12 gives the results computed by TENE-THNH,
combining TENE and THNH formulation. In this CFIE, the resonant
peaks or discontinuity in the plots are not seen. All the four figures
in Fig. 12 shock well wi combining TENE and THNH formulation. In this CFIE, the resonant
peaks or discontinuity in the plots are not seen. All the four figures
in Fig. 12 check well with the PMCHW and Mie solution for the
conjugation currents, th peaks or discontinuity in the plots are not seen. All the four figures
in Fig. 12 check well with the PMCHW and Mie solution for the
equivalent currents, the far fields, and RCS. The results of CFIE
formulation when it per in Fig. 12 check well with the PMCHW and Mie solution for the equivalent currents, the far fields, and RCS. The results of CFIE formulation when it neglects one term as discussed before are shown from Fig. 13 to Fig. 16. A equivalent currents, the far fields, and RCS. The results of CFIE
formulation when it neglects one term as discussed before are shown
from Fig. 13 to Fig. 16. All numerical results do not show the resonant
problem and agre formulation when it neglects one term as discussed before are shown
from Fig. 13 to Fig. 16. All numerical results do not show the resonant
problem and agree well with the PMCHW solution for the currents and
Mis solution f from Fig. 13 to Fig. 16. All numerical results do not show the resonant
problem and agree well with the PMCHW solution for the currents and
Mie solution for the far field and RCS exhibiting a small difference. But
the resu problem and agree well with the PMCHW solution for the currents and
Mie solution for the far field and RCS exhibiting a small difference. But
the results of TENE-TH are most accurate among them for both the
far field and t Mie solution for the far field and RCS exhibiting a small difference. But the results of TENE-TH are most accurate among them for both the far field and the RCS.

Figure 11. THNH res 0, $f_H = -1$, $g_H = 1$).

Since the exact solution of the far field or RCS for a cube and a
day are not known, we use the sode WIDI, D as a reference for Since the exact solution of the far field or RCS for a cube and a cylinder are not known, we use the code WIPL-D as a reference for comparison. To shock the validity of this solution, we need to compare Since the exact solution of the far field or RCS for a cube and a cylinder are not known, we use the code WIPL-D as a reference for comparison. To check the validity of this solution, we need to compare the solution from W cylinder are not known, we use the code WIPL-D as a reference for comparison. To check the validity of this solution, we need to compare the solution from WIPL-D with the exact solution. Fig. 17 compares the WIPL D and the comparison. To check the validity of this solution, we need to compare
the solution from WIPL-D with the exact solution. Fig. 17 compares
the WIPL-D and the Mie solutions for a sphere with a good agreement.
The number of the solution from WIPL-D with the exact solution. Fig. 17 compares
the WIPL-D and the Mie solutions for a sphere with a good agreement.
The number of unknowns is 2,400 in the computation using WIPL-D. Table 6 summarizes the averaged difference of the normalized far field The number of unknowns is 2,400 in the computation using WIPL-D.
Table 6 summarizes the averaged difference of the normalized far field
and monostatic RCS for the formulations which do not exhibit the
research problem. Fr Table 6 summarizes the averaged difference of the normalized far field
and monostatic RCS for the formulations which do not exhibit the
resonant problem. From Table 6, we find that the TENE-NH has the
smallest difference and monostatic RCS for the formulations which do not exhibit the resonant problem. From Table 6, we find that the TENE-NH has the smallest difference in the θ -component of the far field, but the difference in the ϕ resonant problem. From Table 6, we find that the TENE-NH has the smallest difference in the θ -component of the far field, but the difference in the ϕ -component of the far field is relatively large.

Figure 12. TENE-THN:
1, $f_H = -1$, $g_H = 1$). 1, $f_H = -1$, $g_H = 1$).
As a second example, we consider a dielectric cube, 1 m on a side,

As a second example, we consider a dielectric cube, 1 m on a side, centered about the origin shown in Fig. 18. There are eight divisions along each direction, respectively. This results in a total of 768 patches. As a second example, we consider a dielectric cube, 1 m on a side,
centered about the origin shown in Fig. 18. There are eight divisions
along each direction, respectively. This results in a total of 768 patches
and 1.15 centered about the origin shown in Fig. 18. There are eight divisions
along each direction, respectively. This results in a total of 768 patches
and 1,152 edges. The z-directed electric current and the y-directed
magnetic along each direction, respectively. This results in a total of 768 patches and $1,152$ edges. The *z*-directed electric current and the *y*-directed magnetic current, as indicated by arrows on a side face of the cube in and $\overline{1,152}$ edges. The z-directed electric current and the y-directed magnetic current, as indicated by arrows on a side face of the cube in Fig. 18, are observed. The computed currents are compared with those of the magnetic current, as indicated by arrows on a side face of the cube in
Fig. 18, are observed. The computed currents are compared with those
of the PMCHW formulation and the far fields and the monostatic RCS
are compared wi Fig. 18, are observed. The computed currents are compared with those
of the PMCHW formulation and the far fields and the monostatic RCS
are compared with the WIPL-D solutions. The number of unknowns
is 2.400 in the comput of the PMCHW formulation and the far fields
are compared with the WIPL-D solutions. T
is 2,400 in the computation using WIPL-D.
Eig 10 shows the results of PMCHW Frace is the Highlands of PMCHD.

Fig. 19 shows the results of PMCHW. The far field and the

Fig. 19 shows the results of PMCHW. The far field and the

some well with WIPL D solutions in the entire frequency position

is 2,400 in the computation using WIPL-D.
Fig. 19 shows the results of PMCHW. The far field and the
RCS agree well with WIPL-D solutions in the entire frequency region.

Figure 13. TENE-TH 1
1, $f_H = -1$, $g_H = 0$).

Figures 20 and 21 show the results for EFIE and MFIE, respectively.
Figures 20 and 21 show the results for EFIE and MFIE, respectively. Figures 20 and 21 show the results for EFIE and MFIE, respectively.
The basis function of the magnetic currents in the EFIE is different
from that in PMCHW, and so we cannot compare them in Fig. 20(b). Figures 20 and 21 show the results for EFIE and MFIE, respectively.
The basis function of the magnetic currents in the EFIE is different
from that in PMCHW, and so we cannot compare them in Fig. 20(b).
Similarly, the basis The basis function of the magnetic currents in the EFIE is different
from that in PMCHW, and so we cannot compare them in Fig. 20(b).
Similarly, the basis function of the electric currents in the MFIE is from that in PMCHW, and so we cannot compare them in Fig. 20(b).
Similarly, the basis function of the electric currents in the MFIE is
different from that in PMCHW, and therefore we do not compare them
in Fig. 21(c). The Similarly, the basis function of the electric currents in the MFIE is
different from that in PMCHW, and therefore we do not compare them
in Fig. 21(a). The electric current in Fig. 20(a) and the magnetic
gunnari in Fig. 2 different from that in PMCHW, and therefore we do not compare them
in Fig. 21(a). The electric current in Fig. 20(a) and the magnetic
current in Fig. 21(b) agree well with those of PMCHW except near
recover frequencies 21 in Fig. 21(a). The electric current in Fig. 20(a) and the magnetic current in Fig. 21(b) agree well with those of PMCHW except near resonant frequencies 212, 335, and 367 MHz. It is clearly seen that there are discontinui current in Fig. 21(b) agree well with those of PMCHW except near
resonant frequencies 212, 335, and 367 MHz. It is clearly seen that
there are discontinuities at the resonant frequencies in the far field and
 PCS RCS. there are discontinuities at the resonant frequencies in the far field and RCS.
Figures 22 and 23 show the numerical results for TENE and

Figure 14. TENE-NH
1, $f_H = 0$, $g_H = 1$). 1, $f_H = 0$, $g_H = 1$).
THNH, respectively. As discussed earlier, Figures 20 and 22 are results

THNH, respectively. As discussed earlier, Figures 20 and 22 are results
derived from only the electric field and Figures 21 and 23 are results THNH, respectively. As discussed earlier, Figures 20 and 22 are results
derived from only the electric field and Figures 21 and 23 are results
derived from only the magnetic field. For this reason, spurious peaks
in the so derived from only the electric field and Figures 21 and 23 are results
derived from only the magnetic field. For this reason, spurious peaks
in the solutions are observed at the internal resonant frequencies. It
is interes derived from only the magnetic field. For this reason, spurious peaks
in the solutions are observed at the internal resonant frequencies. It
is interesting to note that the electric current for the EFIE and the
magnetic cu in the solutions are observed at the internal resonant frequencies. It
is interesting to note that the electric current for the EFIE and the
magnetic current for the MFIE agree well with the PMCHW solution
than these of T is interesting to note that the electric current for the EFIE and the magnetic current for the MFIE agree well with the PMCHW solution than those of TENE and THNH, even though g_m is used as expansion magnetic current for the MFIE agree well with the PMCHW solution
than those of TENE and THNH, even though g_m is used as expansion
function for the magnetic current in EFIE and for the electric current in
MEIE formulatio than those of TENE and THNH, even though \underline{g}_m is used as expansion
function for the magnetic current in EFIE and for the electric current in
MFIE formulations, which violate the property of the currents. Fig. 24
pres function for the magnetic current in EFIE and for the electric current in MFIE formulations, which violate the property of the currents. Fig. 24 presents the results computed by TENE-THNH, by combining the TENE and THNH formulation. In this CFIE, the resonant peak or

Figure 15. TE-THNH 1
0, $f_H = -1$, $g_H = 1$).

 d discontinuity is not seen. The four figures in Fig. 24 agree well with
the PMCHW formulation for the equivalent currents and the WIPI D discontinuity is not seen. The four figures in Fig. 24 agree well with
the PMCHW formulation for the equivalent currents and the WIPL-D
solution for the far field and BCS. The results of the CEIE with one discontinuity is not seen. The four figures in Fig. 24 agree well with
the PMCHW formulation for the equivalent currents and the WIPL-D
solution for the far field and RCS. The results of the CFIE with one
term dramad are s the PMCHW formulation for the equivalent currents and the WIPL-D solution for the far field and RCS. The results of the CFIE with one term dropped are shown in Figures 25 to 28. All numerical results do solution for the far field and RCS. The results of the CFIE with one
term dropped are shown in Figures 25 to 28. All numerical results do
not exhibit the resonant problem and agree well with the PMCHW
formulation for the formulation for the currents and the WIPL-D solution for the far field
and BCS are the vertex is a small difference. It is important to
and BCS are though there is a small difference. It is important to not exhibit the resonant problem and agree well with the PMCHW
formulation for the currents and the WIPL-D solution for the far field
and RCS even though there is a small difference. It is important to
note that the magnet formulation for the currents and the WIPL-D solution for the far field
and RCS even though there is a small difference. It is important to
note that the magnetic current of the TENE-TH shows an excellent
agreement with th and RCS even though there is a small difference. It is important to note that the magnetic current of the TENE-TH shows an excellent agreement with the PMCHW solution in Fig. 25(b). Also the far field and BCS of the TENE note that the magnetic current of the TENE-TH shows an excellent agreement with the PMCHW solution in Fig. $25(b)$. Also the far field and RCS of the TENE-NH in Fig. $26(c)$ and (d) are the most accurate. Table 7 summarizes the averaged difference between the normalized

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Figure 16. NE-THNH 1, $f_H = -1$, $g_H = 1$).

Figure 17. WIPL results for a dielectric sphere.

Table 6. Averaged difference of the normalized far field and Table 6. Averaged difference of the normalized far field and
monostatic RCS between Mie and the numerical solution for the
dielectric sphere in Fig. 2. Table 6. Averaged differends monostatic RCS between Midielectric sphere in Fig. 2.

Formulation	mV Δe_{θ}	$\rm (mV$ Δe_{ϕ}	
PMCHW		0.8	0.34
TENE-THNH	5.3	16.1	0.40
TENE-TH	4.1	6.4	0.30
TENE-NH	3.5	12.9	0.28
TE-THNH	8.7	7.2	0.55
NE-THNH	5.7	21.5	0.52

^z- and ^y-directed arrows represent position and direction of sampled currents J and M , respectively.

far field and the monostatic RCS for the PMCHW and five different
far field and the monostatic RCS for the PMCHW and five different
CEIE formulations. If we look at Table 7, TENE NH has the smallest far field and the monostatic RCS for the PMCHW and five different CFIE formulations. If we look at Table 7, TENE-NH has the smallest difference in both θ , and ϕ component of the for field and BCS. far field and the monostatic RCS for the PMCHW and five difference in both θ - and ϕ -component of the far field and RCS.

Figure 19. PMCHW results for a dielectric cube.

Figure 20. EFIE results for a dielectric cube.

Figure 21. MFIE results for a dielectric cube.

Table 7. Averaged difference of the normalized far field and Table 7. Averaged difference of the normalized far field and
monostatic RCS between WIPL-D and numerical solution for the Table 7. Averaged differences
monostatic RCS between N
dielectric cube in Fig. 18.

Formulation	(mV) Δe_{θ}	(mV) Δe_{ϕ}	(dBm^2)
PMCHW	8.2	10.9	0.65
TENE-THNH	9.5	23.7	0.50
TENE-TH	11.6	16.3	0.59
TENE-NH	6.4	9.5	0.35
TE-THNH	9.0	20.1	0.52
NE-THNH	12.0	18.1	0.68

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Figure 22. The $g_H = 0$.

 $A = 0$.
As a final example, we present the numerical results for a finite
strip sulindar with a radius of 0.5 m and a height 1 m, septemed As a final example, we present the numerical results for a finite dielectric cylinder with a radius of 0.5 m and a height 1m, centered at the evising as shown in Fig. 20. We subdivide the evising integrated As a final example, we present the numerical results for a finite
dielectric cylinder with a radius of 0.5 m and a height 1 m, centered
at the origin, as shown in Fig. 29. We subdivide the cylinder into
four tworty four a dielectric cylinder with a radius of 0.5 m and a height 1 m, centered
at the origin, as shown in Fig. 29. We subdivide the cylinder into
four, twenty-four, and eight divisions along r, ϕ , and z directions,
respectively at the origin, as shown in Fig. 29. We subdivide the cylinder into
four, twenty-four, and eight divisions along r , ϕ , and z directions,
respectively. This represents a total of 720 patches with 1,080 edges.
The z d four, twenty-four, and eight divisions along r , ϕ , and z directions, respectively. This represents a total of 720 patches with 1,080 edges. The *z*-directed electric current and the ϕ -directed magnetic current a respectively. This represents a total of 720 patches with 1,080 edges.
The z-directed electric current and the ϕ -directed magnetic current
are observed at a location indicated by the arrows in Fig. 29. The
computed cur The z-directed electric current and the ϕ -directed magnetic current
are observed at a location indicated by the arrows in Fig. 29. The
computed currents are compared with those of the PMCHW and the
forfields and managt are observed at a location indicated by the arrows in Fig. 29. The computed currents are compared with those of the PMCHW and the far fields and monostatic RCS are compared with the WIPL-D solution.
The number of unknowns computed currents are compared with those of the PMCHW and the far fields and monostatic RCS are compared with the WIPL-D solution.
The number of unknowns is 2,688 in the computation using WIPL-D.
Fig. 20 shows the results far fields and monostatic RCS are compared with the WIPL-D solution.
The number of unknowns is 2,688 in the computation using WIPL-D.
Fig. 30 shows the results for the PMCHW. The far field and RCS

Figure 23. THI
-1, $g_H = 1$). -1, $g_H = 1$).
agree well with the WIPL-D solution in the entire frequency region.

agree well with the WIPL-D solution in the entire frequency region.
Figs. 31 and 32 show the results of EFIE and MFIE, respectively. agree well with the WIPL-D solution in the entire frequency region.
Figs. 31 and 32 show the results of EFIE and MFIE, respectively.
The basis function for the magnetic currents in EFIE is different from
that in the PMCHW Figs. 31 and 32 show the results of EFIE and MFIE, respectively.
The basis function for the magnetic currents in EFIE is different from
that in the PMCHW, and so we do not compare them in Fig. 31(b).
Similarly, the basis The basis function for the magnetic currents in EFIE is different from that in the PMCHW, and so we do not compare them in Fig. 31(b). Similarly, the basis function of the electric currents in the MFIE is that in the PMCHW, and so we do not compare them in Fig. 31(b).
Similarly, the basis function of the electric currents in the MFIE is
different from that in the PMCHW, and so we do not compare them
in Fig. 22(c). The elec Similarly, the basis function of the electric currents in the MFIE is
different from that in the PMCHW, and so we do not compare them
in Fig. 32(a). The electric current in Fig. 31(a) and the magnetic
gument in Fig. 22(b) different from that in the PMCHW, and so we do not compare them
in Fig. 32(a). The electric current in Fig. 31(a) and the magnetic
current in Fig. 32(b) agree well with those of the PMCHW except at
the resense frequencies in Fig. 32(a). The electric current in Fig. 31(a) and the magnetic current in Fig. 32(b) agree well with those of the PMCHW except at the resonant frequencies. It is clearly seen that there are discontinuities in the figu current in Fig. 32(b) agree well with those of the PMCHW except at
the resonant frequencies. It is clearly seen that there are discontinuities
in the figures near the resonant frequencies 230, 328, and 366 MHz in
the fan the resonant frequencies. It is clearly seen that there are discontinuities in the figures near the resonant frequencies 230 , 328 , and 366 MHz in the far field and RCS.

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Figure 24. TENE-THN
1, $f_H = -1$, $g_H = 1$).

Figures 33 and 34 show the numerical results of TENE and THNH,
Figures 33 and 34 show the numerical results of TENE and THNH, Figures 33 and 34 show the numerical results of TENE and THNH,
respectively. Figures 31 and 33 are the results derived from only the
algebric field and Figures 22 and 24 are results derived from only the Figures 33 and 34 show the numerical results of TENE and THNH,
respectively. Figures 31 and 33 are the results derived from only the
electric field and Figures 32 and 34 are results derived from only the
magnetic field. Fo respectively. Figures 31 and 33 are the results derived from only the electric field and Figures 32 and 34 are results derived from only the magnetic field. For this reason, spurious peaks in the solution are absented at t electric field and Figures 32 and 34 are results derived from only the magnetic field. For this reason, spurious peaks in the solution are observed at the resonant frequencies. It is interesting to note that the electric c magnetic field. For this reason, spurious peaks in the solution are observed at the resonant frequencies. It is interesting to note that the electric current for the EFIE and the magnetic current for the MFIE formulation agree well with the PMCHW solution than those the electric current for the EFIE and the magnetic current for the MFIE formulation agree well with the PMCHW solution than those of TENE and THNH, even though g_m is used as expansion function MFIE formulation agree well with the PMCHW solution than those
of TENE and THNH, even though g_m is used as expansion function
for the magnetic current in the EFIE and the electric current in the
MEIE formulations, which of TENE and THNH, even though \underline{g}_m is used as expansion function
for the magnetic current in the EFIE and the electric current in the
MFIE formulations, which is not the appropriate expansion function
for each curren for the magnetic current in the $\overline{EFE}}^m$ and the electric current in the MFIE formulations, which is not the appropriate expansion function for each current. Fig. 35 shows the results computed by TENE-THNH,

Figure 25. TENE-TH
1, $f_H = -1$, $g_H = 0$).

by combining the TENE and the THNH formulation. For this CFIE,
the guy is recovert posts are not seen. All the four figures in Fig. 25 by combining the TENE and the THNH formulation. For this CFIE,
the spurious resonant peaks are not seen. All the four figures in Fig. 35
sheek well with BMCHW and WIDI, D solutions. The results of the by combining the TENE and the THNH formulation. For this CFIE,
the spurious resonant peaks are not seen. All the four figures in Fig. 35
check well with PMCHW and WIPL-D solutions. The results of the
CEIE formulation with the spurious resonant peaks are not seen. All the four figures in Fig. 35
check well with PMCHW and WIPL-D solutions. The results of the
CFIE formulation with one term neglected are shown from Figures 36
to 30. All of the check well with PMCHW and WIPL-D solutions. The results of the CFIE formulation with one term neglected are shown from Figures 36 to 39. All of the numerical results do not exhibit the spurious resonant posts and agree we CFIE formulation with one term neglected are shown from Figures 36 to 39. All of the numerical results do not exhibit the spurious resonant peaks and agree well with the PMCHW for the currents and the WIPL-
D solution for to 39. All of the numerical results do not exhibit the spurious resonant
peaks and agree well with the PMCHW for the currents and the WIPL-
D solution for the far field and RCS. Table 8 summarizes the averaged
differences peaks and agree well with the PMCHW for the currents and the WIPL-
D solution for the far field and RCS. Table 8 summarizes the averaged
differences of the normalized far field and the monostatic RCS for
PMCHW and five CEI D solution for the far field and RCS. Table 8 summarizes the averaged differences of the normalized far field and the monostatic RCS for PMCHW and five CFIE formulations. If we look carefully at Table 8, expect for the BM differences of the normalized far field and the monostatic RCS for PMCHW and five CFIE formulations. If we look carefully at Table 8, except for the PMCHW, we observe that TENE-THNH has a small difference in the 4 componen PMCHW and five CFIE formulations. If we look carefully at Table 8, except for the PMCHW, we observe that TENE-THNH has a small difference in the θ -component of the far field, yet the difference in the ϕ -component of the far field is relatively large. If we consider the

(c) Normalized far field
 Figure 26. TENE-NH results for a dielectric cube $(f_E = 1, g_E = 1, f_H = 0, g_H = 1)$ **Figure 26.** TENE-NI
1, $f_H = 0$, $g_H = 1$).

averaged difference of both the far field and RCS, TENE-NH seems to
he the most escure to averaged difference of b
be the most accurate. **6. CONCLUSION**

A set of coupled integral equations is used to analyze scattering from 3-D dielectric objects.
3-D dielectric objects. The integral equation formulations are derived
wing the equivalence principle and utilizing the continuity conditions A set of coupled integral equations is used to analyze scattering from
3-D dielectric objects. The integral equation formulations are derived
using the equivalence principle and utilizing the continuity conditions
on the f 3-D dielectric objects. The integral equation formulations are derived
using the equivalence principle and utilizing the continuity conditions
on the fields. To obtain a numerical solution, we employ the MoM
in conjunction using the equivalence principle and utilizing the continuity conditions
on the fields. To obtain a numerical solution, we employ the MoM
in conjunction with the planar triangular patch basis function. The
EELE and MELE for on the fields. To obtain a numerical solution, we employ the MoM
in conjunction with the planar triangular patch basis function. The
EFIE and MFIE formulations give valid solutions except at frequencies,
which correspond t in conjunction with the planar triangular patch basis function. The EFIE and MFIE formulations give valid solutions except at frequencies, which correspond to an internal resonant frequency of the scatterer, are

(c) Normalized far field
 Figure 27. TE-THNH results for a dielectric cube $(f_E = 1, g_E = 0, f_H = -1, g_H = 1)$ **Figure 27.** TE-THNH
0, $f_H = -1$, $g_H = 1$). 0, $f_H = -1$, $g_H = 1$).
compared with two other formulations TENE and THNH, which also

compared with two other formulations TENE and THNH, which also
exhibit resonance problems. In order to obtain CFIE, we introduce four
testing soefficients in the combination of the TENE and THNH, As a compared with two other formulations TENE and THNH, which also
exhibit resonance problems. In order to obtain CFIE, we introduce four
testing coefficients in the combination of the TENE and THNH. As a
result we have eight exhibit resonance problems. In order to obtain CFIE, we introduce four
testing coefficients in the combination of the TENE and THNH. As a
result, we have eight different cases of the CFIE formulation of which
only two form testing coefficients in the combination of the TENE and THNH. As a
result, we have eight different cases of the CFIE formulation of which
only two formulations are not affected by internal resonances. We also
present givit result, we have eight different cases of the CFIE formulation of which
only two formulations are not affected by internal resonances. We also
present sixteen possible cases of the CFIE by dropping one of terms in
the testi only two formulations are not affected by internal resonances. We also
present sixteen possible cases of the CFIE by dropping one of terms in
the testing and check to see if all the sixteen formulations give valid
solution present sixteen possible cases of the CFIE by dropping one of terms in the testing and check to see if all the sixteen formulations give valid solutions. The important point to note is that not all possible CFIE formulatio the testing and check to see if all the sixteen formulations give valid
solutions. The important point to note is that not all possible CFIE
formulations eliminate the internal resonance problem. When we use
CEIE with othe solutions. The important point to note is that not all possible CFIE formulations eliminate the internal resonance problem. When we use CFIE with other formulations, for example, to analyze scattering from composite struct formulations eliminate the internal resonance problem. When we use
CFIE with other formulations, for example, to analyze scattering from
composite structures with conductors and dielectrics, TENE-THNH,
TENE TH or TENE NH m CFIE with other formulations, for examp
composite structures with conductors a
TENE-TH, or TENE-NH may be used.

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Figure 28. NE-THNH
1, $f_H = -1$, $g_H = 1$).

Figure 29. Triangle surface patching of a dielectric cylinder (radius 0.5 m and height 1 m). z - and ϕ -directed arrows represent position and direction of campled survents I and M, respectively. **Figure 29.** Triangle surface patching of a dielectric 0.5 m and height 1 m). z - and ϕ -directed arrows repredirection of sampled currents <u>J</u> and <u>M</u>, respectively.

Figure 30. PMCHW results for a dielectric cylinder.

Table 8. Averaged difference of the normalized far field and Table 8. Averaged difference of the normalized far field and
monostatic RCS between WIPL-D and numerical solution for the
dialectric sulinder in Fig. 20 Table 8. Averaged differen
monostatic RCS between WII
dielectric cylinder in Fig. 29.

APPENDIX A.

In this Appendix, it is shown how to evaluate some of the integrals. We drop the subscript v representing the medium. Note that while In this Appendix, it is shown how to evaluate some of the integrals.
We drop the subscript v representing the medium. Note that while
computing the elements of the matrix, the appropriate material
parameters should be i We drop the subscript v representing the medium. Note to computing the elements of the matrix, the appropriate parameters should be included in evaluating the integrals. parameters should be included in evaluating the integrals.
A.1. Integrals (26), (73), and (74)

A.1. Integrals (26), (73), and (74)
First, we consider the integral (26). Substituting (17) into (26), we get
four tarms. We define the summation energies as **A.1. Integrals (20), (73), and (74)**
First, we consider the integral (26). Substituting (17)
four terms. We define the summation operator as

$$
A_{mn} = A_{mn}^{++} + A_{mn}^{+-} + A_{mn}^{-+} + A_{mn}^{--} \equiv \sum_{p,q} A_{mn}^{pq} \tag{A1}
$$

Figure 31. EFIE results for a dielectric cylinder.

where

$$
A_{mn}^{pq} = \frac{1}{4\pi} \int_S \underline{f}_m^p(\underline{r}) \cdot \int_S \underline{f}_n^q(\underline{r}') G(\underline{r}, \underline{r}') dS' dS \tag{A2}
$$

and p and q can be either + or −. If the integral on the unprimed and p and q can be either + or -. If the integral on the unprimed
variable is evaluated by approximating the integrand by the respective
values at the control of the testing triangle T_p^p (A) becomes and p and q can be either + or -. If the integral on the unprivariable is evaluated by approximating the integrand by the respection values at the centroid of the testing triangle T_m^p , (A2) becomes

$$
A_{mn}^{pq} = \frac{l_m l_n}{16\pi} \rho_m^{cp} \cdot \frac{1}{A_n^q} \int_{T_n^q} \rho_n^q \frac{e^{-jkR_m^p}}{R_m^p} dS'
$$
 (A3)

where

$$
R_m^p = |\underline{r}_m^{cp} - \underline{r}'| \tag{A4}
$$

Figure 32. MFIE results for a dielectric cylinder.

Figure 32. MFIE results for a dielectric cylinder.

and r_{m}^{cp} is the position vector of the centeroid of the triangle T_{m}^{p} .

Equation (72) is some as (26) or (A1) and (74) may be evaluated and \underline{r}_m^{cp} is the position vector of the centeroid of the triangle T_m^p .
Equation (73) is same as (26) or (A1), and (74) may be evaluated
by replacing c_p^{cp} with $\hat{\epsilon} \times c_p^{cp}$ in (A2) and \underline{r}_m^{cp} is the position vector of the
Equation (73) is same as (26) or (A1)
by replacing $\underline{\rho}_m^{cp}$ with $\hat{n} \times \underline{\rho}_m^{cp}$ in (A3). by replacing $\underline{\rho}_{m}^{cp}$ with $\hat{n} \times \underline{\rho}_{m}^{cp}$ in (A3).
A.2. Integrals (29), (75), and (76)

A.2. Integrals (29) , (75) , and (76)
By substituting (18) into (29) , we get

$$
B_{mn} = B_{mn}^{++} + B_{mn}^{+-} + B_{mn}^{-+} + B_{mn}^{-} \equiv \sum_{p,q} B_{mn}^{pq} \tag{A5}
$$

where

$$
B_{mn}^{pq} = \frac{1}{4\pi} \int_S \nabla_S \cdot \underline{f}_m^p(\underline{r}) \int_S \nabla'_S \cdot \underline{f}_n^q(\underline{r}') G(\underline{r}, \underline{r}') dS' dS.
$$
 (A6)

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Figure 33. TENE re
1, $f_H = 0$, $g_H = 0$).

 $A_{\text{pproximating the integrand by the respective values at the centroid}$
Approximating the integrand by the respective values at the centroid Approximating the integrand by the respector the testing triangle T_m^p , (A6) becomes

$$
B_{mn}^{pq} = \frac{l_m l_n}{4\pi} \frac{1}{A_n^q} \int_{T_n^q} \frac{e^{-jkR_m^p}}{R_m^p} dS' \tag{A7}
$$

 $B_{mn}^{pq} = \frac{1}{4\pi} \frac{1}{A_n^q} \int_{T_n^q} \frac{1}{R_m^p} dS$ (A7)
where R_m^p is given by (A4). Equation (75) is same as (18) or (A5).
Equation (76) is evaluated through where R_m^p is given by (A4). Equation
Equation (76) is evaluated through

$$
B_{mn}^g = \sum_{p,q} B_{mn}^{pq,g} \tag{A8}
$$

Figure 34. THNH rest $0, f_H = -1, g_H = 1$.

where

re
\n
$$
B_{mn}^{pq,g} = \frac{1}{4\pi} \int_S \hat{n} \times \underline{f}_m^p(\underline{r}) \cdot \int_S \nabla_S' \cdot \underline{f}_n^q(\underline{r}') \nabla' G(\underline{r}, \underline{r}') dS' dS.
$$
 (A9)

Approximating the integrand by the respective values at the centroid Approximating the integrand by the respective the testing triangle T_m^p , (A9) becomes

$$
B_{mn}^{pq,g} = \frac{l_m l_n}{8\pi} \hat{n} \times \underline{\rho}_m^p \cdot \frac{1}{A_n^q} \int_{T_n^q} \nabla' \frac{e^{-jkR_m^p}}{R_m^p} dS'
$$
 (A10)
The evaluation of the potential integrals in (A3), (A7), and (A10) may
be compiled out by using the numerical methods are
sially developed for

 8π and 8π and $A_n^T J T_n^q$ and K_m^T
The evaluation of the potential integrals in (A3), (A7), and (A10) may
be carried out by using the numerical methods specially developed for The evaluation of the potenti
be carried out by using the n
triangular regions in [11–14].

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Figure 35. TENE-THNH result, $g_E = 1$, $f_H = -1$, $g_H = 1$).

A.3. Integrals (35), (77), and (78)

Using a vector identity [8], (35) is given by

$$
C_{mn} = \frac{1}{2} \int_S \underline{f}_m(\underline{r}) \cdot \hat{n} \times \{\hat{n} \times \underline{f}_n(\underline{r})\} dS = -\sum_{p,q} \frac{1}{2} \int_S \underline{f}_m^p \cdot \underline{f}_n^q dS. \tag{A11}
$$

 $C_{mn} = 2 \int_S \frac{f_m(\Sigma) \cdot h \wedge (h \wedge f_n(\Sigma))}{\Sigma} d\Sigma = \frac{1}{\Sigma} 2 \int_S \frac{f_m}{\Sigma} \cdot \frac{f_n d\Sigma}{\Sigma}$. (A11)
The integral of (A11) can be computed analytically and the result is The integral of given by $[9]$

$$
\frac{1}{2} \int_{S} \underline{f}_{m}^{p} \cdot \underline{f}_{n}^{q} dS = \pm \frac{l_{m} l_{n}}{8A} \left\{ \frac{3}{4} | \underline{r}_{c} |^{2} + \frac{1}{12} (| \underline{r}_{1} |^{2} + | \underline{r}_{2} |^{2} + | \underline{r}_{3} |^{2}) \right\}
$$

Figure 36. TENE-TH r
1, $f_H = -1$, $g_H = 0$).

$$
-{\underline{r}}^c \cdot ({\underline{r}}_m + {\underline{r}}_n) + {\underline{r}}_m \cdot {\underline{r}}_n \bigg\} \qquad (A12)
$$

 $-\underline{r}^c \cdot (\underline{r}_m + \underline{r}_n) + \underline{r}_m \cdot \underline{r}_n$ (A12)
where \underline{r}_1 , \underline{r}_2 , and \underline{r}_3 are the position vectors for the three vertices of
the triangle T^p or \overline{T}^q \underline{r}^c is the centroid of the triangle T^p where \underline{r}_1 , \underline{r}_2 , and \underline{r}_3 are the position vectors for the three vertices of
the triangle T_m^p or T_n^q , \underline{r}^c is the centroid of the triangle T_m^p or T_n^q , and
A is the area of T^p $\begin{bmatrix} r &$ where \underline{r}_1 , \underline{r}_2 , and \underline{r}_3 are the position vectors for the three vertices of
the triangle T_m^p or T_n^q , \underline{r}^c is the centroid of the triangle T_m^p or T_n^q , and
A is the area of T_m^p . $\underline{r}_$ the triangle T_m^p or \overline{T}_n^q , \underline{r}^c is the centroid of the triangle T_m^p or T_n^q , and
 A is the area of T_m^p . \underline{r}_m and \underline{r}_n are the position vectors for the free

vertex of the triangles T_m^p A is the area of T_m^p . T_m and T_n are the position vectors for the free
vertex of the triangles T_m^p and T_n^q , respectively. We note that if the
field point does not lie on the triangle T_n^q , i.e., $T \notin T_n^q$, t $C_{mn}^{pq} = 0$. In (A12), the sign is positive when p and q are same and parties of the triangles T_m^p and T_n^q , respectively. We note that if the eld point does not lie on the triangle T_n^q , i.e., $\underline{r} \notin T_n^q$, the result is $\begin{array}{c} p^n q = 0. \end{array}$ In (A12), the sign is positive when p field point does no
 $C_{mn}^{pq} = 0$. In (A12

negative otherwise.

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Figure 37. TENE-NH
1, $f_H = 0$, $g_H = 1$).

Using the summation operator, (77) is given by

$$
C_{mn}^f = \frac{1}{2} \int_S \underline{f}_m(\underline{r}) \cdot \hat{n} \times \underline{f}_n(\underline{r}) dS = \sum_{p,q} \frac{1}{2} \int_S \underline{f}_m^p \cdot \hat{n} \times \underline{f}_n^q dS. \tag{A13}
$$

By evaluating the element of (A13) analytically, the result is given by

$$
\frac{1}{2} \int_{S} \underline{f}_{m}^{p} \cdot \hat{n} \times \underline{f}_{n}^{q} dS = \pm \frac{l_{m} l_{n}}{8A} \hat{n} \cdot [(\underline{r}_{m} - \underline{r}_{n}) \times \underline{r}^{c} - (\underline{r}_{m} \times \underline{r}_{n})]. \tag{A14}
$$

Figure 38. TE-THNH r
0, $f_H = -1$, $g_H = 1$).

By using a vector identity [8], (78) may be written as

$$
C_{mn}^g = \frac{1}{2} \int_S \hat{n} \times \underline{f}_m(\underline{r}) \cdot \hat{n} \times \underline{f}_n(\underline{r}) dS = \sum_{p,q} \frac{1}{2} \int_S \underline{f}_m^p(\underline{r}) \cdot \underline{f}_n^q(\underline{r}) dS \tag{A15}
$$

where the integral is same as in (A12).

A.4. Integrals (37), (51), (79), and (80)

A.4. Integrals (37), (51), (79), and (80)
By using the summation operator, we may write terms of (37) as

sing the summation operator, we may write terms of (37) as
\n
$$
D_{mn}^{pq} = \frac{1}{4\pi} \int_{S} \underline{f}_{m}^{p}(\underline{r}) \cdot \int_{S} \hat{n}' \times \underline{f}_{n}^{q}(\underline{r}) \times \nabla' G(\underline{r}, \underline{r}') dS' dS
$$
\n(A16)

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Figure 39. NE-THNH r
1, $f_H = -1$, $g_H = 1$).

1, $fH = -1$, $gH = 1$).
where \hat{n}' denotes the unit vector normal to the triangle T_n^q . Thus,
whetituting (17b) into (416) we obtain where \hat{n}' denotes the unit vector normal to
substituting (17b) into (A16), we obtain

$$
D_{mn}^{pq} = \frac{l_m l_n}{16\pi A_m^p A_n^q} \int_{T_m^p} \rho_m^p \cdot \int_{T_n^q} (\hat{n}' \times \rho_n^q) \times \underline{R}(1 + jkR) \frac{e^{-jkR}}{R^3} dS' dS.
$$
\n(A17)

This integral may be computed using a Gaussian quadrature scheme This integral may be computed using a Gaussian quadrature scheme
over the unprimed and primed coordinates numerically. Other integrals
of (51), (70), and (80) may be evaluated in a similar fashion This integral may be computed using a Gaussian quadrature over the unprimed and primed coordinates numerically. Other of (51), (79), and (80) may be evaluated in a similar fashion.

A.5. Integrals (40), (46), (50), (54), (72), and (89)

We consider (40) for $v = 1$ and apply a centroid testing, yielding

$$
\int_{S} \underline{f}_{m}(\underline{r}) \cdot \underline{E}^{i}(\underline{r}) dS = \int_{S} \underline{f}_{m}^{+}(\underline{r}) \cdot \underline{E}^{i}(\underline{r}) dS + \int_{S} \underline{f}_{m}^{-}(\underline{r}) \cdot \underline{E}^{i}(\underline{r}) dS
$$
\n
$$
= \frac{l_{m}}{2} \{ \underline{\rho}_{m}^{c+} \cdot \underline{E}^{i}(\underline{r}_{m}^{c+}) + \underline{\rho}_{m}^{c-} \cdot \underline{E}^{i}(\underline{r}_{m}^{c-}) \}. \tag{A18}
$$

Other integrals may be evaluated in a similar fashion.

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