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Kaluza-Klein Theory with Torsion confined to the Extra-dimension

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Here we consider a variant of the 5 dimensional Kaluza-Klein theory within the framework of Einstein-Cartan formalism that includes torsion. By imposing a set of constraints on torsion and Ricci rotation coefficients, we show that the torsion components are completely expressed in terms of the metric. Moreover, the Ricci tensor in 5D corresponds exactly to what one would obtain from torsion-free general relativity on a 4D hypersurface. The contributions of the scalar and vector fields of the standard K-K theory to the Ricci tensor and the affine connections are completely nullified by the contributions from torsion. As a consequence, geodesic motions do not distinguish the torsion free 4D space-time from a hypersurface of 5D space-time with torsion satisfying the constraints. Since torsion is not an independent dynamical variable in this formalism, the modified Einstein equations are different from those in the general Einstein-Cartan theory. This leads to important cosmological consequences such as the emergence of cosmic acceleration.

In any attempt to link fundamental matter fields with intrinsic spin to gravity, it becomes necessary to extend the Riemannian space-time to include *torsion*, defined to be the antisymmetric part of affine connection. The resulting theory of gravity, known as the Einstein-Cartan theory[1] treats the metric and torsion as two independent geometrical characteristics of space-time [2].

Historically, beginning with the Kaluza-Klein (KK) theory, there has been a great interest in introducing extra dimensions of space-time to unify gravity with elementary particle interactions. In the KK theory, the scalar and vector fields, which are the extra dimensional components of the metric tensor contribute to the affine connection and the Ricci tensor and hence modify their values from the corresponding values in 4D space-time[3]. The contribution of these fields to the Einstein tensor are normally interpreted as gravity induced matter.

In this work, we incorporate torsion into 5D KK theory[4]. The inclusion of torsion introduces free parameters in the affine connection and the Ricci tensor in addition to the contributions from extra dimensional metric components that occur in the torsion free KK theory. In this paper we impose a minimal set of conditions so as to restrict torsion to purely extra dimensional components and determine all its components in terms of the metric. Interestingly, the imposed conditions lead to a complete cancellation between the modifications induced by the extra dimensional metric components and the contributions from the torsion. Thus the Ricci tensor in 5D space-time in the resulting formalism is exactly the Ricci tensor in a torsion free 4D space-time. In the second part of the paper, we apply the action principle to derive the equations of motion of this formalism. The modified Einstein equations thus derived are finally applied to Robertson-Walker cosmology that leads to a novel expansion history for the universe (see figure 1).

To describe the constraints to be imposed, we start, for the sake of completeness, with a brief overview on the relationship between tetrads, torsion and affine connection.

Setting up the framework: Since the constraints we propose to impose can be more simply stated with reference to a locally flat inertial system (allowed by the equivalence principle), we proceed to define and collect together from reference [5], the relevant standard relations we need in both the coordinate and the inertial frames. Let (i, j, k, \dots) and (A, B, \dots) denote coordinate and inertial frame indices respectively and $\hat{e}_i = \partial_i$ and $\hat{\theta}^i = dx^i$ be the basis of the tangent and dual spaces at each point in space-time. We define the corresponding inertial basis to be $\hat{e}_A = e^i_A \hat{e}_i$ and $\hat{\theta}^A = e^A_i \hat{\theta}^i$, where the vielbeins e^i_A and e^B_j satisfy the orthonormality conditions,

$$e^i_A e^j_A = \delta^j_i; \quad e^i_A e^j_B = \delta^A_B. \quad (1)$$

By definition, the metric in the inertial frame is Minkowskian η_{AB} , and the metric tensor in the coordinate system is $g_{ij} = e^A_i e^B_j \eta_{AB}$ and $g^{ij} = e^i_A e^j_B \eta^{AB}$. The covariant derivative operator ($\tilde{\nabla}$) can be defined in terms of the coordinate basis, or equivalently in terms of the inertial frame basis,

$$\tilde{\nabla}_{\hat{e}_i} \hat{e}_j = \tilde{\Gamma}^k_{ij} \hat{e}_k, \quad \tilde{\nabla}_{\hat{e}_A} \hat{e}_B = \omega^C_{AB} \hat{e}_C, \quad (2)$$

where $\tilde{\Gamma}^i_{jk}$ and ω^A_{BC} are the affine and the Ricci rotation coefficients respectively. The relationship between these two quantities follows from the transformation laws between the coordinate frame(\hat{e}_i) and inertial frame(\hat{e}_A),

$$\omega^A_{BC} = e^i_B (\tilde{\nabla}_{\hat{e}_i} e^j_C) e^A_j. \quad (3)$$

The affine connection by itself is not a tensor, but its antisymmetric part, the torsion, is a tensor. ¹

$$T^i_{jk} = \tilde{\Gamma}^i_{jk} - \tilde{\Gamma}^i_{kj} = 2\tilde{\Gamma}^i_{[jk]} \quad (4)$$

¹ Throughout this paper, square brackets enclosing indices denote the conventional anti-symmetrization, while the regular brackets enclosing the indices denotes symmetrization.

Again, using the transformation laws between the coordinate and the inertial frames, we have

$$\mathbb{T}^i_{.jk} = e_j^B e_k^C e_A^i \mathbb{T}^A_{.BC}. \quad (5)$$

Furthermore, with the standard assumption of metric compatibility, namely $\nabla_{\hat{e}_i} g_{jk} = 0$, we obtain

$$\hat{\Gamma}^i_{.jk} = \hat{\Gamma}^i_{.jk} + K^i_{.jk}, \quad (6)$$

where $\hat{\Gamma}^i_{.jk}$ is the Christoffel connection,

$$\hat{\Gamma}^i_{.jk} = \left\{ j^i_k \right\} = \frac{1}{2} \mathbf{g}^{im} [\partial_j \mathbf{g}_{km} + \partial_k \mathbf{g}_{jm} - \partial_m \mathbf{g}_{jk}], \quad (7)$$

and $K^i_{.jk}$ is the contorsion tensor.

$$K^i_{.jk} = \frac{1}{2} [\mathbb{T}^i_{.jk} + \mathbb{T}^i_{.kj} + \mathbb{T}^i_{.j\dot{k}}]. \quad (8)$$

Constructing the 5D space-time: We shall now focus on applying the above formalism to a five dimensional space-time. Consider a foliation of the 5D space-time in terms of a family of 4D hypersurfaces, which are parametrized by the coordinate system $\{x^\mu\}$, where (μ, ν, \dots) denote the coordinate indices on these hypersurfaces. Let x^5 denote the parametrization of the family, and 5 denote the corresponding coordinate index. The hypersurface coordinates $\{x^\mu\}$ together with x^5 will then span the entire 5D space-time. Let the metric and its inverse on each of the hypersurface be $g_{\mu\nu}$ and $g^{\mu\nu}$ respectively, which can in principle depend on the x^5 coordinate. Let $\{e_a^\mu, e_\mu^a\}$ denote the tetrad system on these hypersurfaces satisfying the orthonormality relations $e_a^\mu e_\mu^b = \delta_a^b$ and $e_\mu^a e_\nu^a = \delta_\mu^\nu$. Here (a, b, \dots) denote the tetrad indices on these 4D hypersurfaces. The metric on the hypersurface is then given by $g_{\mu\nu} = e_\mu^a e_\nu^b \eta_{ab}$ and its inverse $g^{\mu\nu} = e_\mu^a e_\nu^b \eta^{ab}$.

We will now construct the vielbeins in the 5D space-time by extending the tetrad system on the 4D hypersurfaces. We take the components of the 5D vielbeins to be $e_{.A}^i = (e_{.a}^\mu, e_{.5}^\mu, e_{.a}^5, e_{.5}^5)$ and $e_i^A = (e_\mu^a, e_\mu^5, e_a^5, e_5^5)$. The index $\dot{5}$ corresponds to the fifth dimension of the inertial frame. The orthonormality relations in 5D (eq. 1) immediately leads to ²

$$\begin{aligned} e_{.5}^\mu &= 0, \quad e_{.a}^5 = -e_{.a}^\mu A_\mu, \quad e_{.5}^5 = \Phi^{-1}, \\ e_5^a &= 0, \quad e_\mu^{\dot{5}} = A_\mu \Phi, \quad e_5^{\dot{5}} = \Phi. \end{aligned} \quad (9)$$

The metric in the 5D is then given by $\mathbf{g}_{ij} = e_i^A e_j^B \eta_{AB}$ and $\mathbf{g}^{ij} = e_i^A e_j^B \eta^{AB}$.

$$\begin{aligned} \mathbf{g}_{\mu\nu} &= g_{\mu\nu} + \epsilon A_\mu A_\nu \Phi^2, \quad \mathbf{g}_{\mu 5} = \epsilon A_\mu \Phi^2, \quad \mathbf{g}_{55} = \epsilon \Phi^2, \\ \mathbf{g}^{\mu\nu} &= g^{\mu\nu}, \quad \mathbf{g}^{\mu 5} = -A^\mu, \quad \mathbf{g}^{55} = A_\lambda A^\lambda + \epsilon \Phi^{-2}. \end{aligned} \quad (10)$$

The raising and lowering of indices on the vector field is done w.r.t the 4D metric $g_{\mu\nu}$. The parameter $\epsilon = \pm 1$ denotes whether the extra dimension is space-like or time-like. Note that the induced metric on the hypersurfaces (induced by the 5D geometry), $g_{\mu\nu} + \epsilon A_\mu A_\nu \Phi^2$ is different from the 4D metric $g_{\mu\nu}$ on them, but quite evidently related by a gauge transformation.

We will now impose a set of constraints on torsion and the Ricci rotation coefficients consistent with Cartan's structure equation [5] that relates torsion and connection coefficients. With a minimal modification of the standard general relativity in mind, we chose the set of constraints [6] so that the torsion components tangential to the 4D hypersurface vanish (see also [7]), while leaving non-vanishing torsion components completely determined in terms of the metric components of the 5D space-time. In the absence of torsion, the metric compatibility condition serves to evaluate the connection coefficients (Christoffel symbols, eq. 7) in terms of the metric components. In the presence of torsion, the imposed constraints can be viewed as an extension to the metric compatibility condition in the spirit that they together serve to determine the connection coefficients in terms of the metric components. Furthermore, the imposed constraints are covariant conditions on the 4D hypersurface. That is, the conditions are form invariant with respect to within-hypersurface coordinate transformations (that do not mix μ, ν with 5).

$$\text{Condition 1 : } \mathbb{T}^a_{.BC} = 0$$

Using eq. 5 and noting that $e_{.5}^\mu = 0$, we find $\mathbb{T}^{\mu}_{.ik} = 0$. This implies that the only nonzero components of torsion are $\mathbb{T}^5_{.ik}$. To determine these, we impose the following condition on the Ricci rotation coefficients,

$$\text{Condition 2 : } \omega_{.BC}^{\dot{5}} = 0$$

This condition along with metric compatibility implies $\omega_{.B\dot{5}}^A = 0$. We can now use eq. 3 to write,

$$\omega_{.B\dot{5}}^A = e_{.B}^i (\tilde{\nabla}_{\hat{e}_i} e_{.5}^j) e_j^A = e_{.B}^i (\partial_i e_{.5}^j + \tilde{\Gamma}_{.ik}^j e_{.5}^k) e_j^A = 0 \quad (11)$$

Since $e_{.5}^\mu = e_{.5}^a = 0$, the above equation implies,

$$\tilde{\Gamma}_{.i5}^\mu = 0, \quad \tilde{\Gamma}_{.i5}^5 = -e_5^{\dot{5}} \partial_i e_5^{\dot{5}} \quad (12)$$

Using the above equations along with eq.6, we can express the contorsion in terms of the Christoffel symbols $\hat{\Gamma}$. and the vielbeins,

$$\begin{aligned} K_{.i5}^\mu &= -\hat{\Gamma}_{.i5}^\mu, \\ K_{.i5}^5 &= -\left(\hat{\Gamma}_{.i5}^5 + e_5^{\dot{5}} \partial_i e_5^{\dot{5}} \right) = -\hat{\Gamma}_{.i5}^5 + J_i, \end{aligned} \quad (13)$$

² There exists another class of vielbeins that satisfy the orthonormality relations. But they are essentially related to eq. 9 by gauge. Here, we choose to work with the vielbeins given by eq. 9 to make the formalism readily comparable to the existing Kaluza-Klein literature.

where $J_i \equiv \Phi^{-1} \partial_i \Phi$.

Note that the above equations give only a subset of the components K^i_{jk} , but along with eq. 8, they are sufficient to determine all the components of the torsion. The torsion thus obtained can in turn be substituted in eq. 8 to determine the remaining components of contorsion. The components of torsion are

$$\begin{aligned} T^{\mu}_{ij} &= T^5_{55} = 0, \\ T^5_{\mu\nu} &= 2\partial_{[\mu} A_{\nu]} + 2J_{[\mu} A_{\nu]}, \\ T^5_{\mu 5} &= J_{\mu} - \partial_5 A_{\mu} - A_{\mu} J_5, \end{aligned} \quad (14)$$

In addition to yielding the non-vanishing components of torsion in terms of the metric components, the solution to eq. 13 also yields the following condition on the metric on the 4D hypersurfaces.

$$\partial_5 g_{\mu\nu} = 0. \quad (15)$$

This implies all the hypersurfaces in the foliating family have the same 4D metric. To place things in perspective, we observe that in the standard Kaluza-Klein theory, the assumption of cylindrical condition makes all the quantities, namely $g_{\mu\nu}$, A_{μ} and Φ independent of x^5 . Whereas in our formulation, the constraints automatically imply $g_{\mu\nu}$ is independent of x^5 , while A_{μ} and Φ could still depend on x^5 .

With all the K^i_{jk} determined from eqns. 14 and 8, we now use eq. 6 to calculate all the affine connection coefficients.

$$\begin{aligned} \tilde{\Gamma}^{\lambda}_{55} &= \tilde{\Gamma}^{\lambda}_{\nu 5} = \tilde{\Gamma}^{\lambda}_{5\nu} = 0, \\ \tilde{\Gamma}^5_{\mu\nu} &= \nabla_{\mu} A_{\nu} + J_{\mu} A_{\nu}, \\ \tilde{\Gamma}^5_{5\mu} &= \partial_5 A_{\mu} + J_5 A_{\mu}, \\ \tilde{\Gamma}^5_{\mu 5} &= J_{\mu}, \quad \tilde{\Gamma}^5_{55} = J_5, \quad \tilde{\Gamma}^{\lambda}_{\mu\nu} = \Gamma^{\lambda}_{\mu\nu}. \end{aligned} \quad (16)$$

Here $\Gamma^{\lambda}_{\mu\nu}$ corresponds to the Christoffel symbols (analogous to eq. 7) obtained from torsion free 4D space-time with metric $g_{\mu\nu}$. Note that the components of 5D Christoffel symbols along the hypersurface coordinates is not equal to the Christoffel symbols calculated on 4D spacetime, that is, $\tilde{\Gamma}^{\lambda}_{\mu\nu} \neq \Gamma^{\lambda}_{\mu\nu}$. The symbol ∇_{μ} corresponds to the covariant derivative operator on the torsion-free 4D space-time, where the Christoffel symbols are exactly the affine connection coefficients.

Substituting the above connection coefficients in the Ricci tensor defined by

$$\tilde{R}_{ik} = \partial_k \tilde{\Gamma}^j_{ji} - \partial_j \tilde{\Gamma}^j_{ki} + \tilde{\Gamma}^j_{km} \tilde{\Gamma}^m_{ji} - \tilde{\Gamma}^j_{jm} \tilde{\Gamma}^m_{ki}, \quad (17)$$

we find

$$\tilde{R}_{\mu\nu} = R_{\mu\nu}, \quad \tilde{R}_{\mu 5} = \tilde{R}_{5\mu} = \tilde{R}_{55} = 0. \quad (18)$$

Here $R_{\mu\nu}$ represents the Ricci tensor on the torsion-free 4D space-time. It also follows that the 5D Ricci scalar is exactly the same as the Ricci scalar in the torsion free

4D space time, that is $\tilde{R} = R$. We also note, in general, the presence of torsion makes the Ricci tensor non-symmetric, but the constraints we have imposed on the torsion leaves the Ricci tensor symmetric.

It is straightforward to see that the formalism and the results obtained thus far are not specific to 4 and 5 dimensions, they can be generalized to any arbitrary D and D+1 dimensions. We shall now consider some implications of the formalism with respect to geodesic equations and solutions to Einstein equations.

Geodesic Equations: The 5D geodesic equations split into

$$\begin{aligned} \ddot{x}^5 + \tilde{\Gamma}^5_{\mu\nu} \dot{x}^{\mu} \dot{x}^{\nu} + \left(\tilde{\Gamma}^5_{\mu 5} + \tilde{\Gamma}^5_{5\mu} \right) \dot{x}^{\mu} \dot{x}^5 + \tilde{\Gamma}^5_{55} (\dot{x}^5)^2 &= 0, \\ \ddot{x}^{\lambda} + \Gamma^{\lambda}_{\mu\nu} \dot{x}^{\mu} \dot{x}^{\nu} &= 0 \end{aligned} \quad (19)$$

We note that the components of the geodesic equations along the hypersurface are exactly the same as the geodesic equations in the torsion free 4D space-time. This is in contrast with the conventional Kaluza Klein theory where the 4D geodesic equations are modified by the presence of the fields A_{μ} and Φ . In this formalism, the presence of torsion completely nullifies the effect of these fields in the 4D geodesic equations. Furthermore, it is worth noting that the geodesic of a particle can be confined to a 4D hypersurface by requiring $\dot{x}^5 = 0$. From eq. 19, we see that this requires the additional condition,

$$\tilde{\Gamma}^5_{\mu\nu} = \nabla_{\mu} A_{\nu} + J_{\mu} A_{\nu} = 0. \quad (20)$$

If this condition can be satisfied, then there will be no observable difference, as far as a test particle is concerned, whether we live in a torsion free 4D space-time or on a hypersurface within the 5D space-time with torsion. However, it is apparent that this condition is a strong constraint requiring the vector and scalar fields satisfy the above equation for a given 4D metric. It is conceivable that for some 4D metrics, no choice of vector and scalar fields would satisfy the above constraint. In such cases, the particle would be free to move in the x^5 direction unless constrained by an external force or if the fifth co-ordinate is compact and small as usually assumed in most adaptations of the Kaluza-Klein theory.

Einstein's equations: To obtain the equations of motion, we need to vary the action with respect to the independent dynamical fields of the theory. In general, if one includes torsion in the theory, torsion and metric are two independent dynamical fields and consequently one needs to vary the action with respect to both these variables [1, 4]. In our approach, with torsion determined in terms of the metric, metric components are the only dynamical variables. Taking the action to be

$S = \int \tilde{R} \sqrt{-\mathbf{g}} d^5x$, its variation yields ³

$$\int \left[\tilde{R} \delta \sqrt{-\mathbf{g}} + \tilde{R}_{ik} \sqrt{-\mathbf{g}} \delta \mathbf{g}^{ik} + \mathbf{g}^{ik} \sqrt{-\mathbf{g}} \delta \tilde{R}_{ik} \right] d^5x \quad (21)$$

One can set this variation to zero to obtain the modified Einstein equations. The first two terms give rise to the usual symmetric Einstein tensor. In the absence of torsion, the third term becomes a boundary integral that contributes nothing to the equation. While in the presence of torsion, the third term can be shown to contribute

$$\int \left[T^m_{\cdot jm} \mathbf{g}^{ik} \delta \tilde{\Gamma}^j_{\cdot ki} - T^m_{\cdot km} \mathbf{g}^{ik} \delta \tilde{\Gamma}^j_{\cdot ji} + T^m_{\cdot kj} \mathbf{g}^{ik} \delta \tilde{\Gamma}^j_{\cdot mi} \right] \sqrt{-\mathbf{g}} d^5x$$

Treating the variation in the torsion and metric components independent, one can obtain the Einstein-Cartan equations derived by Hehl et.al [1]. However, by expressing $\delta \tilde{\Gamma}$ purely in terms of variation of the 5D metric components (from eq. 16), it turns out that the above expression can be simplified to

$$\int H_{\mu\nu} \delta g^{\mu\nu} \sqrt{-\mathbf{g}} d^5x \quad (22)$$

where

$$\begin{aligned} H_{\mu\nu} &\equiv \nabla_{(\mu} B_{\nu)} - (\nabla \cdot B) g_{\mu\nu} + J_{(\mu} B_{\nu)} - (J \cdot B) g_{\mu\nu}, \\ B_\mu &\equiv T^5_{\cdot \mu 5} = J_\mu - \partial_5 A_\mu - A_\mu J_5, \end{aligned} \quad (23)$$

are interpreted as tensors in torsion free 4D space. Since eq. 22 is the only additional term that contributes to the variation of action, $H_{\mu\nu}$ is essentially the modification to the standard torsion free Einstein tensor. The modified Einstein equations then turns out to be

$$R_\mu^\nu - \frac{1}{2} R \delta_\mu^\nu + H_\mu^\nu = \Sigma_\mu^\nu \quad (24)$$

$$-A^\alpha R_{\mu\alpha} - A^\alpha H_{\mu\alpha} = \Sigma_\mu^5 \quad (25)$$

$$-\frac{1}{2} R = \Sigma_5^5 \quad (26)$$

Here Σ is the stress tensor that one would obtain when matter fields are included in the Lagrangian prior to variation of action. Its 4D components will be the observed stress energy tensor and can be identified with the stress energy tensor of the usual 4D torsion free general relativity. Its conservation immediately yields

$$\nabla_\nu \Sigma_\mu^\nu = 0 \implies \nabla_\nu H_\mu^\nu = 0. \quad (27)$$

In general, the components Σ_μ^5 and Σ_5^5 cannot be obtained from observations on the 4D hypersurface. Hence, without an explicit 5D matter Lagrangian specifying

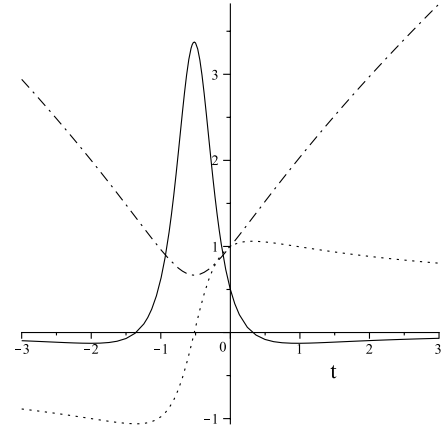


FIG. 1: The dashed-dot curve denotes $a(t)$, the dotted curve denotes $\dot{a}(t)$, and the solid curve denotes $\ddot{a}(t)$. The time axis is in units of H_o^{-1} .

these components, it is not possible to solve equations 25 and 26. Thus, we are left with just equations 24 and 27 to solve for the metric components $g_{\mu\nu}$, A_μ and Φ .

Robertson-Walker cosmology: To illustrate an application of the formalism, let us consider spatially flat homogenous and isotropic universe with metric

$$ds^2 = -dt^2 + a^2(t) (dx^2 + dy^2 + dz^2). \quad (28)$$

The assumption of homogeneity and isotropy of the 4D geometry requires that $A_\mu = (A_t, 0, 0, 0)$ and A_t and Φ are functions of t and not the spatial coordinates. To further simplify, we shall also assume that the fields do not depend on x^5 . From eq. 14, these constraints imply that $B_\mu = J_\mu$ and the only non vanishing component of J_μ is J_t , and of A_μ is A_t . Applying the conservation equation (eq. 27) yields

$$(i) \dot{\Phi} = 0, J_t = 0, \text{ or } (ii) \Phi = \dot{a}(t), J_t = \ddot{a}/\dot{a}. \quad (29)$$

Case (i) yields $H_{\mu\nu} = 0$, and eq. 24 yields the usual Friedman equation along with matter conservation.

$$(\dot{a}/a)^2 = \frac{8\pi}{3} \rho.$$

Case (ii) yields non vanishing $H_{\mu\nu}$, which when applied to eq. 24 gives the modified Friedman equation

$$(\dot{a}/a)^2 + (\ddot{a}/a) = \frac{8\pi}{3} \rho, \quad (30)$$

along with matter conservation which implies $\rho a^3 = \rho_o$ is a constant in a matter dominated universe.

To solve the equation, we specify initial conditions at the current instant of time, $a = 1$, $\dot{a} = H_o$, the Hubble

³ We are using geometric units where $G = c = 1$. We also note that the action in general will include a matter Lagrangian term which would also contribute to the variation.

constant and $\ddot{a} = -q_o H_o^2$, where q_o is the current deceleration parameter. These conditions can be used to calculate ρ_o , the current matter density (including dark matter). Taking $q_o = -0.5$, which is consistent with the current observations [?], the solution to eq. 30 is plotted in figure 1. From the dashed-dot curve (scale factor), note that the universe started expanding at $t = -0.518 H_o^{-1}$, from a size of $a = 2/3$, prior to which it was in a contracting phase. This is in contrast to the solution of the usual Friedmann equations which yields a big bang ($a = 0$) at $t = -0.667 H_o^{-1}$. From the solid curve, note that the acceleration is currently positive but decreasing and would become negative beyond $t = 0.319 H_o^{-1}$. This is qualitatively consistent with the analysis of observed data in [8],

but is in sharp contrast with the standard Λ CDM model which predicts that the acceleration would continue to increase for ever. Clearly, more detailed studies are needed to understand the full implications of this formalism on cosmology.

In conclusion, the formalism in this paper provides a general mathematical result pertaining to affine connection and Ricci tensor in Einstein-Cartan theory in higher dimensions. It provides an alternative way to confine gravity from (D+1) to D dimensions, with modifications that can be significant to cosmology.

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