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Black holes with magnetic charge and quantized mass

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Abstract

We examine the issue of magnetic charge quantization in the presence of black holes. It is pointed out that quantization of magnetic charge can lead to the mass quantization for magnetically charged black holes. We also discuss some implications for the experimental searches of magnetically charged black holes.

The purpose of this note is to draw attention to some aspects of the interplay between gravity and the quantization of magnetic charge. The issue we are going to address is not only of fundamental theoretical importance but also can be relevant for the experimental searches of monopoles.

Let us view the problem of magnetic charge from two perspectives. First, the magnetic charge g_{RN} of the extreme Reissner-Nordstrom black hole is related to its mass according to the equation,

$$g_{RN} = \sqrt{G}M, \tag{1}$$

where M is the mass of the black hole. This relation follows from an exact solution of Einstein-Maxwell equations [1,2] , and is a purely classical result with no appeal to quantum mechanics. Similarly, an extension to coupled Einstein- Yang-Mills-Higgs system leads to non-abelian and abelian black holes with magnetic charges [3]. References to earlier and related work are contained in this paper. More recently, generalizations to slowly rotating non-Abelian black holes have been reported by M.S. Volkov and N. Straumann [4]. Second, the Dirac quantization condition [5] reads:

$$g_D = \frac{n\hbar c}{2e}. \tag{2}$$

We use the Gauss system of units where $e^2/\hbar c = \alpha$.

Now, what is the relationship between the Eqs.(1) and (2)? Should we equate the left-hand sides of the above formulas? Or are they totally independent of each other? Surprisingly, this supposedly simple question, to the best of our knowledge, does not have a straightforward answer. What we really obtain here is a dilemma: on the one hand, we can insist that the Dirac quantization condition derived in flat space-time, should be valid also in curved space-time in the presence of a black hole; on the other hand, we may conjecture that there may exist some mechanism of evading the Dirac quantization condition in the regions with strong gravitational fields. Both these alternatives have interesting consequences. Which of them is realized in Nature seems to be an open question at present. For some particular cases it has been claimed that the Dirac quantization condition remains intact in the presence of gravity [6]. However, there is no general proof that we know of, establishing the validity of the Dirac quantization condition in arbitrary curved space-time. Neither there are any models that would definitely establish a possibility of its violation. Therefore, we examine qualitatively each scenario and discuss the consequences.

If we assume that the Dirac quantization condition is valid in the presence of gravity then we can equate the left-hand sides of Eqs.(1) and (2) to obtain the equation

$$M = \frac{n\hbar c}{\sqrt{G}2e} = n\frac{M_{Pl}}{2\sqrt{\alpha}}. \tag{3}$$

This says that the masses of Reissner-Nordstrom black holes are quantized and the minimum possible mass for the extreme magnetically charged black holes is given by

$$M_{min} = 5.85 M_{Pl}, \tag{4}$$

with an equi-spaced mass spectrum analogous to the energy spectrum of a quantum simple harmonic oscillator. Whether such black holes are the final stages of more massive black holes undergoing Hawking radiation and reaching a stable state is an interesting question. Anyway, theory aside, the range of magnetic monopole masses around the point $5.85 M_{Pl}$ should be considered seriously in the monopole search experiments.

Now, let us consider the other alternative, namely, the possible violation of the Dirac quantization condition in the presence of gravity. Not knowing the precise nature of its violation assume that the magnetic charge of a Reissner-Nordstrom black hole obeys only one constraint, that of Eq.(1), being unrestricted by Eq.(2). Thus, we have magnetic monopoles with "dequantized" magnetic charge determined by the mass [7]. Thus there is no restriction on the range of allowed magnetic charges. However, let us confine our attention to charges between zero and one Dirac unit, $g = 68.5e$ and note that very small magnetic charges correspond to the black holes with the mass much less than the Planck mass for which the Compton wavelength $\frac{\hbar}{Mc}$ is longer than the Schwarzschild radius $\frac{2GM}{c^2}$. Therefore, the effects of quantum gravity should be quite important for such black holes. For this reason it would be safer to leave very small magnetic charges from our consideration. A particularly interesting case from the point of view of symmetry between electricity and magnetism, is $g = e$. This equality of magnetic charge of the black hole and the electric charge of the electron would occur for the black hole mass

$$M = \sqrt{\alpha} M_{Pl} = 0.08 M_{Pl} \tag{5}$$

Another special case of interest is the black hole with the Planck mass for which the magnetic charge is

$$g_{Pl} = \frac{e}{\sqrt{\alpha}} = 11.7e, \tag{6}$$

so $g_{Pl} = 1$ in dimensionless units ($\hbar = c = 1$). Black holes with the Planck mass have been called maximons in [8].

In this context, it is interesting to recall that the reported monopole observations of Ehrenhaft [9] corresponded to the values of magnetic charge *less* than the standard Dirac charge of $68.5e$, and hence they were given less credibility on the ground that they violated Dirac quantization condition.

Are there any arguments in favour of the violation of the Dirac quantization condition by the gravitational effects? Originally, the Dirac quantization condition appeared as the necessary condition for the unobservability of the Dirac string. The Dirac string is the line of a singularity in the vector potential describing the monopole. It starts at the monopole location and stretches to infinity. Analytically it can be written, for example, as

$$A_r^D = A_\theta^D = 0, \tag{7a}$$

$$A_\varphi^D = \tfrac{g}{r} \tan \tfrac{\theta}{2}. \tag{7b}$$

The magnetic field corresponding to that vector potential is also singular and has the form [10] (here, $\mathbf{n}$ is the unit vector along the string, i.e. opposite the z-axis):

$$\nabla \times \mathbf{A}^D = \frac{g}{r^3}\mathbf{r} + \mathbf{n} 4\pi g \theta(-z) \delta(x) \delta(y). \tag{8}$$

Note that to obtain the above equation rigorously one has to treat all quantities involved in the sense of distributions rather than regular functions [11]. Therefore, the string makes a singular contribution to the energy momentum tensor of the electromagnetic field. A natural question then arises: can such a string be observed by its gravitational effects? The gravitational field of a gauge string has been studied in [12] (A gauge string has a finite mass μ per unit length whereas a Dirac string has *infinite* line density). It has been found that the spacetime around such a string is locally flat, but globally it is similar to a cone. It means that the range of the polar angle ϕ (measured around the string) is not 2π but $0 \leq \phi \leq 2\pi(1 - 4G\mu)$. This angular deficit has been called a conical singularity. The presence of conical singularity leads to several observable effects: double images of sources located behind the string, anisotropy of the background radiation, and the creation of sheets of matter in the wake of the string. These results strongly suggest that the Dirac string *may* become observable by its gravitational effects *regardless of the value of the magnetic charge*. [1] Thus we are led to conjecture that in the presence of gravitation it may become irrelevant whether the quantization condition for the Dirac monopole is fulfilled or violated. (Of course, an alternative interpretation of the same result would be to claim that the whole concept of the Dirac monopole is inconsistent with gravity.)

Let us now turn to the situation with the t'Hooft-Polyakov type of monopoles. For these monopoles there exist two very different pictures: one uses a gauge with the "hedgehog" configuration of the scalar field; this gauge is completely free of any singularity lines (non-singular gauge). The other approach is very close to the picture of a Dirac monopole because of the presence of a string similar to the Dirac string (string gauge). These two pictures are believed to be physically equivalent because there exist a (singular) gauge transformation from one to the other. Therefore, if we choose to work in the string gauge we can again require the string to be unobservable and thus obtain the quantization condition.

Now, we would like to think of the monopole coupled to gravity. One of the fundamental questions is: will the two gauges remain physically equivalent? If they are then we can use again the same line of argument as above. Consequently, we can again conjecture that the quantization condition may become irrelevant in the presence of gravity. On the other hand, the non-singular gauge and the string gauge may become non-equivalent once the gravity is switched on, in which case we do not know the form the quantization condition takes. In other words, at the present state of our knowledge, arbitrary magnetic charges unconstrained by the Dirac quantization condition seem not to be ruled out by sound theoretical arguments. This should be kept in mind in experimental searches.

We now turn to the analysis of the experimental constraints on the flux of magnetically charged black holes ("black poles" for short). The most stringent constraints come from

[1] Note that there exists some evidence that the string may become visible even in flat space. In [13] the motion of a monopole in the magnetic field of an external electric current was analyzed. Roughly speaking, the physical results depend on how many times the monopole string winds up around the current line. Another configuration where the strings turn out to be a problem has been considered in [14]: a charged particle in the magnetic field of a Dirac monopole *line*. Further problems related to the Dirac quantization condition and existence of strings have been raised recently in the context of quantum field theory with monopoles, see e.g. [15–17].It is not clear whether or not the stringless formalisms of Wu-Yang and others [18–20] based on the fibre bundle theory would help in solving these problems.

the results of non- accelerator experiments devoted to the search of superheavy magnetic monopoles arising in Grand Unified Theories of strong, weak, and electromagnetic (but not including gravitational) interactions. These searches can be divided into two main classes: the first uses the effect of current induction caused by the monopole magnetic field and the second is based upon the ionizing properties of monopoles. The induction experiments give the most direct upper limits on the monopole flux because they are independent of the unknown characteristics of the monopoles such as its mass and velocity. The best upper bounds on the monopole flux F from induction experiments are:

$$F < 4.4 \times 10^{-12} cm^{-2} sr^{-1} s^{-1} \text{ [21]}, \tag{9}$$

$$F < 7.2 \times 10^{-13} cm^{-2} sr^{-1} s^{-1} \text{ [22]} \tag{10}$$

$$F < 5 \times 10^{-14} cm^{-2} sr^{-1} s^{-1} \text{ [23]} \tag{11}$$

These constraints can be taken over for the case of *quantized* black holes regardless of their mass and velocity. As for the *dequantized* black holes, however, the situation is different. The reason is that the magnetic flux through a superconducting loop is quantized and can change only in multiples of magnetic flux quantum $\phi^0 = \frac{1}{2}(4\pi \times \frac{\hbar c}{2e})$. Therefore, it is unclear to what extent the superconductive induction experiments are sensitive to the dequantized magnetic charges. A sufficient condition for a magnetic charge g to be detected in the induction experiment reads

$$4\pi g = \frac{\phi_0}{n_l}, \tag{12}$$

where n_l is a geometric factor which is specific for the loop configuration adopted in a particular experiment. For instance, for the original Cabrera's detector $n_l = 4$ since there are 4 superconducting loops connected in a series. An interesting problem is how to design an experimental setup (or use the existing setups) in such a way that it would be capable of detecting *both* quantized *and dequantized* magnetic charges (including the charges not satisfying Eq. (12)).

Let us now turn to the experiments that use the ionizing properties of monopoles. We consider first the case of quantized black holes, assuming that the black holes with $n \geq 1$ are stable. The ionization experiments are much more sensitive to the theoretical input than the induction experiments. In particular, their sensitivity crucially depends on the monopole's velocity: the monopole would be undetectable if its velocity were below the threshold, somewhere between $10^{-3}c$ and $10^{-4}c$. To find out the typical monopole velocity, we need to find the greater of the two characteristic velocities: the first is the velocity with which the monopole enters the Galaxy ($\sim 10^{-3}c$) and the second is the typical monopole velocity due to its acceleration by the galactic magnetic field, given by

$$v_n \simeq 3 \times 10^{-3} c (\frac{10^{16} GeV}{M_n})^{1/2}. \tag{13}$$

The quantized magnetically charged black holes with masses $M \gtrsim 6 \times 10^{19}$ GeV can be viewed as *ultraheavy* monopoles (as contrasted with *superheavy* GUT monopoles with the mass of the order of 10^{16} GeV). It is readily seen that the total spectrum of the quantized black poles can be divided into two parts: 1)"low-lying", low charge black poles, corresponding to $n \lesssim 600$ and 2)"very heavy ", heavily charged black poles" with $n \gtrsim 600$. For the low-lying black poles the galactic magnetic field is not strong enough to accelerate them, so their typical velocity is expected to be of the order of $10^{-3}c$. Therefore the astrophysical bounds on their flux obtained from the condition of survival of the galactic magnetic field (the Parker bound and its refinements) are *inapplicable* for them. On the contrary, for the superheavy black poles their dynamics is dominated by the galactic magnetic field, so most of them are soon ejected out of the galaxy similar to the case of the ordinary GUT monopoles. Thus, the low-lying black poles with the mass in the interval $(6-3600)M_{Pl}$ and, consequently, the magnetic charge in the range $(1-600)g_1$ seem to be more interesting experimentally. Although the Parker-type bounds do not hold for these poles, their flux can nevertheless be constrained at a similar level, $F \lesssim 10^{-15}cm^{-2}sr^{-1}s^{-1}$, from the requirement that the galactic density of the poles should not be unacceptably high. The best available limits from the ionization detectors on the ultraheavy monopole flux with velocities of the order of $10^{-3}c$ have the same order of magnitude. For instance,

$$F < 5.6 \times 10^{-15}cm^{-2}sr^{-1}s^{-1}\ [24], F < 2.7 \times 10^{-15}cm^{-2}sr^{-1}s^{-1}\ [25] \tag{14}$$

Furthermore, if one takes into account the possibility of monopole catalysis of nucleon decay (Rubakov-Callan effect) [26]) then a somewhat stronger limit can be obtained [27] (it is, however, sensitive to the assumed catalysis cross-section):

$$F < 5 \times 10^{-16}cm^{-2}sr^{-1}s^{-1}. \tag{15}$$

To summarize, we have examined the issue of magnetic charge quantization in the presence of black holes. It was pointed out that quantization of magnetic charge can lead to the mass quantization for magnetically charged black holes. We have also discussed some implications for the experimental searches of magnetically charged black holes.

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