Syracuse University

SURFACE

5-1990

A Lexical Extension of Montague Semantics

William C. Purdy
Syracuse University, wcpurdy@ecs.syr.edu

Recommended Citation

A Lexical Extension of Montague Semantics

William C. Purdy

May 1990

School of Computer and Information Science
Suite 4-116
Center for Science and Technology
Syracuse, New York 13244-4100

(315) 443-2368

A Lexical Extension of Montague Semantics

William C. Purdy*

March 1988
(revised September 1989)

Abstract

This paper presents a model theory of lexical semantics that is compatible with theories in the Montagovian tradition. Lexical expressions are modeled as subsets or "subspaces" in a "semantic space".

A unique representation is defined for subspaces of the semantic space. This unique representation is called the "normal form" of the lexical denotation. A Boolean algebra of normal forms is developed, in which lexical entailment is Boolean inclusion.

The presentation in the body of the paper is informal, making use of examples to illustrate the theory and to indicate the range of applicability. Formal definitions and proofs in support of the presentation are given in the Appendix.

*School of Computer and Information Science, Syracuse University

1

1. Introduction

This paper presents a theory of lexical semantics that is compatible with theories in the Montagovian tradition [21, 2, 13, 12]. Like Montagovian semantics, it is model-theoretic. Lexical expressions are modeled as subsets or "subspaces" in a "semantic space."

Since Montagovian theories treat most lexical items as nonlogical constants having primitive denotations, the theory presented here can be viewed as an extension of those theories. The theory is not only relevant to Montagovian semantics, however. For example, it can also be viewed as an extension of automatic reasoning theories employing resolution with sorts (e.g., [22]). In this role, the theory provides a representation in which the subsort and supersort relations are intrinsic.

The presentation in the body of the paper is informal, making use of examples to illustrate the theory and to indicate the range of applicability. Formal definitions and proofs in support of the presentation are given in the Appendix.

2. A Model of Lexical Semantics

The theory is introduced by an example from a traditional domain: English words for kinship. The kinship vocabulary and its definition, shown in Figure 1, are taken from Nida [17]. The elements of the vocabulary are listed at the top of the table. "Diagnostic components," properties that distinguish elements of the vocabulary each from the other, appear along the left side of the table. The body of the table indicates which diagnostic components characterize each vocabulary element. This example is restricted to consanguineal kinship (c-kinship). However, partial consanguineal relations and affinal relations can be dealt with similarly [18].

C-kinship can be modeled by a relational structure. For example, the denotation of **father** is defined by the expression[1] $\text{father}(x,y) \leftrightarrow \text{male}(x) \wedge \text{prec}(x,y) \wedge \text{L0}(x,y)$ where $\text{prec}(x,y)$ asserts that x is of the generation preceding that of y and $\text{L0}(x,y)$ asserts a direct lineal relation between x and y. If **male** is modified so that $\text{male}(x,y)$ is taken to assert that x is male, and application is defined to distribute over Boolean operations, the above can be written more compactly $\text{father}(x,y) \leftrightarrow (\text{male} \wedge \text{prec} \wedge \text{L0})(x,y)$. If all expressions are so treated, the variable symbols are no longer needed. That is, $\text{father} \leftrightarrow \text{male} \wedge \text{prec} \wedge \text{L0}$ conveys the same information.[2]

A relation R_1 is said to be contained by or included in a relation R_2 if for all pairs (x,y), $R_1(x,y) \rightarrow R_2(x,y)$, or in variable-free form, $R_1 \rightarrow R_2$. To illustrate this, c-kinship can be extended to include the lexical items **self, parent, child, sibling** and **immediate family**, defined as follows:

$\text{self} \leftrightarrow \text{same} \wedge \text{L0}$

$\text{parent} \leftrightarrow \text{prec} \wedge \text{L0}$

$\text{child} \leftrightarrow \text{succ} \wedge \text{L0}$

---

[1]Denotations are written in sans serif type.

[2]The modification of **male** is called *homogenization* by Quine. In terms of Quine's functors, **male** has been replaced by *inv Pad* **male**. Further discussion of homogenization and its role in elimination of variables can be found in [19], pp. 283-288.

3

sibling ↔ same ∧ L1

immediate family ↔ L0 ∨(same ∧ L1)

From the definitions of these new lexical items it can be inferred for example that sister → sibling, i.e., sister is included in sibling. Similarly, it can be inferred that sibling → immediate family, i.e., sibling is included in immediate family.

This suggests a way to model entailment between lexical items. Using componential analysis [17] or semantic field analysis [15] one can identify lexical features that distinguish between members of a set of related lexical items (a "semantic domain" or "semantic field"). C-kinship is an example. The derived relational structure can then model the semantic domain, providing denotations for the lexical features and the lexical items.

The Boolean model cannot express some assertions that can be expressed in first-order logic. For example, using first-order logic one can assert that the parent relation is inverse to the child relation:

$$\forall x \forall y [\text{parent}(x,y) \leftrightarrow \text{child}(y,x)]$$

Or, it can be asserted that the uncle relation entails a brother relation:

$$\forall x \forall y [\text{uncle}(x,y) \rightarrow \exists z [\text{brother}(x,z)]]$$

But the Boolean model has the advantage of simplicity: entailment is simply set inclusion.

Specifically, let H be a set of individuals. The power set $2^{H \times H}$ represents the set of all binary relations on H. Indeed, a binary relation is typically identified with the set of pairs that satisfy it. For example, prec is identified with the set $\{(x,y) \in H \times H | \text{prec}(x,y)\}$.

Let $S \subseteq H \times H$ be a subset of consanguineal pairs such that {prec,same,succ} partitions S. That is,

4

1. $\text{prec} \cup \text{same} \cup \text{succ} = S$

2. $\text{prec} \cap (\text{same} \cup \text{succ}) = \emptyset$

   $\text{same} \cap \text{succ} = \emptyset$

3. $\text{prec} \neq \emptyset$

   $\text{same} \neq \emptyset$

   $\text{succ} \neq \emptyset$

Let $\{L0,L1,L2\}$ and $\{male,female\}$ also partition S.

S can be diagrammed as in Figure 2a or, to suggest a multidimensional space, as in Figure 2b. In this multidimensional space, subspaces or subsets are denotations of c-kinship relations. For example, the subspace $\text{parent} = \text{prec} \cap L0$ is the denotation of **parent**. When the denotation of a lexical item includes several cells (e.g., cousin=L2), this is indicated by labeling each of the cells with that lexical item. Some examples of subspaces are given in Figure 3.

Thus a subspace can be viewed as the extension or meaning of the associated lexical item. Moreover, relations between subspaces can be viewed as relations between meanings. Let $\mathbf{R_1}$ and $\mathbf{R_2}$ be any c-kinship lexical items, and R_1 and R_2 their respective denotations (subspaces). Then $\mathbf{R_1}$ entails $\mathbf{R_2}$ if and only if $R_1 \subseteq R_2$. That is, subspace inclusion can be viewed as entailment or meaning inclusion. Similarly, subspace exclusion (disjointness) can be viewed as contradiction. The intersection of two subspaces can be viewed as the meaning common to the corresponding lexical items. In the multidimensional space, inclusion, exclusion, intersection and the like can be determined quite directly. The examples of Figure 4 illustrate this.

The partitions that subdivide the multidimensional space in the preceding example have an important property that was not made explicit. Residence in any given block of the partition $\{prec,same,succ\}$ does not restrict residence in any block of the

partition {L0,L1,L2}. A similar assertion holds for any subset of the three partitions. This property is called "independence."

More precisely, let $B = \{P_i | i \in I\}$ be a set of partitions of a set S, where $P_i = \{p_i^j | j \in J_i\}$. Then B will be said to be *independent* if and only if for any selection of $j_i \in J_i$, for each $i \in I$, $\bigcap_{i \in I} p_i^{j_i}$ is nonempty.[3]

Independence means that the set of partitions contains no redundancy. Each partition contributes information in every case. If one visualizes the atomic cells of the multidimensional space, independence implies that some individuals occupy every cell. Put another way, no cell represents a logically impossible condition. This is not to be confused with "lexical gaps," which are breaks in a pattern of related lexical items [15, 16]. It may be that a particular cell is the denotation of no lexical item; but it is the denotation of some expression or paraphrase. Thus independence does not imply no lexical gaps; rather it implies no "logical gaps."

An independent set of partitions of a set S will be called a *basis of S*. The partitions of a basis of S define dimensions of S. Their blocks correspond to the coordinate values. Thus each partition can be viewed as a dimension of meaning. The blocks can be viewed as mutually antonymous "primitive" meanings.

Geometrically each block can be thought of as a hyperplane orthogonal to a coordinate axis. These hyperplanes are the simplest subspaces. Next in order of simplicity are those subspaces that can be expressed as the intersection of such hyperplanes, one or the union of several from each dimension.

In the c-kinship space defined previously, prec corresponds to a plane orthogonal to the "generation" axis. The intersection of prec, L0 (a plane orthogonal to the "lineality" axis) and male ∪ female (union of planes orthogonal to the "gender" axis)

---

[3]To simplify the present discussion it is assumed that all partitions as well as all sets of partitions are finite. This assumption is not necessary. Indeed infinite partitions are considered in Section 6. Finiteness is not assumed in the Appendix.

is the subspace previously identified as the extension of **parent**. Such subspaces will be called "elementary subsets." They are analogous to convex subspaces because they can have no "inside corners." But they are not exactly convex subspaces because they need not be connected. Equivalently, a subspace x is an elementary subset if and only if for some reordering of the blocks of each partition, x becomes a rectangular polyhedron. Thus **parent** and **cousin** are elementary subsets. So is **prec**$\cup$**succ**, although not connected. But **immediate family** is not an elementary subset. It has inside corners and so cannot be formed by intersecting sets of planes orthogonal to the coordinate axes.

More precisely, if $B = \{P_i | i \in I\}$ is a basis of S where $P_i = \{p_i^j | j \in J_i\}$, then an *elementary subset of S relative to the basis B* is a subspace x that can be represented $x = \bigcap_{i \in I} \bigcup_{j \in J_i^x} p_i^j$ where $J_i^x \subseteq J_i$. This representation is called the *standard form for x*. The conjunct $\bigcup_{j \in J_i^x} p_i^j$ is called the *i-th component of x*.

Thus the i-th component of an elementary subset is formed by taking the union of some of the planes orthogonal to the i-th coordinate. The elementary subset is the intersection of its components.

An equivalent representation is $x = \bigcap_{i \in I^x} \bigcup_{j \in J_i^x} p_i^j$ where $i \in I^x$ if and only if $J_i^x \neq J_i$. For example, the expression **L0** represents the same elementary subset that (**prec** $\cup$ **same** $\cup$ **succ**) $\cap$ **L0** $\cap$ (**male** $\cup$ **female**) does. This is called the *abbreviated standard form for x*.

It is shown in the Appendix that the standard form for elementary subset x is unique. It follows that the abbreviated standard form is also unique.

The smallest nonempty elementary subsets are the intersections of hyperplanes where exactly one hyperplane is orthogonal to each coordinate axis. These elementary subsets are called *atoms*. For example, **father** = **prec** $\cap$ **L0** $\cap$ **male** is an atom.

	father	mother	uncle	aunt	brother	sister	son	daughter	nephew	niece	cousin
male	X		X		X		X		X		X
female		X		X		X		X		X	X
prec.gen.	X	X	X	X							X
same gen.					X	X					X
succ.gen.							X	X	X	X	X
dir.lin.	X	X					X	X			
once rem.			X	X	X	X			X	X	
twice rem.											X
consang.	X	X	X	X	X	X	X	X	X	X	X
affinal			X	X					X	X	

Figure 1: Definition of Kinship Relations (from Nida)

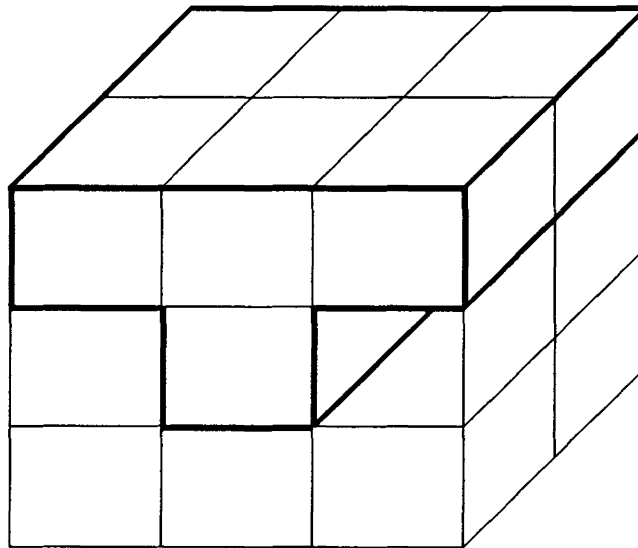|	male			female		
L0	father	self	son	mother	self	daughter
L1	uncle	brother	nephew	aunt	sister	niece
L2	cousin	cousin	cousin	cousin	cousin	cousin
	prec	same	succ	prec	same	succ

(a) Planar Representation
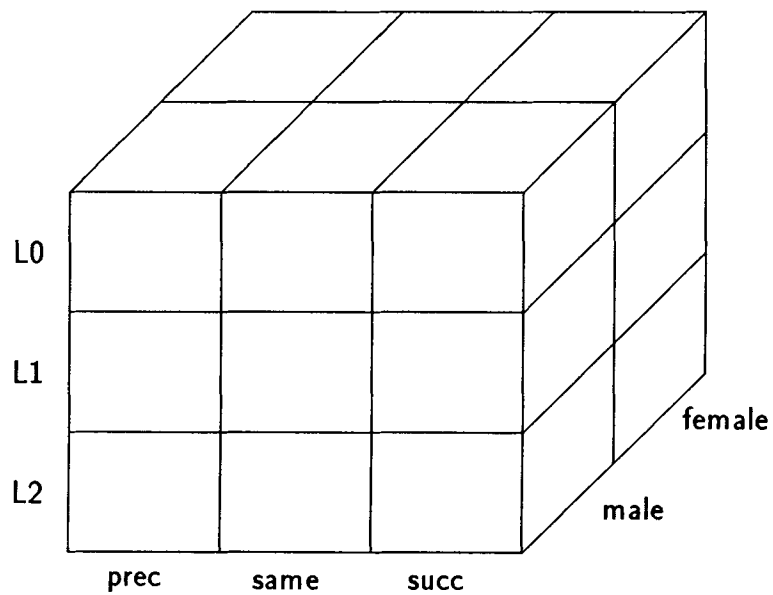
(b) Spatial Representation

Figure 2: C-Kinship as a Multidimensional Space
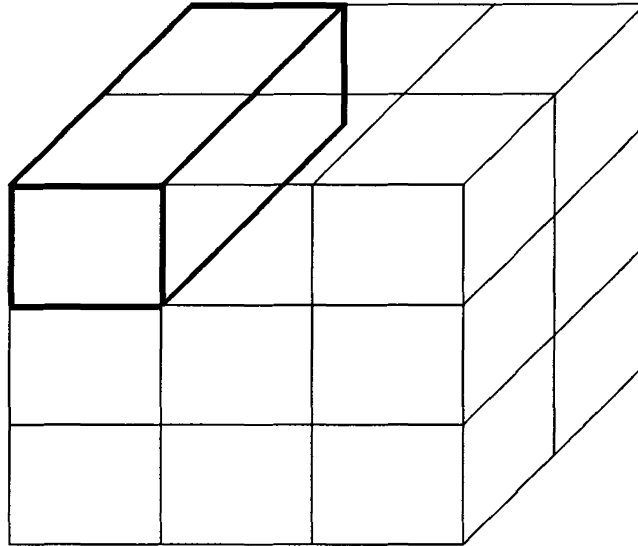
(a) parent

(b) immediate family

Figure 3: Subspaces of the C-Kinship Semantic Space

10

Example 1. **Father** entails **parent**

(1) **father** $\mapsto$ prec $\cap$ L0 $\cap$ male
(2) **parent** $\mapsto$ prec $\cap$ L0
(3) prec $\cap$ L0 $\cap$ male $\subseteq$ prec $\cap$ L0

Example 2. **Child** entails **immediate family**

(1) **child** $\mapsto$ succ $\cap$ L0
(2) **immediate family** $\mapsto$ L0 $\cup$ same $\cap$ L1
(3) succ $\cap$ L0 $\subseteq$ L0 $\subseteq$ L0 $\cup$ same $\cap$ L1

Example 3. **Uncle** entails ¬ **immediate family**

(1) **uncle** $\mapsto$ prec $\cap$ L1 $\cap$ male
(2) **immediate family** $\mapsto$ L0 $\cup$ same $\cap$ L1
(3) (prec $\cap$ L1 $\cap$ male) $\cap$ (L0 $\cup$ same $\cap$ L1)
 = (prec $\cap$ L0 $\cap$ L1 $\cap$ male) $\cup$ (prec $\cap$ same $\cap$ L1 $\cap$ male)
 = 0

Figure 4: Entailment as inclusion

3. A Normal Form

An arbitrary subspace is a union of elementary subsets. Clearly, any subspace is a union of atoms. But in general, there are many distinct sets of elementary subsets each having as its union the same subspace. For example,

$\{\text{prec} \cap \text{L0}, \text{same} \cap \text{L0}, \text{succ} \cap \text{L0}, \text{same} \cap \text{L1}\}$

$\{(\text{prec} \cup \text{succ}) \cap \text{L0}, \text{same} \cap (\text{L0} \cup \text{L1})\}$

$\{\text{L0}, \text{same} \cap \text{L1}\}$

$\{\text{L0}, \text{same} \cap (\text{L0} \cup \text{L1})\}$

are each a set of elementary subsets whose union is **immediate family**. The last set is special however in that each of its members is maximal.

If x is an arbitrary subspace and y is an elementary subset contained in x, then y is *maximal in* x if no other elementary subset z in x properly contains y. That is, if for every elementary subset $z \subseteq x$, $y \subseteq z \subseteq x$ implies $z = y$, then y is maximal in x.[4]

It is shown in the Appendix that if x is an arbitrary subspace the set of elementary subsets that are maximal in x is unique. Thus any subspace is the union of a unique set of maximal elementary subsets, each of which has a unique standard form. The set of maximal elementary subsets of a subspace therefore constitutes a unique representation or *normal form* for that subspace. Consequently each extension or meaning has a normal form.

Continuing the c-kinship example, **immediate family** has the normal form $\{\text{L0}, \text{same} \cap (\text{L0} \cup \text{L1})\}$. Notice that no elementary subset in **immediate family** properly contains either of the elementary subsets in the normal form. Moreover, every elementary subset in **immediate family** is contained in one of the elementary subsets in the normal form.

The normal form of a subspace x will be denoted $\mathcal{N}(x)$.

---

[4]It may be helpful for readers familiar with switching theory to think of "maximum elementary subset" as a generalization of "prime implicant."

Having defined a normal form for subspaces of the multidimensional space of lexical meaning, the next task is to identify useful operations under which the set of normal forms is closed. This will be done by first defining intersection and complement for elementary subsets. Then these operations are generalized to arbitrary subspaces. Finally a union operation is defined. The presentation will continue to be informal. However, the results obtained as well as all subsequent results leading to a Boolean algebra of normal forms are proved in the Appendix.

In the simple case of elementary subsets, geometric intuition may be invoked. Let x and y be elementary subsets with standard forms $\bigcap_{i \in I} \bigcup_{j \in J_i^x} p_i^j$ and $\bigcap_{i \in I} \bigcup_{j \in J_i^y} p_i^j$ respectively. One is easily convinced by geometric considerations that $x \cap y$ is also an elementary subset and moreover that its standard form is $\bigcap_{i \in I} \bigcup_{j \in J_i^x \cap J_i^y} p_i^j$. (See Figure 5 for an example.) That is, intersection of elementary subsets is computed componentwise. For the simple case where x and y are elementary subsets, define $\mathcal{N}(x) \underline{\wedge} \mathcal{N}(y) = \{x\} \underline{\wedge} \{y\} = \{x \cap y\}$.

Now consider the elementary subset $z_i = \bigcup_{j \in (J_i - J_i^x)} p_i^j$. This is the union of hyperplanes, orthogonal to the i-th coordinate axis, that do not intersect the elementary subset x. It is obvious from geometric considerations that $x \cap z_i = 0$ (the *null subspace*). This also follows from the previous result, since for each $i \in I$: $J_i^x \cap (J_i - J_i^x) = \emptyset$. The distributive law holds for the multidimensional space, and therefore $x \cap (\bigcup_{i \in I} z_i) = 0$ as well. Further, $x \cup (\bigcup_{i \in I} z_i) = 1$ (the *unit subspace*, i.e., the denotation of the entire semantic domain under consideration). Thus, $\bigcup_{i \in I} z_i$ is the complement of subspace x. (See Figure 6 for an example.) The complement will be written $-x$. Of course, $-x$ is not in general an elementary subset. But notice that the z_i for $i \in I^x$ are maximal in $-x$ and are irredundant. Therefore, $\{z_i | i \in I^x\} = \mathcal{N}(-x)$. For the special case where x is an elementary subset, define $\sim \mathcal{N}(x) = \{\bigcup_{j \in (J_i - J_i^x)} p_i^j | i \in I^x\}$. Then if x is an elementary subset, $\mathcal{N}(-x) = \sim \mathcal{N}(x)$.

At this point, an intersection operation, $\underline{\wedge}$, and a complement operation, $\sim$, have been defined for elementary subsets.

Next consider arbitrary subspaces x and y with $\mathcal{N}(x) = \{x_1, x_2, \ldots, x_m\}$ and $\mathcal{N}(y) = \{y_1, y_2, \ldots, y_l\}$. Since by definition $x = x_1 \cup x_2 \cup \cdots \cup x_m$ and $y = y_1 \cup y_2 \cup \cdots \cup y_l$, it follows by distributivity that $x \cap y = \bigcup_{1 \leq r \leq m, 1 \leq q \leq l} x_r \cap y_q$. Each of the $x_r \cap y_q$ is an elementary subset. Moreover, the set $\{x_r \cap y_q | 1 \leq r \leq m, 1 \leq q \leq l\}$ contains all the maximal elementary subsets in $x \cap y$. It does not, however, contain *only* the maximal elementary subsets. (For an example, see Figure 7.) Therefore, letting **irr** be the operation that removes subsumed elements, $\mathcal{N}(x \cap y) = \mathbf{irr}\{x_r \cap y_q | 1 \leq r \leq m, 1 \leq q \leq l\}$. Define $\mathcal{N}(x)\underline{\wedge}\mathcal{N}(y) = \mathbf{irr}\{x_r \cap y_q | 1 \leq r \leq m, 1 \leq q \leq l\}$. Then the set of normal forms is closed under $\underline{\wedge}$ and $\mathcal{N}(x \cap y) = \mathcal{N}(x)\underline{\wedge}\mathcal{N}(y)$.

By De Morgan's law, $-x = -x_1 \cap -x_2 \cap \cdots \cap -x_m$, where each $-x_r$ is the complement of an elementary subset, viz., x_r. Applying the result for intersection of normal forms, $\mathcal{N}(-x) = \mathcal{N}(-x_1)\underline{\wedge} \cdots \underline{\wedge}\mathcal{N}(-x_m)$ or $\sim \mathcal{N}(x) = \sim \mathcal{N}(x_1)\underline{\wedge} \cdots \underline{\wedge} \sim \mathcal{N}(x_m)$. Thus $\sim$ is defined for arbitrary subspaces as well as elementary subsets.

Thus the set of normal forms is closed under a complement operation $\sim$ and an intersection operation $\underline{\wedge}$. Next a union operation for normal forms is defined in terms of these operations. Since $x \cup y = -(-x \cap -y)$ by De Morgan's law, $\mathcal{N}(x \cup y) = \sim (\sim \mathcal{N}(x)\underline{\wedge} \sim \mathcal{N}(y))$. Therefore a union operation for normal forms is defined $\mathcal{N}(x)\underline{\vee}\mathcal{N}(y) = \sim (\sim \mathcal{N}(x)\underline{\wedge} \sim \mathcal{N}(y))$.

These results may be summarized as follows. Given a multidimensional space of lexical meaning defined by some basis, the set of normal forms along with operations $\underline{\wedge}$, $\underline{\vee}$ and $\sim$ form a Boolean algebra.

Inclusion between normal forms can be defined: $\mathcal{N}(x) \leq \mathcal{N}(y)$ if and only if $\mathcal{N}(x)\underline{\wedge}\mathcal{N}(y) = \mathcal{N}(x)$. Thus $\mathcal{N}(x) \leq \mathcal{N}(y)$ is equivalent to $x \subseteq y$.

14

Two examples based on c-kinship will illustrate these operations. (See Figure 7.) Each demonstrates computation of a union of subspaces. In both cases the resulting subspace is **immediate family**.
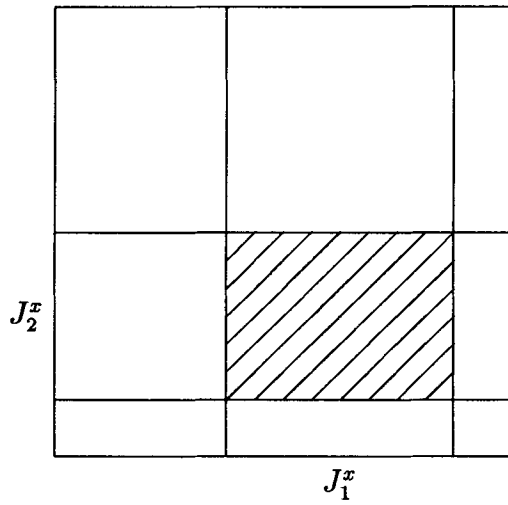
(a) Elementary Subset x
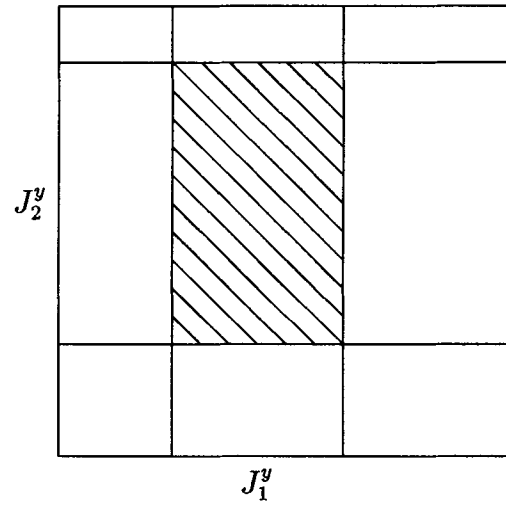
(b) Elementary Subset y
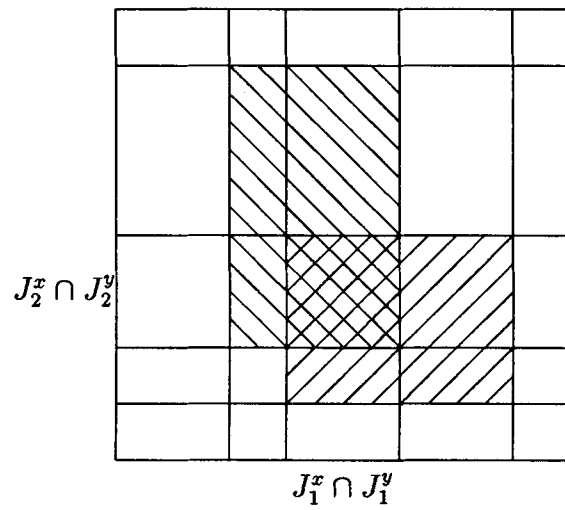
(c) Intersection of x and y

Figure 5: Example of Intersection of Elementary Subsets

16

(a) Elementary Subset x

(b) $z_1 = \bigcup_{j \in (J_1 - J_1^x)} p_i^j$

(d) $-x = z_1 \cup z_2$

(c) $z_2 = \bigcup_{j \in (J_2 - J_2^x)} p_i^j$

Figure 6: Example of Complement of Elementary Subset

Example 1.

Let $\mathcal{N}(x) = \{\mathsf{L0}\}$ and $\mathcal{N}(y) = \{\mathsf{same} \cap \mathsf{L1}\}$

Then $\sim \mathcal{N}(x \cup y) = \{\mathsf{L1} \cup \mathsf{L2}\}\underline{\Delta}\{\mathsf{prec} \cup \mathsf{succ}, \mathsf{L0} \cup \mathsf{L2}\}$

$\qquad\qquad\qquad = \mathbf{irr}\{\mathsf{L2}, (\mathsf{prec} \cup \mathsf{succ}) \cap (\mathsf{L1} \cup \mathsf{L2})\}$

Hence $\mathcal{N}(x \cup y) \quad = \{\mathsf{L0} \cup \mathsf{L1}\}\underline{\Delta}\{\mathsf{L0}, \mathsf{same}\}$

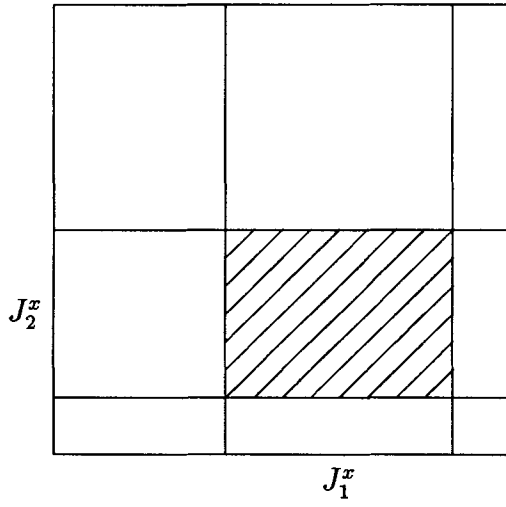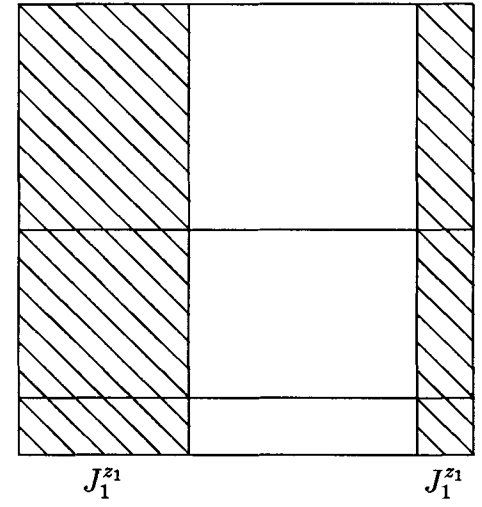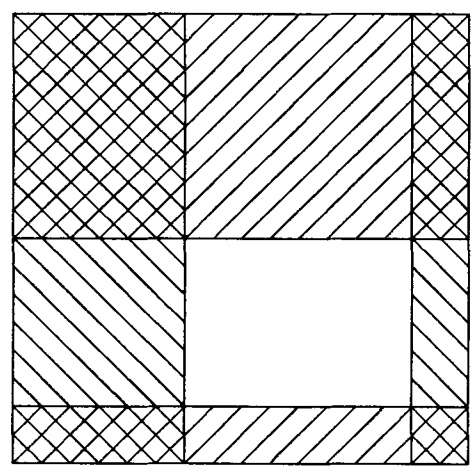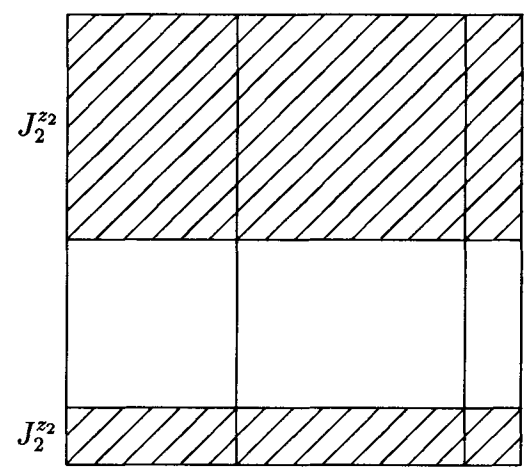$\qquad\qquad\qquad = \mathbf{irr}\{\mathsf{L0}, \mathsf{same} \cap (\mathsf{L0} \cup \mathsf{L1})\}$

$\qquad\qquad\qquad = \{\mathsf{L0}, \mathsf{same} \cap (\mathsf{L0} \cup \mathsf{L1})\}$

The result is the set of maximal elementary subsets of the subspace immediate family.

Example 2.

Let $\mathcal{N}(x) = \{(\mathsf{prec} \cup \mathsf{same}) \cap \mathsf{L0}, \mathsf{same} \cap (\mathsf{L0} \cup \mathsf{L1})\}$ and $\mathcal{N}(y) = \{\mathsf{succ} \cap \mathsf{L0}\}$

Then $\sim \mathcal{N}(x \cup y) = \{\mathsf{succ}, \mathsf{L1} \cup \mathsf{L2}\}\underline{\Delta} \{\mathsf{prec} \cup \mathsf{succ}, \mathsf{L2}\}\underline{\Delta} \{\mathsf{prec} \cup \mathsf{same}, \mathsf{L1} \cup \mathsf{L2}\}$

$\qquad\qquad\qquad = \mathbf{irr}\{\mathsf{L2}, (\mathsf{prec} \cup \mathsf{same}) \cap \mathsf{L2}, (\mathsf{prec} \cup \mathsf{succ}) \cap (\mathsf{L1} \cup \mathsf{L2}),$

$\qquad\qquad\qquad\qquad \mathsf{prec} \cap (\mathsf{L1} \cup \mathsf{L2}), \mathsf{succ} \cap \mathsf{L2}, \mathsf{succ} \cap (\mathsf{L1} \cup \mathsf{L2})\}$

$\qquad\qquad\qquad = \{\mathsf{L2}, (\mathsf{prec} \cup \mathsf{succ}) \cap (\mathsf{L1} \cup \mathsf{L2})\}$

Hence $\mathcal{N}(x \cup y) \quad = \{\mathsf{L0} \cup \mathsf{L1}\}\underline{\Delta} \{\mathsf{same}, \mathsf{L0}\}$

$\qquad\qquad\qquad = \mathbf{irr}\{\mathsf{L0}, \mathsf{same} \cap (\mathsf{L0} \cup \mathsf{L1})\}$

$\qquad\qquad\qquad = \{\mathsf{L0}, \mathsf{same} \cap (\mathsf{L0} \cup \mathsf{L1})\}$

Again the result is the normal form of subspace immediate family.

Figure 7: Boolean Operations on Normal Forms

4. The Lexicon

Given a set of lexical items, such as the words denoting c-kinship, distinguishing lexical features can be determined by linguistic analysis. These lexical features can then be organized into sets whose denotations partition the universe modeling the lexical items. It is possible to select a subset of these partitions that has the property of independence. Such a set is a basis. It structures the universe to yield a multidimensional space. Subspaces of the multidimensional space are uniquely represented by normal forms, for which a Boolean algebra can be defined. The multidimensional space so formed will be called a *semantic space.*

The structure of a semantic space can be encoded using the index sets $\{J_i | i \in I\}$. For example, the standard form (or abbreviated standard form) for an elementary subset x can be encoded as a sequence of binary strings, the i-th string representing J_i^x. The normal form for an arbitrary subspace y can be encoded as the sequence of codes for its maximal elementary subsets in lexical order.

Linguistic analysis provides definitions of the lexical items in terms of (specifically, as Boolean functions of) the lexical features. These definitions can be used to define a mapping from lexical items to normal forms (or codes for the normal forms) of the semantic space. This mapping will be called a *lexicon* for the vocabulary of lexical items.

Let the mapping be denoted v. Then the following definitions can be made. Relative to the basis that defines the semantic space, lexical items x and y are synonymous if and only if $v(x) = v(y)$; x and y are contradictory if and only if $v(x) \underline{\wedge} v(y) = 0$; x entails y if and only if $v(x) \leq v(y)$, that is, if and only if $v(x) \underline{\wedge} v(y) = v(x)$ or equivalently, $v(x) \underline{\wedge} \sim v(y) = 0$.

v can be extended to Boolean expressions over lexical items (of the same type) by

defining $v(x \text{ or } y) = v(x)\underline{\vee}v(y)$, $v(x \text{ and } y) = v(x)\underline{\wedge}v(y)$, and $v(\text{not } x) = \sim v(x)$.

Definition of a lexicon for c-kinship is given in Figure 8.

It is to be noted that the basis selected for construction of the semantic space of lexical meaning will determine the precision of the meanings associated with the lexical items. Therefore, meaning equivalence and meaning inclusion are understood relative the basis. Equivalence or inclusion relative to a given basis may not hold relative to a refinement of that basis. Thus a notion of learning or development is inherent in this theory.

While this approach to lexical semantics seems to have a desirable simplicity, its expressiveness is limited relative to that of first-order logic. For example, logic permits assertions such as $\text{parent}(x, y) \leftrightarrow \text{child}(y, x)$ and $\text{uncle}(x, y) \rightarrow \exists z[\text{brother}(x, z)]$. A semantic space cannot explicitly represent such knowledge. However, as the next definition of c-kinship demonstrates, it is sometimes possible to implicitly represent such knowledge.

Consider a set $S \subseteq H \times H$ comprising three generations of blood kin. For $i = 1, 2, 3$, define:

$Li = \{(x, y) \in S | \text{the join of } x \text{ and } y \text{ in the family tree is a distance } i \text{ from } x\}$

$Ri = \{(x, y) \in S | \text{the join of } x \text{ and } y \text{ in the family tree is a distance } i \text{ from } y\}$

It will be assumed that S is partitioned by $P_1 = \{\text{L0,L1,L2}\}$, $P_2 = \{\text{R0,R1,R3}\}$ and $P_3 = \{\text{male,female}\}$. As a consequence, $B = \{P_1, P_2, P_3\}$ is a basis of S. The semantic space is shown in Figure 9.

This basis defines a space that is better than the first one in several ways. First, the meanings are grouped more simply: cousin occupies just two atoms; immediate family is now an elementary subset, viz., $(\text{L0} \cup \text{L1}) \cap (\text{R0} \cup \text{R1})$. Second, $Li \cap Rj$ is inverse to $Lj \cap Ri$. For example, $\text{L1} \cap \text{R2}$ is the extension of **uncle or aunt**. The inverse c-kinship

relation is **nephew or niece** which has the extension $L2 \cap R1$. Thus knowledge about inverse c-kinship relations is implicit in this semantic space. Third, $Li \cap Rj$ where $i \neq 0 \neq j$ implies the existence of a sibling relation.

The basis defining this space and the underlying linguistic analysis seem to more fully represent the meanings of c-kinship relations. It is likely that a similar circumstance will obtain in most semantic domains. Therefore, the empirical linguistic analysis underlying construction of a lexicon seems to be a procedure requiring experience and good judgment.

For still another basis for c-kinship, see [8], p. 60.

This concludes consideration of the c-kinship domain. It is appropriate to enumerate the conclusions that can be drawn from this first example.

1. For at least certain semantic domains (kinship being one), linguistic analysis can provide lexical features that distinguish between nonsynonymous lexical items in the domain. But a set of lexical features so obtained is not unique. Different analyses can yield different sets of features and a given set might be judged "better" than another for any of a variety of as yet unformalized reasons.

2. It is important to note that the lexical features are *not* lexical items; rather they are logical predicates. Lexical features may be described by English words and phrases. Nonetheless they are not to be identified with these words and phrases. Of course it may happen that a particular lexical item in the domain also describes a lexical feature. In this case the denotation of the lexical item and the denotation of the lexical feature coincide.

3. Subdivision of a semantic space is *not* dependent on the existence of lexical items. Rather the lexical features subdivide the semantic space. The semantic space is *conceptual*. Whether or not a particular concept is lexicalized in En-

glish has no bearing on the existence of a denotation (i.e., subspace) for that concept. Analysis of a domain of lexical items should result in distinguishing features that characterize the essential properties of the domain. Thereafter these lexical features, not the lexical items, are primary. In general, denotations of lexical items only partially populate the semantic space, with paraphrases, possibly quite complex, denoting the "gaps." A failure to recognize this distinction between lexical items (which are basic expressions of the object language) and lexical features (which are expressions of the metalanguage) can generate spurious issues.

4. The lexical features must distinguish between those lexical items intended to be nonsynonymous, but they need not, and should not, be exhaustive. The properties not included among the lexical features might be called *encyclopedic data*. Such data properly resides in an encyclopedia, not a lexicon. This observation is not however to be construed as endorsement of the "minimal description principal" [9].

5. A set of lexical features can be structured into independent sets of mutually antonymous sets (called a basis) which can be modeled as orthogonal partitions of some universe. This model can be viewed as an n-dimensional semantic space. Subspaces of a semantic space can be viewed as denotations of lexical items of the domain as well as of Boolean expressions in these lexical items.

6. Any subspace has a unique representation with respect to the given basis. Therefore any lexical expression has a unique representation for its denotation. This unique representation is called the normal form of the expression.

7. The mapping from lexical expressions to their normal forms is called a lexicon for the semantic domain.

8. The partial order, viz., inclusion, of normal forms corresponds to hyponymy

of the corresponding lexical expressions. Thus synonymy, contradiction and entailment are primitive relations inherent in the lexicon. These relations are relative to the basis that defines the lexicon. They may change with refinement or correction of the basis.

9. It is possible to construct distinct lexicons that are *equivalent* in that they have the same domain and define the same synonymy, contradiction and entailment relations.

$$B = \{P_1, P_2, P_3\}$$
$$P_1 = \{\text{prec, same, succ}\}$$
$$P_2 = \{\text{L0, L1, L2}\}$$
$$P_3 = \{\text{male, female}\}$$

v: **father** $\mapsto$ prec $\cap$ L0 $\cap$ male
   mother $\mapsto$ prec $\cap$ L0 $\cap$ female
   uncle $\mapsto$ prec $\cap$ L1 $\cap$ male
   aunt $\mapsto$ prec $\cap$ L1 $\cap$ female
   brother $\mapsto$ same $\cap$ L1 $\cap$ male
   sister $\mapsto$ same $\cap$ L1 $\cap$ female
   son $\mapsto$ succ $\cap$ L0 $\cap$ male
   daughter $\mapsto$ succ $\cap$ L0 $\cap$ female
   nephew $\mapsto$ succ $\cap$ L1 $\cap$ male
   niece $\mapsto$ succ $\cap$ L1 $\cap$ female
   cousin $\mapsto$ L2
   self $\mapsto$ same $\cap$ L0
   parent $\mapsto$ prec $\cap$ L0
   child $\mapsto$ succ $\cap$ L0
   sibling $\mapsto$ same $\cap$ L1
   immediate family $\mapsto$ L0 $\cup$ same $\cap$ L1

Figure 8: Lexicon for C-Kinship

	male			female		
L0	self	father	gfather	self	mother	gmother
L1	son	brother	uncle	daughter	sister	aunt
L2	grandson	nephew	cousin	gdaughter	niece	cousin
	R0	R1	R2	R0	R1	R2

Figure 9: A Second Basis for C-Kinship

24

5. Extended Bases

Each of the bases considered thus far consists of a single set of partitions. In the general case, a basis consists of several sets of partitions. The former are called *simple*, the latter *extended* bases. In this section, the way in which extended bases arise and their structure will be shown by considering another lexical domain: English verbs of motion. This example will also provide a demonstration that the theory being developed is not limited to nominal domains.

Intransitive verbs combine with noun phrases to form sentences. In Montagovian theory intransitive verbs denote subsets of a universe of individuals. The analysis of this model is taken from Nida [17]. Figure 10 lists the lexical features. The index of a feature in this list will be used as an abbreviation for that feature. For example, "continuous contact with the surface by one then another limb or set of limbs" will be abbreviated "E3a." Further, the common prefix of a set of indexes will be used as an abbreviation for that set. For example, "A" for "{A1a,A1b,A1c,A2,A3}," "A1" for "{A1a,A1b,A1c}," and so on.

The denotations of lexical features are used to construct the basis for a semantic space modeling the domain. This construction uses Boolean relationships that exist between the denotations. In the first example the English descriptions of the lexical features give this information simply and directly. However the present example is more complex and the English descriptions alone are not sufficiently precise. It is necessary to formalize the definitions of the lexical features. Nida's data must be extended.

To make the formalization concise, two properties will be implicitly assumed for all lexical features. First, the denotation of each lexical feature is assumed to be nonempty. That is, no lexical feature is logically impossible. Second, unless explicitly stated differently the denotations of each pair of lexical features are assumed to over-

lap. That is, if X and Y are denotations then $X \cap Y$, $X \cap \overline{Y}$ and $\overline{X} \cap Y$ are nonempty. Therefore it is required to specify explicitly only inclusion, $X \subseteq Y$, and exclusion, $X \cap Y = 0$.

Since basis construction deals with partitions, the following abbreviation will be convenient. $\{X_1, \ldots, X_k\}$ *partitions* Y or Y *is partitioned by* $\{X_1, \ldots, X_k\}$ abbreviates

$X_i \cap Y \neq 0$ for $1 \leq i \leq k$

$X_i \cap X_j = 0$ for $1 \leq i < j \leq k$

$Y \subseteq X_1 \cup \cdots \cup X_k$

That is, the restrictions of the X_i to Y form a partition of Y.

Definition of the lexical features for English verbs of motion can now be completed as follows.

A, B, and G each partition 1, the unit subspace.

$B3 \subseteq G2$.

$Ci \subseteq B1 \cup B2$, for i = 1,2,3, and C partitions $B1 \cup B2$.

$Ci \subseteq A1a \cup A2 \cup A3$, for i = 1,2,3, and C partitions $A1a \cup A2 \cup A3$.

$Di \subseteq A1a \cup A1b \cup A1c$, for i = 1,2,3, and D partitions $A1a \cup A1b \cup A1c$.

$Ei \subseteq A1a \cup A1b \cup A1c$, for i = 1,2,3a,3b, and Ei partitions $A1a \cup A1b \cup A1c$.

$Fi \subseteq E2 \cup E3a \cup E3b$, for i = 1,2,3,4, and F partitions $E2 \cup E3a \cup E3b$.

$Fi \subseteq C1 \cup C2 \cup C3$, for i = 1,2,3,4, and F partitions $C1 \cup C2 \cup C3$.

With this information construction of the basis can begin. A, B and G are partitions of the unit subspace but they do not form a basis because they are not independent. Specifically, B and G are dependent because $B3 \subseteq G2$. A and B, as well as A and G are independent, however, so either pair is a basis for 1. Let A and G be chosen, yielding fifteen atoms $a_1, a_2, \ldots, a_{15}$. $B = \{A, G\}$ is called the *first level basis*.

Next each atom defined by B is examined. Consider atom $a_6 = A1a \cap G2$. a_6 is partitioned by B, C, D, and E. While B and C are not independent, B, D, and E are

and form a basis for a_6. This basis, $B_6 = \{B, D, E\}$ is called a *second level basis*.[5] B_6 defines 36 atoms, $a_{6.1}, \ldots, a_{6.36}$. a_7 and a_8 are treated similarly.

$a_1 = \mathsf{A1a} \cap \mathsf{G1}$ is partitioned by $\{\mathsf{B1,B2}\}$, C, D, and E, which form an independent set. They could therefore form a basis. However, for symmetry with B_6, let $B_1 = \{\{\mathsf{B1,B2}\}, \mathsf{D,E}\}$ be the second level basis for a_1. B_1 defines 24 atoms, $a_{1.1}, \ldots, a_{1.24}$. a_2, a_3, a_{11}, a_{12}, and a_{13} are treated in similar fashion to a_1.

In this way, fifteen second level bases are defined. Then the second level atoms are examined. Consider $a_{1.1}$. It is partitioned by C. Thus $B_{1.1} = \{\mathsf{C}\}$, a *third level basis*. $a_{1.2}$, $a_{1.9}$, $a_{1.10}$, $a_{1.17}$, and $a_{1.18}$ are similar. $a_{1.3}$ is partitioned by C and F. Since these partitions are independent, the third level basis $B_{1.3} = \{\mathsf{C, F}\}$.

The second level basis $B_2 = \{\{\mathsf{B1, B2}\}, \mathsf{D,E}\}$ defines 24 atoms, but they are not subdivided by the lexical features.

Continuing in this manner, an extended basis is constructed. The result is a tree structure, each node being a simple basis. See Figure 11. The internal structure of these simple bases is shown in Figure 12.

The vocabulary and definitions of vocabulary elements in terms of the lexical features are shown in Figure 13.[6] These data immediately determine v, the lexicon mapping. For example,
$v(\text{climb}) = [\mathsf{A1a} \cap \mathsf{G3}] \cap [\mathsf{B1} \cap \mathsf{D1} \cap \mathsf{E3a}] \cap [\mathsf{C2} \cap \mathsf{F1}]$ and
$v(\text{fall}) = [\mathsf{A1c} \cap \mathsf{G2}] \cap [\mathsf{B3} \cap \mathsf{D2} \cap \mathsf{E1}]$.

As in the first example, the particular lexical features chosen for verbs of motion constitute only one possible set, which may not be as good as some other. The lexical features chosen will affect the "quality" of the lexicon. It should be appreciated that

---

[5]More accurately, the restrictions of the elements of B, D and E to a_6 form a basis of a_6.

[6]Minor deviations from Nida's data are indicated. These seem to be required by the descriptions of the lexical features.

selection of a "good" set of lexical features for a given semantic domain is a difficult task. Much of Nida's book is devoted to detailing this task. The difficulty can be illustrated by attempting to add "bounce" to the verbs of motion.

Having a person on a trampoline in mind, one might define bounce as $A1a \cap G3 \cap B2 \cap D1 \cap E1 \cap C2$. However, if one thinks of a ball bouncing on the floor, the definition might be $A1a \cap G3 \cap B3 \cap D2 \cap E1$. But the inclusion of G3 does not permit use of the word to describe a ball bouncing off a wall or ceiling!

One might define $bounce_1$, $bounce_2$ and $bounce_3$ to represent these different senses. But it would be better to admit that the set of lexical features is too limited to accommodate this new lexical item and should be revised. A possible revision, for which no claim to quality is made, is the following.

B. Source of energy

    1. animate source

    2. combination of animate and inanimate sources

    3. inanimate source

B'. Form of energy from inanimate source

    1. potential energy

        a. gravity

        b. elastic

        c. chemical

        d. electrical

    2. kinetic energy

    3. exchange of potential and kinetic energy

In terms of these revised distinguishing properties, the essence of bounce might be rendered as $A1a \cap B'3 \cap D2 \cap (E1 \cup E2)$. Further consideration might reveal this set to be inadequate as well.

Another example of a verb domain is suggested by Lehrer [15]. It is described as difficult, perhaps because of the lack of agreement on its semantics, perhaps because, unlike the verbs of motion, it is abstract. The domain is the English verbs of belief.

In the absence of an analysis of this domain comparable to those used in the previous examples, data was extracted from *Webster's Dictionary of Synonyms* [7]. Since this was carried out by the author of this paper, the expertise that produced analyses of the previous domains cannot be claimed for this one. Nonetheless, it demonstrates application of the theory to an abstract domain.

The universe is again a set of individuals, but the subsets represent abstract rather than physical properties. The lexical features are listed in Figure 14. Using the same abbreviation conventions as for the previous example, the relations between lexical features are the following.

C and D each partition 1, the unit subspace.

$E_i \subseteq C1 \cup D1$, for i = 1,2, and E partitions $C1 \cup D1$.

$A_i \subseteq C2$, for i = 1,2,3,4, and A partitions C2.

$B_i \subseteq C2$, for i = 1,2,3, and B partitions C2.

Based on this definition, an extended basis for the semantic space is the following.

$B = \{C\}$

$B_1 = \{D\} \qquad B_2 = \{A,B,D\}$

$B_{1.1} = \{E\}$

The lexicon mapping is given in Figure 15. The vocabulary includes those words considered by Lehrer along with "premise."

As remarked above, the semantics of this domain does not have general consensus. That given here is only one possible. However the theory is not dependent on the analysis and the data produced by it. Indeed this must be the case if the theory is to allow for distinct instantiations of lexical semantics due to subculture, historical

period, and individual idiosyncrasy (including stages in the development of a language faculty).

It appears that acquisition of information for construction of a semantic space or a lexicon is somewhat difficult. It might be supposed that it is subject to error as well. Therefore if this theory of lexical semantics were realized in a cognitive agent, it is likely that invalid data would occasionally be used in the construction of the lexicon. The resulting errors in the lexicon would require eventual correction. These issues are not addressed in this paper. However, they suggest important directions for extension of the theory. Specifically, how can incremental extension of the semantic space in response to acquisition of new semantic data be accomplished? And how can revision of the semantic space to correct errors resulting from use of invalid data at an earlier time be accomplished without repeating the entire construction?

Based on the examples of this section, the conclusions of the previous section can be extended.

10. Verb domains as well as nominal domains permit a linguistic analysis which can be used to construct a model. Abstract domains, although more difficult to analyze, can also be modeled.

11. In general, a collection of simple bases (organized in a tree structure called an extended basis) is required to define a semantic space modeling a lexical domain.

12. Although a formal algorithm for construction of a basis is not given, it is clear that one could be defined. Important issues not discussed are incremental development of bases, and correction of bases constructed from invalid data.

A. Environment
   1. Surfaces
      a. Supporting
      b. Nonsupporting
      c. Between surfaces on different levels
   2. Air
   3. Water
B. Source of energy
   1. Animate being
   2. Animate being and gravity
   3. Gravity
C. Use of limbs for propulsion
   1. All four limbs
   2. All limbs normally in contact with supporting surface
      (with optional addition of forelimbs for bipeds in climbing)
   3. Forelimbs
D. Points of contact with the surface
   1. Extremities of the limbs
   2. Any parts
   3. Continuous series of points
E. Nature of contact with the surface
   1. No contact during movement
   2. Intermittent contact
   3. Continuous contact
      a. By one and then another limb or set of limbs
      b. By the same or contiguous portion
F. Order of repeated contact between limbs and surface
   1. Alternating
   2. Variable but rhythmic
   3. 1-1-1-1 or 2-2-2-2 or continuous series of short jumps
   4. 1-1-2-2-1-1-2-2
G. Directional orientation
   1. Indeterminant
   2. Down
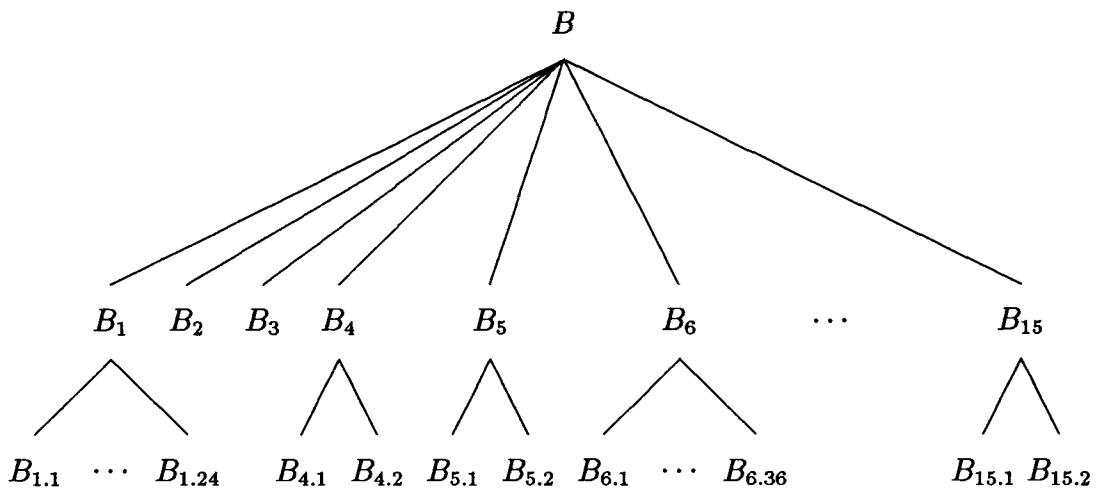   3. Up

Figure 10: Lexical features for verbs of motion.

Figure 11: The extended basis for verbs of motion.

	A1a	A1b	A1c	A2	A3
G1	a_1	a_2	a_3	a_4	a_5
G2	a_6	a_7	a_8	a_9	a_{10}
G3	a_{11}	a_{12}	a_{13}	a_{14}	a_{15}

Basis B

Second level basis B_i, $i = 6, 7, 8$

Second level basis B_i, $i = 1, 2, 3, 11, 12, 13$

Second level basis B_i, $i = 9, 10$

Second level basis B_i, $i = 4, 5, 14, 15$

Third level basis B_i, $i = 6.4, 6.5, \ldots$

Third level basis B_i, $i = 4.1, 4.2, \ldots$

Figure 12: Internal structure of the bases for verbs of motion.

	climb	crawl	dance	fly	fall	hop	jump	roll	run	sink	skip	slide	swim	walk
A1a	x	x	x			x	x	x	x		x	x		x
A1b											x			
A1c					x									
A2				x										
A3													x	
B1	x	x	x	x		x	x		x		x		x	x
B2								x				x		
B3					x						x			
C1		x						*					x	
C2	x		*			x	x	*	x		x			x
C3				x				*						
D1	x	x	x			x	x		x		x			x
D2				x								x		
D3									x					
E1					x		x							
E2		x				x			x		x			
E3a	*	x	x											x
E3b								x				x		
F1	x	x							x					x
F2		x												
F3						x								
F4											x			
G1		x	x	x		x	x	x	x		x	x	x	x
G2					x					x				
G3	x													

Figure 13: Definition of verbs of motion. ("x" indicates Nida's data; "*" indicates deviations from Nida.)

34

A. Source of belief
    1. Perception or experience
    2. Reason or logic
    3. Intuition or self evidence
    4. Trust or faith
B. Strength of belief
    1. Certitude
    2. Partial assent
    3. Doubt
C. Hypothetical component
    1. Present
    2. Absent
D. Deliberate component
    1. Present
    2. Absent
E. Formality
    1. Formal
    2. Informal

Figure 14: Lexical features for verbs of belief.

$$B = \{C\}$$
$$B_1 = \{D\} \qquad B_2 = \{A,B,D\}$$
$$B_{1.1} = \{E\}$$

v: **assume** $\mapsto$ C1
    presume $\mapsto$ C2 $\cap$ (A1 $\cup$ A2) $\cap$ (B1 $\cup$ B2)
    postulate $\mapsto$ C1 $\cap$ D1 $\cap$ E1
    posit $\mapsto$ C2 $\cap$ B1 $\cap$ D1
    premise $\mapsto$ C2 $\cap$ B1 $\cap$ D1) $\cap$ (A1 $\cup$ A2 $\cup$ A3)
    presuppose $\mapsto$ C1 $\cup$ C2 $\cap$ (B1 $\cup$ B2)
    suppose $\mapsto$ C1 $\cap$ D1
    take for granted $\mapsto$ C2 $\cap$ $B1$
    guess $\mapsto$ C2 $\cap$ B2 $\cap$ D1
    know $\mapsto$ C2 $\cap$ B1 $\cap$ (A1 $\cup$ A2 $\cup$ A3)
    think $\mapsto$ C2 $\cap$ (B1 $\cup$ B2) $\cap$ (A1 $\cup$ A2 $\cup$ A3)
    doubt $\mapsto$ C2 $\cap$ B3

Figure 15: Lexicon for verbs of belief

6. Logical Domains

The domains dealt with thus far are empirical in nature. That is, their models are based on empirical data about language use by native speakers. Following the terminology of model theory, these domains will be called *nonlogical domains*. In contrast to these are lexical items that convey the logical structure of English. Examples are determiners and pronouns. These will be called *logical domains*. It is clear that the source of data defining these domains is principally derived from a theoretic understanding of English semantics and the logic of thought.

The first logical domain to be considered is determiners. Syntactically determiners combine with phrases exemplified by common nouns to yield noun phrases. Semantically determiners denote binary relations on the power set of the model universe. Thus if **A** and **B** denote subsets of the universe and **D** is a determiner, DAB asserts that subset A stands in the D relation to subset B. For example, **all men are mortal** denotes **all men mortal**, i.e., **men** stands in the inclusion relation to **mortal**.

Construction of a basis for this domain will be only partial, leaving some subspaces undecomposed. For the subspace of quantifiers, two subbases will be considered.

A broad subdivision of the domain contrasts count and mass determiners, denoting discrete and continuous relations respectively. An extensive catalog of count determiners can be found in [14]. The first level basis B then consists of the single partition $P = \{\text{count}, \text{mass}\}$. Only atom a_1 will be decomposed. But first some properties of count determiners are defined.

If, for all subsets A and B, DAB is equivalent to $DA(B \cap A)$, then **D** is said to be *conservative*. If the assertion remains true when restricted to any universe containing A, then **D** is said to possess the *extension* property. If the truth of DAB depends only upon the numbers of individuals in the subsets defined by A and B (viz., $\mid A - B \mid$,

$| A \cap B |$, $| B - A |$, and $|\sim (A \cup B) |$), then **D** is said to possess the *quantity* property. Count determiners that possess all three properties are called *quantifiers*. These constitute an important and interesting subset. (See [3].)

The quantifiers and nonquantifiers are used to construct a second level basis, viz., $B_1 = \{P_1\}$ where $P_1 = \{\text{quant}, \text{nonquant}\}$. Among the nonquantifiers are the definite determiners: **the, both, that, these**, etc. On one view, these determiners are context indicators that function to restrict the universe [23]. Possessives are also nonquantifiers since they do not satisfy quantity. A subbasis for $a_{1.1}$ will be constructed next.

Quantifiers are subdivided by a number of properties, such as monotonicity (left, right, upward, downward), continuity, first-order definability, symmetry and transitivity [3, 4]. The most important are the monotonicity properties, denoted $\uparrow$ MON, $\downarrow$ MON, MON $\uparrow$, and MON $\downarrow$, which will be used to construct an extended basis for $a_{1.1}$. But first, to achieve independence, $a_{1.1}$ is partitioned into trivial and nontrivial quantifiers. The trivial quantifiers are the empty and universal quantifiers. Therefore $B_{1.1} = \{P_{1.1}\}$ where $P_{1.1} = \{\text{triv}, \text{nontriv}\}$. Then $a_{1.1.2}$ is partitioned into quantifiers that have and those that do not have the *variety* property. A quantifier **D** has the variety property if and only if for nonempty **A** there are **B** and **B'** such that **DAB** and $\neg$**DAB'**; i.e., the second argument makes a difference. Examples of determiners **D** not having the variety property are **there are at least n A**, **there are at most n A**, and **there are exactly n A**. Therefore $B_{1.1.2} = \{P_{1.1.2}\}$, where $P_{1.1.2} = \{\text{var}, \text{nonvar}\}$.

Now $a_{1.1.2.1}$ can be decomposed by $B_{1.1.2.1} = \{P_{1.1.2.1.1}, P_{1.1.2.1.2}\}$, where $P_{1.1.2.1.1} = \{\uparrow$ MON, $\downarrow$ MON, $|$ MON$\}$ and $P_{1.1.2.1.2} = \{$MON $\uparrow$, MON $\downarrow$, MON $|\}$. ($|$ indicates absence of either upward or downward monotonicity.) $P_{1.1.2.1.1}$ and $P_{1.1.2.1.2}$ are independent because each atom defined by $B_{1.1.2.1}$ is populated. For example, $a_{1.1.2.1.1}, \ldots, a_{1.1.2.1.9}$ contain, respectively, denotations of **some, all, most, not all, no, less than half**,

at least m are and at least n are not, at most m are and at most n are not, and **exactly n.**

Finally, consider atom $a_{1.1.2.1.1}$, containing denotations of ↑ MON ↑ quantifiers. These quantifiers are all disjunctions of quantifiers of the form **there are at least m A,** **and at least n of them are B.**

This is enough of a complete extended basis to show that the space of determiners can be partitioned by important properties but that these properties are interrelated in complex ways. An alternative, and in some ways more natural basis for $a_{1.1}$ is defined by the van Benthem "tree of numbers" [4], which can be described as follows. As a consequence of **D** possessing the properties of conservativity, extension and quantity, **DAB** only depends on | A − B | and | A ∩ B |. That is, the truth of **DAB** depends on whether the pair of integers ⟨| A − B |,| A ∩ B |⟩ is appropriate or not. Therefore the quantifiers can be modeled in the universe $\mathbf{N} \times \mathbf{N}$ (the tree of numbers) where $\mathbf{N}$ is the nonnegative integers. Each quantifier denotes a subset of this universe. Let $P_1 = \{\{i\} \times \mathbf{N} \mid i \in \mathbf{N}\}$ and $P_2 = \{\mathbf{N} \times \{j\} \mid j \in \mathbf{N}\}$. Then P_1 and P_2 are independent and form a basis $B = \{P_1, P_2\}$ which will be referred to as the *Bentham basis*.

This semantic space contrasts in several respects with those considered in previous sections. The universes in which previous semantic domains were interpreted could be thought of as individuals or tuples of individuals. Partitions of the universes were finite and unordered. But the lexical items in those domains denote properties of individuals (e.g., people). Determiners, by contrast, denote relations between properties of individuals. It is to be expected therefore that the universe in which determiners are interpreted will be different, and indeed it is. The Benthem universe is an abstract numerical structure. Its partitions are infinite (each indexed by $\mathbf{N}$) and totally ordered. This semantic space represents the essential semantic properties of a special class of relations. The atoms of this semantic space are tuples $\langle m, n \rangle$ of integers. If determiner **D** denotes $\langle m, n \rangle$, then **DAB** can be rendered **exactly n A's are B's**

38

and exactly m A's are not B's.

Examples of subspaces of the semantic space defined by B are the following.

all denotes $(\{0\} \times \mathbf{N}) \cap (\mathbf{N} \times \mathbf{N})$, or equivalently $\{0\} \times \mathbf{N}$.

at least three denotes $\mathbf{N} \times \bigcup_{j \geq 3}\{j\}$.

between five and ten denotes $\mathbf{N} \times \{5, 6, 7, 8, 9, 10\}$.

These are all elementary subsets.

Among quantifiers that are not first-order definable, it is interesting that

an odd number of denoting $\mathbf{N} \times \bigcup_{j \in \mathbf{N}}\{2j + 1\}$,

is an elementary subset, while

more than half which denotes $[\{0\} \times \bigcup_{j \geq 1}\{j\}] \cup [\{1\} \times \bigcup_{j \geq 2}\{j\}] \cup \cdots$,

is not an elementary subset.

The Benthem basis does not differentiate between some lexical items generally considered nonsynonymous however. For example, **all**, **every** and **each** denote the same subspace, viz., $\{0\} \times \mathbf{N}$. But, as Nida ([17], p. 106) points out, these lexical items differ in the component of "distribution." That is, **all men** conveys totality, **every man** conveys partial distribution, and **each man** conveys distribution. Even the latter can be refined to convey more or less distribution as shown by the phrases **each man**, **each of the men** and **each one of the men**.

Let $P_3 = \{\text{total,partially distributed,distributed}\}$ be another partition of the quantifier semantic space. Assume, to permit further illustration of the theory, that P_3 is independent of the Benthem basis. Then $B' = \{P_1, P_2, P_3\}$ is a refinement of the Benthem basis. Thus, for example, in the refined space

all denotes $\{0\} \times \mathbf{N} \times \{\text{total}\}$.

It is likely that other lexical features are also necessary to adequately refine the Benthem basis.

If certain properties of quantifiers, such as monotonicity, are not used to partition the space, they might be ascertained in either of two ways. First, whether a lexical item has a particular property might be inferred from its denotation. For example, assuming the Benthem basis, downward left monotonicity is present if and only if given any point $\langle x, y \rangle$ in the subspace, all points $\langle x', y \rangle$ for $x' \leq x$ and $\langle x, y' \rangle$ for $y' \leq y$ are also in the subspace. Second, the denotation of a lexical item might be used (as a key or pointer) to enter an encyclopedia that would provide the desired information.

A second example of a logical domain is provided by the pronouns. According to Montagovian theory, pronouns function as variables. But an English pronoun differs from a variable of the predicate calculus in that it possesses a *sort* that restricts the context in which it may occur and the binding that it may receive. For example, **he, him,** or **his** can bind only to a noun phrase having masculine gender and singular number. As a result of the binding, the pronoun acquires the denotation of a noun phrase or a possessive determiner, and can play the semantic role of a nominative noun phrase, objective noun phrase, or possessive determiner, respectively.

It must be emphasized that pronouns (like variables) do not denote individuals, sets of individuals or relations between sets of individuals. They are logical entities that can acquire indirect denotations of those kinds through binding to noun phrases. The only properties possessed by pronouns, other than the property of being a pronoun, relate to their sorts. Therefore the universe in which they are interpreted is abstract, consisting of elements that represent various combinations of logical and syntactic sorts.

The relevant lexical features are identified by logical and syntactic analysis. For the domain of personal and possessive pronouns they are listed in Figure 16.

The lexical features are related as follows.

40

A, B and C each partition 1.

$D_i \subseteq A_3 \cap B_1$, for $i = 1, 2, 3$, and D partitions $A_3 \cap B_1$.

A basis and lexicon is constructed as before. The result is shown in Figure 17. The subscripts are used to make homonyms distinct.

Based on these examples of logical domains, the conclusions drawn in previous sections are extended as follows.

13. In addition to the nonlogical domains of nouns and verbs, logical domains such as determiners and pronouns can be accommodated by this theory of lexical semantics.

14. Models for the logical lexical domains are abstract logical or mathematical structures. They may be infinite as well as finite. The partitions may be ordered as well as unordered.

A. Person
    1. First
    2. Second
    3. Third
B. Number
    1. Singular
    2. Plural
C. Case
    1. Nominative
    2. Objective
    3. Possessive
D. Gender
    1. Masculine
    2. Feminine
    3. Neuter

Figure 16: Lexical features for pronouns.

$B = \{A,B,C\}$

$B_7 = \{D\} \qquad B_8 = \{D\} \qquad B_9 = \{D\}$

v: **I** $\mapsto$ A1 $\cap$ B1 $\cap$ C1
   you$_1$ $\mapsto$ A2 $\cap$ B1 $\cap$ C1
   he $\mapsto$ A3 $\cap$ B1 $\cap$ C1 $\cap$ D1
   she $\mapsto$ A3 $\cap$ B1 $\cap$ C1 $\cap$ D2
   it$_1$ $\mapsto$ A3 $\cap$ B1 $\cap$ C1 $\cap$ D3
   we $\mapsto$ A1 $\cap$ B2 $\cap$ C1
   you$_2$ $\mapsto$ A2 $\cap$ B2 $\cap$ C1
   they $\mapsto$ A3 $\cap$ B2 $\cap$ C1
   me $\mapsto$ A1 $\cap$ B1 $\cap$ C2
   you$_3$ $\mapsto$ A2 $\cap$ B1 $\cap$ C2
   him $\mapsto$ A3 $\cap$ B1 $\cap$ C2 $\cap$ D1
   her$_1$ $\mapsto$ A3 $\cap$ B1 $\cap$ C2 $\cap$ D2
   it$_2$ $\mapsto$ A3 $\cap$ B1 $\cap$ C2 $\cap$ D3
   us $\mapsto$ A1 $\cap$ B2 $\cap$ C2
   you$_4$ $\mapsto$ A2 $\cap$ B2 $\cap$ C2
   them $\mapsto$ A3 $\cap$ B2 $\cap$ C2
   my $\mapsto$ A1 $\cap$ B1 $\cap$ C3
   your$_1$ $\mapsto$ A2 $\cap$ B1 $\cap$ C3
   his $\mapsto$ A3 $\cap$ B1 $\cap$ C3 $\cap$ D1
   her$_2$ $\mapsto$ A3 $\cap$ B1 $\cap$ C3 $\cap$ D2
   its $\mapsto$ A3 $\cap$ B1 $\cap$ C3 $\cap$ D3
   our $\mapsto$ A1 $\cap$ B2 $\cap$ C3
   your$_2$ $\mapsto$ A2 $\cap$ B2 $\cap$ C3
   their $\mapsto$ A3 $\cap$ B2 $\cap$ C3

Figure 17: Lexicon for pronouns.

7. Conclusion

A complete lexicon for English would incorporate a variety of lexical domains. Some idea of its extent is conveyed by the semantic domains defined by Nida for classification of lexical meaning of Koine Greek [17]. This classification may not be ideal data for construction of an extended basis. It might yield a poorly structured space as did the initial classification of c-kinship. It does however illustrate the embedding of semantic domains starting with the most general.

A sampling of the highest levels is given below. For more detail one can consult [17] and further references given there.

1. Entities

    (a) Inanimate

        i. Natural

            A. Geographical

            B. Natural substances

            C. Flora and plant products

        ii. Manufactured or constructed

            A. Artifacts

            B. Processed substances

            C. Constructions

    (b) Animate

        i. Animals, birds, insects

        ii. Humans

            A. Generic and distinctions by age and sex

            B. Kinship

C. Groups

D. Body, body parts and body products

iii. Supernatural powers or beings

2. Events

(a) Physical

(b) Physiological

(c) Sensory

(d) Emotive

etc.

3. Abstracts

(a) Time

(b) Distance

(c) Volume

(d) Velocity

etc.

4. Relationals

(a) Spatial

(b) Temporal

(c) Deictic

(d) Logical

etc.

Of course even a space of lexical meaning of the scope suggested by this classification will not express certain kinds of knowledge about lexical entities. This additional knowledge might be called "encyclopedic" information. For example, **father** (in the c-kinship sense) is adequately defined as a relation of direct lineality between persons in which the first person is male and of the generation preceding that of the second person. That the first person is typically also a husband, that a strong emotional bond usually exists between the first and second persons, that the first person is believed to "know best" in matters affecting the second person, etc. can be considered encyclopedic information, neither necessary nor appropriate for a lexicon. If the lexicon is adequate to distinguish the meanings denoted by the lexical items in its vocabulary, then the unique representations provided by the lexicon can serve as references to such encyclopedic knowledge.

Therefore it seems reasonable that a lexicon, as defined here, will be only one component of the total lexical knowledge of a natural language faculty. An encyclopedic knowledge base is another. Other components such as a morphological analyzer (e.g., see [5, 6, 12]) will also be necessary.

The role of lexical knowledge in language understanding is an important one. To see this, consider the simple sentences **Mary loves every man who loves her** and **An actor adores Mary**. These sentences entail **Mary loves an actor**, but only in the presence of lexical knowledge. Using the Logic of Generalized Quantifiers (L(GQ)) [2], this can be demonstrated as follows.

all $\hat{x}[\text{man}(x) \wedge \text{love}(x,m)] \, \hat{y}[\text{love}(m,y)]$	premise
some actor $\hat{y}[\text{adore}(y,m)]$	premise
some actor $\hat{y}[\text{love}(y,m)]$	$v(\textbf{adore}) \leq v(\textbf{love})$ (lexical knowledge)
	some is right upward monotonic
	(lexical knowledge)

some actor $\hat{y}[\text{love}(y,m) \wedge \text{actor}(y)]$	**some** is conservative (lexical knowledge)
all $\hat{x}[\text{actor}(x) \wedge \text{love}(x,m)] \; \hat{y}[\text{love}(m,y)]$	$v(\text{actor}) \leq v(\text{man})$ (lexical knowledge)
	all is left downward monotonic
	(lexical knowledge)
some actor $\hat{y}[\text{love}(m,y)]$	**some** is right upward monotonic
	(lexical knowledge)
	all $AB \Leftrightarrow v(A) \leq v(B)$ (lexical knowledge)

A lexicon as defined here provides entailment (i.e., meaning inclusion) directly. It may provide more. A set of lexical features denoting a partition is similar in many respects to a "contrast set." Grandy [10] argues that membership in a contrast set is an essential part of the meaning of a lexical item and that semantic phenomena such as metaphor and question are best understood in terms of contrast sets. The representation in a semantic space may therefore provide more than direct entailment. It may also provide the kind of contextual data thought to explain the above phenomena.

In addition to an explanatory role, a theory can sometimes provide a basis for implementing the capabilities it purports to explain. It might be of interest to examine the practical aspects of the theory of lexical semantics presented here. Lexicons of the kind described have direct application to mechanical reasoning.

An important issue is the complexity of the structures and operations involved. First it can be stated that construction of a basis for a given semantic domain is NP-hard since the Boolean Satisfiability Problem (SAT) reduces to the problem of basis construction. Two observations are relevant here. First, exponentially complex computations are infeasible only if the size of the input is large. An exponential computation may be more efficient than a polynomial computation on small quantities of data. Semantic domains seem to be small. A similar observation holds for many human

capabilities. Second, the construction of a basis is performed only once (to create a lexicon) or is performed incrementally (to evolve a lexicon).

By the same reasoning, computation of the normal form of an arbitrary lexical expression is NP-hard. A similar observation with regard to size is relevant. With regard to complexity, semantic spaces, as representations for lexical semantics, fare no worse than logic or semantic nets.

However, many useful computations are of polynomial complexity. Witness the L(GQ) example above and the examples of Figure 4. These exemplify the kind of reasoning that occurs routinely in natural language understanding and generation. The simplest kind of reasoning is the traditional syllogistic. In this formalism, **all A are B** if $v(A) \leq v(B)$; and **some A are B** if $v(A) \cap v(B) \neq 0$. An extension of traditional syllogistic reasoning is found in Sommers' Term Calculus [20]. In this logic as well as in the Logic of Generalized Quantifiers monotonicity properties play a central role. The partial order provided by the lexicon directly supports inference based on monotonicity.

There is this difference between semantic spaces and other representations. The independence of the dimensions of a semantic space makes all operations inherently parallel, making this approach well matched to the resources of advanced computer systems.

Appendix

This appendix formalizes the definitions given in the body of the paper, and gives proofs for the claims made there. The first section deals with the semantic space as a model for lexical domains. The second section defines the normal form of subspaces and develops a Boolean algebra of normal forms.

A1. Semantic Space

Let S be a nonempty set and Su be the power set of S. Su is viewed as the set of properties of the members of S.

DEFINITION 1 *Let $\{p^j | j \in J\}$ be a subset of Su and P be a function from 2^J into Su such that for any $J' \subseteq J$, $P(J') := \bigcup \{p^j | j \in J'\}$.[7] If $j \in J$, $P(\{j\})$ is written $P(j)$. P is called a* partition *of S if it satisfies:*

(i) $P(J) = S$

(ii) $P(J') = \emptyset$ iff $J' = \emptyset$

(iii) $\forall J', J'' \in 2^J : P(J') \cap P(J'') = P(J' \cap J'')$

This definition is equivalent to the one used in the body of the paper. It is introduced here because it results in more succinct expressions. However, where convenient the usual notation $P = \{p^j | j \in J\}$ will also be used.

DEFINITION 2 *Let $B = \{P_i | i \in I_B\}$ be a set of partitions of S. A subset x of S is called an* elementary subset of S defined by B *if it can be written $x = \bigcap_{i \in I_B^x} P_i(J_i^x)$ where $I_B^x \subseteq I_B$ is finite.*

---

[7]The notation "$X := Y$" means that X is defined to be equal to Y; "$X :\Leftrightarrow Y$" means that X is defined to be logically equivalent to Y; etc.

Notice that $\bigcap_{i \in I_B^x} P_i(J_i^x) = \bigcap_{i \in I_B} P_i(J_i^x)$ where $\forall i \notin I_B^x : J_i^x := J_i$. The latter form is called the *standard form for x relative to B*. The former form is called the *abbreviated standard form for x relative to B*. The conjunct $P_i(J_i^x)$ is called the ith *component* of x. The set of all elementary subsets defined by B is denoted Es_B.

LEMMA 3 *Let $B = \{P_i | i \in I_B\}$ be a set of partitions of S and $x = \bigcap_{i \in I_B} P_i(J_i^x)$, $y = \bigcap_{i \in I_B} P_i(J_i^y)$ be elementary subsets of S defined by B. Then $x \cap y$ is an elementary subset and $x \cap y = \bigcap_{i \in I_B} P_i(J_i^x \cap J_i^y)$.*

proof: $x \cap y = (\bigcap_{i \in I_B} P_i(J_i^x)) \cap (\bigcap_{i \in I_B} P_i(J_i^y)) = \bigcap_{i \in I_B}(P_i(J_i^x) \cap P_i(J_i^y)) = \bigcap_{i \in I_B} P_i(J_i^x \cap J_i^y)$ by Definition 1. □

Thus intersection of elementary subsets is computed componentwise. Since Es_B is closed under set intersection, it forms a meet semilattice, ordered by set inclusion, denoted **Es$_B$**. It has the zero element $\emptyset$ and the unit element S, denoted 0 and 1 respectively.

Let Su_B be the closure of Es_B under finite set union. Then Su_B forms a lattice, denoted **Su$_B$**. Since it is a sublattice of the subset lattice formed by Su, it is distributive. **Es$_B$** is embedded as a meet semilattice in **Su$_B$**.

DEFINITION 4 *Let $B = \{P_i | i \in I_B\}$ be a set of partitions of S. B is called a basis of S if $\forall x = \bigcap_{i \in I_B^x} P_i(J_i^x) \in Es_B : x = 0$ iff $\exists i \in I_B^x : J_i^x = \emptyset$.*

LEMMA 5 *Let B be a basis of S and $x = \bigcap_{i \in I_B} P_i(J_i^x)$ be a nonzero elementary subset. Let $q \in I_B$ and $r \in J_q$. Then $P_q(r) \cap x \neq 0$ iff $r \in J_q^x$.*

proof: Since B is a basis, $x \neq 0$ iff $\forall i \in I_B : J_i^x \neq \emptyset$. Then $P_q(r) \cap x \neq 0$ iff $\{r\} \cap J_q^x \neq \emptyset$, ie., iff $r \in J_q^x$. □

THEOREM 6 *Let B be a basis of S and $x = \bigcap_{i \in I_B} P_i(J_i^x)$ be a nonzero elementary subset. Then the standard form for x relative to B is unique. It follows that the abbreviated standard form for x is unique as well.*

proof: Suppose that $\bigcap_{i \in I_B} P(J_i^1)$ and $\bigcap_{i \in I_B} P(J_i^2)$ are standard forms for x. Let $q \in I_B$ and $r \in (J_q^1 \oplus J_q^2)$. By Lemma 5, $r \in J_q^1$ iff $P_q(r) \cap x \neq 0$ iff $r \in J_q^2$. Therefore $J_q^1 \oplus J_q^2 = \emptyset$ and the two standard forms are identical. $\square$

LEMMA 7 *Let B be a basis of S and $x = \bigcap_{i \in I_B} P_i(J_i^x)$, $y = \bigcap_{i \in I_B} P_i(J_i^y)$ be nonzero elementary subsets of S defined by B. Then $x \subseteq y$ iff $\forall i \in I_B : J_i^x \subseteq J_i^y$. Equivalently, $x \subseteq y$ iff $I_B^y \subseteq I_B^x \wedge \forall i \in I_B^y : J_i^x \subseteq J_i^y$.*

proof: $x \subseteq y$ iff $x \cap y = x$. $x \cap y = \bigcap_{i \in I_B} P_i(J_i^x \cap J_i^y)$ by Lemma 3. Since the standard form is unique (Theorem 6), $\forall i \in I_B : J_i^x \cap J_i^y = J_i^x$. I.e., $J_i^x \subseteq J_i^y$. $\square$

EXAMPLE. Let $S = \mathbf{N}$, the non-negative integers. Let $P_1 = \{\{i | i = 0, mod4\}, \{i | i = 1, mod4\}, \{i | i = 2, mod4\}, \{i | i = 3, mod4\}\}$ and $P_2 = \{\{i | is\text{-}prime(i)\}, \{i | \neg is\text{-}prime(i)\}\}$. Then P_1 and P_2 are partitions of S. But note that $p_1^1 \cap p_2^1 = 0$ since the conjunction $i = 0, mod4 \wedge is\text{-}prime(i)$ is logically impossible. Thus, while P_1 and P_2 are partitions of S, $\{P_1, P_2\}$ is not a basis of S.

EXAMPLE. Let $S = \mathbf{N}_+$, the positive integers, and let π_i denote the ith prime. Let $B = \{P_i | i \in I_B\}$, where $I_B = \mathbf{N}_+$. Let $P_i = \{p_i^j | j \in J_i\}$ where $J_i = \mathbf{N}$. Let $p_i^j = \{n \in S | divides(\pi_i^j, n) \wedge \neg divides(\pi_i^{j+1}, n)\}$ for $j \neq 0$, and $p_i^0 = \{n \in S | \neg divides(\pi_i, n)\}$. Then B is a basis of S.

If x and y are elements of $\mathbf{Es_B}$, y *covers* x, written $x \prec y$, iff $\forall z \in \mathbf{Es_B} : x < z \leq y$ implies $z = y$. x is an *atom* iff $0 \prec x$.

It is not necessary that atoms exist in $\mathbf{Es_B}$. In the second example above, $\mathbf{Es_B}$ has no atoms.

Let P be a partition of $Y \subseteq S$ and let $X \subseteq Y$. Define the restriction of P to X: $P \uparrow_X (J^1) := P(J^1) \cap X$. Note that $P \uparrow_X$ may fail to be a partition of X because it does not satisfy the conditions of Definition 1. Let $B = \{P_i | i \in I_B\}$ be a basis of Y. Define the restriction of B to X: $B \uparrow_X := \{P_i \uparrow_X | i \in I_B\}$. $B \uparrow_X$ may fail to be a basis of X because some $P_i \uparrow_X$ is not a partition of X or because the condition of Definition 4 is not satisfied.

Let Es_B be the set of elementary subsets defined by basis B and a_1, a_2 be atoms of $\mathbf{Es_B}$. Let B' be a basis of $X \subseteq S$ such that $B' \cap B = \emptyset$. Suppose that $B' \uparrow_{a_1}$ is a basis of a_1 but $B' \uparrow_{a_2}$ is not a basis of a_2. It may be that B' defines properties that are relevant to members of a_1 but inconsistent with members of a_2. For example, properties peculiar to animate entities would be inconsistent if applied to inanimate entities.

Let $B' \uparrow_{a_1} = B_1$. B_1 determines a semilattice of elementary subsets, $\mathbf{Es_{B_1}}$, with unit element a_1. B and B' together determine a combined semilattice $\mathbf{Es_B}$, where $\mathbf{Es_B}$ is embedded in the interval $[0, 1]$ and $\mathbf{Es_{B_1}}$ is embedded in the interval $[0, a_1]$ such that the covering relation is preserved for all nonzero elements.

EXAMPLE. Let $B = \{P_1, P_2\}$, $P_1 = \{NT, T\}$, $P_2 = \{NP, P\}$. Suppose that $B' = \{Q_1, Q_2\}$, where $Q_1 = \{SL, PH\}$ and $Q_2 = \{NTT, TT\}$, and that $B' \uparrow_{a_3}$ and $B' \uparrow_{a_4}$ are bases of a_3 and a_4, respectively. Suppose further that $B' \uparrow_{a_1}$ and $B' \uparrow_{a_2}$ are not bases. The resulting partitions of S form three bases: one *first level* basis and two *second level* bases. They can be diagrammed as shown in Figure 18.[8]

This situation is generalized as follows. Let T be a tree indexing defined in the usual way: (i) $T \subset \mathbf{N_+^*}$, where $\mathbf{N_+}$ denotes the positive integers and $*$ denotes the Kleene closure; (ii) $\alpha, \beta \in \mathbf{N_+^*}$ and $\alpha.\beta \in T$ implies $\alpha \in T$; (iii) $\alpha \in \mathbf{N_+^*}$, $b \in \mathbf{N_+}$ and $\alpha.b \in T$

---

[8]This example is part of an example in [17] dealing with nouns denoting rigid fasteners. The distinguishing properties are: not threaded (NT), threaded (T), not pointed (NP), pointed (P), slot drive (SL), Phillips drive (PH), not threaded to top (NTT) and threaded to top (TT).

implies $\forall c \in \mathbf{N}_+: c < b \Rightarrow \alpha.c \in T$.

Let $\mathcal{B} = \{B_\alpha | \alpha \in T\}$ be a system of bases such that $B = B_\epsilon$ is a basis of S (ϵ denotes the empty string) and $B_{\alpha.b}$ is a basis of $a_{\alpha.b}$, an atom of $\mathbf{Es_{B_\alpha}}$. $\mathcal{B}$ is called an *extended basis* of S.

Define $Es_\mathcal{B} := \bigcup_{\alpha \in T} Es_{B_\alpha}$. Set intersection is given as follows. Let $\alpha, \beta \in T$, $x = \bigcap_{i \in I_{B_\alpha}} P_{\alpha,i}(J_{\alpha,i}^x)$ and $y = \bigcap_{i \in I_{B_\beta}} P_{\beta,i}(J_{\beta,i}^y)$. Then

$$x \cap y := \begin{cases} \bigcap_{i \in I_{B_\alpha}} P_{\alpha,i}(J_{\alpha,i}^x \cap J_{\alpha,i}^y) & \text{if } \alpha = \beta \\ x & \text{if } \alpha = \beta.b.\gamma \text{ and } y \cap a_{\beta.b} = a_{\beta.b} \\ y & \text{if } \beta = \alpha.b.\gamma \text{ and } x \cap a_{\alpha.b} = a_{\alpha.b} \\ 0 & \text{otherwise} \end{cases}$$

Thus $Es_\mathcal{B}$ forms a meet semilattice, denoted $\mathbf{Es_\mathcal{B}}$. As before, $\mathbf{Es_{B_\alpha}}$ is embedded in $[0, a_\alpha]$ such that the covering relation is preserved for all nonzero elements. $a_\epsilon = a$ is taken to be 1; thus $\mathbf{Es_B}$ is embedded in $[0, 1]$.

Let $Su_\mathcal{B}$ be the closure of $Es_\mathcal{B}$ under finite set union. Then $\mathbf{Su_\mathcal{B}}$ is a distributive lattice. The (possibly empty) set A of atoms of $\mathbf{Su_\mathcal{B}}$ consists of atoms defined by bases in $\mathcal{B}$ and not further decomposed. That is, an atom $a_{\alpha.b}$ defined by basis $B_\alpha \in \mathcal{B}$ is an atom of $\mathbf{Su_\mathcal{B}}$ just in case α is maximal in T (i.e., $\alpha.1 \notin T$).

$\mathbf{Su_B}$ can be visualized as a space of dimension equal to the cardinality of I_B. The $P_i(j)$ are coordinate values that define hyperplanes in this space. Each $P_i \in B$ is regarded as a "dimension of meaning". The $P_i(j)$ are mutually antonymous "primitive meanings." Elementary subsets are the elementary concepts, defined by these primitive meanings, from which arbitrarily complex (finite) concepts can be constructed.

A2. Normal Form

In this section a unique representation, or normal form, for elements of $\mathbf{Su}_B$ is defined. Then an algebra of normal forms is defined.

An elementary subset x is *maximal in* $y \in Su_B$ iff $x \subseteq y$ and for any elementary subset z, $x \subseteq z \subseteq y$ implies $z = x$. The properties of maximal elementary subsets will be developed in a lattice (the ideal lattice) in which the elementary subsets are distinguished elements.

DEFINITION 8 *Let* $X \subseteq Es_B$. *The* order ideal *generated by* X *is defined* $I(X) :=$ $\{y \in Es_B - \{0\} | y \subseteq x \text{ for some } x \in X\}$. *If* $X = \{x\}$ *then* $I(X)$ *is* principal *and is written* $I(x)$. *If* X *is finite then* $I(X)$ *is* finitely generated.

Since unions and intersections of order ideals are again order ideals, the set of all order ideals ordered by set inclusion is a lattice. This lattice is called the *ideal lattice* of $\mathbf{Es}_B$. It contains the zero element $\emptyset$ and unit element $Es_B - \{0\}$. The finitely generated ideals of $\mathbf{Es}_B$ form a sublattice, denoted $\mathbf{H}_B$, of the ideal lattice. Since $\mathbf{H}_B$ is a sublattice of $2^{Es_B - \{0\}}$, it is a distributive lattice. $\mathbf{Es}_B$ is embedded as a meet semilattice in $\mathbf{H}_B$ by the mapping $x \mapsto I(x)$.

The next three paragraphs review relevant facts from lattice theory about finite decomposition [1, 11].

Let $\mathbf{L}$ be a lattice. An element $x \in \mathbf{L}$ is *(join) irreducible* iff $\forall y, z \in \mathbf{L}$: $x = y \cup z$ implies either $x = y$ or $x = z$. An expression $x = x_1 \cup \cdots \cup x_k$, where $x_1, \ldots, x_k$ are irreducible, is a *(finite) decomposition* of x. If no x_k can be eliminated, the decomposition is *irredundant*. If x has a decomposition, it has an irredundant decomposition, formed by deleting superfluous elements.

54

Now let $\mathbf{L}$ be a distributive lattice. *If $x \in \mathbf{L}$ is irreducible and $x \leq x_1 \cup \cdots \cup x_k$, where $x_1, \ldots, x_k$ are arbitrary elements, then $x \leq x_q$ for some q, $1 \leq q \leq k$.* Since $\mathbf{L}$ is distributive, $x = x \cap (x_1 \cup \cdots \cup x_k) = x \cap x_1 \cup \cdots \cup x \cap x_k$. Since x is irreducible, $\exists q : x = x \cap x_q$. Thus $x \leq x_q.$

If $x \in \mathbf{L}$ has an irredundant decomposition, it is unique. Suppose x has two distinct irredundant decompositions $x = x_1 \cup \cdots \cup x_k = y_1 \cup \cdots \cup y_l$. Let $x_q \notin \{y_1, \ldots, y_l\}$. Then $x_q \leq y_1 \cup \cdots \cup y_l$ implying $\exists r : x_q \leq y_r$. Similarly, $y_r \leq x_1 \cup \cdots \cup x_k$ which implies $\exists t : y_r \leq x_t$. Thus $x_q \leq y_r \leq x_t$ yielding a contradiction since $t = q$ implies that $x_q = y_r$ and $t \neq q$ implies that x_q is redundant.

Since $\mathbf{H}_B$ and $\mathbf{Su}_B$ are distributive lattices, all the above results apply.

The irreducible elements of $\mathbf{H}_B$ are precisely the principal ideals, i.e., the images of elementary subsets. To see this, consider nonzero ideal $I(X) \in \mathbf{H}_B$ where $X \subseteq Es_B - \{0\}$. Then $z \prec I(X)$ iff $z = I(X) - \{x\}$ for $x \in X$. Therefore $I(X)$ is irreducible iff $I(X) = I(x)$ for $x \in Es_B - \{0\}$, i.e., iff $I(X)$ is principal.

Every element x of $\mathbf{H}_B$ is a finitely generated ideal. Let $x = I(\{x_1, \ldots, x_k\})$. Then $x = I(x_1) \cup \cdots \cup I(x_k)$ is a decomposition of x. By the above results, x has a unique irredundant decomposition. In the sequel it will be assumed that the generators given for an element of $\mathbf{H}_B$ are irredundant and therefore unique.

DEFINITION 9 *Let $x \in \mathbf{H}_B$. The* pseudocomplement *of x is that element $x^* \in \mathbf{H}_B$ such that $\forall y \in \mathbf{H}_B : y \cap x = 0$ iff $y \subseteq x^*$. Thus, if it exists, $x^* := \sup\{y \in \mathbf{H}_B | x \cap y = 0\}$.*

Because of the structure of $\mathbf{H}_B$, the pseudocomplement relative to an interval is useful.

DEFINITION 10 *Let $\mathcal{B}$ be a system of bases with domain T. Let $\alpha = \beta.b \in T$, a_α be*

an atom defined by basis B_β and $x \in [0, a_\alpha]$. Then the pseudocomplement of x in $[0, a_\alpha]$ *is defined* $x_\alpha^* := sup\{y \in [0, a_\alpha] | x \cap y = 0\}$.

LEMMA 11 *Let* $\alpha = b_1.b_2.\cdots.b_m$. *Then* $x^* = \bigcup_{k=1}^m (a_{b_1.\cdots.b_k})_{b_1.\cdots.b_{k-1}}^* \cup x_\alpha^*$. *(Note that b_0 is interpreted as the empty string, ϵ.)*

proof: Let $\alpha = \beta.b$. Then it follows from $sup\{y \in [0, a_\beta] | x \cap y = 0\} = sup\{y \in [0, a_\alpha] | x \cap y = 0\} \cup sup\{y \in [0, a_\beta] | a_\alpha \cap y = 0\}$ that $x_\beta^* = (a_\alpha)_\beta^* \cup x_\alpha^*$. The lemma follows by induction. $\square$

It will now be shown that $\mathbf{H}_B$ is pseudocomplemented.

LEMMA 12 *Every irreducible element of $\mathbf{H}_B$ has a pseudocomplement.*

proof: First consider the pseudocomplement in an interval with a single basis B. Let $I(x)$ be the principal ideal generated by $x = \bigcap_{i \in I_B^x} P(J_i^x) \in Es_B$. Define $z_i := P(J_i - J_i^x) \in Es_B$. Then by Lemma 3, $x \cap z_i = 0$ for all $i \in I_B^x$. Moreover, if $y \in Es_B$ such that $x \cap y = 0$ then $\exists i \in I_B^x : y \subseteq z_i$. Since $\mathbf{Es_B}$ is embedded in $\mathbf{H_B}$ as a meet semilattice, $I(x) \cap I(z_i) = 0$ for all $i \in I_B^x$ also. By distributivity of $\mathbf{H_B}$, $I(x) \cap [\bigcup_{i \in I_B^x} I(z_i)] = 0$.

Let $I(\{y_1, \ldots, y_l\}) \in \mathbf{H_B}$ be an arbitrary nonzero element such that $x \cap y = 0$. By distributivity, $I(x) \cap I(y_r) = 0$ for all $1 \le r \le l$, and hence $x \cap y_r = 0$ in $\mathbf{Es_B}$. Then $\exists i \in I_B^x : y_r \subseteq z_i$. Therefore $\forall r : I(y_r) \subseteq \bigcup_{i \in I_B^x} I(z_i)$, and so $I(\{y_1, \ldots, y_l\}) \subseteq \bigcup_{i \in I_B^x} I(z_i)$. Consequently $\bigcup_{i \in I_B^x} I(z_i)$ is the pseudocomplement of $I(x)$.

The general case is similar. Let $I(x)$ be the principal ideal generated by $\bigcap_{i \in I_{B_\alpha}} P_{\alpha,i}(J_{\alpha,i}^x) \in Es_B$, where $\alpha = b_1.b_2.\cdots.b_m$. Then by Lemma 11, $I(x)^* = \bigcup_{k=1}^m I(a_{b_1.\cdots.b_k})_{b_1.\cdots.b_{k-1}}^* \cup I(x)_\alpha^* = \bigcup_{k=1}^m [\bigcup_{i \in I_{B_{b_1.\cdots.b_{k-1}}}} I(P_{b_1.\cdots.b_{k-1},i}(J_{b_1.\cdots.b_{k-1},i} - J_{b_1.\cdots.b_{k-1},i}^{a_{b_1.\cdots.b_k}}))] \cup [\bigcup_{i \in I_{B_\alpha}^x} I(P_{\alpha,i}(J_{\alpha,i} - J_{\alpha,i}^x))]$. $\square$

THEOREM 13 $\mathbf{H}_\mathcal{B}$ *is a pseudocomplemented lattice.*

proof: Consider an arbitrary $x \in \mathbf{H}_\mathcal{B}$. Let $x = x_1 \cup \cdots \cup x_k$ be its decomposition.

(i) $x \cap (x_1^* \cap \cdots \cap x_k^*) = (x_1 \cup \cdots \cup x_k) \cap (x_1^* \cap \cdots \cap x_k^*) = (x_1 \cap x_1^* \cap \cdots \cap x_k^*) \cup \cdots \cup$

$(x_k \cap x_1^* \cap \cdots \cap x_k^*) = 0$

(ii) Let $y \in \mathbf{H}_\mathcal{B}$ such that $x \cap y = 0$. Then $\forall q : x_q \cap y = 0$ which implies $\forall q : y \subseteq x_q^*$,

ie., $y \subseteq (x_1^* \cap \cdots \cap x_k^*)$. Thus $x^* = x_1^* \cap \cdots \cap x_k^*$. $\square$

EXAMPLE. Let $\mathcal{B} = \{B, B_1, B_2\}$, $B_\alpha = \{P_{\alpha,1}, P_{\alpha,2}\}$ for $\alpha \in \{\epsilon, 1, 2\}$, $P_{\alpha,i} = \{p_{\alpha,i}^1, p_{\alpha,i}^2\}$

for $i \in \{1, 2\}$ and $x = p_1^1 \cap p_2^1 \cap p_{1,1}^1 \cup p_1^1 \cap p_2^2 \cap p_{2,1}^1$ (see Figure 19).

Then $x^* = [p_1^1 \cap p_2^1 \cap p_{1,1}^2 \cup p_1^2 \cup p_2^2] \cap [p_1^1 \cap p_2^2 \cap p_{2,1}^2 \cup p_1^2 \cup p_2^1]$

$= [p_1^1 \cap p_2^1 \cap p_{1,1}^2] \cup [p_1^2] \cup [p_1^2 \cap p_2^1] \cup [p_1^1 \cap p_2^2 \cap p_{2,1}^2] \cup [p_1^2 \cap p_2^2]$.

LEMMA 14 *Every elementary subset of* $\mathbf{Su}_\mathcal{B}$ *has a complement.*

proof: The proof follows that of Lemma 12, with the observation that in $\mathbf{Su}_\mathcal{B}$, with x and z_i as defined there, $x \cup \bigcup_{i \in I_\mathcal{B}^x} z_i = 1$. $\square$

THEOREM 15 $\mathbf{Su}_\mathcal{B}$ *is a Boolean lattice.*

proof: A proof similar to that of Theorem 13, using Lemma 14, shows that every element of $\mathbf{Su}_\mathcal{B}$ has a complement. Since $\mathbf{Su}_\mathcal{B}$ is distributive, complements are unique. $\square$

DEFINITION 16 $\sigma : \mathbf{H}_\mathcal{B} \to \mathbf{H}_\mathcal{B}$ *is defined* $\sigma(x) := \overline{x} := x^{**}$.

That σ is a closure operation on $\mathbf{H}_\mathcal{B}$ can be seen as follows. By Definition 9, (i) $x \subseteq x^{**}$ and (ii) $x \subseteq y \Rightarrow y^* \subseteq x^*$. From (i), $x^* \subseteq x^{***}$; from (i) and (ii), $x^{***} \subseteq x^*$; hence $x^* = x^{***}$. Thus $x \subseteq \overline{x}$, $x \subseteq y \Rightarrow \overline{x} \subseteq \overline{y}$ and $\overline{\overline{x}} = \overline{x}$.

The quotient lattice formed by the closed elements of $\mathbf{H}_B$ with set inclusion as the order is denoted $\mathbf{H}_B/\sigma$. The meet is $x \wedge y = x \cap y$. The join is $x \vee y = (x^* \cap y^*)^*$.

It will now be shown that $\mathbf{H}_B/\sigma \cong \mathbf{Su}_B$.

LEMMA 17 $\phi : \mathbf{H}_B \to \mathbf{Su}_B$ defined $\phi(I(X)) = \bigcup X$ is a homomorphism of $\mathbf{H}_B$ onto $\mathbf{Su}_B$. Moreover, $\phi(I(X)) = 0$ iff $I(X) = 0$ and $\phi(I(X)^*) = \phi(I(X))'$.

proof: (i) If $x \in \mathbf{Su}_B$ then $x = \bigcup X$, where $X \subseteq \mathbf{Es}_B$ is finite. But $I(X) \in \mathbf{H}_B$ and $\phi(I(X)) = x$. Therefore ϕ is onto.

(ii) $\phi(I(X) \cup I(Y)) = \phi(I(X \cup Y)) = \bigcup(X \cup Y) = \bigcup X \cup \bigcup Y = \phi(I(X)) \cup \phi(I(Y))$.

(iii) $\phi(I(X) \cap I(Y)) = \phi(I(Z))$ where $Z = \mathbf{irr}\{x \cap y | x \in X, y \in Y\}$ and $\mathbf{irr}$ reduces a set to its irredundant elements. $\phi(I(Z)) = \bigcup Z = (\bigcup X) \cap (\bigcup Y) = \phi(I(X)) \cap \phi(I(Y))$.

(iv) $I(X) = 0$ implies $X = \emptyset$ implies $\bigcup X = \emptyset$ implies $\phi(I(X)) = 0$. On the other hand, $I(X) \neq 0$ implies $X \neq \emptyset$ implies $\bigcup X \neq \emptyset$ implies $\phi(I(X)) \neq 0$.

(v) To see that $\phi(I(X)^*) = \phi(I(X))'$, let $y \in \mathbf{H}_B$ such that $\phi(y) = \phi(I(X))'$. Then $\phi(I(X) \cap y) = \phi(I(X)) \cap \phi(y) = 0$. By (iv), $I(X) \cap y = 0$ and therefore $y \subseteq I(X)^*$, implying $\phi(y) \subseteq \phi(I(X)^*)$. Since $\phi(I(X)^*) \cap \phi(I(X)) = 0$ implies $\phi(I(X)^*) \subseteq \phi(y)$, it follows that $\phi(I(X)^*) = \phi(I(X))'$. □

THEOREM 18 $\mathbf{H}_B/\sigma \cong \mathbf{Su}_B$. Moreover, if $I(X) \in \mathbf{H}_B/\sigma$ then X is exactly the set of elementary subsets maximal in $\bigcup X \in \mathbf{Su}_B$.

proof: Let ϕ_σ denote ϕ restricted to $\mathbf{H}_B/\sigma$. ϕ_σ is an isomorphism if it is 1:1 and onto. ϕ_σ is onto since ϕ is, and for any $I(X) \in \mathbf{H}_B$, $\phi(I(X)^{**}) = \phi(I(X))'' = \phi(I(X))$. To see that ϕ_σ is 1:1, suppose $\phi(I(X)^{**}) = \phi(I(Y)^{**})$. By Lemma 17, $\phi(I(X)^{**} \cap I(X)^*) = 0$ implies $\phi(I(Y)^{**} \cap I(X)^*) = 0$ implies $I(Y)^{**} \cap I(X)^* = 0$ implies $I(Y)^{**} \subseteq I(X)^{**}$. A symmetrical argument yields $I(X)^{**} \subseteq I(Y)^{**}$. Then $I(X)^{**} = I(Y)^{**}$. Thus $\mathbf{H}_B/\sigma \cong \mathbf{Su}_B$.

Now let $x = \bigcup X \in \mathbf{Su}_B$, and $I(Z) = I(X)^{**} \in \mathbf{H}_B/\sigma$. Suppose $y \in \mathbf{Es}_B$ such that $y \subseteq x$. Then $y \cap x' = 0$ and therefore $\phi(I(y) \cap I(X)^*) = 0$. This implies that $I(y) \cap I(X)^* = 0$ and therefore $I(y) \subseteq I(X)^{**}$. But then $y \in I(X)^{**} = I(Z)$ and hence $\exists z \in Z : y \subseteq z$. Thus the elements of Z are exactly the maximal elementary subsets of $\bigcup Z \in \mathbf{Su}_B$. $\square$

Therefore the set of maximal elementary subsets of any subspace of $\mathbf{Su}_B$ is exactly the unique set of irredundant generators of the corresponding closed order ideal of $\mathbf{H}_B$.

EXAMPLE. Let $B = \{P_i | i = 1, 2\}$, $P_i = \{p_i^j | j = 1, 2, 3\}$, $x = [p_1^2 \cap (p_2^2 \cup p_2^3)] \cup [(p_1^2 \cup p_1^3) \cap p_2^2] \cup [p_1^3 \cap (p_2^1 \cup p_2^2)]$, $y = [p_2^2] \cup [p_1^3 \cap (p_2^2 \cup p_2^3)]$. Then $x \cap y = (p_1^2 \cup p_1^3) \cap p_2^2$ and $x \cup y = [p_1^3] \cup [(p_1^2 \cup p_1^3) \cap (p_2^2 \cup p_2^3)] \cup [p_2^2]$. The elementary subsets forming each union are maximal. Therefore the ideals generated by the elementary subsets in the unions for x and y are in $\mathbf{H_B}/\sigma$. Combining these ideals under the operations $\wedge$ and $\vee$, one can see that the results are the ideals generated by the elementary subsets that are maximal in $x \cap y$ and $x \cup y$, respectively.

DEFINITION 19 *Let $x = \bigcup X \in \mathbf{Su}_B$. Let $\overline{I(X)} = I(x_1) \cup \cdots \cup I(x_k)$ be the irredundant decomposition of $\overline{I(X)} \in \mathbf{H}_B$ into irreducible elements. Then the normal form of x is defined $\mathcal{N}(x) := \{x_1, \ldots, x_k\}$.*

Operations on normal forms are defined to parallel operations of $\mathbf{H}_B/\sigma$.

DEFINITION 20 *Let $x, y \in \mathbf{Su}_B$ with normal forms $\mathcal{N}(x) = \{x_1, \ldots, x_k\}$ and $\mathcal{N}(y) = \{y_1, \ldots, y_l\}$. Then $\mathcal{N}(x) \underline{\wedge} \mathcal{N}(y) := \mathbf{irr}\{x_q \cap y_r | 1 \leq q \leq k, 1 \leq r \leq l\}$.*

Note that Lemma 7 asserts that the operation **irr** involves only componentwise Boolean operations on elementary subsets.

DEFINITION 21 *Let* $x \in \mathbf{Su}_{\mathcal{B}}$. *The* complement *of* $\mathcal{N}(x)$ *is defined as follows.*

(i) If $x = \bigcap_{i \in I_{B_\alpha}} P_{\alpha,i}(J^x_{\alpha,i}) \in Es_{\mathcal{B}}$, *where* $\alpha = b_1.b_2.\cdots.b_m$, *so that* $\mathcal{N}(x) = \{x\}$ *then*

$\sim \mathcal{N}(x) := \bigcup_{k=1}^{m} [\bigcup_{i \in I_{B_{b_1 \cdots b_{k-1}}}} I(P_{b_1 \cdots b_{k-1},i}(J_{b_1 \cdots b_{k-1},i} - J^{a_{b_1 \cdots b_k}}_{b_1 \cdots b_{k-1},i}))] \cup [\bigcup_{i \in I^x_{B_\alpha}} I(P_{\alpha,i}(J_{\alpha,i}$

$-J^x_{\alpha,i}))]$.

(ii) If $x \notin Es_{\mathcal{B}}$ *and* $\mathcal{N}(x) = \{x_1, \ldots, x_k\}$, *then* $\sim \mathcal{N}(x) := \sim \mathcal{N}(x_1) \underline{\wedge} \cdots \underline{\wedge} \sim \mathcal{N}(x_k)$.

DEFINITION 22 *Let* $x, y \in \mathbf{Su}_{\mathcal{B}}$ *with normal forms* $\mathcal{N}(x) = \{x_1, \ldots, x_k\}$ *and* $\mathcal{N}(y) = \{y_1, \ldots, y_l\}$. *Then* $\mathcal{N}(x) \underline{\vee} \mathcal{N}(y) := \sim (\sim \mathcal{N}(x) \underline{\wedge} \sim \mathcal{N}(y))$.

Thus the algebra with universe equal to the set of normal forms of elements of $\mathbf{Su}_{\mathcal{B}}$ and signature $\{\underline{\vee}, \underline{\wedge}, \sim, 0, 1\}$ is a Boolean algebra, the *algebra of normal forms*. This algebra is denoted $\mathbf{C}_{\mathcal{B}} = (C_{\mathcal{B}}, \underline{\vee}, \underline{\wedge}, \sim, 0, 1)$.
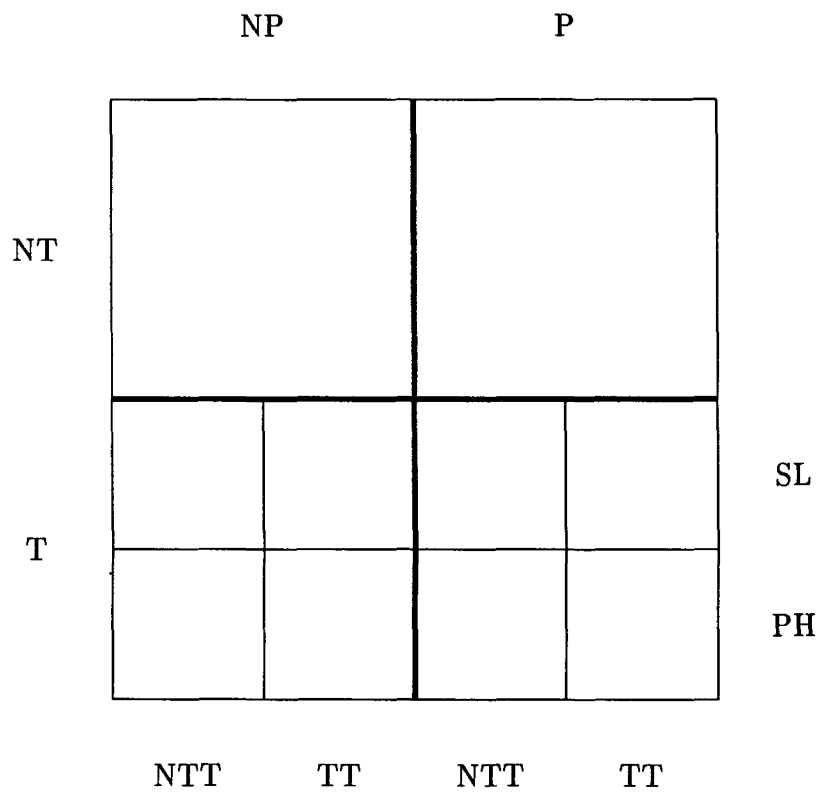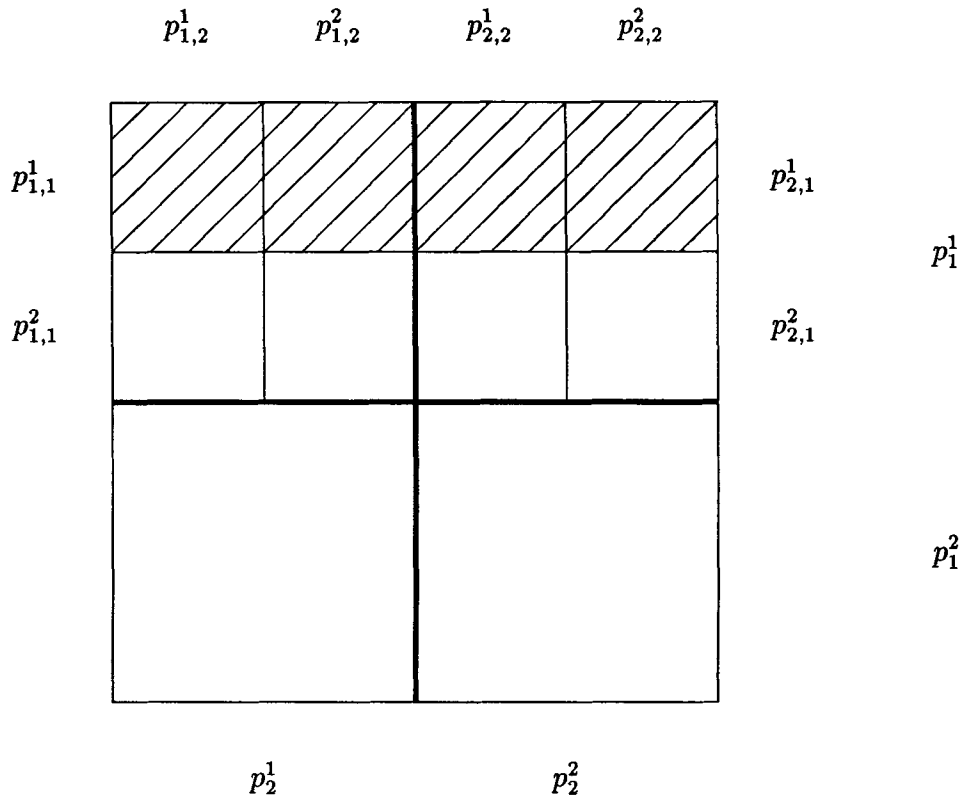
Figure 18: An Example of Embedded Bases.

Figure 19: Example Illustrating Pseudocomplement (x is the shaded area).

References

[1] Aigner, Martin: 1979, *Combinatorial Theory*, Springer-Verlag.

[2] Barwise, Jon and Robin Cooper: 1981, "Generalized Quantifiers and Natural Language," *Linguistics and Philosophy* 4, 159-219.

[3] Benthem, Johan van: 1983, "Determiners and Logic," *Linguistics and Philosophy* 6, 447-478.

[4] Benthem, Johan van: 1984, "Questions about Quantifiers," *Journal of Symbolic Logic* 49, 443-466.

[5] Dowty, David R.: 1978, "Applying Montague's Views on Linguistic Metatheory to the Structure of the Lexicon," in Donka Farkas et al. (eds.), *Parasession on the Lexicon*, Chicago Linguistic Society.

[6] Dowty, David R.: 1979, *Word Meaning and Montague Grammar*, D. Reidel Publishing Company.

[7] Egan, Rose F.: 1951, *Webster's Dictionary of Synomyms*, First Edition, G. & C. Merriam Co., Publishers.

[8] Fillenbaum, Samuel and Amnon Rapoport: 1971, *Structures in the Subjective Lexicon*, Academic Press.

[9] Fillmore, Charles J.: 1978, "On the organization of semantic information in the lexicon," in Donka Farkas et al. (eds.), *Parasession on the Lexicon*, Chicago Linguistic Society.

[10] Grandy, Richard E.: 1987, "In Defense of Semantic Fields," in Ernest LePore (ed.), *New Directions in Semantics*, Academic Press.

[11] Gratzer, George: 1978, *General Lattice Theory*, Birkhauser Verlag.

[12] Hausser, Roland R.: 1984, *Surface Compositional Grammar*, Wilhelm Fink Verlag.

[13] Keenan, Edward L. and Leonard M. Faltz: 1985, *Boolean Semantics for Natural Language*, D. Reidel Publishing Company.

[14] Keenan, Edward L. and Jonathan Stavi: 1986, "A Semantic Characterization of Natural Language Determiners," *Linguistics and Philosophy* 6, 447-478.

[15] Lehrer, A.: 1974, *Semantic Fields and Lexical Structure*, North-Holland Publishing Company.

[16] Lyons, John: 1977, *Semantics*, Volume I, Cambridge University Press.

[17] Nida, Eugene A.: 1975, *Componential Analysis of Meaning*, Mouton.

[18] Purdy, William C.: 1988, *A Lexical Extension of Montague Semantics*, Report CIS-88-1, School of Computer and Information Science, Syracuse University.

[19] Quine, W. V.: 1982, *Methods of Logic*, Fourth Edition, Harvard University Press.

[20] Sommers, Fred: 1982, *The Logic of Natural Language*, Clarendon Press Oxford.

[21] Thomason, Richard H. (ed.): 1974, *Formal Philosophy: Selected Papers of Richard Montague*, Yale University Press.

[22] Walther, Christoph: 1987, *A Many-Sorted Calculus Based on Resolution and Paramodulation*, Pitman.

[23] Westerstahl, Dag: 1985, "Determiners and Context Sets," in Benthem, Johan van (ed.), *Generalized Quantifiers in Natural Language*, Foris Publications.