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Badi H. Baltagi
Syracuse University, bbaltagi@maxwell.syr.edu

Peter Egger
Federal Institute of Technology

Michael Pfaffermayr
University of Innsbruck

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Badi H. Baltagi, Peter Egger,
and Michael Pfafermayr

Center for Policy Research
Maxwell School of Citizenship and Public Affairs
Syracuse University
426 Eggers Hall
Syracuse, New York 13244-1020
(315) 443-3114 | Fax (315) 443-1081
e-mail: ctrpol@syr.edu

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Abstract

This paper proposes a generalized panel data model with random effects and first-order spatially autocorrelated residuals that encompasses two previously suggested specifications. The first one is described in Anselin's (1988) book and the second one by Kapoor, Kelejian, and Prucha (2007). Our encompassing specification allows us to test for these models as restricted specifications. In particular, we derive three LM and LR tests that restrict our generalized model to obtain (i) the Anselin model, (ii) the Kapoor, Kelejian, and Prucha model, and (iii) the simple random effects model that ignores the spatial correlation in the residuals. For two of these three tests, we obtain closed form solutions and we derive their large sample distributions. Our Monte Carlo results show that the suggested tests are powerful in testing for these restricted specifications even in small and medium sized samples.

JEL classification: C23; C12

Keywords: Panel data; Spatially autocorrelated residuals; Maximum- likelihood estimation; Lagrange multiplier; Likelihood ratio

Badi H. Baltagi, Department of Economics and Center for Policy Research, Syracuse University, Syracuse, NY 13244-1020 U.S.A.; bbaltagi@maxwell.syr.edu

Peter Egger: University of Munich and CESifo, Poschingerstr. 5, 81679 Munich, Germany, E-mail: egger@ifo.de

Michael Pfaffermayr: Department of Economics, University of Innsbruck, Universitaetsstrasse 15, 6020 Innsbruck, Austria and Austrian Institute of Economic Research, P.O. Box 91, A-1103 Vienna, Austria; Michael.Pfaffermayr@uibk.ac.at. Michael Pfaffermayr gratefully acknowledges financial support from the Austrian Science Foundation Grant 17028.

1 Introduction[1]

The recent literature on spatial panels distinguishes between two different spatial autoregressive error processes. One specification assumes that spatial correlation occurs only in the remainder error term, whereas no spatial correlation takes place in the individual effects (see Anselin, 1988, Baltagi, Song, and Koh, 2003, and Anselin, Le Gallo, and Jayet, 2008; henceforth referred to as the Anselin model). Another specification assumes that the same spatial error process applies to both the individual and remainder error components (see Kapoor, Kelejian, and Prucha, 2007; henceforth referred to as the KKP model).

While the two data generating processes look similar, they imply different spatial spillover mechanisms. For example, consider the question of firm productivity using panel data. Besides the deterministic components, firms differ also with respect to their unobserved know-how or their managerial ability to organize production processes efficiently. At least over a short time period, this managerial ability may be time-invariant. Beyond that there are innovations that vary from period to period like random firm-specific technology shocks, capacity utilization shocks, etc. Under this scenario, it seems reasonable to assume that firm productivity may be spatially correlated due to spillovers. Such spillovers can occur, e.g., through information flows (transmission of process technologies) embodied in worker flows between firms at local labor markets or through input-output channels (technology requirements and interdependence of capacity utilization). Whereas the Anselin model assumes that spillovers are inherently time-varying, the KKP process assumes the spillovers to be time-invariant as well as time-variant. For example, firms located in the neighborhood of highly productive firms may get time-invariant permanent spillovers affecting their productivity in addition to the time-variant spillovers as in the

[1] We would like to thank the editor Cheng Hsiao, Matthias Koch, Ingmar Prucha and three anonymous referees for their helpful comments and suggestions. Preliminary versions of this paper were presented at the 13th International conference on panel data held in Cambridge, England, and the 23rd annual Canadian econometric study group meeting in Niagara Falls, Canada.

Anselin model. While the Anselin model seems restrictive in that it does not allow permanent spillovers through the individual firm effects, the KKP approach is restrictive in the sense that it does not allow for a differential intensity of spillovers of the permanent and transitory shocks.

This paper introduces a generalized spatial panel model which encompasses these two models and allows for spatial correlation in the individual and remainder error components that may have different spatial autoregressive parameters. We consider a maximum likelihood estimator (MLE) for this more general spatial panel model when the individual effects are assumed to be random. This in turn allows us to test the restrictions on our generalized model to obtain (i) the Anselin model, (ii) the Kapoor, Kelejian, and Prucha model, and (iii) a simple random effects model that ignores the spatial correlation in the residuals. We derive the corresponding LM and LR tests for these three hypotheses and we compare their size and power performance using Monte Carlo experiments.

2 A Generalized Model

Econometric models for panel data with spatial error processes have been proposed by Anselin (1988), Baltagi, Song, and Koh (2003), Kapoor, Kelejian, and Prucha (2007) and Anselin, Le Gallo, and Jayet (2008), to mention a few. A generalized spatial panel data model that encompasses these previous specifications is given as follows:[2]

$$\begin{aligned}
\mathbf{y}_t &= \mathbf{X}_t\boldsymbol{\beta} + \mathbf{u}_1 + \mathbf{u}_{2t}, \quad t = 1, ..., T \\
\mathbf{u}_1 &= \rho_1 \mathbf{W}\mathbf{u}_1 + \boldsymbol{\mu} \\
\mathbf{u}_{2t} &= \rho_2 \mathbf{W}\mathbf{u}_{2t} + \boldsymbol{\nu}_t,
\end{aligned}$$

where the $(N \times 1)$ vector $\mathbf{y}_t$ includes the observations on the dependent variable at time t, with N denoting the number of unique cross-sectional units. The

[2] To avoid index cluttering, we suppress the subscript indicating that the elements of the spatial weights matrix may depend on N and that the dependent variable and the disturbances form triangular arrays.

non-stochastic $(N \times K)$ matrix $\mathbf{X}_t$ gives the observations at time t for a set of K exogenous variables, including the constant. $\boldsymbol{\beta}$ is the corresponding $(K \times 1)$ parameter vector. The disturbance term follows an error component model which involves the sum of two disturbances. The $(N \times 1)$ vector of random variables $\mathbf{u}_1$ captures the time-invariant unit-specific effects and therefore has no time subscript. The $(N \times 1)$ vector of the remainder disturbances $\mathbf{u}_{2t}$ varies with time. Both $\mathbf{u}_1$ and $\mathbf{u}_{2t}$ are spatially correlated with the same spatial weights matrix $\mathbf{W}$, but with different spatial autocorrelation parameters ρ_1 and ρ_2, respectively. The $(N \times N)$ spatial weights matrix $\mathbf{W}$ has zero diagonal elements and its entries are typically declining with distance.

We further assume that the row and column sums of $\mathbf{W}$ are uniformly bounded in absolute value and that ρ_r is bounded in absolute value and independent of N. In case $\mathbf{W}$ is row normalized, the parameter space for ρ_r is a closed interval contained in $(-1, 1)$. Following Lee (2004, p. 1904), we assume for the case where $\mathbf{W}$ is not normalized (or maximum row sum normalized) but its eigenvalues are real, the parameter space for ρ_r is contained in the closed interval $-1/\lambda_{\min} < \rho_r < 1/\lambda_{\max}$ for all N and $r = 1, 2$. $\lambda_{\min}$ is the smallest and $\lambda_{\max}$ is the largest absolute value of the eigenvalues of $\mathbf{W}$. Hence, the spatial weights matrix may be either row normalized or maximum row normalized (see Kelejian and Prucha, 2008). Further, let $\mathbf{A} = \mathbf{I}_N - \rho_1\mathbf{W}$ and $\mathbf{B} = \mathbf{I}_N - \rho_2\mathbf{W}.$ The matrices $\mathbf{A}$ and $\mathbf{B}$ are non-singular for all ρ_r, $r = 1$, 2 in the parameter space and all N.

The elements of $\boldsymbol{\mu}$ are assumed to be independent across $i = 1, ..., N$, and i.i.d $N(0, \sigma^2_\mu)$. The elements of $\boldsymbol{\nu}_t$ are assumed to be independent across i and t and i.i.d $N(0, \sigma^2_\nu)$. Also, the elements of $\boldsymbol{\mu}$ and $\boldsymbol{\nu}_t$ are assumed to be independent of each other. Appendix B provides a more detailed set of assumptions.

Stacking the cross-sections over time yields

$$\begin{aligned}
\mathbf{y} &= \mathbf{X}\boldsymbol{\beta} + \mathbf{u} && (1)\\
\mathbf{u} &= \mathbf{Z}_\mu \mathbf{u}_1 + \mathbf{u}_2 \\
\mathbf{u}_1 &= \rho_1 \mathbf{W}\mathbf{u}_1 + \boldsymbol{\mu} \\
\mathbf{u}_2 &= \rho_2 (\mathbf{I}_T \otimes \mathbf{W})\mathbf{u}_2 + \boldsymbol{\nu},
\end{aligned}$$

where $\mathbf{y} = [\mathbf{y}_1', ..., \mathbf{y}_T']'$, $\mathbf{X} = [\mathbf{X}_1', ..., \mathbf{X}_T']'$, etc., so that the faster index is i and the slower index is t. The unit-specific errors $\mathbf{u}_1$ are repeated in all time periods using the $(NT \times N)$ selector matrix $\mathbf{Z}_\mu = \boldsymbol{\iota}_T \otimes \mathbf{I}_N$. $\boldsymbol{\iota}_T$ is a vector of ones of dimension T and $\mathbf{I}_N$ is an identity matrix of dimension N.

This model encompasses both the KKP model, which assumes that $\rho_1 = \rho_2$, and the Anselin model, which assumes that $\rho_1 = 0$. If $\rho_1 = \rho_2 = 0$, i.e., there is no spatial correlation, this model reduces to the familiar random effects (RE) panel data model; see Baltagi (2008).

Let $\mathbf{A} = (\mathbf{I}_N - \rho_1 \mathbf{W})$ and $\mathbf{B} = (\mathbf{I}_N - \rho_2 \mathbf{W})$, then, under the present assumptions we have

$$\begin{aligned}
\mathbf{u}_1 &= \mathbf{A}^{-1}\boldsymbol{\mu} \sim N(\mathbf{0}, \sigma_\mu^2 (\mathbf{A}'\mathbf{A})^{-1}) && (2)\\
\mathbf{u}_2 &= (\mathbf{I}_T \otimes \mathbf{B}^{-1})\boldsymbol{\nu} \sim N(\mathbf{0}, \sigma_\nu^2 (\mathbf{I}_T \otimes (\mathbf{B}'\mathbf{B})^{-1}).
\end{aligned}$$

The variance-covariance matrix of the spatial random effects panel data model is given by

$$\begin{aligned}
\boldsymbol{\Omega}_u &= E(\mathbf{u}\mathbf{u}') = E[(\mathbf{Z}_\mu \mathbf{u}_1 + \mathbf{u}_2)(\mathbf{Z}_\mu \mathbf{u}_1 + \mathbf{u}_2)'] && (3)\\
&= \sigma_\mu^2 (\mathbf{J}_T \otimes (\mathbf{A}'\mathbf{A})^{-1}) + \sigma_\nu^2 (\mathbf{I}_T \otimes (\mathbf{B}'\mathbf{B})^{-1}) \\
&= (\overline{\mathbf{J}}_T \otimes (T\sigma_\mu^2 (\mathbf{A}'\mathbf{A})^{-1} + \sigma_\nu^2 (\mathbf{B}'\mathbf{B})^{-1})) + \sigma_\nu^2 (\mathbf{E}_T \otimes (\mathbf{B}'\mathbf{B})^{-1}) = \sigma_\nu^2 \boldsymbol{\Sigma}_u,
\end{aligned}$$

where $\boldsymbol{\Sigma}_u$ is defined as $\boldsymbol{\Sigma}_u = (\overline{\mathbf{J}}_T \otimes (\frac{T\sigma_\mu^2}{\sigma_\nu^2} (\mathbf{A}'\mathbf{A})^{-1} + (\mathbf{B}'\mathbf{B})^{-1})) + (\mathbf{E}_T \otimes (\mathbf{B}'\mathbf{B})^{-1})$. This uses the fact that $E[\mathbf{u}_1 \mathbf{u}_2'] = \mathbf{0}$ since $\boldsymbol{\mu}$ and $\boldsymbol{\nu}$ are assumed to be independent. Note that $\mathbf{Z}_\mu \mathbf{Z}_\mu' = \mathbf{J}_T \otimes \mathbf{I}_N$, where $\mathbf{J}_T$ is a matrix of ones of dimension T. Let $\mathbf{E}_T = \mathbf{I}_T - \overline{\mathbf{J}}_T$, where $\overline{\mathbf{J}}_T = \mathbf{J}_T / T$ is the averaging matrix, the last equality replaces $\mathbf{J}_T$ by $T\overline{\mathbf{J}}_T$ and $\mathbf{I}_T$ by $\mathbf{E}_T + \overline{\mathbf{J}}_T$. It is easy to show that the inverse of the

$(NT \times NT)$ matrix $\mathbf{\Omega}_u$ can be obtained from the inverse of matrices of smaller dimension $(N \times N)$ as follows: $\mathbf{\Omega}_u^{-1} = (\bar{\mathbf{J}}_T \otimes (T\sigma_\mu^2(\mathbf{A}'\mathbf{A})^{-1} + \sigma_\nu^2(\mathbf{B}'\mathbf{B})^{-1})^{-1}) + \frac{1}{\sigma_\nu^2}(\mathbf{E}_T \otimes \mathbf{B}'\mathbf{B}) = \frac{1}{\sigma_\nu^2}\mathbf{\Sigma}_u^{-1}$, where

$$\mathbf{\Sigma}_u^{-1} = (\bar{\mathbf{J}}_T \otimes (\tfrac{T\sigma_\mu^2}{\sigma_\nu^2}(\mathbf{A}'\mathbf{A})^{-1} + (\mathbf{B}'\mathbf{B})^{-1})^{-1}) + (\mathbf{E}_T \otimes \mathbf{B}'\mathbf{B}).$$

Also, $\det[\mathbf{\Omega}_u] = \det[T\sigma_\mu^2(\mathbf{A}'\mathbf{A})^{-1} + \sigma_\nu^2(\mathbf{B}'\mathbf{B})^{-1}]\det[\sigma_\nu^2(\mathbf{B}'\mathbf{B})^{-1}]^{T-1}$. We also assume that the inverses $\mathbf{A}^{-1}$, $\mathbf{B}^{-1}$ and $[T\sigma_\mu^2(\mathbf{A}'\mathbf{A})^{-1} + \sigma_\nu^2(\mathbf{B}'\mathbf{B})^{-1}]^{-1}$ have uniformly bounded row and column sums, see Assumption A2 in the Appendix for further details. Under the present assumptions, the log-likelihood function of the general model is given by

$$\begin{aligned} L(\boldsymbol{\beta}, \boldsymbol{\theta}) \quad = \quad & -\tfrac{NT}{2}\ln 2\pi - \tfrac{1}{2}\ln\det[T\sigma_\mu^2(\mathbf{A}'\mathbf{A})^{-1} + \sigma_\nu^2(\mathbf{B}'\mathbf{B})^{-1}] \\ & -\tfrac{T-1}{2}\ln\det[\sigma_\nu^2(\mathbf{B}'\mathbf{B})^{-1}] - \tfrac{1}{2}(\mathbf{y} - \mathbf{X}\boldsymbol{\beta})'\mathbf{\Omega}_u^{-1}(\mathbf{y} - \mathbf{X}\boldsymbol{\beta}), \end{aligned} \tag{4}$$

where $\boldsymbol{\theta} = (\sigma_\nu^2, \sigma_\mu^2, \rho_1, \rho_2)$. The maximum likelihood estimates are obtained by maximizing the log-likelihood function numerically using a constrained quasi-Newton method.[3]

The hypotheses under consideration in this paper are the following:

(1) H_0^A: $\rho_1 = \rho_2 = 0$, and the alternative H_1^A is that at least one component is not zero. The restricted model is the standard random effects (RE) panel data model with no spatial correlation, see Baltagi (2008).

(2) H_0^B: $\rho_1 = 0$, and the alternative is H_1^B: $\rho_1 \neq 0$. The restricted model is the Anselin (1988) spatial panel model with random effects. In fact, the restricted log-likelihood function reduces to the one considered by Anselin (1988, p.154).

(3) H_0^C: $\rho_1 = \rho_2 = \rho$ and the alternative is H_1^C: $\rho_1 \neq \rho_2$. The restricted model is the KKP spatial panel model with random effects.

In the next subsections, we derive the corresponding LM tests for these hypotheses and we compare their performance with the corresponding LR tests

[3] The numerical maximization procedure can be simplified, if one concentrates the likelihood with respect to $\boldsymbol{\beta}$ and σ_ν^2. However, our optimization for the Monte Carlo simulation using MATLAB were quite fast using the constrained quasi-Newton method. Appendix E describes some details on the numerical optimization procedure.

using Monte Carlo experiments.[4] Appendix A describes some general results used to derive the score and information matrix for these alternative models; Appendix B proves the consistency of the ML estimates of the general model; while Appendices C and D provide the derivations of the large sample distributions of these LM tests.

2.1 LM and LR Tests for $H_0^A : \rho_1 = \rho_2 = 0$

The ML estimates under H_0^A are labeled by a tilde and the corresponding restricted parameter vector is indexed by A. The joint LM test statistic for the null hypothesis of no spatial correlation, $H_0^A : \rho_1 = \rho_2 = 0$, is derived in Appendix C and it is given by

$$\widetilde{LM}_A = \frac{1}{2b_A\widetilde{\sigma}_1^4}\widetilde{G}_A^2 + \frac{1}{2b_A(T-1)\widetilde{\sigma}_\nu^4}\widetilde{M}_A^2, \tag{5}$$

where $\widetilde{\sigma}_1^2 = T\widetilde{\sigma}_\mu^2 + \widetilde{\sigma}_\nu^2$, $b_A = tr[(\mathbf{W}' + \mathbf{W})^2]$, $\widetilde{G}_A = \widetilde{\mathbf{u}}'[\bar{\mathbf{J}}_T \otimes (\mathbf{W}' + \mathbf{W})]\widetilde{\mathbf{u}}$, and $\widetilde{M}_A = \widetilde{\mathbf{u}}'[\mathbf{E}_T \otimes (\mathbf{W}' + \mathbf{W})]\widetilde{\mathbf{u}}$. In this case, $\widetilde{\mathbf{u}} = \mathbf{y} - \mathbf{X}\widetilde{\boldsymbol{\beta}}$ denotes the vector of the estimated residuals under H_0^A. The restricted model is the simple random effects (RE) panel data model without any spatial autocorrelation. In fact, $\widetilde{\sigma}_\nu^2 = \frac{\widetilde{\mathbf{u}}'(\mathbf{E}_T\otimes\mathbf{I}_N)\widetilde{\mathbf{u}}}{N(T-1)}$ and $\widetilde{\sigma}_1^2 = \frac{\widetilde{\mathbf{u}}'(\bar{\mathbf{J}}_T\otimes\mathbf{I}_N)\widetilde{\mathbf{u}}}{N}$. Under H_0^A, the $\widetilde{LM}_A$ statistic is asymptotically distributed as χ_2^2 as shown in Appendix C.

One can also derive the corresponding LR test for H_0^A: $\rho_1 = \rho_2 = 0$ as

$$LR_A = 2(L_G - L_A),$$

using the maximized log-likelihood of the general model denoted by L_G and the maximized log-likelihood under H_0^A:

$$L_A = -\frac{NT}{2}\ln 2\pi\widetilde{\sigma}_\nu^2 - \frac{N}{2}\ln\frac{\widetilde{\sigma}_1^2}{\widetilde{\sigma}_\nu^2} - \frac{1}{2}\widetilde{\mathbf{u}}'\widetilde{\boldsymbol{\Omega}}_u^{-1}\widetilde{\mathbf{u}}.$$

This test statistic is likewise asymptotically distributed as χ_2^2.

[4] LM tests for spatial models are surveyed in Anselin (1988, 2001) and Anselin and Bera (1998), to mention a few. For a joint test for the absence of spatial correlation and random effects in a panel data model, see Baltagi, Song, and Koh (2003).

2.2 LM and LR Tests for $H_0^B : \rho_1 = 0$

Under H_0^B: $\rho_1 = 0$, the restricted model is the spatial panel data model with random effects described in Anselin (1988). The corresponding LM test for H_0^B is a conditional test for zero spatial correlation in the individual effects, allowing for the possibility of spatial correlation in the remainder error term, i.e., $\rho_2 \neq 0$. In fact, under H_0^B, the information matrix is block-diagonal with the lower block being independent of $\boldsymbol{\beta}$. Let $\mathbf{d}_\theta$ be the (4×1) score vector referring to the parameter vector $\boldsymbol{\theta} = (\sigma_\mu^2, \sigma_\nu^2, \rho_1, \rho_2)$ and denote the 4×4 lower block of the information matrix by $\mathbf{J}_\theta$. The ML estimates under H_0^B are labeled by a hat. The corresponding estimated residuals are then $\widehat{\mathbf{u}} = \mathbf{y} - \mathbf{X}\widehat{\boldsymbol{\beta}}$. The LM test for H_0^B makes use of the estimated score $\widehat{\mathbf{d}}_\theta = [0, 0, \widehat{d}_{\rho_1}, 0]'$ with

$$\begin{aligned} \widehat{d}_{\rho_1} &= \left.\frac{\partial L}{\partial \rho_1}\right|_{H_0^B} = -\tfrac{1}{2} T \widehat{\sigma}_\mu^2 tr[\widehat{\mathbf{C}}_1 \mathbf{C}_2] + \tfrac{1}{2} \widehat{\sigma}_\mu^2 \widehat{\mathbf{u}}' (\mathbf{J}_T \otimes \widehat{\mathbf{C}}_1 \mathbf{C}_2 \widehat{\mathbf{C}}_1) \widehat{\mathbf{u}} \\ &= \tfrac{1}{2} T \widehat{\sigma}_\mu^2 [(\widehat{\mathbf{u}}' \widehat{\mathbf{G}}_B \widehat{\mathbf{u}}) - \widehat{g}_B], \end{aligned}$$

where $\widehat{\mathbf{C}}_1 = [T\widehat{\sigma}_\mu^2 \mathbf{I}_N + \widehat{\sigma}_\nu^2 (\widehat{\mathbf{B}}'\widehat{\mathbf{B}})^{-1}]^{-1}$ and $\mathbf{C}_2 = (\mathbf{W}' + \mathbf{W})$, $\widehat{\mathbf{G}}_B = (\bar{\mathbf{J}}_T \otimes \widehat{\mathbf{C}}_1 \mathbf{C}_2 \widehat{\mathbf{C}}_1)$, and $\widehat{g}_B = tr[\widehat{\mathbf{C}}_1 \mathbf{C}_2]$. An estimate of the lower (4×4) block of the information matrix $\widehat{\mathbf{J}}_\theta$ under H_0^B is given by[5]

$$\left.\widehat{\mathbf{J}}_\theta\right|_{H_0^B} =$$

$$\begin{bmatrix} \frac{1}{2} tr[\widehat{\mathbf{C}}_3{}^2] + \frac{N(T-1)}{2\widehat{\sigma}_\nu^4} & \frac{T}{2} tr\left[\widehat{\mathbf{C}}_3 \widehat{\mathbf{C}}_1\right] & \frac{T\widehat{\sigma}_\mu^2}{2} tr[\widehat{\mathbf{C}}_3 \widehat{\mathbf{C}}_1 \mathbf{C}_2] & \frac{\widehat{\sigma}_\nu^2}{2} tr[\widehat{\mathbf{C}}_3 \widehat{\mathbf{C}}_1 \widehat{\mathbf{C}}_5] + \frac{(T-1)}{2\widehat{\sigma}_\nu^2} tr[\widehat{\mathbf{C}}_4] \\ \frac{T}{2} tr\left[\widehat{\mathbf{C}}_3 \widehat{\mathbf{C}}_1\right] & \frac{T^2}{2} tr\left[\widehat{\mathbf{C}}_1^2\right] & \frac{T^2 \widehat{\sigma}_\mu^2}{2} tr[\widehat{\mathbf{C}}_1^2 \mathbf{C}_2] & \frac{T\widehat{\sigma}_\nu^2}{2} tr[\widehat{\mathbf{C}}_1^2 \widehat{\mathbf{C}}_5] \\ \frac{T\widehat{\sigma}_\mu^2}{2} tr[\widehat{\mathbf{C}}_3 \widehat{\mathbf{C}}_1 \mathbf{C}_2] & \frac{T^2 \widehat{\sigma}_\mu^2}{2} tr[\widehat{\mathbf{C}}_1^2 \mathbf{C}_2] & \frac{T^2 \widehat{\sigma}_\mu^4}{2} tr[(\widehat{\mathbf{C}}_1 \mathbf{C}_2)^2] & \frac{T\widehat{\sigma}_\mu^2 \widehat{\sigma}_\nu^2}{2} tr[\widehat{\mathbf{C}}_1 \mathbf{C}_2 \widehat{\mathbf{C}}_1 \widehat{\mathbf{C}}_5] \\ \frac{\widehat{\sigma}_\nu^2}{2} tr[\widehat{\mathbf{C}}_3 \widehat{\mathbf{C}}_1 \widehat{\mathbf{C}}_5] + \frac{(T-1)}{2\widehat{\sigma}_\nu^2} tr[\widehat{\mathbf{C}}_4] & \frac{T\widehat{\sigma}_\nu^2}{2} tr[\widehat{\mathbf{C}}_1^2 \widehat{\mathbf{C}}_5] & \frac{T\widehat{\sigma}_\mu^2 \widehat{\sigma}_\nu^2}{2} tr[\widehat{\mathbf{C}}_1 \mathbf{C}_2 \widehat{\mathbf{C}}_1 \widehat{\mathbf{C}}_5] & \frac{\widehat{\sigma}_\nu^4}{2} tr[(\widehat{\mathbf{C}}_1 \widehat{\mathbf{C}}_5)^2] + \frac{(T-1)}{2} tr[\widehat{\mathbf{C}}_4^2] \end{bmatrix},$$

where $\widehat{\mathbf{C}}_3 = (\widehat{\mathbf{B}}'\widehat{\mathbf{B}})^{-1} \widehat{\mathbf{C}}_1$, $\widehat{\mathbf{C}}_4 = (\mathbf{W}'\widehat{\mathbf{B}} + \widehat{\mathbf{B}}'\mathbf{W})(\widehat{\mathbf{B}}'\widehat{\mathbf{B}})^{-1}$ and $\widehat{\mathbf{C}}_5 = (\widehat{\mathbf{B}}'\widehat{\mathbf{B}})^{-1} \widehat{\mathbf{C}}_4$. The LM test for H_0^B is calculated as

$$LM_B = \widehat{\mathbf{d}}_\theta' \widehat{\mathbf{J}}_\theta^{-1} \widehat{\mathbf{d}}_\theta = \widehat{d}_{\rho_1}^2 \widehat{\mathbf{J}}_{33}^{-1}, \tag{6}$$

where $\widehat{\mathbf{J}}_{33}^{-1}$ is the $(3,3)$ element of the inverse of the estimated information matrix $\widehat{\mathbf{J}}_\theta^{-1}$ under H_0^B. This test statistic has no closed form representation, but using similar assumptions and proofs as in the Appendices, this test statistic should be asymptotically distributed as χ_1^2.

[5] Detailed derivations are available form the authors upon request.

The corresponding LR test is based upon the maximized log-likelihood under H_0^B:

$$L_B = -\frac{NT}{2}\ln 2\pi\widehat{\sigma}_\nu^2 - \frac{1}{2}\ln\det(\widehat{\mathbf{C}}_1) + \frac{T-1}{2}\ln\det(\widehat{\mathbf{B}}'\widehat{\mathbf{B}}) - \frac{1}{2}\widehat{\mathbf{u}}'\widehat{\mathbf{\Omega}}_u^{-1}\widehat{\mathbf{u}}.$$

This restricted log-likelihood is the same as that given by Anselin (1988, p. 154).

2.3 LM and LR Tests for $H_0^C : \rho_1 = \rho_2 = \rho$

Under $H_0^C : \rho_1 = \rho_2 = \rho$, the true model is the one suggested by Kapoor, Kelejian, and Prucha (2007). In this case, $\mathbf{B} = \mathbf{A}$ and the parameter estimates under H_0^C are labeled by a bar. The corresponding estimated residuals are given by $\overline{\mathbf{u}} = \mathbf{y} - \mathbf{X}\overline{\boldsymbol{\beta}}$. The score and the information matrix needed for this test are derived in Appendix D. The joint LM test statistic for H_0^C is given by

$$LM_C = \frac{T}{2\overline{b}_C(T-1)\overline{\sigma}_1^4}\overline{G}_C^2, \tag{7}$$

with $\overline{G}_C = \overline{\mathbf{u}}'(\overline{\mathbf{J}}_T \otimes \overline{\mathbf{F}})\overline{\mathbf{u}} - \overline{\sigma}_1^2 tr[\overline{\mathbf{D}}]$, $\overline{\mathbf{F}} = \mathbf{W}'\overline{\mathbf{A}} + \overline{\mathbf{A}}'\mathbf{W}$ and $\overline{\mathbf{D}} = \overline{\mathbf{F}}(\overline{\mathbf{A}}'\overline{\mathbf{A}})^{-1}$. Also, $\overline{b}_C = tr[\overline{\mathbf{D}}^2] - (tr[\overline{\mathbf{D}}])^2/N$, $\overline{\sigma}_1^2 = \frac{\overline{\mathbf{u}}'[\overline{\mathbf{J}}_T \otimes (\overline{\mathbf{A}}'\overline{\mathbf{A}})]\overline{\mathbf{u}}}{N}$ and $\overline{\sigma}_\nu^2 = \frac{\overline{\mathbf{u}}'[\mathbf{E}_T \otimes (\overline{\mathbf{A}}'\overline{\mathbf{A}})]\overline{\mathbf{u}}}{N(T-1)}$. Under H_0^C, the LM_C statistic is asymptotically distributed as χ_1^2 as shown in Appendix D. The LR test is based on the following maximized log-likelihood under H_0^C:

$$L_C = -\frac{NT}{2}\ln 2\pi\overline{\sigma}_\nu^2 - \frac{N}{2}\ln(\frac{\overline{\sigma}_1^2}{\overline{\sigma}_\nu^2}) + \frac{T}{2}\ln\det(\overline{\mathbf{A}}'\overline{\mathbf{A}}) - \frac{1}{2}\overline{\mathbf{u}}'\overline{\mathbf{\Omega}}_u^{-1}\overline{\mathbf{u}}.$$

Kapoor, Kelejian, and Prucha (2007) consider a generalized method of moments estimator, rather than MLE, for their spatial random effects panel data model. L_C is the maximized log-likelihood for the KKP model with normal disturbances.

3 Monte Carlo Results

In the Monte Carlo analysis, we use a simple panel data model that includes one explanatory variable and a constant ($K = 2$)

$$y_{it} = \beta_0 + \beta_1 x_{it} + u_{it}, \quad i = 1, ..., N \text{ and } t = 1, ..., T,$$

where $\beta_0 = 5$ and $\beta_1 = 0.5$. x_{it} is generated by $x_{it} = \zeta_i + z_{it}$, where $\zeta_i \sim i.i.d.$ $U[-7.5, 7.5]$ and $z_{it} \sim i.i.d.$ $U[-5, 5]$ with $U[a, b]$ denoting the uniform distribution on the interval $[a, b]$. The processes ζ_i and z_{it} are assumed to be independent. The individual-specific effects are drawn from a normal distribution so that $\mu_i \sim i.i.d.$ $N(0, 20\theta)$, while for the remainder error we assume $\nu_{it} \sim i.i.d.$ $N(0, 20(1 - \theta))$ with $0 < \theta < 1$. $\theta = \frac{\sigma_\mu^2}{\sigma_\mu^2 + \sigma_\nu^2}$ is the proportion of the total variance due to the heterogeneity of the individual-specific effects. This implies that $\sigma_\mu^2 + \sigma_\nu^2 = 20$.

We generate the spatial weights matrix by allocating observations randomly on a grid of $2N$ squares. Consequently, as the number of observations N increases, the number of squares in the grid grows larger, too. The probability that an observation is located on a particular coordinate is equal for all coordinates on the grid. This results in an irregular lattice, where each observation possesses 3 neighbors on average. The spatial weighting scheme is based on the Queens design and the corresponding spatial weights matrix is normalized so that each row sums to one.

The parameters ρ_1 and ρ_2 vary over the set $\{-0.8, -0.5, -0.2, 0, 0.2, 0.5, 0.8\}$. The cross-sectional and time dimensions are $N = 50$, 100 and $T = 3$, 5, 10, respectively. Lastly, the proportion of the variance due to the random individual effects takes the values $\theta = 0.25$, 0.50, 0.75. In total, this gives 882 experiments. For each experiment, we calculate the three LM and LR tests as derived above, using 2000 replications.[6]

[6] In a few cases, we got negative LR test statistics due to numerical imprecision. These cases occur mainly with the Anselin model at $\rho_1 = 0$. However, this happened in less than 0.5 percent of the Monte Carlo experiments. We drop the corresponding experiments in the subsequent calculations of the size and power of the tests.

===== Tables 1-3 =====

Table 1 reports the frequency of rejections for $N = 50$, $T = 5$, and $\theta = 0.5$ in 2000 replications. This means that $\sigma_\mu^2 = \sigma_\nu^2 = 10$. The size of each test is denoted in bold figures and is not statistically different from the 5% nominal size. The only exception where the LM test might be undersized is for the KKP model, for high absolute values of ρ_1 and ρ_2, both equal to 0.8. The size adjusted power[7] of the LR and LM tests is reasonably high for all three hypotheses considered. The performance of the LM test is almost the same as that of the LR test, except for a few cases. For $H_0^A: \rho_1 = \rho_2 = 0$, when $\rho_1 = -0.5$ and $\rho_2 = 0$, the size adjusted power of the LM test is 61.4% as compared to 64.6% for LR. At $\rho_1 = 0.5$ and $\rho_2 = 0$, the size adjusted power of the LM test is 70% as compared to 66.4% for LR. Similarly, for H_0^B: $\rho_1 = 0$, when $\rho_1 = -0.5$ and $\rho_2 = 0$, the size adjusted power of the LM test is 70.2% as compared to 72.9% for LR. At $\rho_1 = 0.5$ and $\rho_2 = 0$, the size adjusted power of the LM test is 76.7% as compared to 74.6% for LR. For $H_0^C: \rho_1 = \rho_2 = \rho$, when $\rho_1 = -0.5$ and $\rho_2 = 0$, the size adjusted power of the LM test is 66.1% as compared to 68.5% for LR. At $\rho_1 = 0.5$ and $\rho_2 = 0$, the size adjusted power of the LM test is 70.6% as compared to 65% for LR.

Tables 2 and 3 repeat the same experiments but now for $\theta = 0.25$ and 0.75, respectively. These tables show that as we increase θ, we increase the power of these tests. In fact, the power of all three tests is higher, the higher the variance of the individual-specific effect as a proportion of the total variance. For example, for $H_0^A: \rho_1 = \rho_2 = 0$, when $\rho_1 = -0.5$ and $\rho_2 = 0$, the size adjusted power of the LM test increases from 61.4% for $\theta = 0.5$ (in Table 1) to 68% for $\theta = 0.75$ (in Table 3), while the size adjusted power of the LR test increases from 64.6% to 74.8%. Similarly, when $\rho_1 = 0.5$ and $\rho_2 = 0$, the size adjusted power of the LM test increases from 70% for $\theta = 0.5$ to 78.4% for $\theta = 0.75$, while the size adjusted power of the LR test increases from 66.4% to

[7] The size corrected critical level for the test is inferred from the empirical distribution of the test statistic in the Monte Carlo experiments, so that the rejection region under the empirical distribution has the correct nominal size.

77.4%. For H_0^B: $\rho_1 = 0$, when $\rho_1 = -0.5$ and $\rho_2 = 0$, the size adjusted power of the LM test increases from 70.2% for $\theta = 0.5$ to 81% for $\theta = 0.75$, while the size adjusted power of the LR test increases from 72.9% to 83.4%. At $\rho_1 = 0.5$ and $\rho_2 = 0$, the size adjusted power of the LM test increases from 76.7% for $\theta = 0.5$ to 86.6% for $\theta = 0.75$, while the size adjusted power of the LR test increases from 74.6% to 84.9% for LR. For H_0^C : $\rho_1 = \rho_2 = \rho$, when $\rho_1 = -0.5$ and $\rho_2 = 0$, the size adjusted power of the LM test increases from 66.1% for $\theta = 0.5$ to 73% for $\theta = 0.75$, while the size adjusted power of the LR test increases from 68.5% to 74.8%. At $\rho_1 = 0.5$ and $\rho_2 = 0$, the size adjusted power of the LM test increases from 70.6% for $\theta = 0.5$ to 80.4% for $\theta = 0.75$, while the size adjusted power of the LR test increases from 65% to 77.3%.

Things also improve if the number of observations increases. The increase in power is larger when we double N from 50 to 100 as compared to doubling T from 5 to 10.[8] We conclude that the three LM and LR tests perform reasonably well in testing the restrictions underlying the simple random effects model without spatial correlation, the Anselin model and the KKP model in small and medium sized samples.

3.1 Robustness Checks

We also assess the robustness of the proposed LM tests with respect to (i) non-normal errors and (ii) the specification of the spatial weighting matrix. To compare the simulated power functions for normal vs. non-normal errors, we generated the remainder error term first as $\nu_{it} \sim t(5)$ and normalized its variance to 10. Hence, $\theta = 0.5$ holds in this case and the results are comparable to the basic Monte Carlo set-up defined above. This implies that the distribution of the remainder error exhibits heavier tails as compared to the normal distribution but it is still symmetric. Second, we analyzed a skewed error distribution assuming ν_{it} follows a log-normal distribution with variance 10, i.e., $\nu_{it} = \sqrt{10}(e^{\xi} -$

[8] We do not include the corresponding Tables for ($N = 50$, $T = 10$) and ($N = 100$, $T = 5$), for $\theta = 0.25, 0.50,$ and 0.75, in order to save space. However, these tables are available upon request from the authors.

$e^{0.5})/\sqrt{e^2 - e^1}$, where $\xi \sim N(0,1)$. For $N = 50$ and $T = 5$, the Monte Carlo experiments show that there are minor changes in the size adjusted power curves under both error distributions. This holds true for all LM tests considered.

===== Table 4 =====

The non-normality of the remainder error, however, does affect the size of the tests. In Table 4, we focus on the size of the LM and LR tests under alternative distributional assumptions of the error term for $N = 50$, $T = 5$ and $\theta = 0.5$. In the first pair of columns we give the true parameters ρ_1, ρ_2, the second pair of columns summarizes the size of the tests under the assumption that $\nu_{it} \sim t(5)$, in the third pair of columns we assume that ν_{it} follows a log-normal distribution with variance 10. It turns out that both the LM tests and the LR tests are fairly insensitive to the chosen alternative assumptions about the distribution of the disturbances at intermediate levels of ρ_1 and ρ_2. However, the LM tests tend to be somewhat more undersized than the LR tests, especially for $\rho_1 = \rho_2 = 0.8$. With the caveat of the limited experiments we performed, this finding suggests that the LM tests considered are fairly robust to deviations from the assumption of a normally distributed error term.

We also investigated the extent to which the specification of the spatial weighting scheme matters for the size and power of the tests considered. We generated an alternative spatial weighting matrix allowing for a more densely populated grid. In particular, we randomly allocated the observations on the grid so that there are 5 rather than 3 neighbors per observation on average. As expected, the power of the tests is somewhat lower in this case, but still big enough to detect relevant deviations from the null.

4 Conclusions

The recent literature on first-order spatially autocorrelated residuals (SAR(1)) with panel data distinguishes between two data generating processes of the error term. One process described in Anselin (1988) and Anselin, Le Gallo and

Jayet (2008) assumes that only the remainder error component is spatially correlated. In an alternative process put forward by Kapoor, Kelejian, and Prucha (2007) both the individual and remainder components of the disturbances are characterized by the same spatial autocorrelation pattern. This paper formulates a SAR(1) process of the residuals with panel data that encompasses these two processes. In particular, this paper derives three LM tests based upon the more general model, testing its restricted counterparts: the Anselin model, the Kapoor, Kelejian, and Prucha model, and the random effects model without spatial correlation. For the latter two tests, closed-form expressions for the LM statistics can be obtained.

Our Monte Carlo study assesses the small sample performance of the derived tests. We find that the tests are properly sized and powerful even in relatively small samples. The LM tests are easy to calculate and their power is reasonably high for all three tests considered. The power of these LM tests matches that of the corresponding LR tests except in few cases. In general, the power of the tests increases with the relative importance of the individual effects' variance as a proportion of the total variance, as well as with increasing N and T. They are robust to non-normality of the error term and sensitive to the specification of the weight matrix. Hence, these LM and LR tests are recommended for the applied researcher to test the restrictions imposed by the RE model with no spatial correlation, the Anselin model, and the Kapoor, Kelejian, and Prucha model.

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Appendix A: Score and Information Matrix

Below we make use of the following derivatives to obtain the score and the relevant part of the information matrix: [9]

$$\begin{aligned}
\frac{\partial \mathbf{\Omega}_u}{\partial \sigma_\nu^2} &= \overline{\mathbf{J}}_T \otimes (\mathbf{B}'\mathbf{B})^{-1} + (\mathbf{E}_T \otimes (\mathbf{B}'\mathbf{B})^{-1}) = \mathbf{I}_T \otimes (\mathbf{B}'\mathbf{B})^{-1} \\
\frac{\partial \mathbf{\Omega}_u}{\partial \sigma_\mu^2} &= \overline{\mathbf{J}}_T \otimes T(\mathbf{A}'\mathbf{A})^{-1} \\
\frac{\partial \mathbf{\Omega}_u}{\partial \rho_1} &= \overline{\mathbf{J}}_T \otimes T\sigma_\mu^2(\mathbf{A}'\mathbf{A})^{-1}(\mathbf{W}' + \mathbf{W} - 2\rho_1\mathbf{W}'\mathbf{W})(\mathbf{A}'\mathbf{A})^{-1} \\
\frac{\partial \mathbf{\Omega}_u}{\partial \rho_2} &= \mathbf{I}_T \otimes \sigma_\nu^2(\mathbf{B}'\mathbf{B})^{-1}(\mathbf{W}' + \mathbf{W} - 2\rho_2\mathbf{W}'\mathbf{W})(\mathbf{B}'\mathbf{B})^{-1}.
\end{aligned}$$

Appendix B: Identification and Consistency

In the sequel, we use subscript 0 to indicate true parameter values where necessary. First, we state the full set of Assumptions.[10]

A1 (**random effects model**): The model comprises unit-specific random effects denoted by the $(N \times 1)$ vector $\boldsymbol{\mu}$. The elements of $\boldsymbol{\mu}$ are *i.i.d.* $N(0, \sigma_\mu^2)$ with $0 < \sigma_\mu^2 < \infty$. $\boldsymbol{\nu}$ is the vector of remainder errors and its elements are *i.i.d.* $N(0, \sigma_\nu^2)$ with $0 < \sigma_\nu^2 < \infty$. The elements of $\boldsymbol{\mu}$ and $\boldsymbol{\nu}$ are independent of each other.

A2 (**spatial correlation**): (i) Both $\mathbf{u}_1$ and $\mathbf{u}_{2t}$ are spatially correlated with the same $(N \times N)$ spatial weighting matrix $\mathbf{W}$ whose elements may depend on N. $\mathbf{W}$ has zero diagonal elements. (ii) The row and column sums of $\mathbf{W}$ are uniformly bounded in absolute value. (iii) In case $\mathbf{W}$ is row normalized, the parameter space for ρ_r is a closed interval contained in $(-1, 1)$. For the case where $\mathbf{W}$ is not normalized or maximum row sum normalized, but its eigenvalues are real, the parameter space for ρ_r is contained in the closed interval $-1/\lambda_{\min}$

[9] Hartley and Rao (1971) and Hemmerle and Hartley (1973) give a general useful formula that helps in obtaining the score of $\boldsymbol{\theta} = (\sigma_\nu^2, \sigma_\mu^2, \rho_1, \rho_2)'$: $\frac{\partial L}{\partial \theta_r} = -\frac{1}{2} tr\left(\mathbf{\Omega}_u^{-1}\frac{\partial \mathbf{\Omega}_u}{\partial \theta_r}\right) + \frac{1}{2}\mathbf{u}'\left(\mathbf{\Omega}_u^{-1}\frac{\partial \mathbf{\Omega}_u}{\partial \theta_r}\mathbf{\Omega}_u^{-1}\right)\mathbf{u}$, $r = 1, ..., 4$. To derive the relevant part of the information matrix, we use the general differentiation result given in Harville (1977): $J_{rs} = E\left[-\frac{\partial^2 L}{\partial \theta_r \theta_s}\right] = \frac{1}{2} tr\left[\mathbf{\Omega}_u^{-1}\frac{\partial \mathbf{\Omega}_u}{\partial \theta_r}\mathbf{\Omega}_u^{-1}\frac{\partial \mathbf{\Omega}_u}{\partial \theta_s}\right]$.

[10] To avoid index cluttering, we suppress the subscript indicating that the elements of the spatial weights matrix may depend on N and that the dependent variable and the disturbances form triangular arrays.

$< \rho_r < 1/\lambda_{\max}$ for all N and $r = 1, 2$. $\lambda_{\min}$ is smallest and $\lambda_{\max}$ is the largest absolute value of the eigenvalues of $\mathbf{W}$. (iv) The matrices $\mathbf{I}_N - \rho_r \mathbf{W}$ are nonsingular for all ρ_r in the parameter space and their inverses have uniformly bounded row and column sums. Let $\mathbf{A} = \mathbf{I}_N - \rho_1 \mathbf{W}$ and $\mathbf{B} = \mathbf{I}_N - \rho_2 \mathbf{W}$. Then $[T\sigma_\mu^2(\mathbf{A}'\mathbf{A})^{-1} + \sigma_\nu^2(\mathbf{B}'\mathbf{B})^{-1}]^{-1}$ has uniformly bounded row and column sums. (v) The elements of $\mathbf{W}$ are nonnegative, the nonzero elements of $\mathbf{W}$ are bounded away from zero so that $\lambda_{\min} \geq c_{\lambda_{\min}} > 0$ for some positive constant $c_{\lambda_{\min}}$.

A3 (**compactness of the parameter space**): The parameter space $\boldsymbol{\Theta}$ with elements $(\boldsymbol{\beta}, \sigma_\mu^2, \sigma_\nu^2, \rho_1, \rho_2)$ is compact. The true parameter vector (indexed by 0) lies in the interior of $\boldsymbol{\Theta}$.

We note that Assumptions A1 and A2 imply that $\boldsymbol{\Xi} = \{(\phi, \rho_1, \rho_2)|(\sigma_\mu^2, \sigma_\nu^2, \rho_1, \rho_2) \in \boldsymbol{\Theta}\}$ with $\phi = \sigma_\mu^2/\sigma_\nu^2$ is also compact. In the following, the elements of $\boldsymbol{\Xi}$ are denoted by the vector $\boldsymbol{\vartheta}$.

A4 (**identification of $\boldsymbol{\vartheta}$**): For every $\boldsymbol{\vartheta} \in \boldsymbol{\Theta}$, $\boldsymbol{\vartheta} \neq \boldsymbol{\vartheta}_0$, and any $\varepsilon > 0$: $\limsup_{N\to\infty} \sup_{\boldsymbol{\vartheta} \in \overline{\mathbf{N}}_\varepsilon(\boldsymbol{\vartheta}_0)} (-\frac{1}{2} \ln(\frac{1}{NT} tr[\boldsymbol{\Sigma}_u(\boldsymbol{\vartheta}_0)\boldsymbol{\Sigma}_u(\boldsymbol{\vartheta})^{-1}]) - \frac{1}{2NT} \ln[\det \boldsymbol{\Sigma}_u(\boldsymbol{\vartheta})/ \det \boldsymbol{\Sigma}_u(\boldsymbol{\vartheta}_0)]) < 0$, where $\overline{\mathbf{N}}_\varepsilon(\boldsymbol{\vartheta}_0)$ is the complement of an open neighborhood of $\boldsymbol{\vartheta}_0$ of diameter ε.

A5 (identification of $\boldsymbol{\beta}$): The non-random matrix $\mathbf{X}$ has full column rank $K < N$ and its elements are uniformly bounded constants for all N. Further, the non-random matrix $\lim_{N\to\infty}(\frac{1}{NT}\mathbf{X}'\boldsymbol{\Sigma}_u(\boldsymbol{\vartheta}_0)^{-1}\mathbf{X})$ is finite and non-singular.

Consistency of the ML estimates under the general model.

In proving the consistency of MLE, we make use of the following lemmas.

Lemma 1 *Under the maintained assumptions, (i) the row and column sums of* $(\mathbf{A}'\mathbf{A})^{-1}$ *and* $(\mathbf{B}'\mathbf{B})^{-1}$ *are uniformly bounded in absolute value. (ii) the row and column sums of* $\boldsymbol{\Sigma}_u(\boldsymbol{\vartheta})$ *are uniformly bounded. (iii)* $\boldsymbol{\Sigma}_u(\boldsymbol{\vartheta})^{-1}$ *exists.*

Proof. By Assumption A2 the row and column sums of the matrices $\mathbf{W}$, $\mathbf{A}$, $\mathbf{B}$, $\mathbf{A}^{-1}$ and $\mathbf{B}^{-1}$ are uniformly bounded in absolute value. This property is preserved when multiplying those matrices (see Kelejian and Prucha, 2001, p. 241f). Hence, the row and column sums of $(\mathbf{A}'\mathbf{A})^{-1}$ and $(\mathbf{B}'\mathbf{B})^{-1}$ are also uniformly bounded in absolute value, say, by constants c_A and c_B, respectively.

(ii) The row and column sums of $\mathbf{\Sigma}_u(\boldsymbol{\vartheta})$ are uniformly bounded in absolute value by Assumptions A2 and A3. To see this, denote the typical element of $\mathbf{\Sigma}_u(\boldsymbol{\vartheta})$ by σ_{ij}. Then, $max_i \sum_j \sigma_{ij} \leq T\phi c_A + c_B < \infty$ and $max_j \sum_i \sigma_{ij} \leq T\phi c_A + c_B < \infty$.
(iii) Since $\mathbf{\Sigma}_u = (\overline{\mathbf{J}}_T \otimes (\frac{T\sigma_\mu^2}{\sigma_\nu^2}(\mathbf{A}'\mathbf{A})^{-1} + \sigma_\nu^2(\mathbf{B}'\mathbf{B})^{-1}))\ +(\mathbf{E}_T \otimes (\mathbf{B}'\mathbf{B})^{-1})$ and $(\mathbf{B}'\mathbf{B})^{-1}$ exists by Assumption A2, it remains to be shown that $(\frac{T\sigma_\mu^2}{\sigma_\nu^2}(\mathbf{A}'\mathbf{A})^{-1} + \sigma_\nu^2(\mathbf{B}'\mathbf{B})^{-1})$ is invertible. Using the updating formula we have $(\frac{T\sigma_\mu^2}{\sigma^2\nu}(\mathbf{A}'\mathbf{A})^{-1} + (\mathbf{B}'\mathbf{B})^{-1})^{-1} = \mathbf{B}'\mathbf{B} - \mathbf{B}'\mathbf{B}\left(\frac{\sigma_\nu^2}{T\sigma_\mu^2}\mathbf{A}'\mathbf{A} + \mathbf{B}'\mathbf{B}\right)^{-1}\mathbf{B}'\mathbf{B}$. The inverse will exist if $\det(\frac{\sigma_\nu^2}{T\sigma_\mu^2}\mathbf{A}'\mathbf{A} + \mathbf{B}'\mathbf{B}) \neq \mathbf{0}$. Since $\frac{\sigma_\nu^2}{T\sigma_\mu^2} > 0$ and since $\mathbf{A}$ and $\mathbf{B}$ have full rank by Assumption A2, $\mathbf{A}'\mathbf{A}$ and $\mathbf{B}'\mathbf{B}$ are positive definite and $\det(\frac{\sigma_\nu^2}{T\sigma_\mu^2}\mathbf{A}'\mathbf{A} + \mathbf{B}'\mathbf{B}) \geq \det(\frac{\sigma_\nu^2}{T\sigma_\mu^2}\mathbf{A}'\mathbf{A}) + \det(\mathbf{B}'\mathbf{B}) > 0$ (see Abadir and Magnus, 2005, p. 215 and p. 325). ■

Lemma 2 *Under the maintained assumptions, the matrices $\mathbf{\Sigma}_u(\boldsymbol{\vartheta})$ and $\mathbf{\Sigma}_u(\boldsymbol{\vartheta})^{-1}$ are positive definite.*

Proof. Observe that $\det[\mathbf{\Sigma}_u(\boldsymbol{\vartheta})] = \det[T\phi(\mathbf{A}'\mathbf{A})^{-1} + (\mathbf{B}'\mathbf{B})^{-1}]\det[(\mathbf{B}'\mathbf{B})^{-1}]^{T-1}$ and that $\det[T\phi(\mathbf{A}'\mathbf{A})^{-1} + (\mathbf{B}'\mathbf{B})^{-1}] \geq \det[T\phi(\mathbf{A}'\mathbf{A})^{-1}] + \det[(\mathbf{B}'\mathbf{B})^{-1}] > 0$, since $\phi > 0$ and $(\mathbf{A}'\mathbf{A})^{-1}$ as well as $(\mathbf{B}'\mathbf{B})^{-1}$ are positive definite by Assumption A2 (see Abadir and Magnus, 2005, p. 215 and p. 325). Therefore, $\mathbf{\Sigma}_u(\boldsymbol{\vartheta})$ and $\mathbf{\Sigma}_u(\boldsymbol{\vartheta})^{-1}$ are positive definite. ■

The proof of consistency of the maximum likelihood estimates under the general model is based on the concentrated log-likelihood:

$$L^c(\boldsymbol{\vartheta}) = -\tfrac{NT}{2}\ln 2\pi - \tfrac{NT}{2}\ln\widehat{\sigma}_\nu^2(\boldsymbol{\vartheta}) - \tfrac{1}{2}\ln\det\mathbf{\Sigma}_u(\boldsymbol{\vartheta}) - \tfrac{NT}{2},$$

where we make use of the first order conditions for $\boldsymbol{\beta}$ and σ_ν^2:

$$\begin{aligned}
\tfrac{\partial L(\boldsymbol{\beta},\boldsymbol{\theta})}{\partial\boldsymbol{\beta}} &= \tfrac{1}{\sigma_\nu^2}\left(\mathbf{X}'\mathbf{\Sigma}_u(\boldsymbol{\vartheta})^{-1}\mathbf{y} - \mathbf{X}'\mathbf{\Sigma}_u(\boldsymbol{\vartheta})^{-1}\mathbf{X}\boldsymbol{\beta}(\boldsymbol{\vartheta})\right) = \mathbf{0} \\
&\Rightarrow \widehat{\boldsymbol{\beta}}(\boldsymbol{\vartheta}) = \left(\mathbf{X}'\mathbf{\Sigma}_u(\boldsymbol{\vartheta})^{-1}\mathbf{X}\right)^{-1}\mathbf{X}'\mathbf{\Sigma}_u(\boldsymbol{\vartheta})^{-1}\mathbf{y} \\
\tfrac{\partial L(\boldsymbol{\beta},\boldsymbol{\theta})}{\partial\sigma_\nu^2} &= -\tfrac{NT}{2\sigma_\nu^2} + \tfrac{1}{2\sigma_\nu^4}\mathbf{u}(\widehat{\boldsymbol{\beta}}(\boldsymbol{\vartheta}))'\mathbf{\Sigma}_u(\boldsymbol{\vartheta})^{-1}\mathbf{u}(\widehat{\boldsymbol{\beta}}(\boldsymbol{\vartheta})) = 0 \\
&\Rightarrow \widehat{\sigma}_\nu^2(\boldsymbol{\vartheta}) = \tfrac{\mathbf{u}(\widehat{\boldsymbol{\beta}}(\boldsymbol{\vartheta}))'\mathbf{\Sigma}_u(\boldsymbol{\vartheta})^{-1}\mathbf{u}(\widehat{\boldsymbol{\beta}}(\boldsymbol{\vartheta}))}{NT}.
\end{aligned}$$

The non-stochastic counterpart of $L^c(\boldsymbol{\vartheta})$ is defined as

$$E[L(\boldsymbol{\beta}_0,\boldsymbol{\theta})] = -\tfrac{n}{2}\ln 2\pi - \tfrac{NT}{2}\ln\sigma_\nu^2 - \tfrac{1}{2}\ln\left[\det\mathbf{\Sigma}_u(\boldsymbol{\vartheta})\right] - \tfrac{\sigma_{v0}^2}{2\sigma_\nu^2}tr[\mathbf{\Sigma}(\boldsymbol{\vartheta})^{-1}\mathbf{\Sigma}_u(\boldsymbol{\vartheta}_0)]$$

with

$$\begin{aligned}\frac{\partial E[L(\boldsymbol{\beta}_0,\boldsymbol{\theta})]}{\partial \sigma_v^2} &= -\frac{NT}{2\sigma_\nu^{*2}} + \frac{\sigma_{\nu,0}^2}{2\sigma_\nu^{*4}} tr[\boldsymbol{\Sigma}_u(\boldsymbol{\vartheta})^{-1}\boldsymbol{\Sigma}_u(\boldsymbol{\vartheta}_0)] = 0 \\ &\Rightarrow \sigma_\nu^{*2}(\boldsymbol{\vartheta}) = \frac{\sigma_{\nu,0}^2}{NT} tr[\boldsymbol{\Sigma}_u(\boldsymbol{\vartheta})^{-1}\boldsymbol{\Sigma}_u(\boldsymbol{\vartheta}_0)].\end{aligned}$$

From Assumption A5 and Lemma 2 it follows that $\sigma_\nu^{*2}(\boldsymbol{\vartheta})$ is uniformly bounded away from zero by some positive constant. Furthermore,

$$\begin{aligned}Q(\boldsymbol{\vartheta}) &= \underset{\sigma_\nu^2}{max} E[L(\boldsymbol{\beta}_0,\boldsymbol{\theta})] \\ &= -\frac{NT}{2}\ln 2\pi - \frac{NT}{2}\ln \sigma_\nu^{*2}(\boldsymbol{\vartheta}) - \frac{1}{2}\ln\det \boldsymbol{\Sigma}_u(\boldsymbol{\vartheta}) - \frac{NT}{2}.\end{aligned}$$

Theorem 3 *Under Assumptions A1-A5, the maximum likelihood estimates are unique and consistent.*

Proof. To prove consistency, we have to show that $\frac{1}{NT}(L^c(\boldsymbol{\vartheta}) - Q(\boldsymbol{\vartheta}))$ converges uniformly to 0 in probability. Note that $\frac{1}{NT}\left(L^c(\boldsymbol{\vartheta}) - Q(\boldsymbol{\vartheta})\right) = -\frac{1}{2}(\ln\widehat{\sigma}_\nu^2(\boldsymbol{\vartheta})$ $-\ln\sigma_\nu^{*2}(\boldsymbol{\vartheta}))$ and $\mathbf{u}(\widehat{\boldsymbol{\beta}}(\boldsymbol{\vartheta}))'\boldsymbol{\Sigma}_u(\boldsymbol{\vartheta})^{-1}\mathbf{u}(\widehat{\boldsymbol{\beta}}(\boldsymbol{\vartheta})) = \mathbf{u}(\boldsymbol{\beta}_0)'\boldsymbol{\Sigma}_u(\boldsymbol{\vartheta})^{-1}\mathbf{u}(\boldsymbol{\beta}_0) - \mathbf{u}(\boldsymbol{\beta}_0)'\cdot$ $\boldsymbol{\Sigma}_u(\boldsymbol{\vartheta})^{-1}\mathbf{M}(\boldsymbol{\vartheta})\mathbf{u}(\boldsymbol{\beta}_0)= tr[\boldsymbol{\Sigma}_u(\boldsymbol{\vartheta})^{-1}(\mathbf{I}_{NT} - \mathbf{M}(\boldsymbol{\vartheta}))\mathbf{u}(\boldsymbol{\beta}_0)\mathbf{u}(\boldsymbol{\beta}_0)']$.
Now, $\lim_{N\to\infty} E[\widehat{\sigma}_\nu^2(\boldsymbol{\vartheta})-\sigma_\nu^{*2}(\boldsymbol{\vartheta})] = -\lim_{N\to\infty}\frac{1}{NT}E[tr[\boldsymbol{\Sigma}_u(\boldsymbol{\vartheta})^{-1}\mathbf{M}(\boldsymbol{\vartheta})\mathbf{u}(\boldsymbol{\beta}_0)\mathbf{u}(\boldsymbol{\beta}_0)']]$ $= -\lim_{N\to\infty}\frac{\sigma_{\nu,0}^2}{NT}tr[\boldsymbol{\Sigma}_u(\boldsymbol{\vartheta})^{-1}\mathbf{M}(\boldsymbol{\vartheta})\boldsymbol{\Sigma}_u(\boldsymbol{\vartheta}_0)]$. According to Assumption 2 and Lemma 1, the row and column sums of $\boldsymbol{\Sigma}_u(\boldsymbol{\vartheta})^{-1}\mathbf{M}(\boldsymbol{\vartheta})$ and $\boldsymbol{\Sigma}_u(\boldsymbol{\vartheta}_0)$ are uniformly bounded in absolute value and this property is preserved under matrix multiplication. Therefore, the elements of $\boldsymbol{\Sigma}_u(\boldsymbol{\vartheta})^{-1}\mathbf{M}(\boldsymbol{\vartheta})\boldsymbol{\Sigma}_u(\boldsymbol{\vartheta}_0)$ are uniformly bounded by some constant c_M (see also Lemma A.7 in Lee, 2004b) so that $\frac{\sigma_{\nu,0}^2}{NT}tr[\boldsymbol{\Sigma}_u(\boldsymbol{\vartheta})^{-1}\mathbf{M}(\boldsymbol{\vartheta})\boldsymbol{\Sigma}_u(\boldsymbol{\vartheta}_0)] \leq \frac{\sigma_{\nu,0}^2}{NT}Kc_M$ and $\lim_{N\to\infty}\frac{\sigma_{\nu,0}^2}{NT}Kc_M = 0$. The latter follows from the fact that $\boldsymbol{\Sigma}_u(\boldsymbol{\vartheta})^{-1}\mathbf{M}(\boldsymbol{\vartheta})\boldsymbol{\Sigma}_u(\boldsymbol{\vartheta}_0)$ is of rank K.
$\lim_{N\to\infty} Var[\widehat{\sigma}_\nu^2(\boldsymbol{\vartheta})-\sigma_\nu^{*2}(\boldsymbol{\vartheta})] = \lim_{N\to\infty} Var[\frac{1}{NT}tr[\boldsymbol{\Sigma}_u(\boldsymbol{\vartheta})^{-1}\mathbf{M}(\boldsymbol{\vartheta})\mathbf{u}(\boldsymbol{\beta}_0)\mathbf{u}(\boldsymbol{\beta}_0)']]$ $= \lim_{N\to\infty}\frac{2\sigma_{\nu,0}^4}{(NT)^2}tr\left[(\boldsymbol{\Sigma}_u(\boldsymbol{\vartheta})^{-1}\mathbf{M}(\boldsymbol{\vartheta})\boldsymbol{\Sigma}_u(\boldsymbol{\vartheta}_0))^2\right]$ using Lemma (A1) in Kelejian and Prucha (2007, p. 29) and Assumption A1. As a result, $\lim_{N\to\infty}\frac{2\sigma_{\nu,0}^4}{(NT)^2}tr[(\boldsymbol{\Sigma}_u(\boldsymbol{\vartheta})^{-1}\cdot$ $\mathbf{M}(\boldsymbol{\vartheta})\boldsymbol{\Sigma}_u(\boldsymbol{\vartheta}_0))^2] = o(1)$. By Chebyshev's inequality, we conclude that $\widehat{\sigma}_\nu^2(\boldsymbol{\vartheta}) -$ $\sigma_\nu^{*2}(\boldsymbol{\vartheta})$=$o_p(1)$.

Using the mean value theorem it follows that $\ln\widehat{\sigma}_\nu^2(\boldsymbol{\vartheta}) = \ln\sigma_\nu^{*2}(\boldsymbol{\vartheta})+\frac{\widehat{\sigma}_\nu^2(\boldsymbol{\vartheta})-\sigma_\nu^{*2}(\boldsymbol{\vartheta})}{\overline{\sigma}_\nu^2}$ with the constant $\overline{\sigma}_\nu^2(\boldsymbol{\vartheta})$ lying in between $\sigma_\nu^{*2}(\boldsymbol{\vartheta})$. Since $\widehat{\sigma}_\nu^2(\boldsymbol{\vartheta}) - \sigma_\nu^{*2}(\boldsymbol{\vartheta})$=$o_p(1)$,

$\widehat{\sigma}_\nu^2(\boldsymbol{\vartheta})$ will be bounded away from zero uniformly in probability. Accordingly, $\frac{1}{\overline{\sigma}_\nu^2(\boldsymbol{\vartheta})} < \frac{1}{\sigma_\nu^{*2}(\boldsymbol{\vartheta})} + \frac{1}{\widehat{\sigma}_\nu^2(\boldsymbol{\vartheta})} < \infty$. Therefore, we obtain $\underset{\boldsymbol{\vartheta}\in\boldsymbol{\Xi}}{sup}\frac{2}{NT}\left|L^c(\boldsymbol{\vartheta}) - Q(\boldsymbol{\vartheta})\right| = \underset{\boldsymbol{\vartheta}\in\boldsymbol{\Xi}}{sup}\left|\ln\widehat{\sigma}_\nu^2(\boldsymbol{\vartheta}) - \ln\sigma_\nu^{*2}(\boldsymbol{\vartheta})\right| = \underset{\boldsymbol{\vartheta}\in\boldsymbol{\Xi}}{sup}\frac{1}{\overline{\sigma}_\nu^2(\boldsymbol{\vartheta})}\left|\widehat{\sigma}_\nu^2(\boldsymbol{\vartheta}) - \sigma_\nu^{*2}(\boldsymbol{\vartheta})\right| = o_p(1)$.

Secondly, we have to prove the following uniqueness identification condition (see Lee, 2004a). For any $\varepsilon > 0$, $\limsup_{N\to\infty}\sup_{\boldsymbol{\vartheta}\in\overline{\mathbf{N}}_\varepsilon(\boldsymbol{\vartheta}_0)}\frac{1}{NT}(Q(\boldsymbol{\vartheta}) - Q(\boldsymbol{\vartheta}_0)) < 0$, where $\overline{\mathbf{N}}_\varepsilon(\boldsymbol{\vartheta}_0)$ is the complement of an open neighborhood of $\boldsymbol{\vartheta}_0$ of diameter ε. Note, $Q(\boldsymbol{\vartheta}) - Q(\boldsymbol{\vartheta}_0) = -\frac{NT}{2}[\ln\sigma_\nu^{*2}(\boldsymbol{\vartheta}) - \ln\sigma_\nu^{*2}(\boldsymbol{\vartheta}_0)] - \frac{1}{2}\ln[\det\boldsymbol{\Sigma}_u(\boldsymbol{\vartheta})/\det\boldsymbol{\Sigma}_u(\boldsymbol{\vartheta}_0)]$, $\ln\sigma_\nu^{*2}(\boldsymbol{\vartheta}) - \ln\sigma_\nu^{*2}(\boldsymbol{\vartheta}_0) = \ln tr\frac{1}{NT}[\boldsymbol{\Sigma}_u(\boldsymbol{\vartheta}_0)\boldsymbol{\Sigma}_u(\boldsymbol{\vartheta})^{-1}]$ and $\limsup_{N\to\infty}\sup_{\boldsymbol{\vartheta}\in\overline{\mathbf{N}}_\varepsilon(\boldsymbol{\vartheta}_0)}\frac{1}{NT}(Q(\boldsymbol{\vartheta}) - Q(\boldsymbol{\vartheta}_0)) = \limsup_{N\to\infty}\sup_{\boldsymbol{\vartheta}\in\overline{\mathbf{N}}_\varepsilon(\boldsymbol{\vartheta}_0)}(-\frac{1}{2}\ln\frac{1}{NT}tr[\boldsymbol{\Sigma}_u(\boldsymbol{\vartheta}_0)\boldsymbol{\Sigma}_u(\boldsymbol{\vartheta})^{-1}] - \frac{1}{2NT}\ln(\det\boldsymbol{\Sigma}_u(\boldsymbol{\vartheta})/\det\boldsymbol{\Sigma}_u(\boldsymbol{\vartheta}_0))) < 0$ for every $\boldsymbol{\vartheta} \neq \boldsymbol{\vartheta}_0 \in \boldsymbol{\Xi}$ and any $\varepsilon > 0$ by Assumption A4. Accordingly, $\widehat{\boldsymbol{\vartheta}}$ is unique and consistent, since $Q(\boldsymbol{\vartheta})$ is continuous and the parameter space is compact. Lastly, the consistency of $\widehat{\boldsymbol{\beta}}(\widehat{\boldsymbol{\vartheta}})$ is established by the lemma given below. ∎

Lemma 4 *If $\hat{\boldsymbol{\theta}} \xrightarrow{p} \boldsymbol{\theta}_0$ and $\widehat{\phi} \xrightarrow{p} \phi_0$ with $\phi_0 > 0$, then (i) $\frac{1}{NT}\mathbf{X}'\left(\boldsymbol{\Sigma}_u(\widehat{\boldsymbol{\vartheta}})^{-1} - \boldsymbol{\Sigma}_u(\boldsymbol{\vartheta}_0)^{-1}\right)\mathbf{X} \xrightarrow{p} \mathbf{0}$, and (ii) $(NT)^{-1/2}\mathbf{X}'\boldsymbol{\Sigma}_u^{-1}(\widehat{\boldsymbol{\vartheta}})\mathbf{u}(\widehat{\boldsymbol{\vartheta}}) - (NT)^{-1/2}\mathbf{X}'\boldsymbol{\Sigma}_u^{-1}(\boldsymbol{\vartheta}_0)\mathbf{u}(\boldsymbol{\vartheta}_0) \xrightarrow{p} \mathbf{0}$.*

Proof. (i) Let $\boldsymbol{\Sigma}_u(\boldsymbol{\vartheta}) = \left(\overline{\mathbf{J}}_T \otimes \boldsymbol{\Sigma}_1(\boldsymbol{\vartheta})\right) + (\mathbf{E}_T \otimes \boldsymbol{\Sigma}_2(\boldsymbol{\vartheta}))$, where $\boldsymbol{\Sigma}_1(\boldsymbol{\vartheta}) = (T\phi(\mathbf{A}'\mathbf{A})^{-1} + (\mathbf{B}'\mathbf{B})^{-1})$ and $\boldsymbol{\Sigma}_2(\boldsymbol{\vartheta}) = (\mathbf{B}'\mathbf{B})^{-1}$. Define $\boldsymbol{\Upsilon}_1 = (T\phi)^{-1}(\mathbf{A}'\mathbf{A})$ and $\boldsymbol{\Upsilon}_2 = \mathbf{B}'\mathbf{B}$.

Using $\boldsymbol{\Sigma}_1(\boldsymbol{\vartheta})^{-1} = (\boldsymbol{\Upsilon}_1^{-1} + \boldsymbol{\Upsilon}_2^{-1})^{-1} = \boldsymbol{\Upsilon}_2 - \boldsymbol{\Upsilon}_2[\boldsymbol{\Upsilon}_1 + \boldsymbol{\Upsilon}_2]^{-1}\boldsymbol{\Upsilon}_2$ yields $\boldsymbol{\Sigma}_1(\widehat{\boldsymbol{\vartheta}})^{-1} - \boldsymbol{\Sigma}_1(\boldsymbol{\vartheta}_0)^{-1} = \widehat{\boldsymbol{\Upsilon}}_2 - \boldsymbol{\Upsilon}_{2,0} - \widehat{\boldsymbol{\Upsilon}}_2[\widehat{\boldsymbol{\Upsilon}}_1 + \widehat{\boldsymbol{\Upsilon}}_2]^{-1}\widehat{\boldsymbol{\Upsilon}}_2 + \boldsymbol{\Upsilon}_{2,0}[\boldsymbol{\Upsilon}_{1,0} + \boldsymbol{\Upsilon}_{2,0}]^{-1}\boldsymbol{\Upsilon}_{2,0}$ and $\boldsymbol{\Sigma}_2(\widehat{\boldsymbol{\vartheta}})^{-1} - \boldsymbol{\Sigma}_2(\boldsymbol{\vartheta}_0)^{-1} = \widehat{\boldsymbol{\Upsilon}}_2 - \boldsymbol{\Upsilon}_{2,0}$. Define the non-stochastic $N \times K$ matrices $\mathbf{U}$ and $\mathbf{V}$ with uniformly bounded row and column sums to obtain

$$\begin{aligned}
N^{-1}\mathbf{U}'\left(\widehat{\boldsymbol{\Upsilon}}_1 - \boldsymbol{\Upsilon}_{1,0}\right)\mathbf{V} &= N^{-1}\mathbf{U}'[(T\widehat{\phi})^{-1}(\widehat{\mathbf{A}}'\widehat{\mathbf{A}}) - N^{-1}(T\phi_0)^{-1}(\mathbf{A}_0'\mathbf{A}_0)]\mathbf{V} \\
&= \frac{1}{NT}\left(\frac{1}{\widehat{\phi}} - \frac{1}{\phi_0}\right)\mathbf{U}'\mathbf{A}_0'\mathbf{A}_0\mathbf{V} \\
&\quad - \frac{1}{NT\widehat{\phi}}(\widehat{\rho}_1 - \rho_{1,0})\mathbf{U}'(\mathbf{W} + \mathbf{W}')\mathbf{V} \\
&\quad + \frac{1}{NT\widehat{\phi}}(\widehat{\rho}_1^2 - \rho_{1,0}^2)\mathbf{U}'(\mathbf{W}'\mathbf{W})\mathbf{V}.
\end{aligned}$$

Since the elements of $\mathbf{U}'\mathbf{A}_0'\mathbf{A}_0\mathbf{V}$, $\mathbf{U}'(\mathbf{W}+\mathbf{W}')\mathbf{V}$ and $\mathbf{U}'(\mathbf{W}'\mathbf{W})\mathbf{V}$ are uniformly bounded under the maintained assumptions and since $\widehat{\boldsymbol{\vartheta}}$ is a consistent estimator of $\boldsymbol{\vartheta}_0$, we have $plim_{N\to\infty} N^{-1}\mathbf{U}'\left(\widehat{\boldsymbol{\Upsilon}}_1 - \boldsymbol{\Upsilon}_{1,0}\right)\mathbf{V} = \mathbf{0}$. By the same argument, it also holds that $plim_{N\to\infty} N^{-1}\mathbf{U}'\left(\widehat{\boldsymbol{\Upsilon}}_2 - \boldsymbol{\Upsilon}_{2,0}\right)\mathbf{V} = \mathbf{0}$.

To show that $plim_{N\to\infty} N^{-1}\mathbf{U}'[(\widehat{\boldsymbol{\Upsilon}}_1 + \widehat{\boldsymbol{\Upsilon}}_2)^{-1} - (\boldsymbol{\Upsilon}_{1,0} + \boldsymbol{\Upsilon}_{2,0})^{-1}]\mathbf{V} = \mathbf{0}$, it is useful to define $\widehat{\mathbf{E}} = [\widehat{\boldsymbol{\Upsilon}}_1 + \widehat{\boldsymbol{\Upsilon}}_2]$ and $\mathbf{E}_0 = [\boldsymbol{\Upsilon}_{1,0} + \boldsymbol{\Upsilon}_{2,0}]$. Then, we need to show that $plim_{N\to\infty} N^{-1}\mathbf{U}'\{\mathbf{E}_0^{-1} - [\mathbf{E}_0 + (\widehat{\mathbf{E}} - \mathbf{E}_0)]^{-1}\}\mathbf{V} = \mathbf{0}$. We may write $N^{-1}\mathbf{U}'\{\mathbf{E}_0^{-1} - [\mathbf{E}_0 + (\widehat{\mathbf{E}} - \mathbf{E}_0)]^{-1}\}\mathbf{V} = N^{-1}\sum_{k=1}^{\infty}(-1)^{k+1}\mathbf{U}'[\mathbf{E}_0^{-1}(\widehat{\mathbf{E}} - \mathbf{E}_0)]^k\mathbf{E}_0^{-1}\mathbf{V}$, following Horn and Johnson (1985, p. 335). The claim now follows by applying the subsequence argument of Lemma C.6 in Kelejian and Prucha (2008). Next, observe that

$$\tfrac{1}{NT}\mathbf{X}'\boldsymbol{\Sigma}_u^{-1}(\boldsymbol{\vartheta})\mathbf{X} = \tfrac{1}{NT}\sum_{t=1}^{T}\left[\mathbf{X}_t'\boldsymbol{\Upsilon}_2\mathbf{X}_t - \left(\sum_{s=1}^{T}\tfrac{1}{T}\mathbf{X}_s'\boldsymbol{\Upsilon}_2\mathbf{E}^{-1}\boldsymbol{\Upsilon}_2\mathbf{X}_t\right)\right].$$

What remains to be shown is

$$\tfrac{1}{N}\mathbf{X}_s'\widehat{\boldsymbol{\Upsilon}}_2\widehat{\mathbf{E}}^{-1}\widehat{\boldsymbol{\Upsilon}}_2\mathbf{X}_t - \tfrac{1}{N}\mathbf{X}_s'\boldsymbol{\Upsilon}_{2,0}\mathbf{E}_0^{-1}\boldsymbol{\Upsilon}_{2,0}\mathbf{X}_t = -\sum_{k=1}^{7}\Delta_k = o_p(1),$$

where[11]

$$\begin{aligned}
\Delta_1 &= \tfrac{1}{N}\mathbf{X}_s'\left[\left(\boldsymbol{\Upsilon}_{1,0} - \widehat{\boldsymbol{\Upsilon}}_1\right)\mathbf{E}_0^{-1}\boldsymbol{\Upsilon}_{1,0}\right]\mathbf{X}_t\\
\Delta_2 &= \tfrac{1}{N}\mathbf{X}_s'\left[\boldsymbol{\Upsilon}_{1,0}(\mathbf{E}_0^{-1} - \widehat{\mathbf{E}}^{-1})\boldsymbol{\Upsilon}_{1,0}\right]\mathbf{X}_t\\
\Delta_3 &= \tfrac{1}{N}\mathbf{X}_s'\left[\boldsymbol{\Upsilon}_{1,0}\mathbf{E}_0^{-1}(\boldsymbol{\Upsilon}_{1,0} - \widehat{\boldsymbol{\Upsilon}}_1)\right]\mathbf{X}_t\\
\Delta_4 &= -\tfrac{1}{N}\mathbf{X}_s'\left[(\boldsymbol{\Upsilon}_{1,0} - \widehat{\boldsymbol{\Upsilon}}_1)(\mathbf{E}_0^{-1} - \widehat{\mathbf{E}}^{-1})\boldsymbol{\Upsilon}_{1,0})\right]\mathbf{X}_t\\
\Delta_5 &= -\tfrac{1}{N}\mathbf{X}_s'\left[(\boldsymbol{\Upsilon}_{1,0} - \widehat{\boldsymbol{\Upsilon}}_1)\mathbf{E}_0^{-1}(\widehat{\boldsymbol{\Upsilon}}_1 - \boldsymbol{\Upsilon}_{1,0})\right]\mathbf{X}_t\\
\Delta_6 &= -\tfrac{1}{N}\mathbf{X}_s'\left[\boldsymbol{\Upsilon}_{1,0}(\mathbf{E}_0^{-1} - \widehat{\mathbf{E}}^{-1})(\boldsymbol{\Upsilon}_{1,0} - \widehat{\boldsymbol{\Upsilon}}_1)\right]\mathbf{X}_t\\
\Delta_7 &= \tfrac{1}{N}\mathbf{X}_s'\left[(\boldsymbol{\Upsilon}_{1,0} - \widehat{\boldsymbol{\Upsilon}}_1)(\widehat{\mathbf{E}}^{-1} - \mathbf{E}_0^{-1})(\boldsymbol{\Upsilon}_{1,0} - \widehat{\boldsymbol{\Upsilon}}_1)\right]\mathbf{X}_t
\end{aligned}$$

Let $\mathbf{U} = \mathbf{X}_s'$, $\mathbf{X}_s'\boldsymbol{\Upsilon}_{1,0}$ or $\mathbf{X}_s'\boldsymbol{\Upsilon}_{1,0}\mathbf{E}_0^{-1}$ and define $\mathbf{V}$ in a similar way. Then,

[11] Let C and E be conformable matrices. Simple, but tedious derivation shows that $C_0E_0C_0 - CEC = (C_0 - C)E_0C_0 + C_0(E_0 - E)C_0 + C_0E_0(C_0 - C)\ -(C_0 - C)(E_0 - E)C_0 - -(C_0 - C)E_0(C_0 - C)\ -C_0(E_0 - E)(C_0 - C) + (C_0 - C)(E_0 - E)(C_0 - C)$.

it follows that $\Delta_k = o_p(1), k = 1, 2, 3$. Similarly,

$$\begin{aligned}
& N^{-1}\mathbf{U}'(\boldsymbol{\Upsilon}_{1,0} - \widehat{\boldsymbol{\Upsilon}}_1)(\mathbf{E}_0^{-1} - \widehat{\mathbf{E}}^{-1})\boldsymbol{\Upsilon}_{1,0}\mathbf{V} \\
= \; & \tfrac{1}{NT}\left(\tfrac{1}{\widehat{\phi}} - \tfrac{1}{\phi_0}\right)\mathbf{U}'\mathbf{A}_0'\mathbf{A}_0(\mathbf{E}_0^{-1} - \widehat{\mathbf{E}}^{-1})\boldsymbol{\Upsilon}_{1,0}\mathbf{V} \\
& -\tfrac{1}{NT\widehat{\phi}}(\widehat{\rho}_1 - \rho_{1,0})\mathbf{U}'(\mathbf{W} + \mathbf{W}')(\mathbf{E}_0^{-1} - \widehat{\mathbf{E}}^{-1})\boldsymbol{\Upsilon}_{1,0}\mathbf{V} \\
& -N^{-1}\tfrac{1}{NT\widehat{\phi}}(\widehat{\rho}_1^2 - \rho_{1,0}^2)\mathbf{U}'(\mathbf{W}'\mathbf{W})(\mathbf{E}_0^{-1} - \widehat{\mathbf{E}}^{-1})\boldsymbol{\Upsilon}_{1,0}\mathbf{V},
\end{aligned}$$

and $\Delta_k = o_p(1)$, $k = 4, 5, 6$. Lastly,

$$\begin{aligned}
& \tfrac{1}{NT}\left(\tfrac{1}{\widehat{\phi}} - \tfrac{1}{\phi_0}\right)\mathbf{U}'\mathbf{A}_0'\mathbf{A}_0(\mathbf{E}_0^{-1} - \widehat{\mathbf{E}}^{-1})(\boldsymbol{\Upsilon}_{1,0} - \widehat{\boldsymbol{\Upsilon}}_1)\mathbf{V} \\
= \; & \tfrac{1}{NT^2}\left(\tfrac{1}{\widehat{\phi}} - \tfrac{1}{\phi_0}\right)^2\mathbf{U}'\mathbf{A}_0'\mathbf{A}_0(\mathbf{E}_0^{-1} - \widehat{\mathbf{E}}^{-1})\mathbf{A}_0'\mathbf{A}_0\mathbf{V} \\
& -\tfrac{1}{NT^2}\left(\tfrac{1}{\widehat{\phi}} - \tfrac{1}{\phi_0}\right)^2\mathbf{U}'\mathbf{A}_0'\mathbf{A}_0(\mathbf{E}_0^{-1} - \widehat{\mathbf{E}}^{-1})(T\widehat{\phi})^{-1}(\widehat{\rho}_1 - \rho_{1,0})(\mathbf{W} + \mathbf{W}')\mathbf{V} \\
& +\tfrac{1}{NT}\left(\tfrac{1}{\widehat{\phi}} - \tfrac{1}{\phi_0}\right)\mathbf{U}'\mathbf{A}_0'\mathbf{A}_0(\mathbf{E}_0^{-1} - \widehat{\mathbf{E}}^{-1})(T\widehat{\phi})^{-1}(\widehat{\rho}_1^2 - \rho_{1,0}^2)(\mathbf{W}'\mathbf{W})\mathbf{V}.
\end{aligned}$$

So $\Delta_7 = o_p(1)$ and $\frac{1}{NT}\left(\mathbf{X}'\boldsymbol{\Sigma}_u(\widehat{\boldsymbol{\vartheta}})^{-1}\mathbf{X} - \mathbf{X}'\boldsymbol{\Sigma}_u(\boldsymbol{\vartheta}_\mathbf{0})^{-1}\mathbf{X}\right) \xrightarrow{p} \mathbf{0}$.

(ii) Following Kapoor, Kelejian, and Prucha (2007), write

$$\begin{aligned}
N^{-1}\mathbf{U}'\left(\widehat{\boldsymbol{\Upsilon}}_1 - \boldsymbol{\Upsilon}_{1,0}\right)\mathbf{v} \; &= \; N^{-1}\mathbf{U}'[(T\widehat{\phi})^{-1}(\widehat{\mathbf{A}}'\widehat{\mathbf{A}}) - N^{-1}(T\phi_0)^{-1}(\mathbf{A}_0'\mathbf{A}_0)]\mathbf{v} \\
&= \; \tfrac{1}{NT}\left(\tfrac{1}{\widehat{\phi}} - \tfrac{1}{\phi_0}\right)\mathbf{U}'\mathbf{A}_0'\mathbf{A}_0\mathbf{v} \\
& \quad -\tfrac{1}{NT\widehat{\phi}}(\widehat{\rho}_1 - \rho_{1,0})\mathbf{U}'(\mathbf{W} + \mathbf{W}')\mathbf{v} \\
& \quad +\tfrac{1}{NT\widehat{\phi}}(\widehat{\rho}_1^2 - \rho_{1,0}^2)\mathbf{U}'(\mathbf{W}'\mathbf{W})\mathbf{v},
\end{aligned}$$

where $\mathbf{v}$ is an $N \times 1$ random vector with variance-covariance matrix $\boldsymbol{\Omega}_v$. $\boldsymbol{\Omega}_v$ has uniformly bounded elements. For this observe that $E[\mathbf{K}_k\mathbf{v}] = \mathbf{0}$, where $\mathbf{K}_1 = \mathbf{U}'\mathbf{A}_0'\mathbf{A}_0$, $\mathbf{K}_2 = \mathbf{U}'(\mathbf{W} + \mathbf{W}')$, and $\mathbf{K}_3 = \mathbf{U}'(\mathbf{W}'\mathbf{W})$. Note that the terms $(NT)^{-1/2}\mathbf{K}_k\mathbf{v}$ have variance-covariance matrices $(NT)^{-1}\mathbf{K}_k\boldsymbol{\Omega}_v\mathbf{K}_k'$ for $k = 1, 2, 3$. Under the maintained assumptions, these have uniformly bounded row and column sums, and their elements are uniformly bounded in absolute value. Therefore, $(NT)^{-1/2}\mathbf{K}_k\mathbf{v} = O_p(1)$ and, therefore, $N^{-1/2}\mathbf{U}'\left(\widehat{\boldsymbol{\Upsilon}}_1 - \boldsymbol{\Upsilon}_{1,0}\right)\mathbf{v} = o_p(1)$, since $\hat{\boldsymbol{\theta}} \xrightarrow{p} \boldsymbol{\theta}_0$. By the same argument, $N^{-1/2}\mathbf{U}'\left(\widehat{\boldsymbol{\Upsilon}}_2 - \boldsymbol{\Upsilon}_{2,0}\right)\mathbf{v} = o_p(1)$. Using the idea of the proof of $plim_{N\to\infty}N^{-1}\mathbf{U}'[(\widehat{\boldsymbol{\Upsilon}}_1 + \widehat{\boldsymbol{\Upsilon}}_2)^{-1} - (\boldsymbol{\Upsilon}_{1,0} + \boldsymbol{\Upsilon}_{2,0})^{-1}]\mathbf{V} = \mathbf{0}$ used before, one can show that $plim_{N\to\infty}N^{-1/2}\mathbf{U}'[(\widehat{\boldsymbol{\Upsilon}}_1 + \widehat{\boldsymbol{\Upsilon}}_2)^{-1}$

$-(\boldsymbol{\Upsilon}_{1,0}+\boldsymbol{\Upsilon}_{2,0})^{-1}]\mathbf{v}=\mathbf{0}$. Furthermore, using a similar decomposition as above, the claim of the second lemma can be established. ■

Appendix C: LM Test for random effects

The following lemma is useful in proving Theorems 6 and 7 that derive the asymptotic distribution of the LM tests for the random effects model and the KKP model.

Lemma 5 *Assume that Assumptions A1-A2 hold and* $\rho_1=\rho_2=\rho$. *Consider the quadratic form* $Q=(\mathbf{Z}_\mu\mathbf{A}^{-1}\boldsymbol{\mu}+(\mathbf{I}_T\otimes\mathbf{A}^{-1})\boldsymbol{\nu})'\left(\alpha\overline{\mathbf{J}}_T+(1-\alpha)\mathbf{E}_T\otimes\mathbf{H}\right)\cdot(\mathbf{Z}_\mu\mathbf{A}^{-1}\boldsymbol{\mu}+(\mathbf{I}_T\otimes\mathbf{A}^{-1})\boldsymbol{\nu})$, *where* $\mathbf{H}=(\mathbf{W}'\mathbf{A}+\mathbf{A}'\mathbf{W})$ *and* $0\leq\alpha\leq1$ *is a real number. Then,* $E[Q]=(\alpha\sigma_1^2+(1-\alpha)\sigma_\nu^2(T-1))tr[\mathbf{H}(\mathbf{A}'\mathbf{A})^{-1}]$, $Var[Q]=2(\alpha^2\sigma_1^4+(1-\alpha)^2(T-1)\sigma_\nu^4)tr[(\mathbf{H}(\mathbf{A}'\mathbf{A})^{-1})^2]$ *and* $\frac{Q-(\alpha\sigma_1^2+(1-\alpha)\sigma_\nu^2(T-1))tr[\mathbf{H}(\mathbf{A}'\mathbf{A})^{-1}]}{\sqrt{2(\alpha^2\sigma_1^4+(1-\alpha)^2(T-1)\sigma_\nu^4)tr[(\mathbf{H}(\mathbf{A}'\mathbf{A})^{-1})^2]}}$ $\xrightarrow{d}N(0,1)$.

Proof. Inserting $\mathbf{Z}_\mu=(\iota_T\otimes\mathbf{I}_N)$ yields

$$Q=\boldsymbol{\xi}'\begin{bmatrix}\alpha T\mathbf{L} & \alpha\mathbf{L} & .. & \alpha\mathbf{L}\\ \alpha\mathbf{L} & \mathbf{L}((1-\alpha)+\frac{2\alpha-1}{T}) & .. & \mathbf{L}(\frac{2\alpha-1}{T})\\ .. & .. & .. & ..\\ \alpha\mathbf{L} & \mathbf{L}(\frac{2\alpha-1}{T}) & .. & \mathbf{L}((1-\alpha)+\frac{2\alpha-1}{T})\end{bmatrix}\boldsymbol{\xi}$$

$$=\alpha T\boldsymbol{\mu}'\mathbf{L}\boldsymbol{\mu}+2\alpha\sum_{t=1}^T\boldsymbol{\nu}_t'\mathbf{L}\boldsymbol{\mu}+(1-\alpha)\sum_{t=1}^T\boldsymbol{\nu}_t'\mathbf{L}\boldsymbol{\nu}_t+(2\alpha-1)\tfrac{1}{T}\left(\sum_{t=1}^T\boldsymbol{\nu}_t'\right)\mathbf{L}\left(\sum_{t=1}^T\boldsymbol{\nu}_t\right),$$

where $\mathbf{L}=\mathbf{A}'^{-1}\mathbf{H}\mathbf{A}^{-1}$ and $\boldsymbol{\xi}=(\boldsymbol{\mu}',\boldsymbol{\nu}_1',...,\boldsymbol{\nu}_T')'$. It can easily verified that $E[Q]=(\alpha T\sigma_1^2+(1-\alpha)\sigma_\nu^2(T-1))tr[\mathbf{H}(\mathbf{A}'\mathbf{A})^{-1}]$ and that $Var[Q]=(\alpha^2\sigma_1^4+(1-\alpha)^2(T-1)\sigma_\nu^4)tr[(\mathbf{H}(\mathbf{A}'\mathbf{A})^{-1})^2]$. Observe that $\sigma_1^4>0$ by Assumption A1. Now $tr[(\mathbf{H}(\mathbf{A}'\mathbf{A})^{-1})^2]=2tr[\mathbf{A}'^{-1}\mathbf{W}'\mathbf{W}\mathbf{A}^{-1}+(\mathbf{W}\mathbf{A}^{-1})^2]\geq2tr[(\mathbf{W}\mathbf{A}^{-1})^2]$. By Assumption A2, we can write $\mathbf{W}=\mathbf{T}^{-1}\Lambda\mathbf{T}$, where Λ is the diagonal matrix of eigenvalues of $\mathbf{W}$ and $\mathbf{T}$ is the corresponding matrix of eigenvectors. Then, $\mathbf{W}\mathbf{A}^{-1}=\mathbf{T}^{-1}\Lambda\left(\sum\limits_{k=0}^{\infty}\rho^k\Lambda^k\right)\mathbf{T}=\mathbf{T}^{-1}Diag\left(\frac{\lambda_i}{1-\rho\lambda_i}\right)\mathbf{T}$ and $N^{-1}tr((\mathbf{W}\mathbf{A}^{-1})^2)=N^{-1}\sum\limits_{i=0}^{N}\left(\frac{\lambda_i}{1-\rho\lambda_i}\right)^2\geq c_{bC1}>0$, since λ_{max} is bounded away from zero by some positive constant according to Assumption A2. Note also that by Assumption A2 $|\rho_r\lambda_{\max}|<1$. Hence, $Var[Q]$ is bounded away from zero by some positive constant.

The row and column sums of $\mathbf{A}$, $(\mathbf{A}'\mathbf{A})^{-1}$ and $\mathbf{H}$ are uniformly bounded and so are those of $\mathbf{L}$. Since the elements of $\boldsymbol{\xi}$ are independent and normally distributed by Assumption A1, the assumptions of the central limit theorem for linear quadratic forms given as Theorem 1 in Kelejian and Prucha (2001, p. 227) are fulfilled and the claim of the lemma follows. ■

Next, this Appendix derives the LM test for the null hypothesis H_0^A : $\rho_1 = \rho_2 = 0$. Under H_0^A we have $\mathbf{B} = \mathbf{A} = \mathbf{I}_N$. Using the general formulas for the score and the information matrix given above one can show that the corresponding LM test statistic is given by

$$\widetilde{LM}_A = \tfrac{1}{2b_A\widetilde{\sigma}_1^4}\widetilde{G}_A^2 + \tfrac{1}{2b_A(T-1)\widetilde{\sigma}_\nu^4}\widetilde{M}_A^2,$$

where $\widetilde{G}_A = \widetilde{\mathbf{u}}'\left[\overline{\mathbf{J}}_T \otimes (\mathbf{W}' + \mathbf{W})\right]\widetilde{\mathbf{u}}$, $\widetilde{M}_A = \widetilde{\mathbf{u}}'\left[\mathbf{E}_T \otimes (\mathbf{W}' + \mathbf{W})\right]\widetilde{\mathbf{u}}$ and $b_A = tr\left[(\mathbf{W}' + \mathbf{W})^2\right]$.

Theorem 6 **(LM_A)** *Suppose Assumptions A1 - A5 hold and H_0^A: $\rho_1 = \rho_2 = 0$ is true. Then, $\widetilde{LM}_A = \frac{1}{2b_A\widetilde{\sigma}_1^4}\widetilde{G}_A^2 + \frac{1}{2b_A(T-1)\widetilde{\sigma}_\nu^4}\widetilde{M}_A^2$ is asymptotically distributed as χ_2^2.*

Proof. First, use the residuals of the true model $\mathbf{u} = \mathbf{y} - \mathbf{X}\boldsymbol{\beta}_0$ and define $G_A = \mathbf{u}'\mathbf{G}_A\mathbf{u}$ and $M_A = \mathbf{u}'\mathbf{M}_A\mathbf{u}$, where $\mathbf{G}_A = \overline{\mathbf{J}}_T \otimes (\mathbf{W}' + \mathbf{W})$, and $\mathbf{M}_A = \mathbf{E}_T \otimes (\mathbf{W}' + \mathbf{W})$.

(i) We can apply Lemma 5 by setting $\alpha = 1$ and $\mathbf{A} = \mathbf{I}_N$ so that $\mathbf{H} =(\mathbf{W}'+\mathbf{W})$ with $tr[\mathbf{H}] = 0$, because $tr[\mathbf{W}] = 0$. Hence, $E[G_A] = 0$ and $Var[G_A] = 2\sigma_1^4 b_A$ with $b_A = tr[\mathbf{H}^2]$. By Assumption A2 the row and column sums of $\mathbf{H}$ are uniformly bounded. $\sigma_1^2\sqrt{2b_A}$ is bounded way from zero by some positive constant as shown in Lemma 5, so $\frac{G_A}{\sigma_1^2\sqrt{2b_A}} \xrightarrow{d} N(0,1)$.

(ii) Setting $\alpha = 0$ in Lemma 5 implies that $\frac{M_A}{\sigma_\nu^2\sqrt{2(T-1)b_A}} \xrightarrow{d} N(0,1)$.

(iii) Inspection of the proof in Lemma 5 establishes the independence of G_A and M_A. From Lemma 5 it follows that $\frac{\alpha_1'}{\sigma_1^2\sqrt{2b_A}}G_A + \frac{\alpha_2'}{\sigma_\nu^2\sqrt{2(T-1)b_A}}M_A$ with $\frac{\alpha_1'}{\sigma_1^2\sqrt{2b_A}} + \frac{\alpha_2'}{\sigma_\nu^2\sqrt{2(T-1)b_A}} = 1$ is also asymptotically normal and, hence, the vector of quadratic forms $\left[\frac{G_A}{\sigma_1^2\sqrt{2b_A}}, \frac{M_A}{\sigma_\nu^2\sqrt{2(T-1)b_A}}\right]'$ converges to a bivariate normal by the Cramér-Wold device. Consequently, $LM_A = \frac{1}{2b_A\sigma_1^4}G_A^2 + \frac{1}{2b_A(T-1)\sigma_\nu^4}M_A^2$ is asymptotically distributed as χ_2^2.

(iv) Notice that $\frac{1}{\sqrt{NT}}\widetilde{\mathbf{u}}'\mathbf{G}_A\widetilde{\mathbf{u}} - \frac{1}{\sqrt{NT}}\mathbf{u}'\mathbf{G}_A\mathbf{u} = \frac{2}{NT}\mathbf{u}'\mathbf{G}_A\mathbf{X}\sqrt{NT}(\widetilde{\boldsymbol{\beta}}-\boldsymbol{\beta}_0) + (NT)^{-\frac{3}{2}} \cdot \sqrt{NT}(\widetilde{\boldsymbol{\beta}}-\boldsymbol{\beta}_0)'\mathbf{X}'\mathbf{G}_A\mathbf{X}\sqrt{NT}(\widetilde{\boldsymbol{\beta}}-\boldsymbol{\beta}_0)$. Given a $\sqrt{N}$-consistent estimator of $\boldsymbol{\beta}_0$, say $\widetilde{\boldsymbol{\beta}}$ and $\widetilde{\mathbf{u}} = \mathbf{y} - \mathbf{X}\widetilde{\boldsymbol{\beta}}$, we have $\frac{1}{\sqrt{NT}}\widetilde{\mathbf{u}}'\mathbf{G}_A\widetilde{\mathbf{u}} - \frac{1}{\sqrt{NT}}\mathbf{u}'\mathbf{G}_A\mathbf{u} = o_p(1)$, since $\mathbf{X}$ and $\mathbf{G}_A$ are non-stochastic matrices (see Lemma 1 in Kelejian and Prucha, 2001, p. 229). Similarly, $\frac{1}{\sqrt{NT}}\widetilde{\mathbf{u}}'\mathbf{M}_A\widetilde{\mathbf{u}} - \frac{1}{\sqrt{NT}}\mathbf{u}'\mathbf{M}_A\mathbf{u} = o_p(1)$. Further, $(NT)^{-1}2\sigma_1^4 b_A > c_1 > 0$ for some constant c_1 and $(NT)^{-1}2\sigma_\nu^4 \cdot (T-1)b_A > c_\nu > 0$ for some constant c_ν, since $\sigma_\mu^2 > 0$ and $\sigma_\nu^2 > 0$ by Assumption A1 and $0 < c_{b_A} \leq b_A$ by Assumption A2. As shown in Appendix B, $\widetilde{\sigma}_1^2 = \sigma_1^2 + o_p(1)$ and $\widetilde{\sigma}_\nu^2 = \sigma_\nu^2 + o_p(1)$. Then, Theorem 2 of Kelejian and Prucha (2001, p. 230) implies that $\frac{\widetilde{G}_A}{\sqrt{2\widetilde{\sigma}_1^4 b_A^2}} - \frac{G_A}{\sqrt{2\sigma_1^4 b_A^2}} = o_p(1)$ and $\frac{\widetilde{M}_A}{\sqrt{2\widetilde{\sigma}_\nu^4(T-1)b_A}} - \frac{M_A}{\sqrt{2\sigma_\nu^4(T-1)b_A}} = o_p(1)$. Hence, $\widetilde{LM}_A - LM_A = o_p(1)$. ■

Appendix D: LM Test for the KKP Model

To derive the asymptotic distribution of the LM test for H_0^C, it proves useful to re-parameterize the model so that $\rho_1 = \rho_2 + \Delta$ and to test $H_0^B : \Delta = 0$ vs. $H_1^B : \Delta \neq 0$, Under H_0^C, $\mathbf{B} = \mathbf{A}$, $\boldsymbol{\Omega}_u = (\sigma_1^2\overline{\mathbf{J}}_T + \sigma_\nu^2\mathbf{E}_T) \otimes (\mathbf{A}'\mathbf{A})^{-1}$ and $\boldsymbol{\Omega}_u^{-1} = (\frac{1}{\sigma_1^2}\overline{\mathbf{J}}_T + \frac{1}{\sigma_\nu^2}\mathbf{E}_T) \otimes (\mathbf{A}'\mathbf{A})$. Using the general formulas for the score and for the information matrix given above, the LM test statistic can be derived as

$$\overline{LM}_C = \overline{\mathbf{D}}_\theta'\overline{\mathbf{J}}_\theta^{-1}\overline{\mathbf{D}}_\theta = \frac{T}{2\overline{b}_C(T-1)\overline{\sigma}_1^4}\overline{G}_C^2.$$

where $\overline{b}_C = \overline{e}_C - \overline{d}_C^2/N$ and $\overline{G}_C = \overline{\mathbf{u}}'(\overline{\mathbf{J}}_T \otimes \overline{\mathbf{F}})\overline{\mathbf{u}} - \overline{\sigma}_1^2 tr[\overline{\mathbf{D}}]$.

Theorem 7 **(LM_C)** *Suppose Assumptions A1 - A5 hold and H_0^c: $\rho_1 = \rho_2 = \rho$ is true. Let $\overline{\mathbf{F}} = (\mathbf{W}'\overline{\mathbf{A}} + \overline{\mathbf{A}}'\mathbf{W})$, $\overline{\mathbf{D}} = \overline{\mathbf{F}}(\overline{\mathbf{A}}'\overline{\mathbf{A}})^{-1}$, $\overline{b}_C = \overline{e}_C - \overline{d}_C^2/N$, $\overline{d}_C = tr[\overline{\mathbf{D}}]$ and $\overline{e}_C = tr[\overline{\mathbf{D}}^2]$ and $\overline{G}_C = \overline{\mathbf{u}}'(\overline{\mathbf{J}}_T \otimes \overline{\mathbf{F}})\overline{\mathbf{u}} - \overline{\sigma}_1^2 tr[\overline{\mathbf{D}}]$. Then, $\overline{LM}_C = \frac{T}{2\overline{b}_C(T-1)\overline{\sigma}_1^4}\overline{G}_C^2$ is asymptotically distributed as χ_1^2.*

Proof. We will make use of the following first order conditions

$$\left.\frac{\partial L}{\partial \Delta}\right|_{H_0^C} = -\frac{T\sigma_\mu^2}{2\sigma_1^2}tr[\mathbf{D}] + \frac{1}{2}\mathbf{u}'(\frac{T\sigma_\mu^2}{\sigma_1^4}\overline{\mathbf{J}}_T \otimes \mathbf{F})\mathbf{u}$$

$$\left.\frac{\partial L}{\partial \rho_2}\right|_{H_0^C} = -\frac{T}{2}tr[\mathbf{D}] + \frac{1}{2}\mathbf{u}'[(\frac{1}{\sigma_1^2}\overline{\mathbf{J}}_T + \frac{1}{\sigma_\nu^2}\mathbf{E}_T) \otimes \mathbf{F}]\mathbf{u}.$$

From the first order conditions, we obtain

$$\overline{\sigma}_1^2 tr[\overline{\mathbf{D}}]=\tfrac{1}{T}\overline{\mathbf{u}}'(\overline{\mathbf{J}}_T\otimes\overline{\mathbf{F}})\overline{\mathbf{u}}+\tfrac{1}{T}\overline{\mathbf{u}}'(\mathbf{E}_T\otimes\overline{\mathbf{F}})\overline{\mathbf{u}},$$

inserting the ML-estimates denoted by a bar. This gives

$$\tfrac{T-1}{T}\overline{LM}_C=\frac{\left(\frac{T-1}{T}\overline{\mathbf{u}}'(\overline{\mathbf{J}}_T\otimes\overline{\mathbf{F}})\overline{\mathbf{u}}-\frac{1}{T}\overline{\mathbf{u}}'(\mathbf{E}_T\otimes\overline{\mathbf{F}})\overline{\mathbf{u}}\right)^2}{2(tr[\overline{\mathbf{D}}^2]-\frac{1}{N}tr[\overline{\mathbf{D}}]^2)\overline{\sigma}_1^4}.$$

Then,

$$\sqrt{\tfrac{T-1}{T}\overline{LM}_C}=\frac{\frac{T-1}{T}\overline{\mathbf{u}}'(\overline{\mathbf{J}}_T\otimes\overline{\mathbf{F}})\overline{\mathbf{u}}}{\overline{\sigma}_1^2\sqrt{2(tr[\overline{\mathbf{D}}^2]-\frac{1}{N}tr[\overline{\mathbf{D}}]^2)}}-\frac{\frac{\sqrt{T-1}}{T}\overline{\mathbf{u}}'(\mathbf{E}_T\otimes\overline{\mathbf{F}})\overline{\mathbf{u}}}{\overline{\sigma}_\nu^2\sqrt{2(tr[\overline{\mathbf{D}}^2]-\frac{1}{N}tr[\overline{\mathbf{D}}]^2)(T-1)}}.$$

Next, observe that $\overline{\mathbf{u}}-\mathbf{u}=-\mathbf{X}\left(\overline{\boldsymbol{\beta}}-\boldsymbol{\beta}\right)$, where $\mathbf{u}=(\boldsymbol{\iota}_T\otimes\mathbf{A}^{-1})\boldsymbol{\mu}+(\mathbf{I}_T\otimes\mathbf{A}^{-1})\boldsymbol{\nu}$ and

$$(NT)^{-1/2}\overline{\mathbf{u}}'(\overline{\mathbf{J}}_T\otimes\overline{\mathbf{F}})\overline{\mathbf{u}}=(NT)^{-1/2}\overline{\mathbf{u}}'(\overline{\mathbf{J}}_T\otimes(\mathbf{W}+\mathbf{W}'-2\overline{\rho}\mathbf{W}'\mathbf{W}))\overline{\mathbf{u}}=\overline{Q}_{bC1}+\overline{Q}_{bC2}.$$

Following Kelejian and Prucha (2001, Lemma 1), one obtains

$$\begin{aligned}\overline{Q}_{bC1}&=(NT)^{-1/2}\mathbf{u}'(\overline{\mathbf{J}}_T\otimes(\mathbf{W}+\mathbf{W}'))\mathbf{u}+o_p(1)\\ \overline{Q}_{bC2}&=(NT)^{-1/2}\mathbf{u}'(\overline{\mathbf{J}}_T\otimes\mathbf{W}'\mathbf{W})\mathbf{u}+o_p(1).\end{aligned}$$

Notice that

$$\bar{\rho}\overline{Q}_{bC2}-\rho Q_{bC2}=(\bar{\rho}-\rho)\,\overline{Q}_{bC2}-\rho(\overline{Q}_{bC2}-Q_{bC2})=o_p(1).$$

The last equality follows since $\bar{\rho}$ is a consistent estimator and $\overline{Q}_{bC2}=O_p(1)$ by Lemma 5, after setting $\mathbf{H}=\mathbf{W}'\mathbf{W}$ and $\alpha=1$. Therefore,

$$\overline{Q}_{bC1}-Q_{bC1}+2\overline{\rho}\overline{Q}_{bC2}-2\rho Q_{bC2}=o_p(1)$$

Hence, we conclude that $(NT)^{-1/2}\overline{\mathbf{u}}'(\overline{\mathbf{J}}_T\otimes\overline{\mathbf{F}})\overline{\mathbf{u}}=(NT)^{-1/2}\mathbf{u}'(\overline{\mathbf{J}}_T\otimes\mathbf{F})\mathbf{u}+o_p(1)$. Defining $Q_{bC}=\frac{\mathbf{u}'(\overline{\mathbf{J}}_T\otimes\mathbf{F})\mathbf{u}-\sigma_1^2 tr(\mathbf{D})}{\sigma_1^2\sqrt{2\{tr[\mathbf{D}^2]}}$, we obtain $\overline{Q}_{bC}-Q_{bC}=o_p(1)$.

Similarly, $(NT)^{-1/2}\overline{\mathbf{u}}'(\mathbf{E}_T\otimes\overline{\mathbf{F}})\overline{\mathbf{u}}-(NT)^{-1/2}\mathbf{u}'(\mathbf{E}_T\otimes\overline{\mathbf{F}})\mathbf{u}=o_p(1)$. Defining $Q_{wC}=\frac{\mathbf{u}'(\mathbf{E}_T\otimes\mathbf{F})\mathbf{u}-\sigma_\nu^2(T-1)tr[\mathbf{D}]}{\sigma_\nu^2\sqrt{2(T-1)tr[\mathbf{D}^2]}}$, we have $\overline{Q}_{wC}-Q_{wC}=o_p(1)$. Also, the two quadratic forms Q_{bC} and Q_{wC} are independent by Lemma 5. As a result we obtain

$$\begin{aligned}\sqrt{\tfrac{T-1}{T}\overline{LM}_C} &= \left(\frac{(NT)^{-1/2}\sigma_1^2\sqrt{2tr[\mathbf{D}^2]}}{(NT)^{-1/2}\overline{\sigma}_1^2\sqrt{2(tr[\overline{\mathbf{D}}^2]-\frac{1}{N}tr[\overline{\mathbf{D}}]^2)}}\right)\left(\frac{\frac{T-1}{T}(\mathbf{u}'(\overline{\mathbf{J}}_T\otimes\mathbf{F})\mathbf{u}-\sigma_1^2tr(\mathbf{D}))}{\sigma_1^2\sqrt{2tr[\mathbf{D}^2]}}\right)\\
&+\left(\frac{(NT)^{-1/2}\sigma_1^2\sqrt{2tr[\mathbf{D}^2]}}{(NT)^{-1/2}\overline{\sigma}_1^2\sqrt{2(tr[\overline{\mathbf{D}}^2]-\frac{1}{N}tr[\overline{\mathbf{D}}]^2)}}\right)\frac{\frac{T-1}{T}tr(\overline{\mathbf{D}}))}{\sqrt{2tr[\mathbf{D}^2]}}\\
&-\left(\frac{(NT)^{-1/2}\sigma_\nu^2\sqrt{2tr[\mathbf{D}^2](T-1)}}{(NT)^{-1/2}\overline{\sigma}_\nu^2\sqrt{2(tr[\overline{\mathbf{D}}^2]-\frac{1}{N}tr[\overline{\mathbf{D}}]^2)(T-1)}}\right)\left(\frac{\frac{\sqrt{T-1}}{T}\left(\mathbf{u}'(\mathbf{E}_T\otimes\mathbf{F})\mathbf{u}-\sigma_\nu^2(T-1)tr[\mathbf{D}]\right)}{\sigma_\nu^2\sqrt{2(tr[\mathbf{D}^2](T-1)}}\right)\\
&-\left(\frac{(NT)^{-1/2}\sigma_\nu^2\sqrt{2tr[\mathbf{D}^2](T-1)}}{(NT)^{-1/2}\overline{\sigma}_\nu^2\sqrt{2(tr[\overline{\mathbf{D}}^2]-\frac{1}{N}tr[\overline{\mathbf{D}}]^2)(T-1)}}\right)\frac{\frac{T-1}{T}tr[\overline{\mathbf{D}}]}{\sqrt{2tr[\mathbf{D}^2]}}+o_p(1).\end{aligned}$$

Notice that $\overline{\sigma}_1^2 = \sigma_1^2 + o_p(1)$, $\overline{\sigma}_\nu^2 = \sigma_\nu^2 + o_p(1)$ and $\sigma_1^2 > 0$ and $\sigma_\nu^2 > 0$ by Assumption A1. Using $\mathbf{H} = \mathbf{D} = (\mathbf{W}'\mathbf{A} + \mathbf{A}'\mathbf{W})$ in Lemma 5, we conclude that $(NT)^{-1}\sigma_\nu^4 2(tr[\mathbf{D}^2])$ is bounded away from zero by some positive constant. Furthermore, $(NT)^{-1}\overline{\sigma}_\nu^4 2(tr[\overline{\mathbf{D}}^2]-\frac{1}{N}tr[\overline{\mathbf{D}}]^2)(T-1)-(NT)^{-1}\sigma_\nu^4 2tr[\mathbf{D}^2](T-1) = o_p(1)$ and we have $p\lim_{N\to\infty}\left(\frac{(NT)^{-1/2}\sigma_1^2\sqrt{2tr[\mathbf{D}^2]}}{(NT)^{-1/2}\overline{\sigma}_1^2\sqrt{2(tr[\overline{\mathbf{D}}^2]-\frac{1}{N}tr[\overline{\mathbf{D}}]^2)}}\right) = 1$. Therefore,

$$\sqrt{\tfrac{T-1}{T}\overline{LM}_C} = \left(\frac{\frac{T-1}{T}(\mathbf{u}'(\overline{\mathbf{J}}_T\otimes\mathbf{F})\mathbf{u}-\sigma_1^2tr(\mathbf{D})}{\sigma_1^2\sqrt{2tr[\mathbf{D}^2])}}\right)-\left(\frac{\frac{\sqrt{T-1}}{T}\left(\mathbf{u}'(\mathbf{E}_T\otimes\mathbf{F})\mathbf{u}-\sigma_\nu^2(T-1)tr[\mathbf{D}]\right)}{\sigma_\nu^2\sqrt{2(tr[\mathbf{D}^2](T-1)}}\right)+o_p(1)$$

observing that both terms in brackets are $O_p(1)$. Since $\frac{(T-1)^2}{T^2} - \frac{(T-1)}{T^2} = \frac{T-1}{T}$ and $Q_{bC} \xrightarrow{d} N(0,1)$ and $Q_{wC} \xrightarrow{d} N(0,1)$ and are independent by Lemma 5, we obtain $\sqrt{\frac{T-1}{T}LM_C} \xrightarrow{d} N(0, \frac{T-1}{T})$ or $\sqrt{LM_C} \xrightarrow{d} N(0,1)$ using the Cramér-Wold device of Lemma 5. This establishes the claim. ■

Appendix E: Numerical optimization

We use the constrained quasi-Newton method involving the constraints $\sigma_\mu^2 > 0$, $\sigma_\nu^2 > 0$, $-1 < \rho_1 < 1$ and $-1 < \rho_2 < 1$ to estimate the parameters of the four models (the unrestricted model and the three restricted ones: random effects, Anselin, and KKP). The quasi-Newton method calculates the gradient of the log-likelihood numerically. We use the optimization routine *fmincon* available from Matlab which uses the sequential quadratic programming method. This method guarantees super-linear convergence by accumulating second order information regarding the Kuhn-Tucker equations using a quasi-Newton updating procedure. An estimate of the Hessian of the Lagrangian is updated at each iteration using the BFGS formula. All tests are based on the analytically derived formulas for both the gradient and the information matrix, using the estimated parameters.

Table 1: Monte Carlo simulations for size and power of LM and LR tests of the random effects, the Anselin and the Kapoor-Kelejian-Prucha models; share of rejections in 2000 replications (N=50, T=5, σ^2_μ=10, σ^2_ν=10)

		Random effects model		Anselin model		Kelejian-Prucha model	
		H_0^A: ρ_1=0, ρ_2=0		H_0^B: ρ_1=0		H_0^C: ρ_1=ρ_2	
ρ_1	ρ_2	LM_A	LR_A	LM_B	LR_B	LM_C	LR_C
-0.80	-0.80	1.000	1.000	0.938	0.964	**0.039**	**0.041**
-0.80	-0.50	1.000	1.000	0.985	0.992	0.590	0.565
-0.80	-0.20	0.997	0.998	0.989	0.991	0.919	0.922
-0.80	0.00	0.979	0.982	0.989	0.991	0.982	0.985
-0.80	0.20	0.997	0.997	0.989	0.993	0.999	0.999
-0.80	0.50	1.000	1.000	0.972	0.977	1.000	1.000
-0.80	0.80	1.000	1.000	0.925	0.938	1.000	1.000
-0.50	-0.80	1.000	1.000	0.562	0.595	0.172	0.307
-0.50	-0.50	1.000	1.000	0.692	0.711	**0.046**	**0.046**
-0.50	-0.20	0.913	0.925	0.727	0.742	0.318	0.324
-0.50	0.00	0.614	0.646	0.702	0.729	0.661	0.685
-0.50	0.20	0.888	0.886	0.690	0.724	0.868	0.894
-0.50	0.50	1.000	1.000	0.613	0.632	0.985	0.992
-0.50	0.80	1.000	1.000	0.430	0.450	0.999	1.000
-0.20	-0.80	1.000	1.000	0.144	0.153	0.643	0.755
-0.20	-0.50	1.000	1.000	0.175	0.183	0.209	0.231
-0.20	-0.20	0.663	0.669	0.164	0.167	**0.042**	**0.045**
-0.20	0.00	0.130	0.139	0.158	0.169	0.157	0.171
-0.20	0.20	0.696	0.660	0.186	0.203	0.453	0.499
-0.20	0.50	1.000	1.000	0.131	0.142	0.863	0.910
-0.20	0.80	1.000	1.000	0.095	0.097	0.976	0.996
0.00	-0.80	1.000	1.000	**0.043**	**0.058**	0.822	0.899
0.00	-0.50	1.000	1.000	**0.043**	**0.055**	0.501	0.509
0.00	-0.20	0.582	0.574	**0.045**	**0.059**	0.106	0.099
0.00	0.00	**0.043**	**0.053**	**0.049**	**0.058**	**0.054**	**0.059**
0.00	0.20	0.646	0.602	**0.042**	**0.047**	0.133	0.154
0.00	0.50	1.000	1.000	**0.049**	**0.051**	0.595	0.672
0.00	0.80	1.000	1.000	**0.050**	**0.053**	0.898	0.962
0.20	-0.80	1.000	1.000	0.117	0.092	0.962	0.983
0.20	-0.50	1.000	1.000	0.147	0.126	0.818	0.827
0.20	-0.20	0.605	0.593	0.174	0.142	0.402	0.382
0.20	0.00	0.130	0.110	0.148	0.125	0.131	0.111
0.20	0.20	0.686	0.649	0.171	0.140	**0.048**	**0.053**
0.20	0.50	1.000	1.000	0.134	0.116	0.283	0.348
0.20	0.80	1.000	1.000	0.093	0.082	0.798	0.909
0.50	-0.80	1.000	1.000	0.667	0.632	0.999	0.999
0.50	-0.50	1.000	1.000	0.761	0.728	0.989	0.988
0.50	-0.20	0.901	0.889	0.781	0.739	0.903	0.886
0.50	0.00	0.700	0.664	0.767	0.746	0.706	0.650
0.50	0.20	0.934	0.923	0.771	0.750	0.372	0.302
0.50	0.50	1.000	1.000	0.683	0.662	**0.044**	**0.054**
0.50	0.80	1.000	1.000	0.397	0.402	0.434	0.590
0.80	-0.80	1.000	1.000	0.994	0.995	1.000	1.000
0.80	-0.50	1.000	1.000	1.000	1.000	1.000	1.000
0.80	-0.20	1.000	1.000	1.000	1.000	1.000	1.000
0.80	0.00	0.999	0.998	0.999	0.999	0.997	0.996
0.80	0.20	1.000	1.000	1.000	1.000	0.988	0.977
0.80	0.50	1.000	1.000	0.990	0.997	0.781	0.699
0.80	0.80	1.000	1.000	0.847	0.947	**0.033**	**0.062**

Note: Bold figures refer to the size of the test at nominal size of 5%. All other figures refer to the size adjusted power of the tests.

Table 2: Monte Carlo simulations for size and power of LM and LR tests of the random effects, the Anselin and the Kapoor-Kelejian-Prucha models; share of rejections in 2000 replications (N=50, T=5, σ^2_μ=5, σ^2_ν=15)

		Random effects model		Anselin model		Kelejian-Prucha model	
		H_0^A: ρ_1=0, ρ_2=0		H_0^B: ρ_1=0		H_0^C: ρ_1=ρ_2	
ρ_1	ρ_2	LM	LR	LM	LR	LM	LR
-0.80	-0.80	1.000	1.000	0.660	0.757	**0.039**	**0.033**
-0.80	-0.50	1.000	1.000	0.824	0.896	0.443	0.401
-0.80	-0.20	0.987	0.991	0.935	0.952	0.804	0.812
-0.80	0.00	0.896	0.923	0.950	0.963	0.940	0.953
-0.80	0.20	0.956	0.961	0.935	0.947	0.974	0.981
-0.80	0.50	1.000	1.000	0.875	0.902	0.993	0.999
-0.80	0.80	1.000	1.000	0.804	0.838	0.993	0.999
-0.50	-0.80	1.000	1.000	0.301	0.320	0.093	0.175
-0.50	-0.50	1.000	1.000	0.422	0.431	**0.047**	**0.038**
-0.50	-0.20	0.853	0.878	0.496	0.532	0.248	0.262
-0.50	0.00	0.389	0.425	0.489	0.502	0.448	0.484
-0.50	0.20	0.767	0.756	0.504	0.548	0.684	0.743
-0.50	0.50	1.000	1.000	0.378	0.419	0.865	0.920
-0.50	0.80	1.000	1.000	0.306	0.328	0.923	0.989
-0.20	-0.80	1.000	1.000	0.097	0.098	0.316	0.455
-0.20	-0.50	1.000	1.000	0.119	0.112	0.120	0.131
-0.20	-0.20	0.641	0.668	0.108	0.123	**0.044**	**0.042**
-0.20	0.00	0.100	0.111	0.126	0.129	0.123	0.125
-0.20	0.20	0.638	0.605	0.129	0.148	0.291	0.324
-0.20	0.50	1.000	1.000	0.084	0.097	0.588	0.674
-0.20	0.80	1.000	1.000	0.066	0.080	0.733	0.909
0.00	-0.80	1.000	1.000	**0.049**	**0.057**	0.457	0.659
0.00	-0.50	1.000	1.000	**0.046**	**0.058**	0.265	0.304
0.00	-0.20	0.570	0.586	**0.050**	**0.053**	0.076	0.071
0.00	0.00	**0.050**	**0.055**	**0.048**	**0.052**	**0.053**	**0.049**
0.00	0.20	0.627	0.596	**0.039**	**0.039**	0.096	0.119
0.00	0.50	1.000	1.000	**0.050**	**0.047**	0.310	0.413
0.00	0.80	1.000	1.000	**0.050**	**0.045**	0.521	0.753
0.20	-0.80	1.000	1.000	0.073	0.069	0.755	0.866
0.20	-0.50	1.000	1.000	0.104	0.081	0.585	0.613
0.20	-0.20	0.552	0.564	0.091	0.083	0.269	0.257
0.20	0.00	0.084	0.070	0.108	0.082	0.107	0.091
0.20	0.20	0.691	0.660	0.109	0.097	**0.041**	**0.045**
0.20	0.50	1.000	1.000	0.075	0.068	0.199	0.245
0.20	0.80	1.000	1.000	0.071	0.072	0.435	0.629
0.50	-0.80	1.000	1.000	0.468	0.438	0.971	0.989
0.50	-0.50	1.000	1.000	0.565	0.520	0.929	0.936
0.50	-0.20	0.772	0.765	0.586	0.571	0.790	0.754
0.50	0.00	0.505	0.482	0.579	0.557	0.535	0.492
0.50	0.20	0.886	0.873	0.541	0.524	0.252	0.197
0.50	0.50	1.000	1.000	0.325	0.351	**0.039**	**0.053**
0.50	0.80	1.000	1.000	0.182	0.193	0.236	0.322
0.80	-0.80	1.000	1.000	0.984	0.987	1.000	1.000
0.80	-0.50	1.000	1.000	0.993	0.993	1.000	1.000
0.80	-0.20	0.993	0.993	0.992	0.991	0.998	0.997
0.80	0.00	0.988	0.987	0.993	0.993	0.989	0.984
0.80	0.20	0.999	0.999	0.990	0.993	0.959	0.930
0.80	0.50	1.000	1.000	0.846	0.960	0.630	0.525
0.80	0.80	1.000	1.000	0.430	0.644	**0.034**	**0.059**

Note: Bold figures refer to the size of the test at nominal size of 5%. All other figures refer to the size adjusted power of the tests.

Table 3: Monte Carlo simulations for size and power of LM and LR tests of the random effects, the Anselin and the Kapoor-Kelejian-Prucha models; share of rejections in 2000 replications ($N=50$, $T=5$, $\sigma^2_\mu=15$, $\sigma^2_\nu=5$)

		Random effects model		Anselin model		Kelejian-Prucha model	
		H_0^A: $\rho_1=0$, $\rho_2=0$		H_0^B: $\rho_1=0$		H_0^C: $\rho_1=\rho_2$	
ρ_1	ρ_2	LM	LR	LM	LR	LM	LR
-0.80	-0.80	1.000	1.000	0.985	0.994	**0.039**	**0.032**
-0.80	-0.50	1.000	1.000	0.997	0.999	0.642	0.610
-0.80	-0.20	0.999	1.000	0.998	0.999	0.964	0.965
-0.80	0.00	0.986	0.995	0.997	0.998	0.995	0.996
-0.80	0.20	0.998	1.000	0.996	0.998	1.000	1.000
-0.80	0.50	1.000	1.000	0.993	0.997	1.000	1.000
-0.80	0.80	1.000	1.000	0.969	0.975	1.000	1.000
-0.50	-0.80	1.000	1.000	0.727	0.769	0.271	0.408
-0.50	-0.50	1.000	1.000	0.815	0.836	**0.046**	**0.046**
-0.50	-0.20	0.927	0.945	0.814	0.831	0.384	0.370
-0.50	0.00	0.680	0.748	0.810	0.834	0.730	0.748
-0.50	0.20	0.935	0.942	0.811	0.820	0.937	0.952
-0.50	0.50	1.000	1.000	0.755	0.777	0.999	1.000
-0.50	0.80	1.000	1.000	0.589	0.619	1.000	1.000
-0.20	-0.80	1.000	1.000	0.174	0.198	0.788	0.885
-0.20	-0.50	1.000	1.000	0.210	0.235	0.241	0.267
-0.20	-0.20	0.671	0.704	0.231	0.249	**0.049**	**0.051**
-0.20	0.00	0.163	0.189	0.236	0.256	0.176	0.192
-0.20	0.20	0.735	0.732	0.230	0.237	0.509	0.555
-0.20	0.50	1.000	1.000	0.178	0.188	0.934	0.965
-0.20	0.80	1.000	1.000	0.136	0.142	1.000	1.000
0.00	-0.80	1.000	1.000	**0.042**	**0.053**	0.951	0.978
0.00	-0.50	1.000	1.000	**0.035**	**0.042**	0.632	0.652
0.00	-0.20	0.579	0.594	**0.039**	**0.050**	0.129	0.117
0.00	0.00	**0.040**	**0.047**	**0.036**	**0.045**	**0.041**	**0.049**
0.00	0.20	0.645	0.625	**0.039**	**0.048**	0.193	0.222
0.00	0.50	1.000	1.000	**0.048**	**0.053**	0.751	0.804
0.00	0.80	1.000	1.000	**0.049**	**0.053**	0.992	0.998
0.20	-0.80	1.000	1.000	0.178	0.153	0.995	0.998
0.20	-0.50	1.000	1.000	0.182	0.170	0.915	0.921
0.20	-0.20	0.644	0.655	0.196	0.166	0.514	0.480
0.20	0.00	0.153	0.136	0.214	0.189	0.176	0.142
0.20	0.20	0.699	0.673	0.206	0.165	**0.038**	**0.045**
0.20	0.50	1.000	1.000	0.178	0.148	0.414	0.476
0.20	0.80	1.000	1.000	0.120	0.102	0.969	0.990
0.50	-0.80	1.000	1.000	0.794	0.775	1.000	1.000
0.50	-0.50	1.000	1.000	0.850	0.832	0.997	0.997
0.50	-0.20	0.938	0.937	0.860	0.845	0.950	0.944
0.50	0.00	0.784	0.774	0.866	0.849	0.804	0.773
0.50	0.20	0.955	0.950	0.860	0.839	0.452	0.386
0.50	0.50	1.000	1.000	0.828	0.811	**0.040**	**0.056**
0.50	0.80	1.000	1.000	0.635	0.639	0.660	0.786
0.80	-0.80	1.000	1.000	1.000	1.000	1.000	1.000
0.80	-0.50	1.000	1.000	1.000	1.000	1.000	1.000
0.80	-0.20	1.000	1.000	1.000	1.000	1.000	1.000
0.80	0.00	0.999	1.000	1.000	1.000	0.999	0.999
0.80	0.20	1.000	1.000	1.000	1.000	0.991	0.981
0.80	0.50	1.000	1.000	0.999	0.999	0.805	0.728
0.80	0.80	1.000	1.000	0.988	0.994	**0.032**	**0.063**

Note: Bold figures refer to the size of the test at nominal size of 5%. All other figures refer to the size adjusted power of the tests.

Table 4: Monte Carlo simulations for the robustness of the LM and LR tests of the random effects, the Anselin and the Kapoor-Kelejian-Prucha models; share of rejections in 2000 replications (N=50, T=5, σ^2_μ=10, σ^2_ν=10)

			ν_{it} ~ t(5)		ν_{it} ~lognormal(0,10)	
	ρ_1	ρ_2	LM	LR	LM	LR
Random effects model, H_0^A: ρ1=0, ρ2=0	0.00	0.00	0.042	0.053	0.041	0.047
Anselin model, H_0^B: ρ_1=0	0.00	-0.80	0.055	0.066	0.045	0.055
	0.00	-0.50	0.052	0.065	0.042	0.049
	0.00	-0.20	0.045	0.053	0.043	0.047
	0.00	0.00	0.045	0.055	0.032	0.038
	0.00	0.20	0.047	0.055	0.038	0.043
	0.00	0.50	0.045	0.047	0.048	0.050
	0.00	0.80	0.050	0.049	0.039	0.040
Kapoor-Kelejian-Prucha model, H_0^C: ρ_1=ρ_2	-0.80	-0.80	0.036	0.035	0.039	0.040
	-0.50	-0.50	0.049	0.046	0.048	0.048
	-0.20	-0.20	0.048	0.044	0.045	0.048
	0.00	0.00	0.043	0.048	0.035	0.039
	0.20	0.20	0.045	0.051	0.035	0.047
	0.50	0.50	0.038	0.054	0.034	0.051
	0.80	0.80	0.029	0.054	0.029	0.059

Table 5: Monte Carlo simulations for size and power of LM and LR tests of the random effects, the Anselin and the Kapoor-Kelejian-Prucha models; share of rejections in 2000 replications ($N=50$, $T=5$, $\sigma^2_\mu=10$, $\sigma^2_v=10$, n_{it} ~ t(5))

		Random effects model		Anselin model		Kelejian-Prucha model	
		H_0^A: $\rho_1=0$, $\rho_2=0$		H_0^B: $\rho_1=0$		H_0^C: $\rho_1=\rho_2$	
ρ_1	ρ_2	LM	LR	LM	LR	LM	LR
-0.80	-0.80	1.000	1.000	0.924	0.958	**0.036**	**0.035**
-0.80	-0.50	1.000	1.000	0.984	0.993	0.559	0.527
-0.80	-0.20	0.998	0.998	0.993	0.995	0.922	0.922
-0.80	0.00	0.977	0.986	0.991	0.993	0.985	0.988
-0.80	0.20	0.996	0.997	0.989	0.992	0.999	0.999
-0.80	0.50	1.000	1.000	0.975	0.978	1.000	1.000
-0.80	0.80	1.000	1.000	0.927	0.944	1.000	1.000
-0.50	-0.80	1.000	1.000	0.537	0.575	0.180	0.293
-0.50	-0.50	1.000	1.000	0.639	0.684	**0.049**	**0.046**
-0.50	-0.20	0.923	0.932	0.718	0.742	0.329	0.332
-0.50	0.00	0.628	0.657	0.724	0.732	0.663	0.688
-0.50	0.20	0.899	0.885	0.696	0.717	0.880	0.907
-0.50	0.50	1.000	1.000	0.605	0.629	0.984	0.992
-0.50	0.80	1.000	1.000	0.433	0.475	0.998	1.000
-0.20	-0.80	1.000	1.000	0.127	0.134	0.589	0.739
-0.20	-0.50	1.000	1.000	0.142	0.166	0.186	0.210
-0.20	-0.20	0.685	0.689	0.169	0.178	**0.048**	**0.044**
-0.20	0.00	0.128	0.134	0.176	0.181	0.161	0.179
-0.20	0.20	0.692	0.651	0.165	0.179	0.418	0.472
-0.20	0.50	1.000	1.000	0.156	0.169	0.869	0.916
-0.20	0.80	1.000	1.000	0.088	0.105	0.971	0.991
0.00	-0.80	1.000	1.000	**0.055**	**0.066**	0.844	0.904
0.00	-0.50	1.000	1.000	**0.052**	**0.065**	0.532	0.548
0.00	-0.20	0.574	0.576	**0.045**	**0.053**	0.120	0.112
0.00	0.00	**0.042**	**0.053**	**0.045**	**0.055**	**0.043**	**0.048**
0.00	0.20	0.633	0.591	**0.047**	**0.055**	0.167	0.184
0.00	0.50	1.000	1.000	**0.045**	**0.047**	0.617	0.692
0.00	0.80	1.000	1.000	**0.050**	**0.049**	0.934	0.974
0.20	-0.80	1.000	1.000	0.104	0.082	0.964	0.982
0.20	-0.50	1.000	1.000	0.120	0.109	0.823	0.820
0.20	-0.20	0.624	0.610	0.168	0.145	0.436	0.403
0.20	0.00	0.123	0.096	0.144	0.113	0.117	0.102
0.20	0.20	0.715	0.669	0.158	0.129	**0.045**	**0.051**
0.20	0.50	1.000	1.000	0.130	0.110	0.323	0.374
0.20	0.80	1.000	1.000	0.096	0.091	0.803	0.905
0.50	-0.80	1.000	1.000	0.653	0.625	0.998	0.999
0.50	-0.50	1.000	1.000	0.735	0.714	0.991	0.989
0.50	-0.20	0.903	0.896	0.787	0.760	0.921	0.908
0.50	0.00	0.722	0.678	0.788	0.750	0.715	0.674
0.50	0.20	0.939	0.924	0.774	0.743	0.357	0.307
0.50	0.50	1.000	1.000	0.683	0.660	**0.038**	**0.054**
0.50	0.80	1.000	1.000	0.410	0.421	0.445	0.612
0.80	-0.80	1.000	1.000	0.997	0.995	1.000	1.000
0.80	-0.50	1.000	1.000	0.998	0.998	1.000	1.000
0.80	-0.20	1.000	1.000	1.000	1.000	1.000	1.000
0.80	0.00	0.998	0.998	0.998	0.998	0.998	0.996
0.80	0.20	1.000	1.000	0.998	0.998	0.983	0.976
0.80	0.50	1.000	1.000	0.991	1.000	0.774	0.698
0.80	0.80	1.000	1.000	0.843	0.947	**0.029**	**0.054**

Note: Bold figures refer to the size of the test at nominal size of 5%. All other figures refer to the size adjusted power of the tests.

Table 6: Monte Carlo simulations for size and power of LM and LR tests of the random effects, the Anselin and the Kapoor-Kelejian-Prucha models; share of rejections in 2000 replications (N=50, T=5, σ^2_μ=10, σ^2_v=10, ln(n_{it}) ~N(0,1))

		Random effects model		Anselin model		Kelejian-Prucha model	
		H_0^A: ρ_1=0, ρ_2=0		H_0^B: ρ_1=0		H_0^C: ρ_1=ρ_2	
ρ_1	ρ_2	LM	LR	LM	LR	LM	LR
-0.80	-0.80	1.000	1.000	0.924	0.957	**0.039**	**0.040**
-0.80	-0.50	1.000	1.000	0.974	0.987	0.531	0.506
-0.80	-0.20	0.996	0.998	0.987	0.990	0.904	0.915
-0.80	0.00	0.971	0.984	0.993	0.995	0.973	0.978
-0.80	0.20	0.994	0.996	0.994	0.995	0.998	1.000
-0.80	0.50	1.000	1.000	0.972	0.982	1.000	1.000
-0.80	0.80	1.000	1.000	0.933	1.000	1.000	1.000
-0.50	-0.80	1.000	1.000	0.586	0.614	0.170	0.294
-0.50	-0.50	1.000	1.000	0.687	0.721	**0.048**	**0.048**
-0.50	-0.20	0.907	0.933	0.737	0.757	0.336	0.339
-0.50	0.00	0.594	0.657	0.766	0.786	0.646	0.668
-0.50	0.20	0.889	0.896	0.707	0.741	0.868	0.896
-0.50	0.50	1.000	1.000	0.631	0.654	0.980	0.990
-0.50	0.80	1.000	1.000	0.500	1.000	0.995	1.000
-0.20	-0.80	1.000	1.000	0.150	0.165	0.604	0.710
-0.20	-0.50	0.999	0.999	0.179	0.206	0.198	0.220
-0.20	-0.20	0.661	0.713	0.201	0.210	**0.045**	**0.048**
-0.20	0.00	0.126	0.153	0.219	0.230	0.146	0.161
-0.20	0.20	0.670	0.664	0.196	0.211	0.436	0.473
-0.20	0.50	1.000	1.000	0.146	0.166	0.849	0.891
-0.20	0.80	1.000	1.000	0.127	1.000	0.966	1.000
0.00	-0.80	1.000	1.000	**0.045**	**0.055**	0.872	0.920
0.00	-0.50	1.000	1.000	**0.042**	**0.049**	0.561	0.576
0.00	-0.20	0.560	0.606	**0.043**	**0.047**	0.155	0.142
0.00	0.00	**0.041**	**0.047**	**0.032**	**0.038**	**0.035**	**0.039**
0.00	0.20	0.605	0.585	**0.038**	**0.043**	0.192	0.209
0.00	0.50	1.000	1.000	**0.048**	**0.050**	0.670	0.737
0.00	0.80	1.000	1.000	**0.039**	**0.040**	0.940	1.000
0.20	-0.80	1.000	1.000	0.126	0.114	0.957	0.975
0.20	-0.50	0.999	1.000	0.143	0.137	0.820	0.816
0.20	-0.20	0.590	0.620	0.146	0.125	0.410	0.372
0.20	0.00	0.143	0.126	0.195	0.172	0.161	0.134
0.20	0.20	0.686	0.670	0.158	0.138	**0.035**	**0.047**
0.20	0.50	1.000	1.000	0.130	0.117	0.331	0.373
0.20	0.80	1.000	1.000	0.114	1.000	0.834	1.000
0.50	-0.80	1.000	1.000	0.689	0.661	1.000	1.000
0.50	-0.50	1.000	1.000	0.767	0.747	0.991	0.992
0.50	-0.20	0.901	0.902	0.802	0.776	0.916	0.905
0.50	0.00	0.705	0.682	0.814	0.796	0.726	0.683
0.50	0.20	0.939	0.939	0.783	0.769	0.385	0.326
0.50	0.50	1.000	1.000	0.712	0.701	**0.034**	**0.051**
0.50	0.80	1.000	1.000	0.463	1.000	0.477	0.616
0.80	-0.80	1.000	1.000	0.998	0.998	1.000	1.000
0.80	-0.50	1.000	1.000	0.999	0.999	1.000	1.000
0.80	-0.20	1.000	1.000	0.999	0.999	1.000	1.000
0.80	0.00	0.997	0.997	0.999	0.999	0.998	0.997
0.80	0.20	1.000	1.000	0.997	0.999	0.984	0.972
0.80	0.50	1.000	1.000	0.991	0.994	0.761	0.675
0.80	0.80	1.000	1.000	0.864	1.000	**0.029**	**0.059**

Note: Bold figures refer to the size of the test at nominal size of 5%. All other figures refer to the size adjusted power of the tests.

Table A1: Monte Carlo simulations for size and power of LM and LR tests of the random effects, the Anselin and the Kapoor-Kelejian-Prucha models; share of rejections in 2000 replications (N=50, T=10, σ^2_μ=10, σ^2_ν=10)

		Random effects model		Anselin model		Kelejian-Prucha model	
		H_0^A: ρ_1=0, ρ_2=0		H_0^B: ρ_1=0		H_0^C: ρ_1=ρ_2	
ρ_1	ρ_2	LM	LR	LM	LR	LM	LR
-0.80	-0.80	1.000	1.000	0.990	0.994	**0.035**	**0.030**
-0.80	-0.50	1.000	1.000	0.999	0.999	0.662	0.614
-0.80	-0.20	1.000	1.000	0.995	0.998	0.963	0.965
-0.80	0.00	0.987	0.993	0.998	0.999	0.997	0.997
-0.80	0.20	1.000	1.000	0.997	0.999	1.000	1.000
-0.80	0.50	1.000	1.000	0.990	0.994	1.000	1.000
-0.80	0.80	1.000	1.000	0.970	0.979	1.000	1.000
-0.50	-0.80	1.000	1.000	0.681	0.726	0.235	0.421
-0.50	-0.50	1.000	1.000	0.765	0.796	**0.043**	**0.044**
-0.50	-0.20	0.994	0.996	0.761	0.792	0.404	0.411
-0.50	0.00	0.666	0.724	0.787	0.808	0.759	0.791
-0.50	0.20	0.992	0.991	0.748	0.775	0.940	0.956
-0.50	0.50	1.000	1.000	0.718	0.752	0.997	0.999
-0.50	0.80	1.000	1.000	0.597	0.605	1.000	1.000
-0.20	-0.80	1.000	1.000	0.170	0.195	0.780	0.895
-0.20	-0.50	1.000	1.000	0.190	0.207	0.285	0.329
-0.20	-0.20	0.937	0.938	0.193	0.208	**0.036**	**0.039**
-0.20	0.00	0.140	0.161	0.202	0.217	0.187	0.226
-0.20	0.20	0.957	0.949	0.179	0.211	0.554	0.623
-0.20	0.50	1.000	1.000	0.185	0.199	0.940	0.974
-0.20	0.80	1.000	1.000	0.158	0.167	0.998	1.000
0.00	-0.80	1.000	1.000	**0.042**	**0.051**	0.932	0.972
0.00	-0.50	1.000	1.000	**0.040**	**0.049**	0.642	0.684
0.00	-0.20	0.901	0.906	**0.049**	**0.067**	0.156	0.146
0.00	0.00	**0.047**	**0.051**	**0.041**	**0.051**	**0.039**	**0.048**
0.00	0.20	0.936	0.928	**0.045**	**0.054**	0.177	0.211
0.00	0.50	1.000	1.000	**0.039**	**0.041**	0.765	0.835
0.00	0.80	1.000	1.000	**0.042**	**0.041**	0.994	0.999
0.20	-0.80	1.000	1.000	0.142	0.124	0.993	0.997
0.20	-0.50	1.000	1.000	0.156	0.129	0.899	0.910
0.20	-0.20	0.915	0.914	0.162	0.138	0.506	0.494
0.20	0.00	0.137	0.115	0.191	0.160	0.189	0.166
0.20	0.20	0.958	0.949	0.175	0.151	**0.033**	**0.046**
0.20	0.50	1.000	1.000	0.162	0.151	0.425	0.534
0.20	0.80	1.000	1.000	0.129	0.119	0.952	0.988
0.50	-0.80	1.000	1.000	0.781	0.753	1.000	1.000
0.50	-0.50	1.000	1.000	0.813	0.799	0.997	0.998
0.50	-0.20	0.987	0.987	0.826	0.800	0.957	0.949
0.50	0.00	0.763	0.750	0.839	0.821	0.798	0.758
0.50	0.20	0.997	0.996	0.821	0.802	0.434	0.363
0.50	0.50	1.000	1.000	0.797	0.784	**0.049**	**0.065**
0.50	0.80	1.000	1.000	0.595	0.601	0.508	0.741
0.80	-0.80	1.000	1.000	0.999	0.999	1.000	1.000
0.80	-0.50	1.000	1.000	1.000	1.000	1.000	1.000
0.80	-0.20	1.000	1.000	0.998	0.998	1.000	1.000
0.80	0.00	0.999	0.999	0.999	0.999	0.999	0.999
0.80	0.20	1.000	1.000	0.999	0.999	0.995	0.990
0.80	0.50	1.000	1.000	0.999	0.999	0.848	0.777
0.80	0.80	1.000	1.000	0.994	0.996	**0.024**	**0.060**

Note: Bold figures refer to the size of the test at nominal size of 5%. All other figures refer to the size adjusted power of the tests.

Table A2: Monte Carlo simulations for size and power of LM and LR tests of the random effects, the Anselin and the Kapoor-Kelejian-Prucha models; share of rejections in 2000 replications (N=50, T=10, σ^2_μ=5, σ^2_ν=15)

		Random effects model		Anselin model		Kelejian-Prucha model	
		H_0^A: ρ_1=0, ρ_2=0		H_0^B: ρ_1=0		H_0^C: ρ_1=ρ_2	
ρ_1	ρ_2	LM	LR	LM	LR	LM	LR
-0.80	-0.80	1.000	1.000	0.903	0.938	**0.042**	**0.035**
-0.80	-0.50	1.000	1.000	0.978	0.986	0.549	0.538
-0.80	-0.20	1.000	1.000	0.980	0.984	0.909	0.920
-0.80	0.00	0.963	0.976	0.985	0.989	0.979	0.986
-0.80	0.20	0.999	0.999	0.981	0.986	0.999	1.000
-0.80	0.50	1.000	1.000	0.963	0.969	1.000	1.000
-0.80	0.80	1.000	1.000	0.903	0.916	1.000	1.000
-0.50	-0.80	1.000	1.000	0.508	0.520	0.159	0.277
-0.50	-0.50	1.000	1.000	0.639	0.679	**0.041**	**0.037**
-0.50	-0.20	0.986	0.990	0.665	0.680	0.330	0.333
-0.50	0.00	0.539	0.597	0.675	0.691	0.657	0.685
-0.50	0.20	0.973	0.975	0.628	0.664	0.854	0.895
-0.50	0.50	1.000	1.000	0.558	0.572	0.969	0.992
-0.50	0.80	1.000	1.000	0.404	0.421	0.986	0.999
-0.20	-0.80	1.000	1.000	0.146	0.151	0.466	0.666
-0.20	-0.50	1.000	1.000	0.167	0.182	0.168	0.209
-0.20	-0.20	0.934	0.937	0.154	0.168	**0.049**	**0.053**
-0.20	0.00	0.112	0.135	0.154	0.169	0.126	0.145
-0.20	0.20	0.937	0.930	0.145	0.154	0.384	0.453
-0.20	0.50	1.000	1.000	0.134	0.141	0.784	0.875
-0.20	0.80	1.000	1.000	0.092	0.088	0.918	0.981
0.00	-0.80	1.000	1.000	**0.039**	**0.055**	0.778	0.876
0.00	-0.50	1.000	1.000	**0.039**	**0.043**	0.470	0.484
0.00	-0.20	0.901	0.910	**0.048**	**0.056**	0.116	0.100
0.00	0.00	**0.049**	**0.053**	**0.045**	**0.050**	**0.043**	**0.054**
0.00	0.20	0.937	0.932	**0.042**	**0.047**	0.142	0.155
0.00	0.50	1.000	1.000	**0.040**	**0.044**	0.574	0.660
0.00	0.80	1.000	1.000	**0.046**	**0.050**	0.841	0.949
0.20	-0.80	1.000	1.000	0.138	0.117	0.933	0.967
0.20	-0.50	1.000	1.000	0.150	0.139	0.785	0.791
0.20	-0.20	0.910	0.912	0.130	0.106	0.375	0.347
0.20	0.00	0.109	0.098	0.151	0.129	0.148	0.118
0.20	0.20	0.944	0.938	0.135	0.115	**0.043**	**0.052**
0.20	0.50	1.000	1.000	0.123	0.108	0.269	0.335
0.20	0.80	1.000	1.000	0.095	0.077	0.700	0.858
0.50	-0.80	1.000	1.000	0.622	0.591	0.997	1.000
0.50	-0.50	1.000	1.000	0.715	0.702	0.990	0.992
0.50	-0.20	0.980	0.983	0.735	0.700	0.906	0.892
0.50	0.00	0.639	0.624	0.750	0.721	0.715	0.672
0.50	0.20	0.990	0.989	0.727	0.705	0.342	0.286
0.50	0.50	1.000	1.000	0.610	0.587	**0.042**	**0.058**
0.50	0.80	1.000	1.000	0.313	0.309	0.340	0.542
0.80	-0.80	1.000	1.000	0.997	0.996	1.000	1.000
0.80	-0.50	1.000	1.000	0.998	0.997	1.000	1.000
0.80	-0.20	1.000	1.000	0.999	0.997	1.000	1.000
0.80	0.00	0.996	0.996	0.999	0.999	0.998	0.997
0.80	0.20	1.000	1.000	0.999	0.998	0.986	0.977
0.80	0.50	1.000	1.000	0.994	0.993	0.766	0.667
0.80	0.80	1.000	1.000	0.782	0.897	**0.034**	**0.064**

Note: Bold figures refer to the size of the test at nominal size of 5%. All other figures refer to the size adjusted power of the tests.

Table A3: Monte Carlo simulations for size and power of LM and LR tests of the random effects, the Anselin and the Kapoor-Kelejian-Prucha models; share of rejections in 2000 replications (N=50, T=10, σ^2_μ=15, σ^2_ν=5)

		Random effects model		Anselin model		Kelejian-Prucha model	
		H_0^A: ρ_1=0, ρ_2=0		H_0^B: ρ_1=0		H_0^C: ρ_1=ρ_2	
ρ_1	ρ_2	LM	LR	LM	LR	LM	LR
-0.80	-0.80	1.000	1.000	0.997	0.999	**0.040**	**0.039**
-0.80	-0.50	1.000	1.000	0.999	1.000	0.677	0.648
-0.80	-0.20	1.000	1.000	1.000	1.000	0.968	0.971
-0.80	0.00	0.990	0.996	0.998	0.998	0.999	0.999
-0.80	0.20	1.000	1.000	1.000	1.000	1.000	1.000
-0.80	0.50	1.000	1.000	0.999	1.000	1.000	1.000
-0.80	0.80	1.000	1.000	0.994	0.996	1.000	1.000
-0.50	-0.80	1.000	1.000	0.773	0.810	0.288	0.493
-0.50	-0.50	1.000	1.000	0.817	0.861	**0.047**	**0.047**
-0.50	-0.20	0.990	0.995	0.820	0.853	0.435	0.440
-0.50	0.00	0.698	0.753	0.792	0.826	0.768	0.813
-0.50	0.20	0.996	0.996	0.833	0.850	0.960	0.980
-0.50	0.50	1.000	1.000	0.802	0.817	1.000	1.000
-0.50	0.80	1.000	1.000	0.689	0.736	1.000	1.000
-0.20	-0.80	1.000	1.000	0.192	0.218	0.838	0.927
-0.20	-0.50	1.000	1.000	0.220	0.245	0.288	0.330
-0.20	-0.20	0.936	0.940	0.226	0.244	**0.046**	**0.053**
-0.20	0.00	0.159	0.193	0.218	0.228	0.190	0.216
-0.20	0.20	0.956	0.953	0.251	0.260	0.576	0.647
-0.20	0.50	1.000	1.000	0.215	0.234	0.971	0.991
-0.20	0.80	1.000	1.000	0.189	0.203	1.000	1.000
0.00	-0.80	1.000	1.000	**0.045**	**0.055**	0.974	0.991
0.00	-0.50	1.000	1.000	**0.041**	**0.048**	0.716	0.722
0.00	-0.20	0.911	0.917	**0.039**	**0.047**	0.155	0.134
0.00	0.00	**0.046**	**0.050**	**0.045**	**0.054**	**0.044**	**0.055**
0.00	0.20	0.937	0.929	**0.040**	**0.048**	0.212	0.224
0.00	0.50	1.000	1.000	**0.042**	**0.050**	0.809	0.865
0.00	0.80	1.000	1.000	**0.042**	**0.039**	0.999	1.000
0.20	-0.80	1.000	1.000	0.165	0.137	0.999	1.000
0.20	-0.50	1.000	1.000	0.204	0.186	0.955	0.957
0.20	-0.20	0.923	0.926	0.203	0.179	0.547	0.499
0.20	0.00	0.149	0.136	0.193	0.166	0.190	0.148
0.20	0.20	0.953	0.950	0.208	0.174	**0.043**	**0.055**
0.20	0.50	1.000	1.000	0.196	0.164	0.440	0.522
0.20	0.80	1.000	1.000	0.156	0.144	0.988	0.999
0.50	-0.80	1.000	1.000	0.832	0.812	1.000	1.000
0.50	-0.50	1.000	1.000	0.857	0.851	0.999	0.999
0.50	-0.20	0.990	0.991	0.870	0.858	0.969	0.962
0.50	0.00	0.788	0.772	0.860	0.840	0.832	0.801
0.50	0.20	0.993	0.993	0.869	0.854	0.502	0.425
0.50	0.50	1.000	1.000	0.867	0.841	**0.040**	**0.056**
0.50	0.80	1.000	1.000	0.791	0.785	0.697	0.863
0.80	-0.80	1.000	1.000	1.000	1.000	1.000	1.000
0.80	-0.50	1.000	1.000	1.000	1.000	1.000	1.000
0.80	-0.20	1.000	1.000	1.000	1.000	1.000	1.000
0.80	0.00	1.000	1.000	1.000	1.000	1.000	1.000
0.80	0.20	1.000	1.000	1.000	1.000	0.995	0.991
0.80	0.50	1.000	1.000	1.000	1.000	0.862	0.773
0.80	0.80	1.000	1.000	0.999	1.000	**0.038**	**0.070**

Note: Bold figures refer to the size of the test at nominal size of 5%. All other figures refer to the size adjusted power of the tests.

Table A4: Monte Carlo simulations for size and power of LM and LR tests of the random effects, the Anselin and the Kapoor-Kelejian-Prucha models; share of rejections in 2000 replications ($N=100$, $T=5$, $\sigma^2_\mu=10$, $\sigma^2_\nu=10$)

		Random effects model		Anselin model		Kelejian-Prucha model	
		H_0^A: $\rho_1=0$, $\rho_2=0$		H_0^B: $\rho_1=0$		H_0^C: $\rho_1=\rho_2$	
ρ_1	ρ_2	LM	LR	LM	LR	LM	LR
-0.80	-0.80	1.000	1.000	0.997	0.998	**0.047**	**0.046**
-0.80	-0.50	1.000	1.000	1.000	1.000	0.697	0.683
-0.80	-0.20	1.000	1.000	1.000	1.000	0.985	0.985
-0.80	0.00	1.000	1.000	1.000	1.000	1.000	1.000
-0.80	0.20	1.000	1.000	1.000	1.000	1.000	1.000
-0.80	0.50	1.000	1.000	1.000	1.000	1.000	1.000
-0.80	0.80	1.000	1.000	0.994	0.996	1.000	1.000
-0.50	-0.80	1.000	1.000	0.824	0.840	0.417	0.500
-0.50	-0.50	1.000	1.000	0.875	0.886	**0.041**	**0.041**
-0.50	-0.20	0.992	0.993	0.898	0.905	0.500	0.500
-0.50	0.00	0.848	0.866	0.913	0.924	0.873	0.884
-0.50	0.20	0.987	0.987	0.872	0.884	0.983	0.987
-0.50	0.50	1.000	1.000	0.822	0.830	1.000	1.000
-0.50	0.80	1.000	1.000	0.661	0.687	1.000	1.000
-0.20	-0.80	1.000	1.000	0.209	0.221	0.899	0.928
-0.20	-0.50	1.000	1.000	0.241	0.266	0.362	0.382
-0.20	-0.20	0.872	0.885	0.273	0.292	**0.050**	**0.051**
-0.20	0.00	0.175	0.194	0.246	0.279	0.217	0.232
-0.20	0.20	0.878	0.866	0.245	0.257	0.621	0.664
-0.20	0.50	1.000	1.000	0.210	0.218	0.975	0.985
-0.20	0.80	1.000	1.000	0.164	0.173	1.000	1.000
0.00	-0.80	1.000	1.000	**0.041**	**0.050**	0.981	0.990
0.00	-0.50	1.000	1.000	**0.046**	**0.051**	0.781	0.788
0.00	-0.20	0.796	0.801	**0.047**	**0.052**	0.181	0.166
0.00	0.00	**0.048**	**0.054**	**0.049**	**0.049**	**0.047**	**0.052**
0.00	0.20	0.846	0.824	**0.050**	**0.055**	0.228	0.247
0.00	0.50	1.000	1.000	**0.047**	**0.051**	0.859	0.890
0.00	0.80	1.000	1.000	**0.052**	**0.049**	0.998	0.999
0.20	-0.80	1.000	1.000	0.220	0.196	1.000	1.000
0.20	-0.50	1.000	1.000	0.243	0.219	0.975	0.975
0.20	-0.20	0.826	0.827	0.255	0.227	0.652	0.632
0.20	0.00	0.192	0.160	0.260	0.234	0.226	0.196
0.20	0.20	0.911	0.899	0.230	0.209	**0.040**	**0.043**
0.20	0.50	1.000	1.000	0.213	0.194	0.505	0.551
0.20	0.80	1.000	1.000	0.125	0.128	0.984	0.992
0.50	-0.80	1.000	1.000	0.923	0.909	1.000	1.000
0.50	-0.50	1.000	1.000	0.938	0.929	1.000	1.000
0.50	-0.20	0.990	0.988	0.954	0.950	0.993	0.992
0.50	0.00	0.931	0.922	0.954	0.952	0.926	0.910
0.50	0.20	0.995	0.995	0.940	0.934	0.590	0.526
0.50	0.50	1.000	1.000	0.911	0.898	**0.042**	**0.053**
0.50	0.80	1.000	1.000	0.629	0.666	0.750	0.816
0.80	-0.80	1.000	1.000	1.000	1.000	1.000	1.000
0.80	-0.50	1.000	1.000	1.000	1.000	1.000	1.000
0.80	-0.20	1.000	1.000	1.000	1.000	1.000	1.000
0.80	0.00	1.000	1.000	1.000	1.000	1.000	1.000
0.80	0.20	1.000	1.000	1.000	1.000	1.000	1.000
0.80	0.50	1.000	1.000	1.000	1.000	0.923	0.891
0.80	0.80	1.000	1.000	0.984	0.996	**0.042**	**0.060**

Note: Bold figures refer to the size of the test at nominal size of 5%. All other figures refer to the size adjusted power of the tests.

Table A5: Monte Carlo simulations for size and power of LM and LR tests of the random effects, the Anselin and the Kapoor-Kelejian-Prucha models; share of rejections in 2000 replications (N=100, T=5, σ^2_μ=5, σ^2_ν=15)

		Random effects model		Anselin model		Kelejian-Prucha model	
		H_0^A: ρ_1=0, ρ_2=0		H_0^B: ρ_1=0		H_0^C: ρ_1=ρ_2	
ρ_1	ρ_2	LM	LR	LM	LR	LM	LR
-0.80	-0.80	1.000	1.000	0.865	0.921	**0.042**	**0.035**
-0.80	-0.50	1.000	1.000	0.968	0.984	0.547	0.543
-0.80	-0.20	1.000	1.000	0.995	0.996	0.924	0.935
-0.80	0.00	0.990	0.990	0.994	0.994	0.988	0.994
-0.80	0.20	1.000	1.000	0.996	0.998	1.000	1.000
-0.80	0.50	1.000	1.000	0.979	0.985	1.000	1.000
-0.80	0.80	1.000	1.000	0.957	0.967	1.000	1.000
-0.50	-0.80	1.000	1.000	0.472	0.507	0.169	0.259
-0.50	-0.50	1.000	1.000	0.651	0.678	**0.049**	**0.047**
-0.50	-0.20	0.979	0.983	0.714	0.738	0.302	0.320
-0.50	0.00	0.647	0.675	0.735	0.746	0.644	0.670
-0.50	0.20	0.952	0.950	0.681	0.706	0.866	0.893
-0.50	0.50	1.000	1.000	0.592	0.605	0.978	0.988
-0.50	0.80	1.000	1.000	0.482	0.494	0.993	1.000
-0.20	-0.80	1.000	1.000	0.123	0.135	0.564	0.686
-0.20	-0.50	1.000	1.000	0.151	0.154	0.200	0.219
-0.20	-0.20	0.890	0.898	0.176	0.193	**0.054**	**0.053**
-0.20	0.00	0.167	0.180	0.184	0.187	0.141	0.156
-0.20	0.20	0.874	0.866	0.150	0.168	0.366	0.411
-0.20	0.50	1.000	1.000	0.122	0.134	0.788	0.848
-0.20	0.80	1.000	1.000	0.079	0.081	0.926	0.982
0.00	-0.80	1.000	1.000	**0.053**	**0.059**	0.819	0.896
0.00	-0.50	1.000	1.000	**0.041**	**0.049**	0.504	0.508
0.00	-0.20	0.828	0.831	**0.042**	**0.045**	0.121	0.114
0.00	0.00	**0.039**	**0.041**	**0.040**	**0.049**	**0.045**	**0.052**
0.00	0.20	0.876	0.860	**0.042**	**0.046**	0.148	0.150
0.00	0.50	1.000	1.000	**0.045**	**0.041**	0.546	0.610
0.00	0.80	1.000	1.000	**0.054**	**0.050**	0.843	0.925
0.20	-0.80	1.000	1.000	0.137	0.123	0.947	0.974
0.20	-0.50	1.000	1.000	0.149	0.130	0.831	0.845
0.20	-0.20	0.817	0.826	0.181	0.168	0.425	0.401
0.20	0.00	0.154	0.134	0.184	0.157	0.135	0.123
0.20	0.20	0.935	0.928	0.149	0.143	**0.049**	**0.052**
0.20	0.50	1.000	1.000	0.120	0.114	0.257	0.300
0.20	0.80	1.000	1.000	0.091	0.100	0.676	0.806
0.50	-0.80	1.000	1.000	0.752	0.742	0.999	1.000
0.50	-0.50	1.000	1.000	0.829	0.801	0.997	0.998
0.50	-0.20	0.963	0.964	0.848	0.826	0.948	0.949
0.50	0.00	0.796	0.777	0.854	0.833	0.794	0.788
0.50	0.20	0.989	0.985	0.798	0.786	0.422	0.394
0.50	0.50	1.000	1.000	0.579	0.601	**0.035**	**0.043**
0.50	0.80	1.000	1.000	0.288	0.315	0.408	0.533
0.80	-0.80	1.000	1.000	1.000	1.000	1.000	1.000
0.80	-0.50	1.000	1.000	1.000	1.000	1.000	1.000
0.80	-0.20	1.000	1.000	1.000	1.000	1.000	1.000
0.80	0.00	1.000	1.000	1.000	1.000	1.000	1.000
0.80	0.20	1.000	1.000	1.000	1.000	0.997	0.995
0.80	0.50	1.000	1.000	0.982	0.996	0.814	0.763
0.80	0.80	1.000	1.000	0.706	0.871	**0.038**	**0.054**

Note: Bold figures refer to the size of the test at nominal size of 5%. All other figures refer to the size adjusted power of the tests.

Table A6: Monte Carlo simulations for size and power of LM and LR tests of the random effects, the Anselin and the Kapoor-Kelejian-Prucha models; share of rejections in 2000 replications (N=100, T=5, σ^2_μ=15, σ^2_v=5)

		Random effects model		Anselin model		Kelejian-Prucha model	
		H_0^A: ρ_1=0, ρ_2=0		H_0^B: ρ_1=0		H_0^C: ρ_1=ρ_2	
ρ_1	ρ_2	LM	LR	LM	LR	LM	LR
-0.80	-0.80	1.000	1.000	1.000	1.000	**0.037**	**0.043**
-0.80	-0.50	1.000	1.000	1.000	1.000	0.769	0.743
-0.80	-0.20	1.000	1.000	1.000	1.000	0.997	0.997
-0.80	0.00	1.000	1.000	1.000	1.000	0.999	0.999
-0.80	0.20	1.000	1.000	1.000	1.000	1.000	1.000
-0.80	0.50	1.000	1.000	1.000	1.000	1.000	1.000
-0.80	0.80	1.000	1.000	0.999	0.999	1.000	1.000
-0.50	-0.80	1.000	1.000	0.929	0.942	0.510	0.611
-0.50	-0.50	1.000	1.000	0.947	0.960	**0.050**	**0.052**
-0.50	-0.20	0.991	0.992	0.951	0.955	0.547	0.553
-0.50	0.00	0.915	0.931	0.961	0.966	0.913	0.925
-0.50	0.20	0.993	0.994	0.942	0.947	0.995	0.996
-0.50	0.50	1.000	1.000	0.909	0.925	1.000	1.000
-0.50	0.80	1.000	1.000	0.804	0.823	1.000	1.000
-0.20	-0.80	1.000	1.000	0.287	0.306	0.965	0.981
-0.20	-0.50	1.000	1.000	0.298	0.335	0.448	0.461
-0.20	-0.20	0.881	0.892	0.319	0.342	**0.049**	**0.053**
-0.20	0.00	0.238	0.255	0.329	0.352	0.258	0.273
-0.20	0.20	0.912	0.906	0.284	0.300	0.750	0.775
-0.20	0.50	1.000	1.000	0.246	0.280	0.997	0.999
-0.20	0.80	1.000	1.000	0.185	0.204	1.000	1.000
0.00	-0.80	1.000	1.000	**0.046**	**0.049**	0.999	0.999
0.00	-0.50	1.000	1.000	**0.051**	**0.052**	0.862	0.871
0.00	-0.20	0.805	0.808	**0.047**	**0.051**	0.209	0.203
0.00	0.00	**0.044**	**0.051**	**0.043**	**0.047**	**0.049**	**0.052**
0.00	0.20	0.866	0.847	**0.049**	**0.056**	0.283	0.312
0.00	0.50	1.000	1.000	**0.055**	**0.054**	0.947	0.959
0.00	0.80	1.000	1.000	**0.057**	**0.053**	1.000	1.000
0.20	-0.80	1.000	1.000	0.309	0.277	1.000	1.000
0.20	-0.50	1.000	1.000	0.290	0.269	0.991	0.991
0.20	-0.20	0.858	0.858	0.320	0.285	0.739	0.726
0.20	0.00	0.250	0.221	0.316	0.292	0.255	0.232
0.20	0.20	0.913	0.895	0.307	0.277	**0.044**	**0.046**
0.20	0.50	1.000	1.000	0.264	0.247	0.631	0.688
0.20	0.80	1.000	1.000	0.196	0.188	1.000	1.000
0.50	-0.80	1.000	1.000	0.971	0.967	1.000	1.000
0.50	-0.50	1.000	1.000	0.977	0.974	1.000	1.000
0.50	-0.20	0.995	0.995	0.977	0.973	0.997	0.996
0.50	0.00	0.961	0.952	0.976	0.971	0.959	0.952
0.50	0.20	0.996	0.995	0.975	0.972	0.702	0.655
0.50	0.50	1.000	1.000	0.971	0.969	**0.038**	**0.046**
0.50	0.80	1.000	1.000	0.869	0.875	0.924	0.953
0.80	-0.80	1.000	1.000	1.000	1.000	1.000	1.000
0.80	-0.50	1.000	1.000	1.000	1.000	1.000	1.000
0.80	-0.20	1.000	1.000	1.000	1.000	1.000	1.000
0.80	0.00	1.000	1.000	1.000	1.000	1.000	1.000
0.80	0.20	1.000	1.000	1.000	1.000	0.999	0.999
0.80	0.50	1.000	1.000	1.000	1.000	0.949	0.931
0.80	0.80	1.000	1.000	1.000	1.000	**0.039**	**0.046**

Note: Bold figures refer to the size of the test at nominal size of 5%. All other figures refer to the size adjusted power of the tests.