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# A Note on the Application of EC2SLS and EC3SLS Estimators in Panel Data Models

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**A NOTE ON THE APPLICATION OF EC2SLS AND EC3SLS ESTIMATORS IN PANEL DATA MODELS**

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### **Abstract**

Baltagi and Li (1992) showed that for estimating a single equation in a simultaneous panel data model, EC2SLS has more instruments than G2SLS. Although these extra instruments are redundant in White (1986) terminology, they may yield different estimates and standard errors in empirical studies with finite N and T. We illustrate this using the crime data of Cornwell and Trumbull (1994). We show that the standard errors of EC2SLS are smaller than those of G2SLS for this example. In general, we prove that the asymptotic variance of G2SLS differs from that of EC2SLS by a positive semi-definite matrix. Although this difference tends to zero as the sample size tends to infinity, in small samples, this difference may be different from zero and can lead to gains in small sample efficiency. This proof is extended to the system equations 3SLS counterparts.

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Keywords: Instrument Variable; Panel Data.

JEL classification: C13

## A Note on the Application of EC2SLS and EC3SLS Estimators in Panel Data Models

Badi H. Baltagi,\* Long Liu<sup>†</sup>

July 16, 2009

#### Abstract

Baltagi and Li (1992) showed that for estimating a single equation in a simultaneous panel data model, EC2SLS has more instruments than G2SLS. Although these extra instruments are redundant in White's (1986) terminology, they may yield different estimates and standard errors in empirical studies with finite N and T. We illustrate this using the crime data of Cornwell and Trumbull (1994). We show that the standard errors of EC2SLS are smaller than those of G2SLS for this example. In general, we prove that the asymptotic variance of G2SLS differs from that of EC2SLS by a positive semi-definite matrix. Although this difference tends to zero as the sample size tends to infinity, in small samples, this difference may be different from zero and can lead to gains in small sample efficiency. This proof is extended to the system equations 3SLS counterparts.

Key Words: Instrument Variable; Panel Data.

#### 1 EC2SLS vs. G2SLS

Consider a panel data regression model with random error component disturbances

$$
y_{it} = Z'_{it}\beta + u_{it} \quad i = 1, ..., N; \quad t = 1, ..., T
$$
 (1)

where  $u_{it} = \mu_i + \nu_{it}$ , with  $\mu_i \sim \text{iid}(0, \sigma_{\mu}^2)$ ,  $\nu_{it} \sim \text{iid}(0, \sigma_{\nu}^2)$ , and  $Z'_{it}$  is  $1 \times g$  vector of observations on the explanatory variables which includes endogenous variables.  $X_{it}$  is the set of k exogenous instruments and the equation is assumed to be identified. We can rewrite  $(1)$  in vector form as

$$
y = Z\beta + u \tag{2}
$$

where y and u are  $n \times 1$  vectors, Z is a  $n \times g$  vector and X is a  $n \times k$  vector with  $n = NT$ . Balestra and Varadharajan-Krishnakumar (1987) suggested  $\widehat{\beta}_{G2SLS} = (Z^{*'}P_{X^*}Z^*)^{-1}Z^{*'}P_{X^*}y^*$  as an estimator of  $\beta$ where  $P_{X^*} = X^* (X^{*'} X^*)^{-1} X^{*'}$ ,  $X^* = \Omega^{-1/2} X$ ,  $Z^* = \Omega^{-1/2} Z$  and  $y^* = \Omega^{-1/2} y$  with  $\Omega^{-1/2} = \frac{P}{\sigma_1} + \frac{Q}{\sigma_\nu}$ <br>and  $P = I_N \otimes \bar{J}_T$  where  $\bar{J}_T = J_T/T$ ,  $Q = I_{NT} - P$  and  $\sigma_1^2 = T \sigma_\mu^2 + \sigma_\nu^2$ .  $I_N$  is an identity dimension N, and  $\otimes$  denotes Kronecker product.  $J_T$  is a matrix of ones of dimension T. Baltagi (1981) suggested  $\hat{\beta}_{EC2SLS} = (Z^* P_A Z^*)^{-1} Z^* P_A y^*$  as an alternative estimator of  $\beta$  where  $P_A = A (A'A)^{-1} A'$  and  $A = \left[ \tilde{X}, \bar{X} \right]$  with  $\tilde{X} = QX$  and  $\bar{X} = PX$ . Here,  $y^* = \Omega^{-1/2}y$  and  $Z^* = \Omega^{-1/2}Z$  with  $\Omega^{-1/2} = \frac{P}{\sigma_1} + \frac{Q}{\sigma_2}$  and

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 $P = I_N \otimes \bar{J}_T$  where  $\bar{J}_T = J_T/T$ ,  $Q = I_{NT} - P$  and  $\sigma_1^2 = T\sigma_\mu^2 + \sigma_\nu^2$ .  $I_N$  is an identity matrix of dimension N, and  $\otimes$  denotes Kronecker product.  $J_T$  is a matrix of ones of dimension T. Both estimators are consistent and Baltagi and Li (1992) showed that they have the same limiting distribution. To compare the two estimators, Baltagi and Li (1992) explained that  $A = [\bar{X}, \bar{X}]$  spans the set of instruments used by Balestra and Varadharajan-Krishnakumar (1987), i.e.  $X^* = [\tilde{X}/\sigma_\nu + \bar{X}/\sigma_1]$ . In fact, Baltagi and Li (1992) illustrated that  $A = [\tilde{X}, \bar{X}], H = [X^*, \tilde{X}]$  and  $G = [X^*, \bar{X}]$  yield the same projection matrix  $P_A$ , and therefore the same 2SLS estimator given by EC2SLS. Using the results in White (1986), the optimal instrument set is  $X^*$ . Therefore, in White's terminology,  $\widetilde{X}$  in H and  $\overline{X}$  in G are redundant with respect to  $X^*$ . Redundant instruments can be interpreted loosely as additional sets of instruments that do not yield extra gains in asymptotic efficiency; see White (1986) for the strict definition and Baltagi and Li (1992) for the proof in this context. In this note, we show that the asymptotic variance of G2SLS differs from that of EC2SLS by a positive semi-definite matrix. Although this difference tends to zero as the sample size tends to infinity, in small samples, this difference may be different from zero and can lead to gains in small sample efficiency. This is illustrated with an empirical example using the crime data of Cornwell and Trumbull (1994). The intuition comes from the fact that extra instruments may yield lower standard errors in small samples.

We first show:

**Lemma 1**  $P_A P_{X^*} = P_{X^*}$ 

Proof.

$$
P_{\tilde{X}} P_{X^*} = \tilde{X} (\tilde{X}' \tilde{X})^{-1} \tilde{X}' X^* (X^{*'} X^*)^{-1} X^{*'} = \tilde{X} (\tilde{X}' \tilde{X})^{-1} \tilde{X}' (\frac{\tilde{X}}{\sigma_{\nu}} + \frac{\bar{X}}{\sigma_1}) (X^{*'} X^*)^{-1} X^{*'} = \frac{1}{\sigma_{\nu}} \tilde{X} (\tilde{X}' \tilde{X})^{-1} \tilde{X}' \tilde{X} (X^{*'} X^*)^{-1} X^{*'} = \frac{1}{\sigma_{\nu}} \tilde{X} (X^{*'} X^*)^{-1} X^{*'}
$$

using the fact that  $\tilde{X}'\overline{X} = 0$ , since  $QP = 0$ . Also,

$$
P_{\bar{X}}P_{X^*} = \bar{X} (\bar{X}'\bar{X})^{-1} \bar{X}' X^* (X^{*'} X^*)^{-1} X^{*'} = \bar{X} (\bar{X}'\bar{X})^{-1} \bar{X}' \left( \frac{\tilde{X}}{\sigma_{\nu}} + \frac{\bar{X}}{\sigma_1} \right) (X^{*'} X^*)^{-1} X^{*'} = \frac{1}{\sigma_1} \bar{X} (\bar{X}'\bar{X})^{-1} \bar{X}' \bar{X} (X^{*'} X^*)^{-1} X^{*'} = \frac{1}{\sigma_1} \bar{X} (X^{*'} X^*)^{-1} X^{*'}.
$$

The summation of these two equations gives us  $(P_{\tilde{X}}+P_{\bar{X}}) P_{X^*} = \frac{1}{\sigma_{\nu}} \tilde{X}(X^{*\prime}X^*)^{-1} X^{*\prime} + \frac{1}{\sigma_{1}} \bar{X}(X^{*\prime}X^*)^{-1} X^{*\prime} =$  $X^* (X^{*'} X^{*})^{-1} X^{*'} = P_{X^*}$ . Using the result that  $P_A = P_{\tilde{X}} + P_{\bar{X}}$ , see Baltagi (2008, p.123), we get  $P_A P_{X^*} =$  $P_{X^*}$ .

 ${\bf Theorem\ 1} \ \mathit{avar}\left(\sqrt{n}\widehat{\beta}_{G2SLS}\right)-\mathit{avar}\left(\sqrt{n}\widehat{\beta}_{EC2SLS}\right)$  is positive semi-definite, where avar denotes asymptotic variance and  $\hat{n} = NT$ .

**Proof.** It is well known, Baltagi (2008, pp. 121-123), that the asymptotic variance of  $\sqrt{n}\hat{\beta}_{G2SLS}$  is given by  $avar\left(\sqrt{n}\widehat{\beta}_{G2SLS}\right) = p \lim_{n \to \infty} \left(\frac{Z^{*'}P_{X^*}Z^*}{n}\right)$  $\int^{-1}$ , and that of  $\sqrt{n} \hat{\beta}_{EC2SLS}$  is given by  $avar\left(\sqrt{n} \hat{\beta}_{EC2SLS}\right)$  =  $p \lim \left( \frac{Z^{*'} P_A Z^*}{n} \right)$  $\int^{-1}$ . Therefore,  $avar\left(\sqrt{n}\widehat{\beta}_{G2SLS}\right)-avar\left(\sqrt{n}\widehat{\beta}_{EC2SLS}\right)$  is positive semi-definite if  $\left(\frac{Z^{*'}P_AZ^{*}}{n}\right)$  $\overline{ }$ Ξ  $\left(\frac{Z^{*'}P_{X^*}Z^*}{n}\right)$ ) is positive semi-definite. Equivalently, if  $\left(\frac{Z^{*'}(P_A-P_{X*})Z^{*}}{n}\right)$ n  $\hat{\theta}$  is positive semi-definite. The latter holds if  $P_A - P_{X^*}$  is idempotent, which follows from lemma 1. In fact,  $(P_A - P_{X^*})^2 = P_A + P_{X^*} - P_A P_{X^*} P_{X^*}P_A = P_A - P_{X^*}$ .

**Remark 1** When  $n \to \infty$ , the term  $\left(\frac{Z^{*'}(P_A - P_{X^*})Z^*}{n}\right)$ n  $=$  0 for  $Z^{*\prime}(P_A - P_{X^*})Z^*$  a finite quantity. Consequently,  $\beta_{G2SLS}$  and  $\beta_{EC2SLS}$  have the same asymptotic variance. However, when n is not large enough, the term  $\left(\frac{Z^{*'}(P_A-P_{X^*})Z^*}{n}\right)$ n  $\int$  may not converge to zero. This implies that the EC2SLS estimator may be more efficient than the G2SLS estimator in finite samples.<sup>1</sup>

#### 2 EC3SLS vs. E3SLS

These results can be extended to their 3SLS counterparts, see Baltagi (2008). Let us consider a system of  $M$  identified equations:

$$
y = Z\delta + u \tag{3}
$$

where  $y' = (y'_1, \ldots, y'_M)$ ,  $Z = \text{diag}[Z_j]$ ,  $\delta' = (\delta'_1, \ldots, \delta'_M)$  and  $u' = (u'_1, \ldots, u'_M)$ , for  $j = 1, \ldots, M$ . Let X be the matrix of exogenous instruments. Baltagi (1981) suggested  $\hat{\delta}_{EC3SLS} = (Z^{*'}P_BZ^*)^{-1}Z^{*'}P_By^*$  as an estimator of  $\delta$ , where  $B = \left[I_M \otimes \tilde{X}, I_M \otimes \bar{X}\right], \widetilde{X} = QX, \ \bar{X} = PX, \ y^* = \Omega^{-1/2}y, \ Z^* = \Omega^{-1/2}Z$ , with

$$
\Omega^{-1/2} = \Sigma_1^{-1/2} \otimes P + \Sigma_{\nu}^{-1/2} \otimes Q \tag{4}
$$

In this case,  $\mu \sim \text{iid}(0, \Sigma_{\mu} \otimes I_N)$ ,  $\nu \sim \text{iid}(0, \Sigma_{\nu} \otimes I_{NT})$  and  $\Sigma_1 = T\Sigma_{\mu} + \Sigma_{\nu}$ . White's (1986) optimal set of instruments  $C^* = \Omega^{-1/2} (I_M \otimes X) = \Sigma_{\nu}^{-1/2} \otimes \tilde{X} + \Sigma_{1}^{-1/2} \otimes \bar{X}$  yields  $\hat{\delta}_{ESSLS} = (Z^{*\prime}P_{C^*}Z^*)^{-1}Z^{*\prime}P_{C^*}y^*.$ Baltagi and Li (1992) showed that the set of instruments  $B = [I_M \otimes \tilde{X}, I_M \otimes \bar{X}]$  used by Baltagi (1981) spans the set of instruments  $C^* = [\Sigma_{\nu}^{-1/2} \otimes \widetilde{X} + \Sigma_{1}^{-1/2} \otimes \overline{X}]$  needed for E3SLS. In what follows we show that the asymptotic variance of E3SLS differs from that of EC3SLS by a positive semi-definite matrix.

Lemma 2  $P_B = P_{I_M \otimes \tilde{X}} + P_{I_M \otimes \tilde{X}}$ .

Proof.

$$
B = (I_M \otimes A) \text{ where } A = [\tilde{X}, \bar{X}], \text{ so that}
$$
  

$$
P_B = (I_M \otimes P_A) = I_M \otimes (P_{\tilde{X}} + P_{\bar{X}}) = P_{I_M \otimes \tilde{X}} + P_{I_M \otimes \bar{X}}
$$

Lemma 3  $P_B P_{C^*} = P_{C^*}.$ 

Proof.

$$
P_{I_M \otimes \tilde{X}} P_{C^*} = (I_M \otimes P_{\tilde{X}}) \left( \Sigma_{\nu}^{-1/2} \otimes \tilde{X} + \Sigma_{1}^{-1/2} \otimes \bar{X} \right) (C^{*'} C^*)^{-1} C^{*'} = \left( \Sigma_{\nu}^{-1/2} \otimes \tilde{X} \right) (C^{*'} C^*)^{-1} C^{*'}.
$$

<sup>&</sup>lt;sup>1</sup>We thank a referee for pointing this out.

using the fact that  $P_{\tilde{X}}\overline{X} = 0$ , since  $QP = 0$ . Also,

$$
P_{I_M \otimes \bar{X}} P_{C^*} = (I_M \otimes P_{\bar{X}}) \left( \Sigma_{\nu}^{-1/2} \otimes \tilde{X} + \Sigma_{1}^{-1/2} \otimes \bar{X} \right) (C^{*'} C^*)^{-1} C^{*'} = \left( \Sigma_{1}^{-1/2} \otimes \bar{X} \right) (C^{*'} C^*)^{-1} C^{*'}.
$$

The summation of these two equations gives us  $(P_{I_M \otimes \tilde{X}} + P_{I_M \otimes \tilde{X}}) P_{C^*} = (\Sigma_{\nu}^{-1/2} \otimes \tilde{X}) (C^{*\prime} C^*)^{-1} C^{*\prime} +$  $\left(\Sigma_1^{-1/2} \otimes \bar{X}\right) \left(C^{*'}C^*\right)^{-1} C^{*'} = P_{C^*}.$ Using the results in Lemma 2,  $P_B = P_{I_M \otimes \tilde{X}} + P_{I_M \otimes \tilde{X}}$ , we get  $P_B P_{C^*} = P_{C^*}.$ 

**Theorem 2**  $avar\left(\sqrt{n}\delta_{ESSLS}\right) -avar\left(\sqrt{n}\delta_{EC3SLS}\right)$  is positive semi-definite.

**Proof.** It is well known, see Baltagi (2008, pp. 130-132), that the asymptotic variance of  $\sqrt{n}\hat{\delta}_{ESSLS}$  is given by  $avar\left(\sqrt{n}\delta_{ESSLS}\right) = p \lim_{n \to \infty} \left(\frac{Z^{*'}P_{C^*}Z^*}{n}\right)$  $\int^{-1}$ , and that of  $\sqrt{n}\delta_{EC3SLS}$  is given by *avar*  $(\sqrt{n}\delta_{EC3SLS})$  =  $p\lim\left(\frac{Z^{*'}P_BZ^*}{n}\right)$  $\int^{-1}$ . Therefore,  $avar\left(\sqrt{n}\hat{\delta}_{ESSLS}\right)-avar\left(\sqrt{n}\hat{\delta}_{EC3SLS}\right)$  is positive semi-definite if  $\left(\frac{Z^{*'}P_BZ^{*}}{n}\right)$  $\overline{ }$ Ξ  $\left(\frac{Z^{*'}P_{C^*}Z^*}{n}\right)$ ) is positive semi-definite. Equivalently, if  $\left(\frac{Z^{*'}(P_B-P_{C^*})Z^{*}}{n}\right)$ n is positive semi-definite. The latter holds if  $P_B - P_{C^*}$  is idempotent, which follows from lemma 3. In fact,  $(P_B - P_{C^*})^2 = P_B + P_{C^*} - P_B P_{C^*}$  $P_{C^*}P_B = P_B - P_{C^*}$ .

#### 3 Empirical Example:

Cornwell and Trumbull (1994), estimated an economic model of crime using panel data on 90 counties in North Carolina over the period 1981-1987. The empirical model relates the crime rate (which is an FBI index measuring the number of crimes divided by the county population) to a set of explanatory variables which include deterrent variables as well as variables measuring returns to legal opportunities. All variables are in logs except for the regional and time dummies. The explanatory variables consist of the probability of arrest (which is measured by the ratio of arrests to offenses), probability of conviction given arrest (which is measured by the ratio of convictions to arrests), probability of a prison sentence given a conviction (measured by the proportion of total convictions resulting in prison sentences), average prison sentence in days as a proxy for sanction severity, the number of police per capita as a measure of the countyís ability to detect crime, the population density which is the county population divided by county land area, a dummy variable indicating whether the county is in the SMSA with population larger than 50,000, percent minority, which is the proportion of the county's population that is minority or non-white, percent young male which is the proportion of the countyís population that is male and between the ages of 15 and 24, regional dummies for western and central counties, opportunities in the legal sector are captured by the average weekly wage in the county by industry. These industries are: construction, transportation, utilities and communication, wholesale and retail trade, finance, insurance and real estate, services, manufacturing, and federal, state and local government. For a replication of this study, see Baltagi (2006).

Table 1 reports the G2SLS and EC2SLS estimators assuming police per capita and the probability of arrest to be endogenous and instrumenting these with tax rate per capita and a measure of face to face crimes. These are the instruments used by Cornwell and Trumbull (1994) using fixed effects 2SLS. For EC2SLS, all the deterrent variables are negative and significant. The sentence severity variable is insignificant and police per capita is positive and significant. Manufacturing wage is negative and significant and percent minority is positive and significant. G2SLS gives basically the same results as EC2SLS but the standard errors are higher. This is due to the fact that EC2SLS uses more instruments than G2SLS. For the probability of arrest, the standard error is 0.221 for G2SLS as compared to 0.097 for EC2SLS with the consequence of overturning the insignificance of this coefficient at the 5% level. The reduction in standard errors is more than 50% in this case. A Hausman test based on the difference between fixed effects 2SLS and random effects 2SLS yields a Hausman statistic of 19:50 for EC2SLS and 16:45 for G2SLS, both of which are asymptotically distributed as  $\chi^2(22)$  with p-values of 0.614 and 0.793, respectively. These do not reject the null hypothesis that EC2SLS and G2SLS yield consistent estimators.

Acknowledgment: The authors would like to gratefully acknowledge a referee for helpful comments. REFERENCES

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Table 1: EC2SLS and G2SLS Estimates for Crime in North Carolina, 1981-1987 (standard errors in parentheses)