1-14-2011

Polarity-Coincidence-Array based spectrum sensing for multiple antenna cognitive radios in the presence of non-gaussian noise

Thakshila Wimalajeewa
twwewelw@syr.edu

Pramod Varshney
Syracuse University, varshney@syr.edu

Department of Electrical Engineering and Computer Science

Technical Report

SYR-EECS-2011-01 Jan. 14, 2011

Polarity-Coincidence-Array Based Spectrum Sensing for Multiple Antenna Cognitive Radios in the Presence of Non-Gaussian Noise

Thakshila Wimalajeewa, *Member, IEEE* twwewelw@syr.edu
Pramod K. Varshney, *Fellow, IEEE* varshney@syr.edu

ABSTRACT: One of the main requirements of the cognitive radio (CR) systems is the ability to perform spectrum sensing in a reliable manner in challenging environments that arise due to propagation channels which undergo multipath fading and non-Gaussian noise. While most existing literature on spectrum sensing has focused on impairments introduced by additive white Gaussian noise (AWGN), this assumption fails to model the behavior of certain types of noise in practice. In this paper, the use of a non-parametric and easily implementable detection device, namely polarity-coincidence-array (PCA) detector, is proposed for the detection of weak primary signals with a cognitive radio equipped with multiple antennas. Its performance is evaluated in the presence of heavy-tailed noise. The detector performance in terms of the probabilities of detection and false alarm is derived when the communication channels between the primary user transmitter and the multiple antennas at the cognitive radio are AWGN as well as when they undergo Rayleigh fading. From the numerical results, it is observed that a significant performance enhancement is achieved by the PCA detector compared to that of the energy detector with AWGN as well as fading channels as the heaviness of the tail of the non-Gaussian noise increases.

KEYWORDS: Cognitive radio, spectrum sensing, non-Gaussian noise, Rayleigh fading, polarity-coincidence-array detectors

Syracuse University - Department of EECS,
4-206 CST, Syracuse, NY 13244
(P) 315.443.2652 (F) 315.443.2583
http://eecs.syr.edu

Polarity-Coincidence-Array Based Spectrum Sensing for Multiple Antenna Cognitive Radios in the Presence of Non-Gaussian Noise

Thakshila Wimalajeewa *Member, IEEE,* and Pramod K. Varshney *Fellow, IEEE*

Department of Electrical Engineering and Computer Science

Syracuse University, Syracuse, NY 13244.

Email: {twwewelw, varshney}@syr.edu

Abstract

One of the main requirements of the cognitive radio (CR) systems is the ability to perform spectrum sensing in a reliable manner in challenging environments that arise due to propagation channels which undergo multipath fading and non-Gaussian noise. While most existing literature on spectrum sensing has focused on impairments introduced by additive white Gaussian noise (AWGN), this assumption fails to model the behavior of certain types of noise in practice. In this paper, the use of a non-parametric and easily implementable detection device, namely polarity-coincidence-array (PCA) detector, is proposed for the detection of weak primary signals with a cognitive radio equipped with multiple antennas. Its performance is evaluated in the presence of heavy-tailed noise. The detector performance in terms of the probabilities of detection and false alarm is derived when the communication channels between the primary user transmitter and the multiple antennas at the cognitive radio are AWGN as well as when they undergo Rayleigh fading. From the numerical results, it is observed that a significant performance enhancement is achieved by the PCA detector compared to that of the energy detector with AWGN as well as fading channels as the heaviness of the tail of the non-Gaussian noise increases.

Index Terms

Cognitive radio, spectrum sensing, non-Gaussian noise, Rayleigh fading, polarity-coincidence-array detectors

I. Introduction

A recent survey of spectrum utilization has revealed that the actual licensed spectrum is largely under-utilized in both temporal and geographic dimensions [1]. Cognitive radio (CR) systems were first proposed in [2] as a possible solution to the under-utilization of the frequency spectrum. CR systems exploit the under-utilized frequency spectrum efficiently by identifying the existence of spectrum holes. There has been extensive research focusing on signal processing challenges faced in designing and implementing cognitive radios in an efficient manner [3], [4]. Spectrum sensing in challenging environments is one of the most important components of the cognitive radio concept [3], [4]. Spectrum sensing is solely carried out by the secondary system, or the cognitive radio to detect the presence of primary users who use the licensed spectrum. While the secondary users are using the licensed spectrum, they should be able to detect the presence of the primary users when they become active with a high probability and vacate the channel within a certain amount of time. For example, in the IEEE 802.22 standard, the secondary users should detect the primary users such as TV and wireless microphone signals and vacate the channel whin two seconds once they become active [5]. Moreover, it is required that the secondary users detect the primary signals with 0.9 probability of detection and 0.1 probability of false alarm in very low signal-to-noise ratio (SNR) regions such as at -20 dB [5]. Thus, accurate spectrum sensing in the presence of multipath fading channels and in non-Gaussian noise environments is necessary to enable the effective use of cognitive radio networks.

Most common approaches proposed for spectrum sensing in the current literature include, matched filtering, energy detector based sensing, feature based sensing, statistical methods based sensing [5]–[9] to name a few. Matched filtering is the optimal detection scheme when the transmitted signal of the primary user is known to the cognitive radio (secondary user). However, since more and more primary bands are being made available for opportunistic access, a cognitive radio needs receivers for all signal types if matched filtering is used for spectrum sensing. Thus, the implementation complexity of the sensing unit can become very large. A simple technique widely used in spectrum sensing is the energy detector [4]. The main drawback of the energy detector is its susceptibility to uncertainties in the background noise. If certain features of the primary signals can be identified, more accurate and robust detectors can be implemented at the cost of increased complexity. One of the most commonly used detectors in

this category is the cyclostationary detector [7]. However, none of these detectors work well when the cognitive radios operate in the presence of non-Gaussian noise. Although the noise distribution is often assumed to be Gaussian which results in more mathematical tractability, not all noise in practice can be modeled as Gaussian. Non-Gaussian noise impairments may include man-made impulsive noise, co-channel interference from other cognitive radios, interference from ultra-wideband systems, to name a few [10], [11].

Spectrum sensing for cognitive radio networks in the presence of non-Gaussian noise has been addressed by several researchers recently [10], [12], [13]. In [10], a suboptimal L_p norm detector for primary signal detection in the presence of non-Gaussian noise was proposed in which a tunable parameter has to be optimized for the underlying type of noise. The decision statistic of the proposed detector also requires the knowledge of the power of the fading channel gains. In [12], cyclostationarity based detectors, which require statistical information about the primary user signal, were optimized for non-Gaussian noise. In [13], a generalized likelihood ratio test (GLRT) based scheme was presented for detecting primary user signals when the non-Gaussian noise variance is unknown. Implementation of all the detectors for non-Gaussian noise presented in [10], [12], [13] is more complex compared to implementing an energy detector.

Use of multiple antennas is a common technology in current wireless communications systems, and its effectiveness in different aspects is discussed in [14]. Exploiting the spatial diversity via multiple antennas for improving the performance of spectrum sensing has been considered by several authors in the recent literature. In [15], the performance of the energy detector with multiple antennas at the cognitive radios is analyzed in the presence of Gaussian noise. In [16], a multiple antenna OFDM based CR scheme is shown to perform well compared to that with single antenna scheme when using the square law combining energy detector. Generalized likelihood based detectors with multiple antennas when some or all the parameters (of noise and signal) are unknown were derived in [17] where the authors have assumed that the noise and the primary user signals are Gaussian. However, the efficient use of multiple antenna systems for primary signal detection in the presence of non-Gaussian noise has not received much attention in the recent CR literature.

In this paper, we consider the application of a non-parametric detection device named polarity-coincidence-

array (PCA) detectors to detect the presence of the primary signal in the presence of non-Gaussian (heavy-tailed) noise when a secondary user (cognitive radio) is equipped with multiple antennas. PCA detector is non-parametric in the sense that the decision statistic and the threshold do not depend on the primary user signal and noise characteristics. Also, the PCA detector is easily implementable. The 2-channel version of the PCA detector, called polarity coincidence correlator (PCC), was considered in [18] assuming equal channel gains. PCA detectors to detect a common random signal received at an array of sensors (with equal channel gains) have been addressed in [19] where the author has proposed several decision statistics. The performance based on the output SNR is derived in the region of low SNR. In [20], analysis on array detectors was presented in which the performance is given in terms of asymptotic relative efficiency (ARE) where ARE is defined in terms of the efficacy. However, a comprehensive analysis on the performance of the PCA detector in terms of probability of detection error, the comparison with energy detectors, the behavior of the detector as the heaviness of the tail of non-Gaussian noise varies, and the performance analysis when the communication channels undergo fading have not been considered in the literature. The use of PCA detector in multi-antenna cognitive radios used for weak primary signal detection in the presence of non-Gaussian (heavy-tailed) noise is considered in this work.

Our major contributions are: (i). Derive the performance measures in terms of probabilities of false alarm and detection for general non-Gaussian noise models and signal distributions which satisfy the assumptions given in section II-C when the communication channels between the primary user transmitter and the multiple antennas at the secondary user are AWGN as well as when they undergo multipath fading. (ii). Derive the asymptotic relative efficiency of the PCA detector relative to the energy detector with equal gain combining, (which is the optimal detector with AWGN channels) for weak signal detection and analyze the behavior of ARE analytically when the number of antennas and the heaviness of the tail of the non-Gaussian noise vary when the communication channels are AWGN. In particular, we show that the ARE is not monotonically increasing with the number of antennas as the heaviness of the tail of the non-Gaussian noise increases and derive the optimal number of antennas to be used to achieve the maximum ARE in such scenarios. (iii). In the presence of fading channels, the performance of the PCA detector is evaluated in closed-form in terms of the probabilities of the false alarm and detection. The performance of the PCA detector is compared to that with the energy detector with equal gain combining.

We show that the performance of the PCA detector is much superior to that of the energy detector as the heaviness of the tail of the non-Gaussian noise increases for AWGN as well as fading channels.

The remainder of the paper is organized as follows. Section II presents the observation, primary user signal and noise models, and the assumptions. In section III, the performance of the PCA detector is derived in terms of the probabilities of detection and false alarm when the communication channels are AWGN. Further, the performance of the PCA detector is compared to that of the energy detector in terms of the asymptotic relative efficiency. In section IV, the performance of the PCA detector is investigated when the communication channels between the primary user and the multiple antennas at the CR undergo fading. Performance results are shown in section V and concluding remarks are given in section VI.

II. Observation, Primary User Signal and Noise Models and Assumptions

A. Observation model

Assume that the cognitive radio (secondary user) has M antennas. The received observation vector at the multi-antenna cognitive radio (CR) from the primary user at time n under each hypothesis (primary user absent/present) is given by

$$\begin{aligned} \mathcal{H}_0: \ \mathbf{x}[n] &= \mathbf{v}[n], \ n = 0, 1, \cdots, N-1 \\ \mathcal{H}_1: \ \mathbf{x}[n] &= s[n]\mathbf{h} + \mathbf{v}[n], \ n = 0, 1, \cdots, N-1 \end{aligned} \tag{1}$$

where $\mathbf{x}[n] = [x_1[n], x_2[n], \cdots, x_M[n]]^T$ is the received signal vector, $\mathbf{h} = [h_1, h_2, \cdots, h_M]^T$ is the vector that corresponds to channel fading coefficients, $s[n]$ is the primary user signal and $\mathbf{v}[n] = [v_1[n], v_2[n], \cdots, v_M[n]]^T$ is the noise vector at time n, respectively. The primary user signal $s[n]$ is assumed to be random, independent and identically distributed (iid) over n. The elements of the noise vector $\mathbf{v}[n]$ are assumed to be iid across channels and time n. Denote σ_v^2 and σ_s^2 to be the variances, μ_v^4 and μ_s^4 to be the fourth moments of noise and signal, respectively. We also let $C_0 = \frac{\mu_v^4}{\sigma_v^4}$ and $C_1 = \frac{\mu_s^4}{\sigma_s^4}$ be the ratios between fourth moment and the square of the second moment of the noise and signal, respectively.

B. Noise model

Although the additive noise is often assumed to be Gaussian, there are many situations for which the Gaussian noise model does not fit well. For example, in modeling urban and man-made RF noise,

low frequency atmospheric noise, certain types of ultra-wideband (UWB) interference, the Gaussian noise assumption is not appropriate [11]. In this paper, we consider the noise is non-Gaussian; i.e. the probability density function of $\mathbf{v}[n]$ is given by $f_{\mathbf{v}[n]}(\mathbf{v}[n]) = \prod_{k=1}^{M} f_{v_k[n]}(v_k[n])$ where $f_{v_k[n]}(v_k[n])$ is a non-Gaussian pdf for $k = 1, 2, \cdots, M$. Specific non-Gaussian models used for performance analysis which are relevant for CRs, are discussed in the following.

1) Generalized Gaussian (GG) noise model: GG noise model is widely used to characterize non-Gaussian noise such as, atmospheric and impulsive noise [21], [22]. A random variable X is said to be distributed as GG, if it has the following pdf.

$$f_X(x) = \frac{1}{2\Gamma\left(1+\frac{1}{\beta}\right)A(\beta,\sigma)} e^{-\left|\frac{x}{A(\beta,\sigma)}\right|^{\beta}} \tag{2}$$

where $\beta, \sigma > 0$, $A(\beta,\sigma) = \left[\frac{\sigma^2\Gamma(\frac{1}{\beta})}{\Gamma(\frac{3}{\beta})}\right]^{1/2}$ is a scaling factor that allows $\mathrm{var}(X) = \sigma^2$, β is the shaping parameter which is used to model heavy-tailed ($0 < \beta < 2$) and short-tailed ($\beta > 2$) noise, and $\Gamma(x) = \int_0^\infty t^{x-1}e^{-t}dt$ is the Gamma function. Laplacian noise and Gaussian noise are contained in GG noise as special cases when $\beta = 1$ and $\beta = 2$, respectively. In [11], it is stated that the GG noise with $\beta \approx 0.5$, can be used to model certain impulsive atmospheric noise. Since the pdf in (2) is symmetric around zero, the odd moments of X are zeros and $F_X(0) = \frac{1}{2}$. The even moments are given by $\mathbb{E}\{X^r\} = \mu_x^r = \left[\frac{\sigma^2\Gamma(\frac{1}{\beta})}{\Gamma(\frac{3}{\beta})}\right]^{r/2} \frac{\Gamma(\frac{r+1}{\beta})}{\Gamma(\frac{1}{\beta})}$ for $r = 2, 4, \cdots$, where $\mathbb{E}\{.\}$ denotes the mathematical expectation. For the GG noise model, we have $C_0 = \frac{\mu_v^4}{\sigma_v^4} = \left[\frac{\Gamma(\frac{1}{\beta})}{\Gamma(\frac{3}{\beta})}\right]^2 \frac{\Gamma(\frac{5}{\beta})}{\Gamma(\frac{1}{\beta})}$.

2) Gaussian mixture (GM) noise: GM noise is used in modeling man-made noise, impulsive noise, and certain types of UWB interference [10], [23]. A random variable X has a GM distribution if the pdf of X is,

$$f_X(x) = \sum_{l=1}^{L} \frac{c_l}{\sqrt{2\pi\sigma_l^2}} e^{-\frac{x^2}{2\sigma_l^2}} \tag{3}$$

where we assume the mean is zero, $0 < c_l < 1$, $\sum_{l=1}^{L} c_l = 1$. Since the pdf given in (3) is symmetric around zero, it satisfies the assumptions in II-C. For the GM noise model, we consider an important special case in this paper, $\epsilon-$ mixture model, where $c_1 = 1-\epsilon$, $c_2 = \epsilon$, $L = 2$, and $\sigma_2^2 = \kappa\sigma_1^2$ for $\kappa > 1$ and $0 < \epsilon < 1$ with the resulting pdf

$$f_X(x) = (1-\epsilon)\mathcal{N}(x; 0, \sigma_1^2) + \epsilon\mathcal{N}(x; 0, \kappa\sigma_1^2), \tag{4}$$

where $\mathcal{N}(x;\mu,\sigma^2)$ denotes the random variable x is distributed as Gaussian with mean zero and the variance σ^2. ϵ- mixture noise model is a popular model used to characterize the behavior of heavy-tailed noise and well approximates the Middleton's Class A noise model [24]. The latter term in (4) models the impulsive noise where impulsive noise occurs with a probability of ϵ and has a variance κ times greater than the Gaussian noise with variance σ_1^2 (first component in (4)). It was shown that the typical values of parameters of the ϵ-mixture model which approximates the Middleton's Class A noise model are in the range of $\epsilon \in [0.01, 0.33]$ and $\kappa \in [20, 10000]$ [11], [12], [25]. For ϵ- mixture noise model, $C_0 = \frac{\mu_v^4}{\sigma_v^4} = \frac{3((1-\epsilon)+\kappa^2\epsilon)}{((1-\epsilon)+\kappa\epsilon)^2}$.

C. Assumptions

Throughout the paper, we make the following assumptions on the signal and noise models. (i). The probability distribution functions of the noise and the signal, F_v and F_s have zero median (the pdfs of signal and noise are symmetric around zero); i.e. $F_v(0) = \frac{1}{2}$ and $F_s(0) = \frac{1}{2}$. Thus the odd moments of the signal and noise are zero. (ii). $F_v(x) = 1 - F_v(-x)$. (iii). $\int x^2 dF_v(x) < \infty$, and $\int x^2 dF_s(x) < \infty$.

III. Primary Signal Detection with AWGN Channels

In this section, we consider the problem of primary signal detection when the communication channels between the primary user transmitter and multiple antennas at cognitive radio are AWGN.

A. Optimal detector with Gaussian inputs

In the absence of fading, we have $\mathbf{h} = [1, 1, \cdots, 1]^T$. If the signal and noise are assumed to be Gaussian such that $\{s[n]\}_{n=0}^{N-1}$ is an iid Gaussian sequence with mean zero and the variance σ_s^2, $\mathbf{v}[n] \sim \mathcal{N}(\mathbf{0}, \sigma_v^2 \mathbf{I}_M)$ where $\mathbf{I}_M$ is the $M \times M$ identity matrix, it can be shown [17] that the optimum Neyman-Pearson (NP) detector has the following test statistic (which has the form of an energy detector after equal gain combining):

$$T_{ED} = \sum_{n=0}^{N-1} \left(\sum_{k=1}^{M} x_k[n] \right)^2 . \tag{5}$$

B. Performance of the energy detector of the form (5)

Exact closed-form expressions for the probabilities of detection and false alarm exist for the energy detector (5) when the noise and signal are Gaussian [17]. However, in the following, we consider the performance of the energy detector (5) for arbitrary signal and noise distributions which satisfy the assumptions in subsection II-C, using central limit theorem (CLT) when the number of samples is sufficiently large since it is easy to compare with the rest of the results presented in the paper. Let $\tilde{y}[n] = \left(\sum_{k=1}^{M} x_k[n]\right)^2$. Then it can be shown that (see Appendix A), $\mathbb{E}\{\tilde{y}[n]|\mathcal{H}_0\} = M\sigma_v^2$, $\mathbb{E}\{\tilde{y}[n]|\mathcal{H}_1\} = M(\sigma_v^2 + M\sigma_s^2)$, $\text{var}\{\tilde{y}[n]|\mathcal{H}_0\} = M\mu_v^4 + (2M^2 - 3M)\sigma_v^4$, and $\text{var}\{\tilde{y}[n]|\mathcal{H}_1\} = M\mu_v^4 + M^4\mu_s^4 + (2M^2 - 3M)\sigma_v^4 + 4M^3\sigma_v^2\sigma_s^2 - M^4\sigma_s^4$. For sufficiently large N, CLT states that the random variable T_{ED} in (5) is Gaussian under two hypotheses, where,

$$\begin{aligned} \mathcal{H}_0 : \quad & T_{ED} \sim \mathcal{N}(N\mathbb{E}\{\tilde{y}[n]|\mathcal{H}_0\}, N\text{var}\{\tilde{y}[n]|\mathcal{H}_0\}) \\ \mathcal{H}_1 : \quad & T_{ED} \sim \mathcal{N}(N\mathbb{E}\{\tilde{y}[n]|\mathcal{H}_1\}, N\text{var}\{\tilde{y}[n]|\mathcal{H}_1\}). \end{aligned} \tag{6}$$

The probability of false alarm of the NP detector is then given by

$$p_f = Q\left(\frac{\tau_e - N\mathbb{E}\{\tilde{y}[n]|\mathcal{H}_0\}}{\sqrt{N\text{var}\{\tilde{y}[n]|\mathcal{H}_0\}}}\right) \tag{7}$$

where τ_e is the threshold of the detector and $Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty e^{\frac{-t^2}{2}}dt$. To keep the probability of false alarm under a value α, the threshold of the detector is chosen as $\tau_e = Q^{-1}(\alpha)\sqrt{N\text{var}\{\tilde{y}[n]|\mathcal{H}_0\}} + N\mathbb{E}\{\tilde{y}[n]|\mathcal{H}_0\}$ $= Q^{-1}(\alpha)\sqrt{NM(\mu_v^4 + (2M-3)\sigma_v^4)} + NM\sigma_v^2$. Then the probability of detection of the $\alpha-$ level NP detector is given by

$$p_d = Q\left(\frac{Q^{-1}(\alpha)\sqrt{V_0} - \sqrt{N}M^2\sigma_s^2}{\sqrt{V_1}}\right), \tag{8}$$

where $V_0 = \text{var}\{\tilde{y}[n]|\mathcal{H}_0\}$, and $V_1 = \text{var}\{\tilde{y}[n]|\mathcal{H}_1\}$. It is noted that, the implementation of this detector requires the knowledge of the second and fourth moments of noise since the false alarm probability depends on these parameters. When the noise and signal are Gaussian as considered in subsection III-A, the probability of detection of the α-level NP detector reduces to,

$$p_d = Q\left(\frac{Q^{-1}(\alpha) - \sqrt{\frac{N}{2}}M\gamma_0^2}{1 + M\gamma_0^2}\right) \tag{9}$$

where $\gamma_0^2 = \frac{\sigma_s^2}{\sigma_v^2}$.

As stated in subsection III-A, the energy detector of the form (5) is the optimal detector when the signal and noise are Gaussian. When the noise is non-Gaussian, the energy detector would not give the best performance. As discussed later in detail in the simulation results section and can be seen from the Fig. 5, the probability of detection of the energy detector degrades as the noise becomes more and more impulsive. To find the test statistic which yields the optimal performance when the noise is non-Gaussian, the knowledge of the signal and noise distributions may be required. Implementation of the energy detector may require certain parameters related to the noise and the primary user signal. Further, as discussed in [26], a poor performance is achieved by the energy detector at very low SNR regions. Thus, in the following we consider a non-parametric approach for the primary signal detection by CRs in the presence of non-Gaussian noise.

C. Polarity coincidence array (PCA) detector

Polarity coincidence correlator (PCC) is a non-parametric two input detection device which outperforms the energy detector in certain situations when the inputs are non-Gaussian [18]. PCC detector is attractive due to its simplicity of implementation and efficiency compared to the energy detector especially in the presence of heavy-tailed noise. The 2-channel PCC detector computes the following test statistic:

$$T_{PCC} = \sum_{n=0}^{N-1} u(x_1[n]x_2[n]) \tag{10}$$

where $u(.)$ is the unit step function. An extension of 2-channel PCC for an array of sensors, is the Polarity coincidence array (PCA) detector [19].

Let $\zeta[n]$ be the difference between the number of channels having the most prevalent sign (positive or negative) at time n and half the number of channels, which will result in, $\zeta[n] = \left|\frac{1}{2}\sum_{k=1}^{M} \mathrm{sgn}(x_k[n])\right|$ where $\mathrm{sgn}(x)$ is 1 if $x \geq 0$ and -1 if $x < 0$. The test statistic of the PCA detector has the form of, $T_{PCA} = \sum_{n=0}^{N-1} g(\zeta[n])$ where $g(.)$ is a monotonic function of $\zeta[n]$.

In this paper, we consider the PCA detector which computes the test statistic [19],

$$T_{PCA} = \sum_{n=0}^{N-1} (\zeta[n])^2 \tag{11}$$

When $M = 2$, it can be seen that the test statistic in (11) and the test statistic for PCC in (10) are equivalent.

D. Performance of the PCA detector

Let $y[n] = (\zeta[n])^2 = \frac{1}{4}\left(\sum_{k=1}^{M} \text{sgn}(x_k[n])\right)^2$. Since $\{s[n]\}$ for $n = 0, 1, \cdots, N-1$ are assumed to be iid, it can be seen that the test statistic T_{PCA} in (11) is a sum of N iid random variables and thus can be approximated as a Gaussian random variable when the number of samples N is large enough.

Mean and the variance of the random variable $y[n]$ under the two hypotheses are given by (see Appendix B for the derivation): $\mathbb{E}\{y[n]|H_0\} = \frac{M}{4}$, $\mathbb{E}\{y[n]|H_1\} = \frac{M}{4}(1 + (M-1)(2p_1 - 1))$, $\text{var}\{y[n]|H_0\} = \frac{M}{8}(M-1)$, and

$$\begin{aligned}\text{var}\{y[n]|H_1\} &= \frac{M}{16}\left[2(M-1) + 4(M-1)(M-2)(2p_1-1) - M(M-1)^2(2p_1-1)^2\right. \\ &\quad \left. +(M-1)(M-2)(M-3)(2p_2-1)\right] \end{aligned} \tag{12}$$

where $p_1 = \int (1 - 2F_v(s) + 2F_v^2(s))dF_s(s)$ and

$$p_2 = \int \left(1 - 4F_v(s) + 12F_v(s)^2 - 16F_v(s)^3 + 8F_v(s)^4\right) dF_s(s). \tag{13}$$

Thus, it can be seen that the decision statistic (11) is distributed as, $T_{PCA}|\mathcal{H}_j \sim \mathcal{N}(\mu_j, \sigma_{t_j}^2)$ where $\mu_j = N\mathbb{E}\{y[n]|H_j\}$ and $\sigma_{t_j}^2 = N\text{var}\{y[n]|H_j\}$ for $j = 0, 1$.

The probabilities of false alarm and detection are given by $p_{fa} = Pr(T_{PCA} > \tau_p|\mathcal{H}_0) = Q\left(\frac{\tau_p - \frac{NM}{4}}{\sqrt{NM(M-1)/8}}\right)$ and $p_d = Pr(T_{PCA} > \tau_p|\mathcal{H}_1) = Q\left(\frac{\tau_p - \frac{NM}{4}(1+(M-1)(2p_1-1))}{\sqrt{N\text{var}\{y[n]|H_1\}}}\right)$ where $\text{var}\{y[n]|H_1\}$ is given in (12) and τ_p is the threshold of the detector. If the false alarm probability is constrained to be less than α, the probability of detection of the PCA detector is given by

$$p_d = Q\left(\frac{Q^{-1}(\alpha)\sqrt{\frac{M(M-1)}{8}} - \sqrt{N}\frac{M}{4}(M-1)(2p_1-1)}{\sqrt{\text{var}\{y[n]|H_1\}}}\right).$$

It is worth mentioning that the test statistic of the PCA detector (11) is easily computable since it requires to compute the sign of the received signal at each channel followed by simple arithmetic operations. Further, to compute the threshold to keep the probability of false alarm under a desired value of the PCA detector it only requires the knowledge of the number of antennas and time samples. These factors make the PCA detector exceedingly simple to implement.

E. Weak signal detection and asymptotic relative efficiency

We further analyze the performance of the PCA detector for the two noise models considered in the low SNR region: That is when $\frac{\sigma_s^2}{\sigma_v^2} << 1$. With the assumptions on signal and noise as in subsection II-C, it can be shown [18] that p_1 is approximated by $p_1 \approx \frac{1}{2} + 2F_v'^2(0)\sigma_s^2$ and p_2 can be approximated by (expanding $F_v(s)$ and its powers in a Maclaurin series), $p_2 \approx \frac{1}{2} + 8F_v'^4(0)\mu_s^4$, when the SNR is small, where $F_v'(s)$ is the first order derivative of $F_v(s)$ with respect to s. Then the probability of detection of the PCA detector as $\frac{\sigma_s^2}{\sigma_v^2} << 1$ is given by

$$Q\left(\frac{Q^{-1}(\alpha)\sqrt{\frac{M(M-1)}{8}} - \sqrt{N}\frac{M}{4}(M-1)4F_v'^2(0)\sigma_s^2}{\sqrt{\tilde{V}_1}}\right), \tag{14}$$

where $\tilde{V}_1 = \frac{M}{16}(2(M-1)+8(M-1)(M-2)F_v'^2(0)\sigma_s^2+16(M-1)(M-2)(M-3)F_v'^4(0)\mu_s^4-16M(M-1)^2F_v'^4(0)\sigma_s^4)$.

In the weak signal detection problem, it can be seen that the asymptotic probability of detection of the PCA detector converges to the following as the number of antennas increases:

$$\lim_{M\to\infty} p_d \to Q\left(-\sqrt{\frac{N}{C_1-1}}\right) \tag{15}$$

where $C_1 = \frac{\mu_s^4}{\sigma_s^4}$ as defined before. It is interesting to note that, when the signal is Gaussian such that $C_1 = 3$, the asymptotic performance (as M increases) of the optimal detector with Gaussian noise given in (9) also converges to the same limit as given in (15). This implies that, as observed in the optimal detector with Gaussian signal and Gaussian noise, the asymptotic (in terms of M) performance of the PCA detector with non-Gaussian noise is ultimately limited by the number of time samples N.

For the weak signal detection problem, the performance of the PCA detector with non-Gaussian noise for multiple antenna cognitive radio is compared with the energy detector of the form (5) in terms of the asymptotic relative efficiency (ARE). When two statistical tests T_a and T_b require sample sizes N_a and N_b to achieve the same probability of detection given the same probability of false alarm, the ARE of the test T_a with respect to the test T_b, ARE_{T_a,T_b}, is defined as , $ARE_{T_a,T_b} \triangleq \lim_{N_a,N_b\to\infty} \frac{N_b}{N_a}$.

Lemma 1: When SNR is small, the asymptotic relative efficiency of the PCA detector with respect to the energy detector (5) is given by

$$ARE_{PCA,ED} = \frac{M-1}{M^2}(C_0 + (2M-3))8F_v'^4(0)\sigma_v^4 \tag{16}$$

where $C_0 = \frac{\mu_v^4}{\sigma_v^4}$ *as defined before.*

Proof: See Appendix C. ■

It is noted that the ARE in (16) is not a monotonically non-decreasing function of M in general. In fact it can be shown that, when $C_0 < 5$, (16) is monotonically non-decreasing with M but not when $C_0 > 5$ (see Appendix D). When $C_0 > 5$, it can be shown that (letting first derivative of (16) equals to zero) the optimal number of antennas M which results in the maximum $ARE_{PCA,ED}$ is given by

$$M_o = 2\left[\frac{C_0 - 3}{C_0 - 5}\right], \ for \ C_0 > 5 \tag{17}$$

where $[x]$ denotes the nearest integer to the real number x. For example when the noise is Gaussian ($\beta = 2$ in (2)), $C_0 = 3$, and $ARE_{PCA,ED}$ in (16) becomes, $ARE^G_{PCA,ED} = \frac{M-1}{M} 16 F_v'^4(0)\sigma_v^4$ which is a monotonically non decreasing function of M. Thus, the maximum achievable $ARE_{PCA,ED}$ with multiple antennas when the noise is Gaussian is $ARE^{G,\max}_{PCA,ED} = \lim_{M\to\infty} ARE^G_{PCA,ED} = 16F_v'^4(0)\sigma_v^4 = 0.4053$. On the other hand, when the noise is double exponential such that $\beta = 1$ in (2) and $C_0 = 6$, $ARE_{PCA,ED}$ does not increase monotonically as the number of antennas increases, and the maximum $ARE_{PCA,ED}$ over M is achieved when $M_o = 6$ which is given by $ARE^{\max}_{PCA,ED} = 4.1667$. The behavior of the maximum achievable $ARE_{PCA,ED}$ as the tail of the non-Gaussian noise varies is further discussed in the numerical results section.

The following lemma states the maximum achievable $ARE_{PCA,ED}$ for weak signal detection using multiple antennas at CR.

Lemma 2: The maximum achievable $ARE_{PCA,ED}$ *at the CR with multiple antennas is given by*

$$ARE^{max}_{PCA,ED} = \begin{cases} 16F_v'^4(0)\sigma_v^4 & if \ C_0 < 5 \\ \frac{M_o-1}{M_o^2}(C_0 + (2M_o - 3))8F_v'^4(0)\sigma_v^4 & if \ C_0 > 5 \end{cases} \tag{18}$$

where M_0 *is as given in (17).*

Proof: When $C_0 < 5$, since $ARE_{PCA,ED}$ is monotonically non-decreasing with M , the maximum achievable $ARE_{PCA,ED}$ is obtained when $M \to \infty$. Thus for $C_0 < 5$, $ARE^{max}_{PCA,ED} = \lim_{M\to\infty} ARE_{PCA,ED} = 16F_v'^4(0)\sigma_v^4$. For $C_0 > 5$, the maximum $ARE_{PCA,ED}$ is achieved when $M = M_0$ where M_0 is as given in (17), resulting (18). ■

Irrespective of the value of C_0, the $ARE_{PCA,ED}$ converges to a constant value as the number of antennas

M increases and this value is the maximum possible $ARE_{PCA,ED}$ if $C_0 < 5$ but not the maximum when $C_0 > 5$ (for heavy-tailed noise).

Remark 1: From (17), it can be seen that the optimal number of antennas which yields the maximum possible $ARE_{PCA,ED}$ approaches 2 as C_0 gets larger. Large C_0 for both considered noise models implies heavy-tailed noise. Thus, it can be seen that as the heaviness of the tail of the noise increases, the best possible performance gain of the PCA detector compared to that with the energy detector in terms of $ARE_{PCA,ED}$ over M is achieved when $M = 2$.

IV. Primary Signal Detection with Rayleigh Fading Channels

In this section, we consider the performance of the PCA detector when the communication channels between the primary user and the multiple antennas at CRs undergo Rayleigh fading.

A. Performance of the PCA detector with fading channels

Let $y[n] = \frac{1}{4}\left(\sum_{k=1}^{M} z_k[n]\right)^2$ as before where now, $\mathcal{H}_0 :\ x_k[n] = v_k[n]$, and $\mathcal{H}_1 :\ x_k[n] = h_k s[n] + v_k[n]$ for $k = 1, 2, \cdots, M$ and $z_k[n] = sign(x_k[n])$. Then it can be shown that, $\mathbb{E}\{y[n]|H_0\} = \frac{M}{4}$, $\mathbb{E}\{y[n]|H_1\} = \frac{1}{4}\left(M + \sum_{i\neq j}(2p_1^{ij} - 1)\right)$, $\text{var}\{y[n]|H_0\} = \frac{M}{8}(M-1)$, and

$$\text{var}\{y[n]|H_1\} = \frac{1}{16}\left[2M(M-1) + 4(M-2)\sum_{i\neq j}(2p_1^{ij} - 1) + \sum_{i\neq j\neq l\neq m}(2p_2^{ijlm} - 1) - \left(\sum_{i\neq j}(2p_1^{ij} - 1)\right)^2\right] \quad (19)$$

where $p_1^{ij} = \int 1 - F_v(h_i s) - F_v(h_j s) + 2F_v(h_i s)F_v(h_j s)dF_S(s)$, and

$$\begin{aligned} p_2^{ijlm} = &\int [1 - (F_v(h_i s) + F_v(h_j s) + F_v(h_l s) + F_v(h_m s)) \\ + &\ 2(F_v(h_i s)F_v(h_j s) + F_v(h_i s)F_v(h_l s) + F_v(h_i s)F_v(h_m s) + F_v(h_j s)F_v(h_l s) \\ + &\ F_v(h_j s)F_v(h_m s) + F_v(h_l s)F_v(h_m s)) + 4(F_v(h_i s)F_v(h_j s)F_v(h_l s) + F_v(h_i s)F_v(h_j s)F_v(h_m s) \\ + &\ F_v(h_i s)F_v(h_l s)F_v(h_m s) + F_v(h_j s)F_v(h_l s)F_v(h_m s)) + 8F_v(h_i s)F_v(h_j s)F_v(h_l s)F_v(h_m s)]dF_S(s) \end{aligned}$$

Conditioned on $\mathbf{h}$, the test statistic T_{PCA} in (11) is a sum of iid random variables under each hypothesis. Thus, the probability of detection while maintaining probability of false alarm under α conditioned on $\mathbf{h}$ is given by

$$p_{d|\mathbf{h}} = Q\left(\frac{Q^{-1}(\alpha)\sqrt{\frac{M(M-1)}{8}} - \sqrt{N}\frac{1}{4}\sum_{i\neq j}(2p_1^{ij} - 1)}{\sqrt{\text{var}\{y[n]|H_1\}}}\right) \quad (20)$$

where $\mathrm{var}\{y[n]|H_1\}$ is as given in (19). Computing the average probability of detection over $\mathbf{h}$ requires M- fold integration of $p_{d|\mathbf{h}}$ in (20). Thus, in the following we consider several special cases for weak signal detection; i.e. $\frac{h_i h_j h_k h_l \sigma_s^2}{\sigma_v^2} \ll 1$ for $i \neq j \neq k \neq l$ so that the higher order terms can be neglected. Then, p_1^{ij} can be approximated as, $p_1^{ij} \approx \frac{1}{2} + 2h_i h_j F_v'^2(0)\sigma_s^2$.

1) Weak signal detection when $M = 2$: Under the assumption of weak signal detection, the probability of detection for $M = 2$ given $\mathbf{h}$, is given by

$$p_d = \mathbb{E}_{\tilde{h}}\left\{ Q\left(\frac{\frac{1}{2}Q^{-1}(\alpha) - \sqrt{N}2\tilde{h}F_v'^2(0)\sigma_s^2}{\sqrt{\frac{1}{4} - 4\tilde{h}^2 F_v'^4(0)\sigma_s^4}} \right) \right\}$$

where $\tilde{h} = h_1 h_2$. When h_j's are iid Rayleigh random variables with the pdf, $f_{h_j}(h) = \frac{h}{\sigma_R^2} e^{-\frac{h^2}{2\sigma_R^2}}$, it can be shown that the distribution of $\tilde{h}$ is given by [27], $f_{\tilde{h}}(\tilde{h}) = \frac{\tilde{h}}{\sigma_R^2} K_0\left(\frac{\tilde{h}}{\sigma_R}\right)$ where K_0 is the modified Bessel function of the second kind. Then the average probability of detection for $M = 2$ is given by

$$\bar{p}_d = \int_0^\infty Q\left(\frac{\frac{1}{2}Q^{-1}(\alpha) - \sqrt{N}2\tilde{h}F_v'^2(0)\sigma_s^2}{\sqrt{\frac{1}{4} - 4\tilde{h}^2 F_v'^4(0)\sigma_s^4}} \right) \frac{\tilde{h}}{\sigma_R^2} K_0\left(\frac{\tilde{h}}{\sigma_R}\right) d\tilde{h} \tag{21}$$

which requires only a single integration.

2) Weak signal detection when M is large: Note that when the SNR is small, the probability of detection of PCA detector given in (20) can be approximated as,

$$p_d \approx \mathbb{E}_{\mathbf{h}}\left\{ Q\left(\frac{Q^{-1}(\alpha)\sqrt{\frac{M(M-1)}{8}} - \sqrt{N}F_v'^2(0)\sigma_s^2 \sum\limits_{i\neq j} h_i h_j}{\sqrt{\frac{M(M-1)}{8} + (M-2)F_v'^2(0)\sigma_s^2 \sum\limits_{i\neq j} h_i h_j}} \right) \right\} \tag{22}$$

The argument of the Q-function in (22) depends on $\mathbf{h}$ via the sum $T_h = \sum\limits_{i\neq j} h_i h_j$. Let $\tilde{T}_h = \sum\limits_{i\neq j, i<j} h_i h_j$. Then $T_h = 2\tilde{T}_h$. It can be seen that, $\tilde{T}_h$ is in general a sum of dependent random variables having $\tilde{M} = M(M-1)/2$ elements. The central limit theorem exists for dependent random variables under certain conditions. In [28], it is stated that the sum of m-dependent sequence of random variables with finite third absolute moment converges to a normal random variable if the number of elements in the sequence is large enough. It is noted that for moderate values of M, we have large enough $\tilde{M}$ such that the central limit theorem can be applied. For sufficiently large $\tilde{M}$ ($\tilde{M} \to \infty$)we have the following results:

Lemma 3: The sequence in the sum $\tilde{T}_h$ is m-dependent with $m = \frac{M^2-3M+4}{2}$. When $\tilde{M}$ is large, $\tilde{T}_h$ has a limiting normal distribution with mean, $\mu_h = \frac{\pi M(M-1)}{4}\sigma_R^2$ and the variance, $\sigma_h^2 = M(M-1)\left(1 - \frac{\pi}{4}\right)\left(2 + \pi(M - \frac{3}{2})\right)\sigma_R^4$.

Proof: See Appendix E. ■

Then the average probability of detection (22) when $\tilde{M}$ is large enough, can be approximated as

$$p_d \approx \int Q\left(\frac{Q^{-1}(\alpha)\sqrt{\frac{M(M-1)}{8}} - \sqrt{N}F_v'^2(0)\sigma_s^2 2\tilde{T}_h}{\sqrt{\frac{M(M-1)}{8} + (M-2)F_v'^2(0)\sigma_s^2 2\tilde{T}_h}}\right)\frac{1}{\sqrt{2\pi\sigma_h^2}}e^{-\frac{(\tilde{T}_h-\mu_h)^2}{2\sigma_h^2}}d\tilde{T}_h \tag{23}$$

which requires only a single integration. It is verified in subsection V-B that this approximation closely matches with the simulation results with finite values of M used in practice (which result relatively large $\tilde{M}$).

To compare the performance of the PCA detector in the presence of fading, we consider the following energy based detector schemes which are commonly used when communication channels undergo fading. Energy detection with maximal ratio combining (MRC) is the optimal scheme for the problem given in (1) for detecting Gaussian signals in the presence of Gaussian noise [17] when the communication channels between the primary user transmitter and the multiple antennas at CRs undergo fading. When the signal and noise are Gaussian, the threshold of this detector can be found numerically involving only a single integration. However, when the noise is non-Gaussian it can be shown that the computation of the threshold requires M-fold integration which might be difficult when M is large (Details are omitted due to space limitation).

B. Energy detection with equal gain combining

Implementing the energy detector with MRC requires the channel state information at the cognitive radio, and computationally difficult integrations when the noise is non-Gaussian. Equal gain combining (EGC) is a simpler technique which equally weights the signals on each channel. With equal gain combining, the decision statistic has the form, $T_{EGC} = \sum_{n=0}^{N-1}\tilde{y}[n]$ where $\tilde{y}[n] = \left(\sum_{k=1}^{M} x_k[n]\right)^2$. Then we have, $\mathbb{E}\{\tilde{y}[n]|\mathcal{H}_0\} = M\sigma_v^2$, $\mathbb{E}\{\tilde{y}[n]|\mathcal{H}_1\} = M\sigma_v^2 + \bar{h}^2\sigma_s^2$, $\text{var}\{\tilde{y}[n]|\mathcal{H}_0\} = M\mu_v^4 + M(2M-3)\sigma_v^4$, $\text{var}\{\tilde{y}[n]|\mathcal{H}_1\} = M(C_0 + (2M-3))\sigma_v^4 + (C_1-1)\bar{h}^4\sigma_s^4 + 4M\bar{h}^2\sigma_v^2\sigma_s^2$ where $\bar{h} = \sum_{k=1}^{M} h_k$.

Then the probability of detection of the α-level NP detector is given by

$$\bar{p}_d = \mathbb{E}_{\bar{h}}\left\{Q\left(\frac{Q^{-1}(\alpha)\sqrt{M(C_0+(2M-3))} - \sqrt{N}\bar{h}^2\gamma_0^2}{\sqrt{M(C_0+(2M-3)) + (C_1-1)\gamma_0^4\bar{h}^4 + 4M\gamma_0^2\bar{h}^2}}\right)\right\} \tag{24}$$

where $\gamma_0^2 = \frac{\sigma_s^2}{\sigma_v^2}$ as before. Finding a closed-form expression for the pdf of $\bar{h}$ in general is difficult. Thus, we consider the following special cases and approximations to evaluate the integral in (24).

1) M=2: When $M = 2$, pdf of $\bar{h}$ is available in closed-form [29] which is given by $f(t) = \frac{t}{2}e^{-t^2/2} + \sqrt{\pi}\left(\frac{1}{2} - Q(t/\sqrt{2})\right)(t^2/2 - 1)e^{-t^2/4}$ where $t = \frac{\bar{h}}{\sigma_R}$. Thus, the expectation in (24) can be evaluated with a single integration when $M = 2$.

2) $M > 2$: For general M, a widely used approximation for the pdf of $\bar{h}$ is given in [30], $f_{\tilde{\bar{h}}}(\tilde{\bar{h}}) = \frac{t^{2M-1}e^{-\frac{t^2}{2b}}}{2^{M-1}b^M(M-1)!}$ where $\tilde{\bar{h}} = \bar{h}/\sigma_R$, $t = \frac{\tilde{\bar{h}}}{\sqrt{M}}$, $b = \frac{\sigma_R^2}{M}[(2M-1)!!]^{1/M}$, $(2M-1)!! = (2M-1)(2M-3)...3.1$. Then the average probability of detection (24) can be approximated as,

$$\bar{p}_d \approx \int Q\left(\frac{Q^{-1}(\alpha)\sqrt{M(C_0 + (2M-3))} - \sqrt{N}\sigma_R^2\tilde{\bar{h}}^2\gamma_0^2}{\sqrt{M(C_0 + (2M-3)) + (C_1 - 1)\gamma_0^4\sigma_R^4\tilde{\bar{h}}^4 + 4M\gamma_0^2\sigma_R^2\tilde{\bar{h}}^2}}\right).f_{\tilde{\bar{h}}}(\tilde{\bar{h}})d\tilde{\bar{h}}.$$

V. Performance Analysis

In this section, we display the performance of the PCA detector and the energy detector for the two types of non-Gaussian noise models considered in the paper.

A. Asymptotic relative efficiency of PCA detector compared to ED with AWGN channels

In this subsection, we evaluate the performance gain achieved by the PCA detector over the energy detector for weak signal detection in terms of $ARE_{PCA,ED}$ when the communication channels between the primary user transmitter and the multiple antennas at the CR are AWGN. Figure 1 shows $ARE_{PCA,ED}$ for the GG noise model when the number of antennas varies for different values of β (and corresponding value of C_0 is also shown in Figures). In Fig. 1, β varies in the range $.6-2$. When β decreases beyond 0.6, the behavior of $ARE_{PCA,ED}$ with M is similar to that with $\beta = 0.6$ (where the maximum $ARE_{PCA,ED}$ is achieved when $M \to 2$) and not shown in the figure for clarity. As mentioned earlier, when $\beta < 2$, the GG noise model represents heavy-tailed noise (impulsive noise). It can be seen that for small values of β (in the region $\beta < 2$), performance of the PCA detector compared to that of the energy detector has a significant performance improvement over a wide range of M (number of antennas) in terms of $ARE_{PCA,ED}$. In particular, from the Lemma 1, it can be easily shown that the maximum achievable $ARE_{PCA,ED}$ with multiple antennas for GG noise model is always greater than 1 when $\beta <\approx 1.4$. Thus for a wide classes of non-Gaussian impulsive noise (e.g. Laplace noise when $\beta = 1$, certain impulsive atmospheric noise $\beta \approx 0.5$ [11]), the PCA detector with multiple antennas outperforms the energy detector in a large scale. Thus it is seen that the decision statistic based on a function of sign information of the

received signal in multiple channels seem to be a better choice in detecting primary signal when the noise becomes more impulsive compared to detecting the primary signal detection based on the total energy collected during a given time interval.

Moreover, it can be observed that when β decreases, the optimal number of antennas which yields the maximum achievable $ARE_{PCA,ED}$ approaches 2 (Fig. 2 (a)). This implies that when β is small, (i.e. the heaviness of the noise tail is large) the maximum performance gain in terms of $ARE_{PCA,ED}$ achieved by multiple antenna PCA detector approaches the value achieved with the 2-channel PCC detector. However, for moderate values of β (but still less than 2), by increasing the number of antennas, the performance of the PCA detector has a considerable performance gain over the PCC ($M = 2$) detector in terms of the $ARE_{PCA,ED}$; i.e., the maximum achievable $ARE_{PCA,ED}$ is achieved with more than 2 antennas. For example, when $\beta = 1$, $ARE_{PCA,ED} = 3.5$ for $M = 2$ and the maximum achievable $ARE_{PCA,ED}$ with $M_o = 6$ antennas is 4.1667.

It can also be observed that for light-tailed noise ($\beta > 2$), the PCA detector does not show a better performance compared to that with the energy detector irrespective of the number of antennas are employed (Fig. 2 (b)). More specifically, it can be shown analytically that when $\beta >\approx 1.4$, the maximum achievable $ARE_{PCA,ED}$ for GG noise model with multiple antennas is always less than 1; that is in this region of β, energy detector is more effective compared to that with the PCA detector. This implies that, use of sign information of the observed signal in different channels of the multiple antenna system does not provide sufficient information to better detect the presence of a primary signal when the noise is light-tailed. It is worth adding a comment for Gaussian noise ($\beta = 2$). As mentioned earlier, the energy detector is optimal for Gaussian noise but the implementation of that detector requires the knowledge of the noise variance. It can be seen from the Lemma 1 that the ARE with $M = 2$ is 0.2026 while the maximum achievable ARE with multiple antennas is $ARE^{G,\max}_{PCA,ED} = 0.4053$ for Gaussian noise. Thus the $ARE_{PCA,ED}$ can be improved at most by a factor of approximately 2 by employing multiple antennas more than two at the CR receiver compared to $M = 2$ when the noise is Gaussian. Even though this maximum value is still less than 1, PCA detector with multiple antennas would be a good choice for even Gaussian noise since that is the price to pay for the non-parametric property of the PCA detector.

In Fig. 3, the maximum achievable $ARE_{PCA,ED}$ with different parameter values of ϵ- mixture noise

model in (4) is depicted. As explained in subsection II-B, with ϵ- mixture model, in the region of small values of ϵ and as κ increases, it represents the heavy-tailed impulsive noise. From Fig. 3, it can be seen that in this region of ϵ and κ the maximum achievable $ARE_{PCA,ED}$ with multiple antenna PCA detector coincides with the one achieved by the 2-channel PCC detector.

B. Performance of the PCA detector with GG noise model with Rayleigh fading

In this subsection, the performance of the PCA detector is investigated when the communication channels undergo Rayleigh fading.

In Fig. 4, the probability of detection of the PCA and energy detectors versus average SNR for different number of antennas is shown for the GG noise model for $\beta = 1$ (i.e. double exponential/Laplacian noise) when the communication channels undergo Rayleigh fading. The average SNR is given by $SNR = \frac{2\sigma_R^2\sigma_s^2}{\sigma_v^2}$. Unless specified otherwise, for all the figures we assume that the signal is Gaussian with mean zero and variance σ_s^2. We compare the analytical results derived in (23) for large $\tilde{M} = M(M-1)/2$ and in (21) for $M = 2$ for the PCA detector in weak signal detection to that is obtained via simulations for Laplacian noise. For the simulations, in each iteration we consider 10^4 sets of M-length vectors of iid Rayleigh random variables, and 10^4 sets of $M \times N$ matrices consisting of iid Laplace random variables, for assumed M and N values. The results are averaged over 50 iterations. In Fig. 4, the sample size $N = 1024$, the probability of false alarm $\alpha = 0.1$, and plots correspond to three different values of M. From Fig. 4, it can be seen that the analytical approximations derived under the assumption of weak signal detection closely match with the simulation results for $M = 2$ and relatively large $\tilde{M}$, in the low SNR region. This further validates the applicability of the central limit theorem (CLT) for dependent random variables as used in subsection IV-A2. From Fig. 4, it can be seen that as the average SNR increases, the performance gain achieved by using multiple antennas compared to that with $M = 2$ also increases. Also, it can be seen that by increasing the number of antennas (e.g. $M = 10$ in the Figure) it is possible to have the probability of detection approach 1 at relatively low SNR values. Further, the performance gain achieved by the PCA detector compared to the energy detector is also depicted in Fig. 4.

In Fig. 5, the performance of the PCA detector is shown as the heaviness of the noise tail in the GG noise model varies for different numbers of antennas. In Fig. 5, the results are based on the numerical

integrations of (21) and (23) for $M = 2$, and $M = 10$, respectively for PCA detector. For the energy detector with EGC, the results are based on numerical integrations as discussed in subsections IV-B1 for $M = 2$ and IV-B2 for $M = 10$, respectively. Further in Fig. 5, we let $N = 1024$, $\alpha = 0.1$, and $SNR = -20$ dB. It can be seen from Fig. 5 that as the heaviness of the tail of the GG noise increases (β decreases), the performance of the energy detector decreases while the performance of the PCA detector is greatly improved. Thus, the performance gain of the PCA detector over the energy detector increases significantly as β decreases. It is interesting to see that for relatively large antenna sizes the probability of detection of the PCA detector is significantly outperforms the energy detector when $\beta <\approx 1.4$ with fading channels, as observed with AWGN channels in terms of asymptotic relative efficiency. It should be noted that the PCA detector basically computes the number of channels having the most prevalent sign. Under hypothesis $\mathcal{H}_0$ (signal is absent), approximately half of the channels will have the same sign since the noise pdf is assumed to be symmetric around zero. When a common random signal is present in each channel under $\mathcal{H}_1$, the number of channels with the same sign as the signal increases. This distinguishability between two hypotheses seems to be more significant as the heaviness of the tail of the non-Gaussian noise increases as well as the number of channels increases, resulting in better detectability of the signal. On the other hand, the energy detector is the optimal detector in detecting Gaussian signals corrupted by Gaussian noise (i.e. $\beta = 2$ in the GG noise model) and as the non-Gaussianity of the noise increases, poor detection performance is achieved by the energy detector. Fig. 5 clearly illustrates the effectiveness of the use of the PCA detector in heavy-tailed GG noise compared to the energy detector.

In Fig. 6, the receiver operating characteristics (ROC) curves for the PCA detector and the energy detector for different values of β are shown for $N = 1024$, $M = 2$ and two different values of SNR (-20 dB and -10 dB). It further shows that at small β values (i.e. with more heavy-tailed noise), PCA detector's performance is much better even in very small SNR regions compared to the energy detector.

In the above analysis, it was assumed that the number of samples is large but fixed at $N = 1024$. In the next experiment, we investigate the performance of the PCA detector as the number of samples varies. In Fig. 7, the performance of the PCA and energy detectors is shown as the number of samples varies for $\alpha = 0.1$, $SNR = -15dB$ and $\beta = 0.8$. It can be seen that, for the assumed parameters, the gain in terms of the number of samples required to achieve the same probability of detection is higher for fewer

number of antennas compared to that with larger number of antennas. This was analyzed in the latter part of section III-E for AWGN channels, where it was shown that as the heaviness of the tail of the noise increases, the maximum achievable $ARE_{PCA,ED}$ is obtained when $M = 2$. This phenomenon was further illustrated in subsection V-A for AWGN channels in terms of $ARE_{PCA,ED}$.

In Fig. 8, we analyze the performance of energy detectors with EGC and maximal ratio combining (MRC) for non-Gaussian noise at low SNR region. With MRC, the decision statistic is given by $T_{MRC} = \sum_{n=0}^{N-1} \left(\sum_{k=1}^{M} h_k x_k[n] \right)^2$. In Fig. 8, we let $N = 1024$ and numerical results are obtained for Laplacian noise (with $\beta = 1$). Fig. 8(a) corresponds to ROC curves for $M = 2$ while Fig. 8(b) corresponds to $M = 10$. It can be observed that, the energy detector with MRC is not a good candidate for the primary signal detection at low SNR region in the presence of non-Gaussian noise compared to that with energy detector with EGC.

C. Performance of PCA detector with GM noise model with Rayleigh fading

In this subsection, we investigate the performance of the PCA detector when the noise is modeled using the ϵ- mixture model as described in (4). In Figures 9 and 10, the probability of detection of PCA and energy detectors versus ϵ is shown for $M = 2$ and for $M = 10$, respectively when $SNR = -20dB$ and $\alpha = 0.1$ as κ varies. It can be seen that as the parameter κ increases (≥ 20) the performance of the PCA detector is improved significantly compared to the energy detector in the range of ϵ in which the ϵ- mixture model characterizes the heavy-tailed noise, except a very small region closer to 0.01. As ϵ increases from 0.01 towards 0.33, i.e., as the probability that the impulsive event occurs increases as explained in subsection 4, the performance gain achieved by the PCA detector over the energy detector becomes more significant. On the other hand, the performance of the energy detector degrades as the heaviness of the tail of ϵ- mixture model increases. Figs. 9 and 10 clearly exhibit that the PCA detector performs significantly better compared to the energy detector when the additive noise at CR is modeled as heavy-tailed ϵ- mixture noise.

VI. Summary

In this paper, we investigated the potential use of polarity-coincidence array (PCA) detectors for spectrum sensing by multiple antenna cognitive radios in the presence of non-Gaussian, heavy-tailed

noise. Closed-form expressions for the performance measures in terms of the probabilities of detection and false alarm were derived when the communication channels between the primary user and the multiple antennas at the cognitive radio are AWGN as well as undergo Rayleigh fading.

With AWGN channels, the performance gain of the PCA detector over that of the widely used energy detector is evaluated analytically in terms of the asymptotic relative efficiency, $ARE_{PCA,ED}$. It was shown that for impulsive noise, there is an optimal number of antennas (≥ 2) which yields the maximum achievable $ARE_{PCA,ED}$. Moreover, it was observed that as the noise becomes more and more impulsive, the number of antennas which results the maximum achievable $ARE_{PCA,ED}$ reaches two. Thus depending on the specific parameters of the non-Gaussian noise model, the CR designer will be able to select the number of antennas to be used at the CR receivers to achieve the maximum performance gain over energy detectors in terms of ARE when the communication channels are AWGN.

In the presence of fading channels, the performance of the PCA detector is compared to that with the energy detector after equal gain combining, in terms of the probability of detection (keeping the probability of false alarm under a certain threshold). With fading channels, it was shown that the PCA detector performs significantly better compared to the energy detector in low SNR regions when the heaviness of the tail of the non-Gaussian noise exceeds a certain value and the performance gain becomes more significant as the heaviness increases. From the results presented in the paper it can be seen that the use of multiple antennas at cognitive radios, together with the large performance gain achieved compared to the energy detector and the ease of implementation make the PCA detectors a useful approach for spectrum sensing when the CRs operate in the presence of impulsive/heavy-tailed noise and when the communication channels are AWGN as well as undergo fading.

APPENDIX A

We have $\tilde{y}[n] = \left(\sum_{k=1}^{M} x_k[n]\right)^2 = \sum_{k=1}^{M} x_k^2[n] + 2\sum_{k\neq j, k<j} x_k[n]x_j[n]$. Based on the observation vector in (1) under two hypothesis, when $\mathbf{h} = [1, 1, \cdots, 1]^T$, we have

$$\mathbb{E}\{\tilde{y}[n]|\mathcal{H}_0\} = \sum_{k=1}^{M} \mathbb{E}\{v_k^2[n]\} + 2\sum_{k\neq j, k<j} \underbrace{\mathbb{E}\{v_k[n]v_j[n]\}}_{=0} = M\sigma_v^2 \tag{25}$$

$$\begin{aligned}\mathbb{E}\{\tilde{y}[n]|\mathcal{H}_1\} &= \sum_{k=1}^{M}\mathbb{E}\{(s[n]+v_k[n])^2\}+2\sum_{k\neq j,k<j}\mathbb{E}\{(s[n]+v_k[n])(s[n]+v_j[n])\}\\ &= M(\sigma_v^2+\sigma_s^2)+M(M-1)\sigma_s^2 = M(M\sigma_s^2+\sigma_v^2) \end{aligned} \tag{26}$$

To find the variance $\mathrm{var}\{\tilde{y}[n]|\mathcal{H}_j\}$, we need to find $\mathbb{E}\{\tilde{y}[n]^2|\mathcal{H}_j\}$ for $j=0,1$. Note that we can write, $(\tilde{y}[n])^2 = \sum_{k=1}^{M} x_k^4[n] + 2\sum_{i\neq j} x_i^2[n]x_j^2[n] + \sum_{(k\neq l)\neq i\neq j} x_k[n]x_l[n]x_i[n]x_j[n] + 2\sum_{k=1}^{M} x_k^2[n]\sum_{l\neq j} x_l[n]x_j[n]$. Thus we have

$$\mathbb{E}\{\tilde{y}[n]^2|\mathcal{H}_0\} = M\mu_v^4 + 2M(M-1)\sigma_v^4 + M(M-1)\sigma_v^4 = M\mu_v^4 + 3M(M-1)\sigma_v^4. \tag{27}$$

To find $\mathbb{E}\{\tilde{y}[n]^2|\mathcal{H}_1\}$, we need to find following quantities:

1) $\mathbb{E}\{x_k^4[n]|\mathcal{H}_1\} = \mathbb{E}\{(s[n]+v_k[n])^4\} = \mu_s^4 + 6\sigma_s^2\sigma_v^2 + \mu_v^4$,
2) $\mathbb{E}\{x_k^2[n]x_j^2[n]|\mathcal{H}_1\} = \mathbb{E}\{(s[n]+v_k[n])^2(s[n]+v_j[n])^2\} = \mu_s^2 + 2\sigma_v^2\sigma_s^2 + \sigma_v^4$ for $k\neq j$,
3) $\mathbb{E}\{x_k^3[n]x_j[n]|\mathcal{H}_1\} = \mathbb{E}\{(s[n]+v_k[n])^3(s[n]+v_j[n])\} = \mu_s^4 + 3\sigma_v^2\sigma_s^2$ for $k\neq j\neq l$,
4) $\mathbb{E}\{x_k^2[n]x_j[n]x_l[n]|\mathcal{H}_1\} = \mathbb{E}\{(s[n]+v_k[n])^2(s[n]+v_j[n])(s[n]+v_l[n])\} = \mu_s^4 + \sigma_v^2\sigma_s^2$ for $k\neq j\neq l$,
5) $\mathbb{E}\{x_k x_j[n]x_l[n]x_m[n]|\mathcal{H}_1\} = \mathbb{E}\{(s[n]+v_k[n])(s[n]+v_j[n])(s[n]+v_l[n])(s[n]+v_m[n])\} = \mu_s^4$ for $k\neq j\neq l\neq m$.

Then we have

$$\begin{aligned}\mathbb{E}\{\tilde{y}[n]^2|\mathcal{H}_1\} &= M(\mu_s^4+6\sigma_s^2\sigma_v^2+\mu_v^4)+3M(M-1)(mu_s^2+2\sigma_v^2\sigma_s^2+\sigma_v^4)+4M(M-1)(\mu_s^4+3\sigma_v^2\sigma_s^2)\\ &+ 6M(M-1)(M-2)(\mu_s^4+\sigma_v^2\sigma_s^2)+M(M-1)(M-2)(M-3)\mu_s^4\\ &= M^4\mu_s^4+M\mu_v^4+3M(M-1)\sigma_v^4+6M^3\sigma_v^2\sigma_s^2. \end{aligned} \tag{28}$$

APPENDIX B

Let $z_k[n] = \mathrm{sgn}(x_k[n])$. Then $y[n] = \frac{1}{4}(\sum_{k=1}^{M} z_k[n])^2 = \frac{1}{4}[\sum_{k=1}^{M} z_k[n]^2 + \sum_{k\neq l} z_k[n]z_l[n]]$ and

$$(y[n])^2 = \frac{1}{16}\left(\sum_{k=1}^{M} z_k[n]\right)^4 = \frac{1}{16}\left[\sum_{k=1}^{M} z_k^4 + 2\sum_{k\neq l} z_k^2 z_l^2 + \sum_{(k\neq l)\neq(i\neq j)} z_k z_l z_i z_j + 2\sum_{k=1}^{M} z_k^2 \sum_{l\neq j} z_l z_j\right]. \tag{29}$$

It was shown in [19] that under $\mathcal{H}_0$, $\mathbb{E}\{y[n]|\mathcal{H}_0\} = \frac{M}{4}$ and $\mathbb{E}\{y[n]^2|\mathcal{H}_0\} = \frac{1}{16}\left(M + 2(M^2-M) + M(M-1)\right) = \frac{1}{16}\left(3M^2-2M\right)$ resulting in $\mathrm{var}\{y[n]|H_0\} = \frac{M}{8}(M-1)$. In the following, we compute the mean and the

variance of $y[n]$ under $\mathcal{H}_1$. We have, $\mathbb{E}\{y[n]|\mathcal{H}_1\} = \frac{1}{4}\left[\sum_{k=1}^{M}\mathbb{E}\{z_k[n]^2|\mathcal{H}_1\} + \sum_{k\neq l}\mathbb{E}\{z_k[n]z_l[n]|\mathcal{H}_1\}\right]$. It can be shown that, $\mathbb{E}\{z_k[n]^2|\mathcal{H}_1\} = 1$ and $\mathbb{E}\{z_k z_l|\mathcal{H}_1\} = 2p_1 - 1$ where $p_1 = pr(x_k[n] > 0, x_l[n] > 0|\mathcal{H}_1 \ or \ x_k[n] < 0, x_l[n] < 0|\mathcal{H}_1)$ which can be found as,

$$p_1 = \int((1 - F_v(-s))^2 + (F_s(-s))^2)dF(s) = \int(1 - 2F_v(s) + 2F_v^2(s))dF_s(s) \tag{30}$$

where $F_x(.)$ is the probability distribution function of the random variable x. (30) can be computed if the signal and noise distributions are known. Then we have, $\mathbb{E}\{y[n]|\mathcal{H}_1\} = \frac{M}{4}[1 + (M-1)(2p_1 - 1)]$. To find the variance of $y[n]$ under $\mathcal{H}_1$, it is required to compute the expectations of all the terms in (29). We have, $\mathbb{E}\{z_k^4|\mathcal{H}_1\} = 1$, $\mathbb{E}\{z_k^2 z_l^2|\mathcal{H}_1\} = 1$, $\mathbb{E}\{z_k z_l^3|\mathcal{H}_1\} = 2p_1 - 1$, $\mathbb{E}\{z_k^2 z_l z_i|\mathcal{H}_1\} = 2p_1 - 1$, $\mathbb{E}\{z_k z_l z_i z_j|\mathcal{H}_1\} = 2p_2 - 1$ for $k, l, i, j = 1, 2, \cdots, M$, where

$$\begin{aligned} p_2 = \ & Pr(x_k > 0, x_l > 0, x_i > 0, x_j > 0|\mathcal{H}_1 \ or \ x_k < 0, x_l < 0, x_i < 0, x_j < 0|\mathcal{H}_1 \ or \\ & x_k > 0, x_l > 0, x_i < 0, x_j < 0|\mathcal{H}_1 \ or \ x_k > 0, x_l < 0, x_i < 0, x_j > 0|\mathcal{H}_1 \ or \\ & x_k > 0, x_l < 0, x_i > 0, x_j < 0|\mathcal{H}_1 \ or \ x_k < 0, x_l < 0, x_i > 0, x_j > 0|\mathcal{H}_1 \ or \\ & x_k < 0, x_l > 0, x_i > 0, x_j < 0|\mathcal{H}_1 \ or \ x_k < 0, x_l > 0, x_i < 0, x_j > 0|\mathcal{H}_1) \end{aligned}$$

and reduces to the following after a simple manipulation:

$$p_2 = \int\left(1 - 4F_v(s) + 12F_v(s)^2 - 16F_v(s)^3 + 8F_v(s)^4\right)dF_s(s).$$

Thus, we have

$$\begin{aligned} \mathrm{var}\{y[n]|H_1\} = \frac{1}{16}\big[&M + 3M(M-1) + 4M(M-1)(2p_1 - 1) + 6M(M-1)(M-2)(2p_1 - 1) \\ &+ M(M-1)(M-2)(M-3)(2p_2 - 1) - M^2(1 + (M-1)(2p_1 - 1))\big] \end{aligned}$$

which reduces to (12) after a simple manipulation.

APPENDIX C

Proof of Lemma 1: By equating the probabilities of detection of the PCA detector under the weak signal assumption (14) and of the energy detector (8), we have

$$\frac{\sqrt{V_1}Q^{-1}(\alpha)\sqrt{\frac{M(M-1)}{8}}}{\sqrt{N_p}} - \sqrt{V_1}M(M-1)F_v'^2(0)\sigma_s^2 = \frac{\sqrt{\tilde{V}_1}Q^{-1}(\alpha)\sqrt{V_0}}{\sqrt{N_p}} - \sqrt{\frac{N_e}{N_p}}\sqrt{\tilde{V}_1}M^2\sigma_s^2 \tag{31}$$

where V_0 and V_1 are defined just after (8) while $\tilde{V}_1$ is given after (14), and N_p and N_e denotes the number of time samples with PCA and energy detector, respectively which are required to yield the same probability of detection. When $N_p \to \infty$, we have, $\frac{N_e}{N_p} = \frac{V_1(M-1)^2 F_v'^4(0)}{\tilde{V}_1 M_2}$. Substituting for V_1 and $\tilde{V}_1$ and letting $\frac{\sigma_s^2}{\sigma_v^2} \to 0$, and letting $\left(\frac{N_e}{N_p}\right)_{N_p,N_e\to\infty} = ARE_{PCA,ED}$ we get the result in (16).

APPENDIX D

We prove that $ARE_{PCA,ED}$ given in (16) is monotonically non-decreasing function of M if $C_0 < 5$ by proving that the derivative of $ARE_{PCA,ED}$ with respect to M is always positive if $C_0 < 5$. The ARE of PCA detector with respect to the energy detector is given in (16). Differentiating (16) with respect to M results in, $\frac{d(ARE)}{dM} = \frac{8F_v'^4(0)\sigma_v^4}{M^2}\left(\left(5 - \frac{6}{M}\right) - C_0\left(1 - \frac{2}{M}\right)\right)$. $\frac{dARE}{dM}$ is positive when $M = 2$. For $M > 2$, for $\frac{dARE}{dM}$ to be positive, the following condition should be satisfied: $C_0 < \frac{5-\frac{6}{M}}{1-\frac{2}{M}}$. It can be easily seen that the quantity $\frac{5-\frac{6}{M}}{1-\frac{2}{M}}$ is monotonically decreasing with M for $M > 2$. Thus, for $\frac{dARE}{dM}$ to be always positive, C_0 should be less than the minimum value of $\frac{5-\frac{6}{M}}{1-\frac{2}{M}}$ which equals to 5.

APPENDIX E

A sequence of random variables $X_1, X_2, \cdots$, is m-dependent if $(X_1, \cdots, X_r)$ is always independent of $(X_s, X_{s+1}, \cdots)$ for $s - r > m$ [28]. In such a sequence if m or more consecutive X's are removed, the remaining two portions of the sequence are independent. Consider the elements in the sequence of the sum $\tilde{T}_h$, are arranged such that $(h_1h_2, h_1h_3, \cdots, h_1h_{M-1}, h_1h_M, h_2h_3, \cdots, h_2h_m, \cdots, h_{M-1}h_M)$. Note that there are $\tilde{M} = \frac{M(M-1)}{2}$ elements in the sequence. Then at a maximum of $\frac{M(M-1)}{2} - (M-2)$ elements are removed from the sequence at any point, the remaining two portions of the sequence are independent (since h_j's are independent random variables for $j = 1, 2, \cdots, M$). Thus the sequence in $\tilde{T}_h$ is m-dependent with $m = \frac{M(M-1)}{2} - (M-2) = \frac{M^2-3M+4}{2}$.

Computing mean and variance of $\tilde{T}_h$: Since h_j's are iid Rayleigh random variables for $j = 1, 2, \cdots, M$, $\mathbb{E}\{h_kh_j\} = \mathbb{E}\{h_k\}\mathbb{E}\{h_j\} = \sigma_R\sqrt{\frac{\pi}{2}}.\sigma_R\sqrt{\frac{\pi}{2}} = \sigma_R^2\frac{\pi}{2}$ for any $j \neq k$. Thus, $\mathbb{E}\{\tilde{T}_h\} = \frac{M(M-1)}{2}.\sigma_R^2\frac{\pi}{2} = \frac{\pi M(M-1)}{4}\sigma_R^2$. Based on the covariance matrix of the elements of the sequence in $\tilde{T}_h$,

$$\mathrm{var}(\tilde{T}_h) = \frac{M(M-1)}{2}\mathrm{var}(h_kh_j) + M(M-1)(M-2)\mathrm{cov}(h_kh_j, h_kh_i),\ k \neq i \neq j. \tag{32}$$

We have, $\mathrm{var}(h_k h_j) = \mathbb{E}\{h_k^2\}\mathbb{E}\{h_j^2\} - (\mathbb{E}\{h_k\}\mathbb{E}\{h_j\})^2 = 4\sigma_R^4 - \left(\frac{\pi}{2}\sigma_R^2\right)^2 = \left(4 - \frac{\pi^2}{4}\right)\sigma_R^4$. $\mathrm{cov}(h_k h_j, h_k h_i)$ for $k \neq i \neq j$ can be computed as,

$$\begin{aligned}\mathrm{cov}(h_k h_j, h_k h_i) &= \mathbb{E}\{h_k^2 h_j h_i\} - \mathbb{E}\{h_k h_j\}\mathbb{E}\{h_k h_i\} = \mathbb{E}\{h_k^2\}\mathbb{E}\{h_j\}\mathbb{E}\{h_i\} - (\mathbb{E}\{h_k\})^2\mathbb{E}\{h_j\}\mathbb{E}\{h_i\} \\ &= 2\sigma_R^2\left(\sqrt{\frac{\pi}{2}}\sigma_R\right)^2 - \left(\sqrt{\frac{\pi}{2}}\sigma_R\right)^4 = \left(1 - \frac{\pi}{4}\right)\pi\sigma_R^4. \qquad (33)\end{aligned}$$

REFERENCES

[1] F. C. Commission, "Spectrum Policy Task Force," *Rep. ET Docket no. 02-135*, Nov. 2002.

[2] J. Mitola and G. Q. Maguire, "Cognitive radio: making software radios more personal," *IEEE Personal Commun.*, vol. 6, no. 4, pp. 13–18, 1999.

[3] S. Haykin, "Cognitive radio: brain-empowered wireless communications,," *IEEE Journal on Selected Areas of Commun.*, vol. 23, no. 2, pp. 201–220, 2005.

[4] A. Ghasemi and E. S. Sousa, "Spectrum sensing in cognitive radio networks: requirements, challenges and design tradeoffs," *IEEE Commun. Magazine*, vol. 46, no. 4, pp. 32–39, April 2008.

[5] Y. Zeng, Y.-C. Liang, A. T. Hoang, and R. Zhang, "A review on spectrum sensing for cognitive radio: Challenges and solutions," *EURASIP Journal on Advances in Signal Processing*, vol. 2010, Article ID 381465, 2010.

[6] H. S. Chen, W. Gao, and D. G. Daut, "Signature based spectrum sensing algorithms for ieee 802.22 wran," in *IEEE Int. Conf. on Communications (ICC 07)*, Glasgow, Scotland, June 2007, pp. 6487–6492.

[7] N. Han, S. H. Shon, J. O. Joo, and J. M. Kim, "Spectral correlation based signal detection method for spectrum sensing in IEEE 802.22 WRAN systems," in *Proc. of 8th Int. Conf. on Advanced Commun. Technology*, Phoenix Park, South Korea, Feb. 2006.

[8] M. Maida, J. Najim, P. Bianchi, and M. Debbah, "Performance analysis of some eigen-based hypothesis tests for collaborative sensing," in *Proc. of IEEE Workshop on Statistical Signal Processing*, Cardiff,Wales, UK, Sept. 2009.

[9] Y. H. Zeng and Y.-C. Liang, "Spectrum-sensing algorithms for cognitive radio based on statistical covariances," *IEEE Trans. on Vehicular Technology*, vol. 58, no. 4, pp. 1804–1815, 2009.

[10] F. Moghimi, A. Nasri, and R. Schober, "Lp-norm spectrum sensing for cognitive radio networks impaired by non-gaussian noise," in *IEEE Global Communications Conference (GLOBECOM)*, Honolulu, Hawaii, USA, Dec. 2009, pp. 1–6.

[11] S. A. Kassam, *Signal Detection in Non-Gaussian Noise*. New York: Springer-Verlag, 1988.

[12] J. Lunden, S. A. Kassam, and V. Koivunen, "Robust nonparametric cyclic correlation-based spectrum sensing for cognitive radio," *IEEE Trans. on Signal Processing*, vol. 58, no. 1, pp. 38–52, Jan. 2010.

[13] S. J. Zahabi and A. A. Tadaion, "Local spectrum sensing in non-gaussian noise," in *17th Int. Conf. on Telecommunications (ICT 2010)*, Mar. 2010, pp. 843–847.

[14] A. Goldsmith, *Wireless Communications*. Cambridge University Press, 2005.

[15] A. Pandharipande and J. P. M. G. Linnartz, "Perfromcane analysis of primary user detection in multiple antenna cognitive radio," in *IEEE Int. Conf. on Communications (ICC 07)*, Glasgow, Scotland, June 2007.

[16] V. Kuppusamy and R. Mahapatra, "Primary user detection in ofdm based mimo cognitive radio," in *3rd Int. Conf. on Cognitive Radio Oriented Wireless Networks and Communications (CrownCom 2008)*, May 2008, pp. 1–5.

[17] A. Taherpour, M. Nasiri-Kenari, and S. Gazor, "Multiple antenna spectrum sensing in cognitive radios," *IEEE. Trans. on Wireless Commun.*, vol. 9, no. 2, pp. 814 – 823, Feb. 2010.

[18] S. S. Wolff, J. B. Thomas, and T. R. Williams, "The polarity-coincidence correlator: A nonparametric detection device," *IRE. Trans. on Information Theory*, pp. 5–9, Mar. 1962.

[19] M. Kanefsky, "Detection of weak signals with polarity coincidence arrays," *IEEE. Trans. on Information Theory*, pp. 260–268, Apr. 1966.

[20] S. A. Kassam and J. B. Thomas, "Array detectors for random signals in noise," *IEEE. Trans. on Sonics and Ultrasonics*, vol. 23, no. 2, pp. 107–112, Mar. 1976.

[21] Y. Chen and N. C. Beaulieu, "Novel low complexity estimators for the shape parameter of the generalized gaussian distribution," *IEEE Trans. on Vehicular Technology*, vol. 58, no. 4, p. 20672071, May 2009.

[22] G. Gonzalez-Farias, J. A. D. Molina, and R. M. Rodrguez-Dagnino, "Efficiency of the approximated shape parameter estimator in the generalized gaussian distribution," *IEEE Trans. on Vehicular Technology*, vol. 58, no. 8, pp. 4214–4223, Oct. 2009.

[23] D. Middleton, "Statistical-physical models of manmade radio noise parts i and ii," in *U.S. Dept. Commerce Office Telecommun*, Apr. 1974 and 1976.

[24] ——, "Non-gaussian noise models in signal processing for telecommunications: new methods and results for class A and class B noise models," *IEEE. Trans. on Information Theory*, vol. 45, no. 4, pp. 1129–1149, May 1999.

[25] K. S. Vastola, "Threshold detection in narrow-band non-Gaussian noise," *IEEE Trans. Communications*, vol. COM-32, pp. 134–139, Feb. 1984.

[26] T. Ycek and H. Arslan, "A survey of spectrum sensing algorithms for cognitive radio applications," *IEEE Commun. Surveys and Tutorials*, vol. 11, no. 1, pp. 116–160, 2009.

[27] J. Salo, H. M. El-Sallabi, and P. Vainikainen, "The distribution of the product of independent rayleigh random variables," *IEEE Trans. on Antennas and Propagation*, vol. 54, no. 2, pp. 639–643, Feb. 2006.

[28] D. A. Fraser, *Nonparametric Methods in Statistics*. John Wiley Sons, Inc. London, 1957.

[29] F. Altman and W. Sichak, "A simplified diversity communication system for beyond-the-horizon links," *IRE. Trans. on Communications Systems*, vol. 4, no. 1, pp. 50–55, Mar. 1956.

[30] J. Hu and N. C. Beaulieu, "Accurate simple closed-form approximations to rayleigh sum distributions and densities," *IEEE Communications Lett.*, vol. 9, no. 2, pp. 109–111, Feb. 2005.

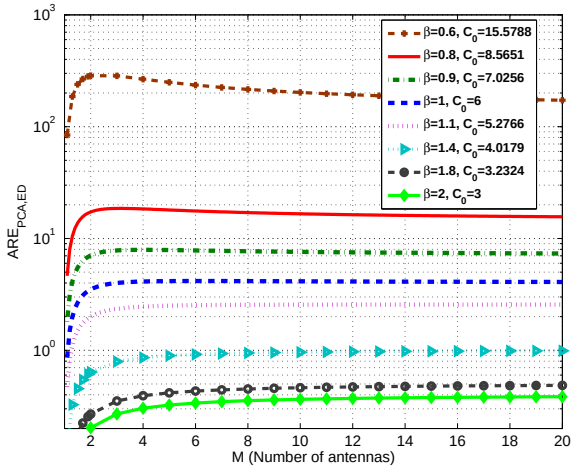

Fig. 1. ARE of PCA detector compared to energy detector given in (5) for the GG noise model

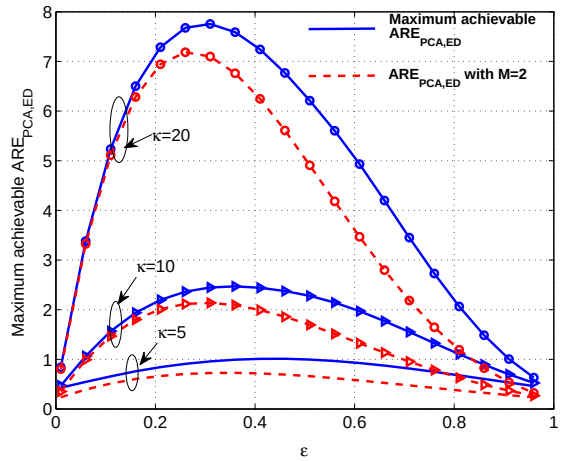

Fig. 3. Maximum achievable $ARE_{PCA,ED}$ with multiple antennas for the GM noise model

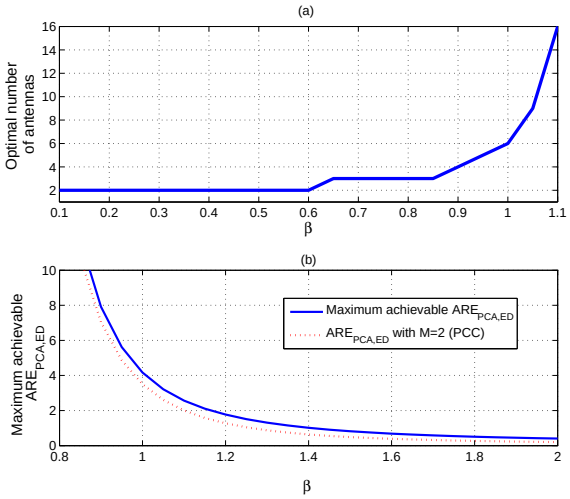

Fig. 2. (a). Optimal number of antennas which yields the maximum possible $ARE_{PCA,ED}$ for $C_0 > 5$ (b). Maximum achievable $ARE_{PCA,ED}$ with multiple antennas, with GG noise model

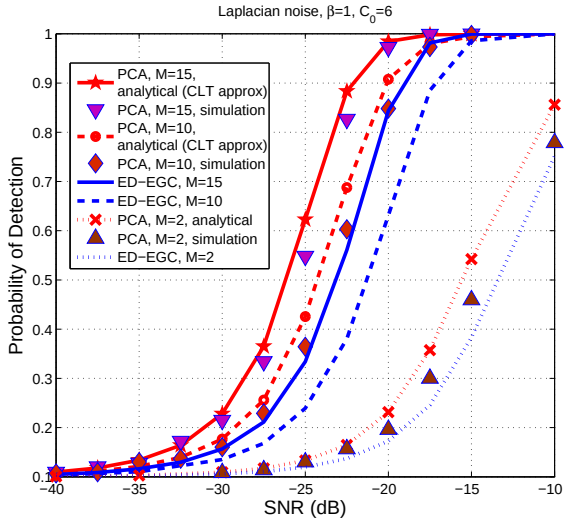

Fig. 4. Probability of detection Vs. SNR with Laplacian noise, $N = 1024$, $\alpha = 0.1$

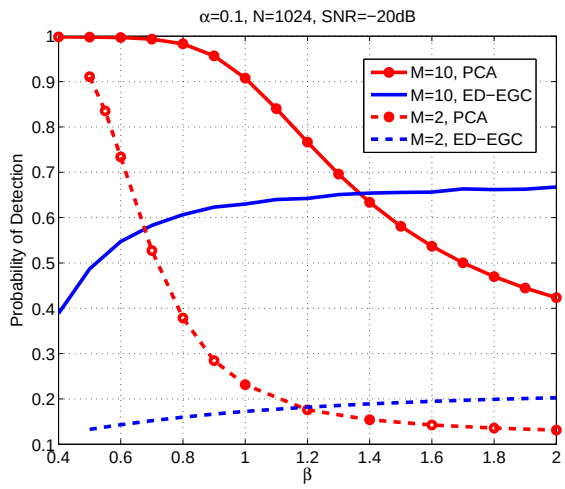

Fig. 5. Probability of detection Vs. β for the GG noise model, $N = 1024$, $\alpha = 0.1$, $SNR = -20dB$

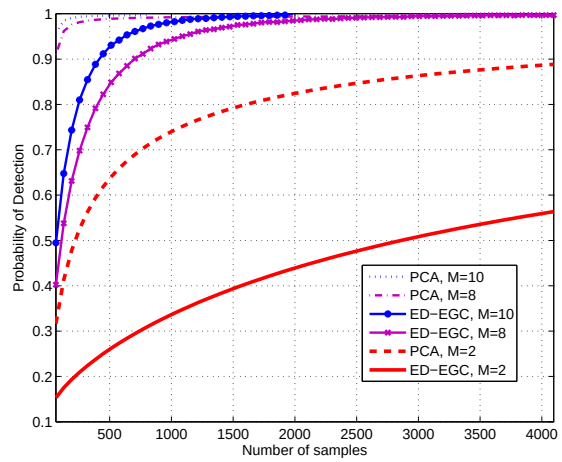

Fig. 7. Probability of detection Vs. number of samples for the GG noise model, $\beta = 0.8$, $SNR = -15dB$, $\alpha = 0.1$

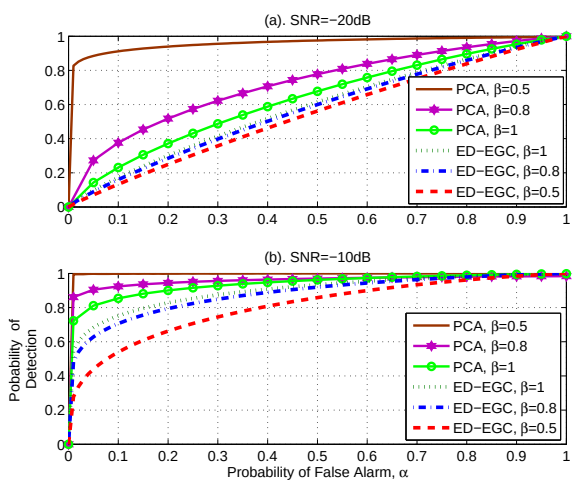

Fig. 6. ROC curves for the GG noise model, $N = 1024$, $M = 2$: (a) $SNR = -20dB$ (b). $SNR = -10dB$

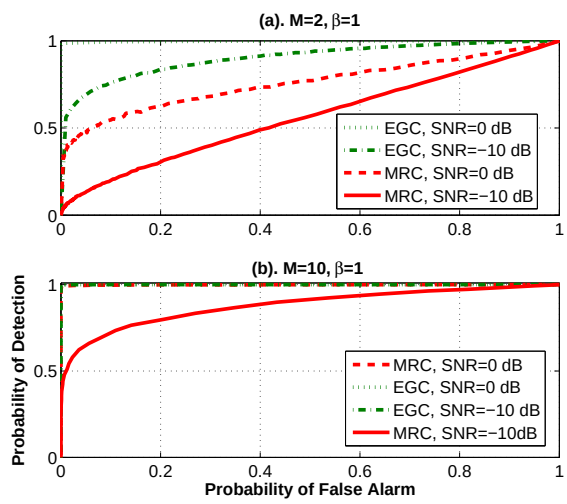

Fig. 8. ROC curves for energy detector with Equal Gain Combining (EGC) and Maximal Ratio Combining (MRC) for Laplacian noise; $\beta = 1$, $N = 1024$

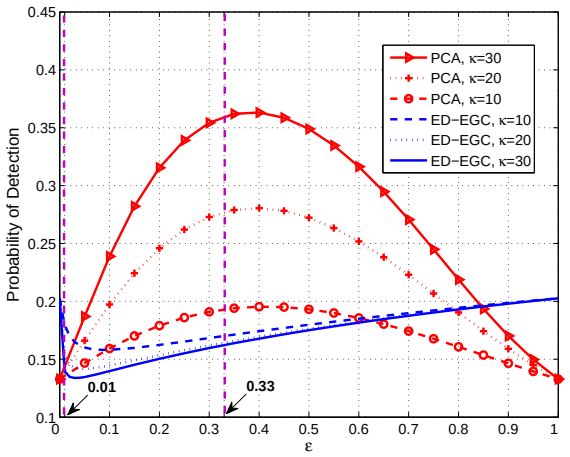

Fig. 9. Probability of Detection Vs. ϵ for the GM noise model, $M = 2$, $SNR = -20dB$, $\alpha = 0.1$

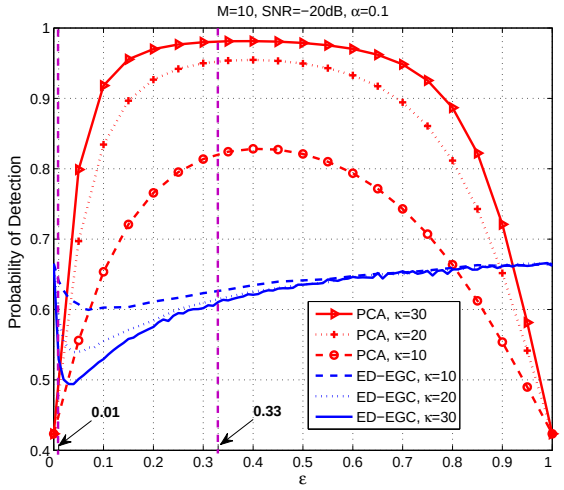

Fig. 10. Probability of Detection Vs. ϵ for the GM noise model, $M = 10$ $SNR = -20dB$, $\alpha = 0.1$