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Philipp Kornreich
Syracuse University

Bart Farell
Syracuse University

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Department of Electrical Engineering and Computer Science

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Single Eye or Camera with Depth Perception

Philipp Kornreich pkornrei@syr.edu, Syracuse University, EECS Dept.

Bart Farell bfarell@syr.edu, Syracuse University, Dept. of Biomedical and
Chemical Engineering

ABSTRACT: An imager that can measure the distance from each pixel to the point on the object that is in focus at the pixel is described here. This is accomplished by the use of short lightguide sections combined with each pixel light sensor. In the eye the rods and cones are the fiber like lightguide sections. The lens selects the object point who's range is to be determined at the particular pixel. The lens reproduces the light pattern of the object point at the image point with the addition of a phase proportional to the distance from object point to image point. This is the input to the photoconducting lightguide. The light guide has contacts along its length. The total oscillating photo current is an exponential function of the product of the range times the loss coefficient, times the ratio of the group velocity of the lightguide to the velocity of light, times the range.

KEYWORDS: Ranging, Three Dimensional Vision, Imaging, Passive LIDAR, LIDAR

Syracuse University - Department of EECS,
4-206 CST, Syracuse, NY 13244
(P) 315.443.2652 (F) 315.443.2583
<http://eecs.syr.edu>

SINGLE EYE OR CAMERA WITH DEPTH PERCEPTION

by

Philipp Kornreich

EECS Department

Syracuse University

Syracuse, NY 13244

Tel: 315 443 4447 Office 315 443 9712 Lab

FAX 315 443 2583

e-mail: pkornrei@syr.edu

and

Bart Farell

Institute of Sensory Research - Department of Biomedical and Chemical Engineering

621 Skytop Road

ISR Building

Syracuse University

Syracuse, NY 13244

Tel: 315 443 9717 Office 315 443 9714 Lab

FAX 315 443 1187

e-mail: bfarell@syr.edu

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I INTRODUCTION

The mechanism of an imager where each pixel can measure the distance from the pixel to the point on the object that is in focus at the pixel is discussed here. That is, each pixel is capable of measuring range to a point on the object.

At the 2006 and 2007 autonomous vehicle race¹ it was found that information of the three dimensional world obtained from pairs of binocular cameras was insufficient. This information was augmented by a rotating LIDAR² (**L**ight **D**etection **A**nd **R**anging) that provided range information. We think that each rod and cone sensor system in the animal eye has ranging capability. All the elements for ranging are present in each pixel sensor system of the animal eye. Thus we think that humans have depth perception capability with each eye. One can build an electronic camera with the same capability.

It is interesting to note that herbivore like horses have eyes on the side of the head. Thus, each eye can see a large area that the other eye can not see. It is

reasonable to assume that the horse can judge distances in those areas separately with each eye.

Fig. 1, A number of different light waves of different wavelength will all have the same phase relation at the image point as was the case at the object with the addition of a phase proportional to the distance L from object point to image point

A LIDAR provides range information by sending out a light pulse. The time between the pulse transmission and the time of arrival of the reflection from the target back at the LIDAR is a measure of the distance to the target. The LIDAR's pulse modulated signal consists of light of many different wavelength. At each wavelength there is a phase component that is proportional to the propagation time from target to receiver. Thus the range information is carried by the phases of the different wavelength components of the LIDAR signal..

Fig. 2. Graded index photoconducting detector lightguide with electric contacts along its length. The contacts integrate the light signal along the length of the lightguide. The insulator support structure of the leads is not shown. The index of refraction of the lightguide has a maximum at its center. The amplitude of the light at the pixel is detected separately at the top of the lightguide. The light amplitude information of the phase detection process is eliminated by dividing the signal from the lightguide contacts by the amplitude detector signal. The semiconductor waveguide can be epitaxially grown on a transparent substrate such as GaN.

There are various passive ranging systems using non coherent light. All measure the distance to a single target similar to a LIDAR. None perform ranging for individual pixels as is proposed here^{4.5.6}

In the passive imaging system discussed here broad band light instead of a light pulses is used. The range information is carried in the phases of the different wavelength components of the broad band detected light, see Fig. 1.

The **lens selects the object point** who's range is to be determined at a particular pixel. All the different wavelength light emitted by an object point and passing through a convex lens will have the same phase again at a point on the image surface as they had at the object with the addition of a phase proportional to the distance from the object point to the image point. At this point the object point will be in focus. At all other points on the image surface the light from this object point sums to zero. The input to the lightguides is located in the focal surface. Thus, the light input to the lightguides has the range phase information.

The **lightguide facilitates the range measurement**. The impute signal to each lightguide at a pixel has a phase factor that is equal to the product of the distance from the object point to the image point times the light frequency divided by the speed of light. The material of the lightguide is photoconducting. The number of charge carriers generated at any point in the lightguide structure is proportional to the light intensity at that location. The light intensity in the lightguide is equal to the products of the electric field and magnetic flux density of the light electromagnetic wave divided by the permeability of free space.. The light signal and thus the light electric field and the light magnetic flux density have a wide bandwidth. The product of the light electric field and light magnetic flux density oscillates at the sum and difference frequencies. The electric current generated by the charge carriers oscillates at the difference frequency. The contacts along the lightguide, as shown in Fig. 2, integrate the photo current over the length of the lightguide. This integral multiplied by the input phase factor that carries the range information forms an offset difference frequency sink function times the lightguide transfer function. This lightguide transfer function varies only gradually with difference frequency. The offset is equal to the product of the loss coefficient times the group velocity of the lifghtguide. The integral of the offset sink function over the difference frequency can be evaluated by the residue theorem. The result of the integration over the difference frequency is a decaying exponential with exponent equal to the range times the lightguide loss coefficient times the group velocity of the lightguide divided by the speed of light.

The lightguide has several parameters that can be adjusted for optimum operation. These parameters are the lightguide length, the light confining factor, and the losses of the lightguide. In this example a graded index light guide is used. Here the light confinement factor is the index of refraction grading parameter.

The photoconductor used to form the lightguide has to be capable of producing a very wide band photo current signal of about 8 GHz. The photo current circuit must also be capable of detecting a signal over a wide bandwidth, of about 8 GHz. The output signal is proportional to the amplitude of this high frequency signal. Thus there must be a second stage in the circuit that detects the amplitude of the high frequency signal.

The light amplitude information at the pixel is eliminated by dividing the lightguide detector signal by an amplitude detector signal. This leaves the phase information which carries the range information. The amplitude of the light at the pixel has to be detected separately as shown in Fig. 2. The light amplitude detector consists of a very short section of the photo conducting light guide located at its input end, see Fig. 2. Note from Fig. 2 that all the leads to the various contacts of the structure are near the input end of the light guide. This is similar to the construction of the animal eye where the light sensors are on the back surface of an array of blood vessels and nerve leads.

the larger the loss the wider difference frequency bandwidth is required for the sink function. But the larger the loss the more signal is transmitted by the lightguide function resulting from the integral over the lightguide contact length.

These systems have to **function with incoherent “white light”**. However, most white light has a coherence length of several wavelengths, otherwise lenses would not work. Each light packet, throughout which light is coherent, is sufficient for this task.

In a conventional camera the light amplitude is detected with semiconductor light detectors. The color information is conventionally obtained by using three different light sensors at each pixel. Each sensor is sensitive to one of the three colors carmine red, green or blue.

To simplify the calculation a lightguide structure where the light is confined to its center without the necessity of a cladding layer was selected. Such a structure is the Graded Index lightguide. This eliminates the necessity of calculating the boundary conditions between the guiding and cladding region.

A graded index lightguide can be fabricated using a compound semiconductor such as $\text{Al}_x\text{Ga}_{1-x}\text{N}$. The center of the lightguide would be GaN with an index of refraction of

2.429. The composition of the compound semiconductor would be gradually varied so that the outside layers of the lightguide would be AlN with an index of refraction of about 2.0. This structure could be grown on a AlN substrate.

II SUMMARY OF RESULTS

Since even the calculation of this simplified model are lengthy a summary of results is presented here. It spares the reader from having to go through the lengthy calculations.

The eye or camera lens generates a signal at the pixel lightguide input that contains a phase factor $\exp\left(j\frac{\omega L}{c}\right)$ that is a function of the distance L from the pixel to the point at the image that is in focus at the pixel. Here ω is the light frequency and c is the speed of light, see Fig. 2

The light electric field $\mathbf{E}$ in the lightguide is a function of the phase factor $\exp\left(j\frac{\omega L}{c}\right)$ times a function of traveling wave exponentials $\exp[-jp(\omega)z - \alpha z]$. Here $p(\omega)$ is a wave vector, α is the loss constant and z is a position along the lightguide length. The complex conjugate of the light magnetic flux density $\mathbf{B}^*$ in the lightguide is also a function of the phase factor $\exp\left(-j\frac{\Omega L}{c}\right)$ times terms like $\exp[jp(\Omega)z - \alpha z]$.

The local density of light induced charge carriers in the photoconducting lightguide is proportional to the light intensity or Poynting's vector $\mathbf{S} = \frac{1}{\mu_0} \mathbf{E}(\omega) \times \mathbf{B}^*(\Omega)$. The light

intensity $\mathbf{S}$ is a function of the sum $\Omega + \omega$ and the difference $\psi = \omega - \Omega$ frequencies.

Thus, the light intensity is a function of the phase factor $\exp\left(j\frac{\psi L}{c}\right)$ times terms of the form $\exp\{-j[p(\Omega) - p(\omega)]z - 2\alpha z\}$. The total detected photo current is proportional to the integral of the light intensity over the length of the lightguide contacts, see Fig. 2.

This integral is of the form of a sink function $\frac{\exp\left(j\frac{\psi L}{c}\right)}{p(\omega) - p(\Omega) - j2\alpha}$ times a function of terms of the form $\exp\{-j[p(\Omega) - p(\omega)]h - 2\alpha h\}$ where h is the length of the lightguide. Since the sink function diminishes rapidly for values of the difference frequency ψ that are small compared to the oscillating frequencies ω of the light, the

wave vector $p(\Omega = \omega - \psi)$ can be expanded to first order in ψ . One obtains for the

sink function $\frac{\exp\left(j\frac{\psi L}{c}\right)}{\frac{dp(\omega)}{d\omega}\psi - j2\alpha}$ and for the other terms $\exp\left\{-j\frac{dp(\omega)}{d\omega}\psi h - 2\alpha h\right\}$. Note

that the group velocity of an electromagnetic wave in the lightguide $v_G = \frac{d\omega}{dp(\omega)}$.

Thus the sink function and the other terms reduce respectively to $\frac{\exp\left(j\frac{\psi L}{c}\right)}{\psi - j2v_G\alpha}$ and

$\exp\left[j\frac{\psi h}{v_G} - 2\alpha h\right]$ The equation containing the sink function and the other terms is next

integrated over all values of the difference frequency. Since the sink function diminishes rapidly with frequency this integral can be extended from minus to plus infinity and the Residue theorem can be used to evaluate the integral. The result is a

decaying exponential function of the range L , $\exp\left[-\alpha\frac{v_G(\omega)}{c}L\right]$ The other terms

reduce to $\exp(-4\alpha h)$. This result must, again, be integrated over the frequency ω .

The net result is a photo current approximately proportional to a decaying exponential function of the range L as shown in Fig's 5, 6, 7, and 8,.

III DETECTOR CIRCUIT

The electrical signal generated by the light guide is proportional to the total number of charge carriers in the lightguide. The total number of charge carriers in the light guide is proportional to the total light power in the light guide. The light power density per unit volume, Poynting's vector, is equal to the cross product of the light electric field and the light magnetic field. Both the light electric field and the light magnetic field oscillate at the same range of frequencies. The light power density has a component oscillating at the difference of the light electric field and light magnetic field oscillating frequencies. The range information is proportional to this difference frequency as will be shown in section 4c. Thus the detector circuit has to be able to detect this difference frequency. In order to be able to detect a sufficient range the circuit has to be able to detect frequencies up to about 8 GHz.

The light detection circuit consists of two parts, a High Frequency circuit and a lower frequency signal circuit. This is shown in Fig. 2.

IV IMAGING

A conventional analysis of the focusing process of a lens is performed here⁴. The light electromagnetic field vectors are approximated by a scalar electric field. The Fresnel approximation is used for calculating the scalar light electric field in the imaging system⁴. The scalar light electric field $u_1(\mathbf{r}_1)$ at the input plane of the lens, shown in Fig. 3, due to light emitted by a point at $\mathbf{r}_o$ on the object is:

$$u_1(\mathbf{r}_1) \approx \frac{\omega}{jca} \int_{\text{Object Area}} u_o(\mathbf{r}_o) \exp \left\{ j\omega \left[t - \frac{a}{c} - \frac{(\mathbf{r}_1 - \mathbf{r}_o) \cdot (\mathbf{r}_1 - \mathbf{r}_o)}{2ca} \right] \right\} dA_o \quad (4.1)$$

Here the radial frequency ω of the light rather than its wavelength λ is used because an integration over the light frequency bandwidth will be performed at a later stage in the calculation. The scalar light electric field $u_2(\mathbf{r}_1)$ at the output plane of the lens, shown in Fig. 3, is:

$$u_2(\mathbf{r}_1) = u_1(\mathbf{r}_1) \exp \left[j \frac{\omega r_1^2}{2cf} \right] \quad (4.2)$$

where f is the focal length of the lens. Next, the scalar light electric field $u_3(\mathbf{r}_2)$ on the curved image surface, shown in Fig. 3 is calculated. This is the light input to the waveguide system at the image surface.

$$u_3(\mathbf{r}_2) \approx \frac{\omega}{jcb} \int_{\text{Lens Area}} u_2(\mathbf{r}_1) \exp \left\{ -j\omega \left[\frac{b}{c} + \frac{(\mathbf{r}_2 - \mathbf{r}_1) \cdot (\mathbf{r}_2 - \mathbf{r}_1) - r_2^2}{2cb} \right] \right\} dA_1 \quad (4.3)$$

Fig. 3, Schematic imaging system. In order to simplify the analysis it is assumed the lens has a square mask. The detector fiber lightguide sections are located on a curved surface similar to the retina of the animal eye.

The image surface has a radius of curvature equal to the distance b from the lens to the image surface, see Fig 3. The last term r_2^2 is due to the curvature of the image surface. By substituting equation 4.1 into equation 4.2 and the resulting equation into equation 4.3 one obtains for the image scalar light electric field $u_3(r_2)$.

$$u_3(r_2) = - \frac{\omega^2}{c^2 ab} \int_{\text{Lens Area}} dA_1 \int_{\text{Object Area}} dA_o u_o(r_o) \exp \left\{ j\omega \left[t - \frac{a+b}{c} - \frac{(r_1 - r_o) \cdot (r_1 - r_o)}{2ca} + \frac{r_1^2}{2cf} - \frac{(r_2 - r_1) \cdot (r_2 - r_1) - r_2^2}{2cb} \right] \right\} \quad (4.4)$$

Multiplying out and collecting terms:

$$u_3(\mathbf{r}_2) = -\frac{\omega^2}{c^2 ab} \exp\left[j\omega\left(t - \frac{a+b}{c}\right)\right] \times \int_{\text{Lens Area}} dA_1 \int_{\text{Object Area}} dA_o \exp\left\{-j\frac{\omega}{2c}\left[\frac{r_o^2}{a} + r_1^2\left(\frac{1}{a} - \frac{1}{f} + \frac{1}{b}\right) - 2\mathbf{r}_1 \cdot \left(\frac{\mathbf{r}_o}{a} + \frac{\mathbf{r}_2}{b}\right)\right]\right\} \quad (4.5)$$

For an imaging system the bracket $\left(\frac{1}{a} - \frac{1}{f} + \frac{1}{b}\right)$ is set to zero. It is the relation between the object distance a , the image distance b , and the focal length f of the lens for the object to be in focus.

$$\frac{1}{a} - \frac{1}{f} + \frac{1}{b} = 0 \quad (4.6)$$

Next, the integral over the lens aperture will be examined using, for simplicity, rectangular coordinates.

$$-\frac{\omega^2}{c^2 ab} \int_{\text{Lens area}} dA_1 \exp\left[-j\frac{\omega \mathbf{r}_1}{ca} \cdot \left(\mathbf{r}_o + \frac{a\mathbf{r}_2}{b}\right)\right] = -\frac{\omega^2}{c^2 ab} \int_{-\frac{p}{2}}^{\frac{p}{2}} dx_1 \int_{-\frac{p}{2}}^{\frac{p}{2}} dy_1 \exp\left[-j\frac{\omega x_1}{ca} \left(x_o + \frac{ax_2}{b}\right)\right] \exp\left[-j\frac{\omega y_1}{ca} \left(y_o + \frac{ay_2}{b}\right)\right] \quad (4.7)$$

Defining the following transformation of variables:

$$\begin{aligned} \text{a) } \xi &= \frac{\omega x_1}{ca} & \text{b) } x_1 &= \frac{ca}{\omega} \xi \\ \text{c) } \zeta &= \frac{\omega y_1}{ca} & \text{d) } y_1 &= \frac{ca}{\omega} \zeta \end{aligned} \quad (4.8)$$

where ξ and ζ have dimensions of reciprocal length or wave vector. The area increment dA_1 is:

$$\text{a) } dA_1 = dx_1 dy_1 \quad \text{b) } dA_1 = \frac{c^2 a^2}{\omega^2} d\xi d\zeta \quad (4.9)$$

The corresponding limits of integration are:

$$\text{a) At } x_1 \pm \frac{p}{2} \quad \xi = \pm \frac{\omega p}{2ca} \quad \text{b) At } y_1 \pm \frac{p}{2} \quad \zeta = \pm \frac{\omega p}{2ca} \quad (4.10)$$

By substituting equations 4.8, 4.9 and 4.10 into equation 4.7 one obtains:

$$\begin{aligned} & - \frac{\omega^2}{c^2 ab} \int_{\text{Lens area}} dA_1 \exp \left[-j \frac{\omega r_1}{ca} \bullet \left(\mathbf{r}_o + \frac{a \mathbf{r}_2}{b} \right) \right] = \\ & - \frac{a}{b} \int_{-\frac{\omega p}{2ca}}^{\frac{\omega p}{2ca}} d\xi \int_{-\frac{\omega p}{2ca}}^{\frac{\omega p}{2ca}} d\zeta \exp \left[-j \xi \left(x_o + \frac{ax_2}{b} \right) \right] \exp \left[-j \zeta \left(y_o + \frac{ay_2}{b} \right) \right] \end{aligned} \quad (4.11)$$

Typical values for the parameters used in equation 4.11 are:

$$b = 0.0225 \text{ m}$$

$$f = 0.022 \text{ m}$$

$$a = 0.5 \text{ m to } 1.5 \text{ m, here } a \approx 1.25 \text{ m}$$

$$p = 0.01 \text{ m}$$

where the wavelength λ is equal to 520 nm

The limits of integration $\frac{\omega p}{2cb}$ have a value of $2.685121927 \times 10^6 \text{ m}^{-1}$. The

parentheses $\left(\frac{bx_o}{a} + x_2 \right)$ typically have values of 0.00509 m, where x_o and x_2 where

assumed to have a value of $\frac{p}{2} = 0.005 \text{ m}$. For values of $\xi \left(\frac{bx_o}{a} + x_2 \right)$ greater than

20π the oscillation will sum to zero. At $\xi \left(\frac{bx_o}{a} + x_2 \right) = 20\pi$ radians $\xi = 12344.17546$

m^{-1} which is much less than the limit of the integration of $2.685121927 \times 10^6 \text{ m}^{-1}$. Thus the integrals can be approximated by delta functions:

$$- \frac{\omega^2}{c^2 ab} \int_{\text{Lens area}} dA_1 \exp \left[-j \frac{\omega r_1}{ca} \bullet \left(\mathbf{r}_o + \frac{a \mathbf{r}_2}{b} \right) \right] \approx - \frac{a}{b} \delta \left(x_o + \frac{ax_2}{b} \right) \delta \left(y_o + \frac{ay_2}{b} \right) \quad (4.12)$$

By substituting equations 4.6 and 4.12 into equation 4.5 one obtains for the scalar electric field at the image point:

$$u_3(\mathbf{r}_2) \approx - \frac{a}{b} \exp \left\{ j\omega \left[t - \frac{(a+b)}{c} \right] \right\} \int_{\text{Objec area}} dA_o u_o(\mathbf{r}_o) \exp \left[-j \frac{\omega r_o^2}{2ca} \right] \delta \left(\mathbf{r}_o + \frac{a \mathbf{r}_2}{b} \right) \quad (4.13)$$

By integrating one obtains:

$$u_3(\mathbf{r}_2) \approx - \frac{a}{b} u_o \left(- \frac{a \mathbf{r}_2}{b} \right) \exp \left[j\omega \left(t - \frac{b+a}{c} - \frac{a r_2^2}{2b^2 c} \right) \right] \quad (4.14)$$

The light electric field $u_3(\mathbf{r}_3)$ is the input to the sensor light lightguides. Indeed, the lens inverts the image as is indicated by the negative sign of the variable $\frac{a \mathbf{r}_2}{b}$ in the function u_o describing the image. The image is demagnified by the ratio $\frac{b}{a}$ and intensified by the ration $\frac{a}{b}$. The phase depends on the distance $b + a + \frac{a r_2^2}{2b^2}$ from the object point to the image point. The lens recreates the phases at an image point that the light had at the object point except for the additional range dependent phase $\omega \left(t - \frac{b+a}{c} - \frac{a r_2^2}{2b^2 c} \right)$.

Equation 4.14 can be expressed in a more compact form as:

$$u_3(\mathbf{r}_2) \approx U_o \exp \left[j\omega \left(t - \frac{L}{c} \right) \right] \quad (4.15)$$

where:

$$\text{a) } L = b + a + \frac{ar_2^2}{2b^2} \quad \text{and} \quad \text{b) } U_o = -\frac{a}{b}u_o \left(-\frac{ar_2}{b} \right) \quad (4.16)$$

and where it was dispensed with the harmonic time dependence. This is the electric field input to the lightguide,

V GRADED INDEX DETECTOR LIGHTGUIDE

5a) The Electromagnetic Field in the lightguide

To simplify the calculation a lightguide structure where the light is confined to its center without the necessity of a cladding layer was selected. Such a structure is the Graded Index lightguide. This eliminates the necessity of calculating the boundary conditions between the guiding and cladding region.

Defining the variable $E_o(\omega)$ from equation 4.16 in order to simplify the calculation process;

$$E_o = U_o \exp\left(j\frac{\omega L}{c}\right) \quad (5.1)$$

The electric field vector $\mathbf{E}$ in the lightguide has the following form:

$$\mathbf{E} = E_o(\omega) \left[\hat{\mathbf{a}}_x y - \hat{\mathbf{a}}_y x \right] \exp\left(-\frac{x^2 + y^2}{2\eta^2}\right) \left[\exp(-j\eta z - \alpha z) + R \exp(j\eta z + \alpha z) \right] \quad (5.2)$$

Here p is the longitudinal wave vector component or propagation constant, α is a loss parameter and $\frac{1}{\eta^2}$ is a transverse wave vector component. The electric field vector satisfies the Gauss - Maxwell equation. The corresponding magnetic flux density pseudo vector $\mathbf{B}$ can be obtained from electric field vector by the use of the Faraday - Maxwell equation.

,

$$\mathbf{B} = E_o(\omega) \exp\left(-\frac{x^2 + y^2}{2\eta^2}\right) \left[\hat{\mathbf{a}}_x x + \hat{\mathbf{a}}_y y \right] \frac{p - j\alpha}{\omega} \times \\ \left\{ \left[\exp(-jpz - \alpha z) - R \exp(jpz + \alpha z) \right] - \right. \\ \left. j \hat{\mathbf{a}}_z \frac{1}{\omega} \left(2 - \frac{x^2 + y^2}{\eta^2} \right) \left[\exp(-jpz - \alpha z) + R \exp(jpz + \alpha z) \right] \right\} \quad (5.3)$$

The dispersion relation can be obtained by the use of the Ampere - Maxwell equation:

$$\text{a) } \nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} + \mathbf{J} \quad \text{b) } \nabla \times \mathbf{B} = j \frac{n^2 \omega}{c^2} \left[1 - \beta^2 (x^2 + y^2) \right] \mathbf{E} + \frac{n}{c} \gamma \mathbf{E} \quad (5.4)$$

where n is an index of refraction constant, β is the index of refraction grading factor of the detector lightguide, γ is a parameter associated with losses though in the lightguide. By substituting equations 5.2 and 5.3 into equation 5.4b and multiplying by $-j$ one obtains:

$$\frac{n^2 \omega^2}{c^2} - \frac{n^2 \omega^2 \beta^2}{c^2} (x^2 + y^2) - p^2 + 2j\alpha p + \alpha^2 = \frac{4}{\eta^2} - \frac{x^2 + y^2}{\eta^4} + j \frac{n\omega}{c} \gamma \quad (5.5)$$

Collecting terms of equal powers of x and y as well as real and imaginary terms:

$$\text{a) } \frac{n^2 \omega^2}{c^2} - \frac{4}{\eta^2} - p^2 + \alpha^2 = 0 \quad \text{b) } \frac{n^2 \omega^2 \beta^2}{c^2} = \frac{1}{\eta^4} \quad \text{c) } 2\alpha p = \frac{n\omega}{c} \gamma \quad (5.6)$$

By solving equation 4.6b for $\frac{1}{\eta^2}$ and equation 5.6c for α and substituting the result into equation 5.6a a quadratic equation for p^2 is obtained.

$$p^4 - \left(\frac{n^2 \omega^2}{c^2} - \frac{4n\omega\beta}{c} \right) p^2 - \frac{n^2 \omega^2 \gamma^2}{4c^2} = 0 \quad (5.7)$$

By solving for p^2 one obtains:

$$p^2 = \frac{n^2 \omega^2}{2c^2} \left[1 - \frac{4c\beta}{n\omega} + \sqrt{\left(1 - \frac{4c\beta}{n\omega}\right)^2 + \frac{c^2}{n^2 \omega^2} \gamma^2} \right] \quad (5.8)$$

Here $\beta = 9.5 \times 10^6$ per m is the index of refraction grading factor, an index of refraction n of 3.54, and the center frequencies ω spectrum portion is $3.435040988 \times 10^{15}$ radians per second for a wavelength of 548.3636364 nm. This gives for $\frac{4c\beta}{n\omega}$ of 0.9368479567. Assuming γ has a value of .40 per m one obtains $\frac{c^2 \gamma^2}{n^2 \omega^2} = 9.616068434 \times 10^{-13}$. Thus, this quantity can be neglected. With this assumption the wave vector p of equation 5.8 and the group velocity v_G are:

$$\begin{aligned} \text{a) } p &\approx \pm \sqrt{\frac{n^2 \omega^2}{c^2} - \frac{4n\beta\omega}{c}} & \text{b) } v_G &\approx \frac{c}{n} \frac{\sqrt{1 - \frac{4c\beta}{n\omega}}}{1 - \frac{2c\beta}{n\omega}} \end{aligned} \quad (5.9)$$

5b) Boundary conditions:

It is assumed that the lightguide is terminated in another structure. The electric field at the end of the waveguide, at $z = h$ is:

$$\exp[-jp(\omega)h - \alpha h] + R(\omega)\exp[jp(\omega)h + \alpha h] = T \quad (5.10)$$

The x and y components of the magnetic flux density pseudo vector must be continuous at $z = h$.

$$\frac{p(\omega) - j\alpha}{\omega} \left\{ \exp[-jp(\omega)h - \alpha h] - R(\omega)\exp[jp(\omega)h + \alpha h] \right\} = \frac{k - j\zeta}{\omega} T(\omega) \quad (5.10)$$

where it is assumed that the magnetic flux density of the terminating structure has a form similar to the magnetic flux density of equation 5.3. Solving equations 5,9 and 5.10 for $R(\omega)$

$$R(\omega) = \frac{\frac{p(\omega) - j\alpha}{k - j\zeta} - 1}{\frac{p(\omega) - j\alpha}{k - j\zeta} + 1} \exp[-2jp(\omega)h - 2\alpha h] \quad (5.11)$$

5b) Light Power in the Detector

The density of light induced charge carriers in the detector is proportional to the local light power in the detector. The total electric $\mathbf{E}_T$ field in the photoconductor waveguide integrated over the bandwidth B_w is

$$\mathbf{E}_T = \frac{U_o}{B_w} \int_{\omega_o - \frac{B_w}{2}}^{\omega_o + \frac{B_w}{2}} d\omega \exp\left(j\frac{\omega L}{c}\right) [\hat{\mathbf{a}}_x y - \hat{\mathbf{a}}_y x] \exp\left(-\frac{x^2 + y^2}{2\eta^2(\omega)}\right) \times \{\exp[-jp(\omega)z - \alpha z] + R(\omega)\exp[jp(\omega)z + \alpha z]\} \quad (5.12)$$

where E_o has been replaced by equation 5.1. The total magnetic flux density in the photoconductor waveguide integrated over the bandwidth B_w is:

$$\mathbf{B}_T = \frac{U_o}{B_w} \int_{\omega_o - \frac{B_w}{2}}^{\omega_o + \frac{B_w}{2}} d\Omega \exp\left(j\frac{\Omega L}{c}\right) \exp\left(-\frac{x^2 + y^2}{2\eta^2}\right) \left\{ (\hat{\mathbf{a}}_x x + \hat{\mathbf{a}}_y y) \left(\frac{n}{c} - j\frac{\alpha}{\Omega} \right) \times \{\exp[-jp(\Omega)z - \alpha z] - R(\Omega)\exp[jp(\Omega)z + \alpha z]\} - j\hat{\mathbf{a}}_z \frac{1}{\Omega} \left[2 - \frac{x^2 + y^2}{\eta^2(\Omega)} \right] \{\exp[-jp(\Omega)z - \alpha z] + R(\Omega)\exp[jp(\Omega)z + \alpha z]\} \right\} \quad (5.13)$$

The charge carriers that provide the signal current depend on the power density or Poynting's vector $\mathbf{S}$ in the lightguide.

$$\mathbf{S} = \frac{1}{\mu_o} \mathbf{E}_T \times \mathbf{B}_T^* \quad (5.14)$$

By substituting equations 5.12 and 5.13 into equation into equation 5.14 one obtains for the light power density in the waveguide:

$$\begin{aligned}
\mathbf{S} = & \frac{|U_o|^2}{\mu_o B_w^2} \int_{\omega_o - \frac{B_w}{2}}^{\omega_o + \frac{B_w}{2}} d\omega \int_{\omega_o - \frac{B_w}{2}}^{\omega_o + \frac{B_w}{2}} d\Omega \exp\left[j \frac{(\omega - \Omega)L}{c}\right] \exp\left(-\frac{x^2 + y^2}{2\theta^2}\right) \times \\
& \left\langle j \left[\hat{\mathbf{a}}_x x + \hat{\mathbf{a}}_y y \right] \frac{1}{\Omega} \left[\frac{x^2 + y^2}{\eta^2(\Omega)} - 2 \right] \left\{ \exp[-jp(\omega)z - \alpha z] + R(\omega) \exp[jp(\omega)z + \alpha z] \right\} \times \right. \\
& \left\{ \exp[jp(\Omega)z - \alpha z] + R^*(\Omega) \exp[-jp(\Omega)z + \alpha z] \right\} + \\
& \hat{\mathbf{a}}_z (x^2 + y^2) \left(\frac{n}{c} - j \frac{\alpha}{\Omega} \right) \left\{ \exp[-jp(\omega)z - \alpha z] + R(\omega) \exp[jp(\omega)z + \alpha z] \right\} \times \\
& \left. \left\{ \exp[jp(\Omega)z - \alpha z] - R^*(\Omega) \exp[-jp(\Omega)z + \alpha z] \right\} \right\rangle
\end{aligned} \tag{5.15}$$

where:

$$\frac{1}{\theta^2} = \frac{1}{\eta^2(\omega)} + \frac{1}{\eta^2(\Omega)} \tag{5.16}$$

The total number of light generated charge carriers is proportional to the total light power W . The light power $W\left(\frac{L}{c}\right)$ in the detector is the integral over the volume of the power density.:

$$W\left(\frac{L}{c}\right) = \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy \int_0^h dz \hat{\mathbf{a}}_s \cdot \mathbf{S} \tag{5.17}$$

where $\hat{\mathbf{a}}_s$ is a unit vector perpendicular to an area.. Next, substituting equation 4.15 into equation 5.17 and integrating over x and y . For simplicity the integration over x and y is assumed to be from $-\infty$ to ∞ . This simplifying assumption assumes that the index gradient concentrates the light electromagnetic field in the middle of the lightguide..

$$\begin{aligned}
dW\left(\frac{L}{c}\right) &= \frac{2\sqrt{2\pi}|U_o|^2}{\mu_o B_w^2} \int_{\omega_o - \frac{B_w}{2}}^{\omega_o + \frac{B_w}{2}} d\omega \int_{\omega_o - \frac{B_w}{2}}^{\omega_o + \frac{B_w}{2}} d\Omega \exp\left[j\frac{(\omega - \Omega)L}{c}\right] \theta^3 \times \\
&\left(\frac{n}{c} - j\frac{\alpha}{\Omega}\right) \left\{ \exp[-jp(\omega)z - \alpha z] + R(\omega) \exp[jp(\omega)z + \alpha z] \right\} \times \\
&\left\{ \exp[jp(\Omega)z - \alpha z] - R^*(\Omega) \exp[-jp(\Omega)z + \alpha z] \right\}
\end{aligned} \tag{5.18}$$

Multiplying out:

$$\begin{aligned}
dW\left(\frac{L}{c}\right) &= \frac{2\sqrt{2\pi}|U_o|^2}{\mu_o B_w^2} \int_{\omega_o - \frac{B_w}{2}}^{\omega_o + \frac{B_w}{2}} d\omega \int_{\omega_o - \frac{B_w}{2}}^{\omega_o + \frac{B_w}{2}} d\Omega \exp\left[j\frac{(\omega - \Omega)L}{c}\right] \theta^3 \times \\
&\left(\frac{n}{c} - j\frac{\alpha}{\Omega}\right) \left\{ \exp[-jp(\omega)z + jp(\Omega)z - 2\alpha z] + R(\omega) \exp[jp(\omega)z + jp(\Omega)z] - \right. \\
&R^*(\Omega) \exp[-jp(\omega)z - jp(\Omega)z] + R(\omega)R^*(\Omega) \exp[jp(\omega)z - jp(\Omega)z + 2\alpha z] \left. \right\}
\end{aligned} \tag{5.19}$$

Integrating over z over the length of the lightguide

$$\begin{aligned}
dW\left(\frac{L}{c}\right) &= -j \frac{2\sqrt{2\pi}|U_o|^2}{\mu_o B_w^2} \int_{\omega_o - \frac{B_w}{2}}^{\omega_o + \frac{B_w}{2}} d\omega \int_{\omega_o - \frac{B_w}{2}}^{\omega_o + \frac{B_w}{2}} d\Omega \exp\left[j\frac{(\omega - \Omega)L}{c}\right] \theta^3 \times \\
&\left(\frac{n}{c} - j\frac{\alpha}{\Omega}\right) \left\langle \frac{1 - \exp\{-j[p(\omega) - p(\Omega)]h - 2\alpha h\}}{p(\omega) - p(\Omega) - j2\alpha} + \frac{\exp\{j[p(\omega) + p(\Omega)]h\} - 1}{p(\omega) + p(\Omega)} - \right. \\
&R^*(\Omega) \frac{1 - \exp\{-j[p(\omega) + p(\Omega)]h\}}{p(\omega) + p(\Omega)} - R(\omega)R^*(\Omega) \frac{\exp\{j[p(\omega) - p(\Omega)]h + 2\alpha h\} - 1}{p(\omega) - p(\Omega) - j2\alpha} \left. \right\rangle
\end{aligned} \tag{5.20}$$

Since the reciprocal of the difference in wave vectors is much larger than the reciprocal of the sum of the wave vectors one can neglect terms with the sum of the frequencies in the denominator.

$$\begin{aligned}
W\left(\frac{L}{c}\right) = & -j \frac{2\sqrt{2\pi} |U_o|^2}{\mu_o B_w^2} \int_{\omega_o - \frac{B_w}{2}}^{\omega_o + \frac{B_w}{2}} d\omega \int_{\omega_o - \frac{B_w}{2}}^{\omega_o + \frac{B_w}{2}} d\Omega \theta^3 \frac{\exp\left[j \frac{(\omega - \Omega)L}{c}\right]}{p(\omega) - p(\Omega) - j2\alpha} \times \\
& \left(\frac{n}{c} - j\frac{\alpha}{\Omega}\right) \left\{1 - \exp\left\{-j[p(\omega) - p(\Omega)]h - 2\alpha h\right\} - \right. \\
& \left. R(\omega)R^*(\Omega) \exp\left\{j[p(\omega) - p(\Omega)]h + 2\alpha h\right\} + R(\omega)R^*(\Omega)\right\}
\end{aligned} \tag{5.21}$$

Making a transformation of variables:

$$\begin{aligned}
a) \quad \psi &= \omega - \Omega & b) \quad \Omega &= \omega - \psi & c) \quad -B_w < \psi \leq B_w
\end{aligned} \tag{5.22}$$

The sink function in equation 5.21 goes to zero for relatively small values of the difference frequency ψ . By substituting the transformation of variables into equation 5.21 and expanding $p(\Omega=\omega-\psi)$ to first order in the difference frequency ψ one obtains:

$$\begin{aligned}
W\left(\frac{L}{c}\right) = & -j \frac{2\sqrt{2\pi} |U_o|^2}{\mu_o B_w^2} \int_{\omega_o - \frac{B_w}{2}}^{\omega_o + \frac{B_w}{2}} d\omega \frac{d\omega}{dp(\omega)} \int_{-B_w}^{B_w} d\psi \theta^3 \frac{\exp\left(j \frac{\psi L}{c}\right)}{\psi - j2 \frac{d\omega}{dp(\omega)} \alpha} \times \\
& \left(\frac{n}{c} - j\frac{\alpha}{\Omega}\right) \left\{1 - \exp\left[j \frac{dp(\omega)}{d\omega} \psi h - 2\alpha h\right] - \right. \\
& \left. R(\omega)R^*(\Omega) \left\{\exp\left[-j \frac{dp(\omega)}{d\omega} \psi h + 2\alpha h\right] + 1\right\}\right\}
\end{aligned} \tag{5.23}$$

From equation 5.11 one obtains the following expression by using the transformation of variables of equation 5.22 and expanding $p(\Omega)$ to first order in the difference frequency ψ :

$$R(\omega)R^*(\Omega) = G(\omega, \Omega) \exp\left[2j \frac{dp(\Omega)}{d\Omega} \psi h - 4\alpha h\right] \tag{5.24}$$

where:

$$G(\omega, \Omega) = \frac{[p(\omega) - k(\omega) - j\alpha + j\zeta][p(\Omega) - k(\Omega) + j\alpha - j\zeta]}{[p(\omega) + k(\omega) - j\alpha - j\zeta][p(\Omega) + k(\Omega) + j\alpha + j\zeta]} \quad (5.25)$$

By substituting equation 5.24 into equation 5.23 one obtains:

$$W\left(\frac{L}{c}\right) = -j \frac{2\sqrt{2\pi} |U_o|^2}{\mu_o B_w^2} \int_{\omega_o - \frac{B_w}{2}}^{\omega_o + \frac{B_w}{2}} d\omega \frac{nv_G}{c} \int_{-B_w}^{B_w} d\psi \theta^3 \frac{\exp\left(j \frac{\psi L}{c}\right)}{\psi - j2\alpha v_G} \times$$

$$\left(1 - j \frac{\alpha c}{\Omega n}\right) \left\{1 - [1 + G(\omega, \Omega)] \exp\left(j \frac{\psi h}{v_G} - 2\alpha h\right) - \right. \quad (5.26)$$

$$\left. G(\omega, \Omega) \exp\left(2j \frac{\psi h}{v_G} - 4\alpha h\right)\right\}$$

where v_G is the group velocity calculated in equation 5.9b. The integral over the difference frequency for large values of the bandwidth B_w can be approximated by an integral minus to plus infinity in frequency. The resulting integral can be extended around the contour in the complex difference frequency plane of Fig. 4.

$$\oint d\psi \left\{1 - [1 + G(\omega, \Omega)] \exp\left(j \frac{\psi h}{v_G} - 2\alpha h\right) - G(\omega, \Omega) \exp\left(2j \frac{\psi h}{v_G} - 4\alpha h\right)\right\} \times$$

$$\frac{\exp\left(j \frac{\psi L}{c}\right)}{\psi - j2\alpha v_G} = \{1 - [1 + G] \exp(-4\alpha h) - G \exp(-8\alpha h)\} \exp\left(-2\alpha \frac{v_G}{c} L\right) \quad (5.27)$$

By substituting equation 5.27 into equation 5.26 for the integral over the difference frequency one obtains for the range function $W\left(\frac{L}{c}\right)$.

$$W\left(\frac{L}{c}\right) = -j \frac{2\sqrt{2\pi} |U_o|^2}{\mu_o B_w^2} \{1 - [1 + G] \exp(-4\alpha h) - G \exp(-8\alpha h)\} \times \quad (5.28)$$

$$\int_{\omega_o - \frac{B_w}{2}}^{\omega_o + \frac{B_w}{2}} d\omega \left(\frac{c}{2n\beta\omega}\right)^{\frac{3}{2}} \frac{nv_G(\omega)}{c} \exp\left[-2\alpha \frac{v_G(\omega)}{c} L\right]$$

where equation 5.6b was used for θ^3 and the group velocity v_G is given by equation 5.9b.

The light signal $W\left(\frac{L}{c}\right)$ oscillates at the difference frequency which can be as large as 5.8888×10^{14} Hz. However, it is sufficient for the detector circuit to have a bandwidth of about 10 GHz to produce a adequate accurate signal.

Fig. 4. Contour for integrating over the difference frequency integral of equation 5.27. The function in the integral goes to zero in the limit of very large imaginary part σ of the complex frequency ψ . Thus, the integral around the contour is equal to the integral along the real axis.

Since the group velocity $v_G(\omega)$ is a function of the radial frequency ω the integral of equation has to be integrated numerically.

The normalized output signal from equation 5.28 as a function of the range L is shown in graphs of Fig's 5, 6, 7, and 8. All range plots are to the same scale. The following lightguide parameters were used in this model:

Light guide length	$h = 2.5 \mu\text{m}$
Index of refraction of lightguide	$n = 3.54$
Graded index confinement factor	$\beta = 9.5 \times 10^6 \text{ per m}$
Absorption constant	$\alpha = 20 \text{ per m}$
Light wavelength range	520 nm to 580 nm

Fig. 5. Output signal as a function of range from 200 mm to 201 mm calculated from equation 5.28.

Fig. 6. Output signal as a function of range from 20 cm to 30 cm calculated from equation 5.28.

Fig. 7. Output signal as a function of range from 0.2 m to 1.2 m calculated from equation 5.28. Note that the output signal is indeed approximately a decaying exponential as predicted by equation 5.28.

Fig. 8. The logarithm of the output signal as a function of range from 1 m to 10 m calculated from equation 5.28.

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