

1999

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## Recommended Citation

Wu, Lin and Varshney, Pramod K., "Performance Analysis of CSMA and BTMA Protocols in Multihop Networks: Part II - Multiple Channel Case" (1999). *Electrical Engineering and Computer Science*. 25.

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# **Performance Analysis of CSMA and BTMA Protocols in Multihop Networks: Part II -- Multiple Channel Case**

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## **Abstract**

Busy tone multiple access protocols have been used in multihop networks to reduce the effect of the hidden terminal problem. This paper demonstrates another approach to reduce the effect of the hidden terminal problem namely the use of multiple channel schemes. A protocol that uses both the busy tone and the multiple channel techniques achieves the best performance. Using a Markov chain model and an approximation, the throughput performance of the multiple channel non-persistent CSMA protocol and the multiple channel conservative BTMA protocol in a large network is evaluated and compared. The results show that the multichannel CSMA and BTMA schemes exhibit a better performance over their single channel counterparts in a multihop network.

## 1. Introduction

In a companion paper[1], we have analyzed the performance of the nonpersistent CSMA, conservative BTMA and idealistic destination based BTMA protocols in large multihop networks. It was assumed that around each terminal, a single channel was available for data transmission (a separate channel for the transmission of the busy tone was also assumed to be available). For single hop networks, it has been shown that multiple channel schemes outperform their single channel counterparts. Multichannel schemes for multihop networks have not been considered. In this paper, we analyze the performance of multichannel CSMA and BTMA schemes in large multihop networks and determine the achievable performance enhancement when multichannel scheme are used.

Multichannel multiaccess protocols have been considered in the literature[2-8]. The results have shown that the throughput of networks increases when the bandwidth is split into several narrow band subchannels. An important advantage of the multichannel mode is that the network can increase or decrease its capability by adding or deleting channels, and channels can be allocated to a network on a demand assignment basis. The RF spectrum can be better utilized than in the single channel case[9]. Another advantage of the multichannel mode is that each subchannel can be utilized better than in the single composite channel[2], especially when they use CSMA protocols. This is due to the well known property of CSMA protocols that their efficiency increases as the ratio of propagation delay to packet transmission time decreases. In the multichannel mode, each user transmits on a slower speed channel, thereby increasing the packet transmission time. The ratio of propagation delay, which is only a function of the distance, to packet transmission time is, therefore, smaller on each of the subchannels than it is on the wide bandwidth channel. So far, most of the multiple channel CSMA (M-CSMA) protocols have been presented for single hop networks. The reason that relatively few papers have discussed the M-CSMA protocols in multihop networks is that the multichannel architecture requires each user in a multihop network to have multiple transceivers to implement simultaneous transmission capability on multiple channels, which is too expensive for most of the applications in multihop networks. Shacham and others in [10-12] have discussed a multichannel protocol for a multihop network with an architecture

called the receiver-directed scheme. The receiver-directed scheme allows the network to operate on multiple channels without increasing the hardware required for a single channel network. But the broadcast capability of the single channel network, which allows a terminal to reach all its neighbors with a single transmission, no longer exists in this receiver-directed scheme. In this paper, we assume that the network under consideration have the required capability to be able to employ any multichannel protocol. With this assumption, we can investigate the performance of multichannel schemes in multihop networks and, if appropriate, these schemes can be implemented.

Since the hidden terminal effect is the key problem for the application of CSMA protocols in multihop networks, we consider the use of the multichannel scheme as another approach to solve the hidden terminal problem in addition to the busy tone mode. The analysis demonstrates that the multiple channel CSMA protocol has a better performance than its single channel counterpart and single channel BTMA protocols in multihop networks. Finally, we analyze the multichannel conservative busy-tone (MC-BTMA) protocol. The MC-BTMA protocol has a better space reuse capability than the single channel C-BTMA protocol, and maintains the desirable property of the single channel C-BTMA protocol, namely, a short vulnerable period caused by the hidden terminals. Numerical results show that both multichannel and busy-tone techniques can reduce the hidden terminal effect and the protocol with the combination of both techniques achieves the best performance.

In Section 2, we briefly discuss some concepts from the theory of Markov processes that are directly related to our development. In Section 3, we present the multiple channel network model and notation. In Section 4, the throughput of the multichannel non-persistent CSMA protocol is evaluated. The multichannel conservative BTMA protocol is presented and analyzed in Section 5. Finally, numerical results are presented and discussed in Section 6.

## 2. Preliminaries

Before we analyze the performance of the multichannel CSMA and BTMA protocols, we present some basic results on Markov processes that will be found useful in the analysis.

### Definition 1:

If a stochastic process  $\{X(t), t \in (0, \infty)\}$  with state space  $E = \{0, 1, 2, \dots\}$ , has the property that there exist time points at which the process restarts itself, this process is called a regenerative process. In other words, for a regenerative process, there exists a time  $T$  with probability one, such that the continuation of the process beyond  $T$  is a probabilistic replica of the whole process starting at zero.  $\square$

Define the limiting probability  $P_j$  as:

$$P_j = \lim_{t \rightarrow \infty} P\{X(t) = j\}, \quad j \in E.$$

Referring to the time between two regeneration points as a cycle, the limiting probability  $P_j$  can be computed as given in the following theorem [13].

### Theorem 1

If  $T$  has an absolutely continuous component (that is, it has a density on some interval), and  $E[T] < \infty$ , then,

$$P_j = E[\text{Amount of time in state } j \text{ during one cycle}] / E[\text{time of one cycle}]$$

for all  $j \geq 0$ .  $\square$

### Definition 2:

If a stochastic process which makes transitions from state to state in accordance with a Markov chain, and if the process is also a regenerative process, the stochastic process is called a Markov regenerative process.

For Markov regenerative processes, the limiting probability  $P_j$  can be computed as follows[16].

### Theorem 2

Let  $P(j)$  be the steady state probability for state  $j$  ( $P(j)$  equals the (long run) proportion of transitions which are into state  $j$ ),  $D_j$  be the mean time spent in state  $j$  per transition, and  $P_j$  be the limiting probability ( $P_j$  equals the (long run) probability that the process is in state  $j$ ). If the markov chain is positive recurrent and irreducible, then:

$$P_j = \frac{P(j) D_j}{\sum_i P(i) D_i} . \quad (1)$$

According to the definition[1], the throughput of the channel, TH, is defined to be the aggregate average amount of data that is transported through the channel in unit time. Hence, it equals the fraction of time in which the channel is engaged in the successful transmission of packets. Therefore, if we denote the successful transmission state as  $S$ , then, the throughput TH is equal to the limiting probability that channel is in state  $S$ . That is,  $TH = P_S$ .

In the rest of this paper, we will use this theorem to evaluate the throughputs of the multiple channel CSMA and BTMA protocols.

### 3 Multiple Channel Multihop Networks and M-CSMA Protocols

The multihop network model and notation used in this paper is the same as the one used in [1] except that the broadcast channel in this network is partitioned into  $M$  identical subchannels each with bandwidth  $W_s$ , respectively. All the terminals in this network are able to transmit on one

channel and receive packets on other channels simultaneously. Let  $W$  be the total available bandwidth, then,  $W$  can be expressed as

$$W = \sum_{i=1}^M W_s = MW_s \quad (2)$$

Let  $T$  be the packet transmission time over the single broadcast channel with bandwidth  $W$ , and  $T_s$  be the packet transmission time over the subchannel with bandwidth  $W_s$ , then, we have

$$T_s = MT \quad (3)$$

For the sake of simplicity, we assume that  $T = 1$ . Then,  $T_s = M$ . As for the slotted non-persistent CSMA protocols used in [1], the duration of one slot in the slotted M-CSMA protocol, which is denoted as  $a$ , is set equal to the one way propagation delay. Let  $\tau_s$  be the number of slots in a packet transmission time or in a transmission period (TP), then we have  $\tau_s = \frac{M}{a}$ . Recall that  $\tau = \frac{1}{a}$ , thus

$$\tau_s = M\tau. \quad (4)$$

Next we introduce the multiple channel non-persistent CSMA protocol. In a single channel network, with the nonpersistent CSMA protocol, when a terminal becomes ready, it senses the channel. If the channel is sensed to be idle, it transmits its packet. Otherwise, it goes to the retransmission mode. In a multiple channel network using the M-CSMA protocol, a ready terminal will act in the same way, that is, sense the channel and then act. But it has more choices in that it has to select which channel to sense and transmit its packet on. Referring to [2], two schemes are considered.

1. Random choice (RC) scheme: In the RC scheme, a ready terminal chooses a channel randomly for sensing. If the chosen channel is sensed to be idle, it transmits its packet. Otherwise, it goes to the retransmission mode. The resulting protocol is called the M-CSMA-RC protocol.
2. Idle channel choice(IC) scheme: In the IC scheme, a ready terminal senses all the channels and then randomly chooses one among those that are sensed to be idle to transmit its packet on. If none of the  $M$  channels are sensed to be idle, it goes to the retransmission mode. The resulting protocol is called the M-CSMA-IC protocol.

In the next section, we evaluate the throughput performance of the above two protocols in multi-hop networks.

## 4 Throughput Evaluation of the M-CSMA-RC and M-CSMA-IC protocols in Multihop Networks

### 4.1 Throughput Performance of the M-CSMA-RC Protocol

Referring to Fig. 1, designate the terminal under consideration as  $x$ , and the channels around  $x$  as  $CH_i(x)$ ,  $i=1,2,\dots,M$ . Using the same approach as in [1][2], assume that the  $M$  subchannels are independent, and that the behavior of each terminal in each slot is also independent. In each slot, terminal  $x$  transmits its packet with probability  $p'$  and does not transmit with probability  $1-p'$ . Here  $p'$  is the transmission rate from a terminal per slot and it is defined to be

$$\begin{aligned} p' &= \sum_{i=1}^M \text{Prob}(\text{terminal } x \text{ is ready}) \times \text{Prob}(\text{it chooses channel } CH_i(x)) \times \\ &\quad \text{Prob}(\text{channel } CH_i(x) \text{ is idle in a slot}) \\ &= p \cdot \sum_{i=1}^M \frac{1}{M} \text{Prob}(\text{channel } CH_i(x) \text{ is idle in a slot}) \end{aligned}$$

Since all of the subchannels are identical and each terminal in the hearing area of  $x$ , denoted as  $N(x)$ , chooses a subchannel with equal probability, each subchannel has the same probability of being idle in a slot, which is denoted as  $P_{cI}$ . Hence,  $p'$  can be expressed as

$$p' = pP_{cI} \tag{5}$$

Now we determine the probability  $P_{cI}$ . Similar to the discussion in [1], each subchannel around terminal  $x$ ,  $CH_i(x)$ , can be modeled by a two-state Markov chain as shown in Fig. 2. The durations of the states in this Markov chain are:

$$D_B = M \tag{6}$$

$$D_I = a \tag{7}$$



Assume that at the beginning of slot  $n$ ,  $\text{CH}_i(x)$  is in state I, at the beginning of the next slot,  $\text{CH}_i(x)$  will go to state I with probability  $P_{cII}$ , and will stay in state I with probability  $1-P_{cII}$ . The transition probability  $P_{cII}$  can be determined by

$$P_{cII} = \sum_{i=0}^{\infty} \text{Prob}(\text{there are } i \text{ terminals in } N(x)) \times \\ \text{Prob}(\text{none of them transmits on } \text{CH}_i(x))$$

The probability that a terminal in  $N(x)$  transmits on  $\text{CH}_i(x)$  is equal to  $\frac{p'}{M}$ . Hence,

$$p_{cII} = \sum_{i=0}^{\infty} \left(1 - \frac{p'}{M}\right)^i \text{Prob}(\text{there are } i \text{ terminals in } N(x)) \\ = \sum_{i=0}^{\infty} \left(1 - \frac{p'}{M}\right)^i \frac{(\lambda\pi R^2)^i}{i!} e^{-\lambda\pi R^2} = e^{-\frac{p'}{M}N} \quad (8)$$

From the Markov chain, we have

$$P_c(I) = P_{cII}P_c(I) + P_c(B)$$

$$P_c(B) = 1 - P_c(I)$$

$$\text{Hence, } P_c(I) = \frac{1}{2 - P_{cII}}. \quad (9)$$

Therefore, from the theorem given in Section 2, the limiting probability  $P_{cI}$  can be expressed as,

$$P_{cI} = \frac{D_I P_c(I)}{(1 - P_c(I)) D_B + D_I P_c(I)} = \frac{a}{\left(1 - e^{-\frac{p'}{M}N}\right) M + a} \quad (10)$$

Substituting (10) in (5),  $p'$  can be expressed as

$$p' = \frac{ap}{\left(1 - e^{-\frac{p'}{M}N}\right) M + a} \quad (11)$$

Now we determine the limiting probability that terminal  $x$  has a successful transmission in a slot, which is equal to the throughput by definition. The transmission states of terminal  $x$  can be modeled by a three-state Markov chain shown in Fig. 3. The durations of the states in this chain are

$$D_S = D_C = M, \quad (12)$$

$$D_I = a. \quad (13)$$

Starting from state I, terminal x may leave state I during the next slot with probability  $p'$ . Thus, the transition probability  $P_{II}$  is given by

$$P_{II} = 1 - p' = 1 - \frac{ap}{\left(1 - e^{-\frac{p'}{M}N}\right)^{M+a}} \quad (14)$$

From the Markov chain,  $P(I) = P(I)P_{II} + P(S) + P(C)$ . Thus, the steady state probability  $P(I)$  can be expressed by:

$$P(I) = \frac{1}{2 - P_{II}} = \frac{1}{1 + p'} \quad (15)$$

Now we determine the transition probability  $P_{IS}$ . Let  $P_{IS}(r)$  denote the transition probability when terminal x is sending a packet to terminal y, where r is the distance between x and y, which is shown in Fig. 1. Based on the condition for successful transmission given in [1] and using the same notation, we have:

$$\begin{aligned} P_{IS}(r) &= \sum_{i=1}^M \text{Prob.}(x \text{ transmits in a slot}) \times \text{Prob.}(it \text{ chooses channel } i) \times \\ &\quad \text{Prob.}(y \text{ does not transmit in a slot on channel } i) \times \text{Prob.}(terminals \text{ in} \\ &\quad \text{C}(r) \text{ do not transmit in a slot on channel } i \mid r) \times \\ &\quad \text{Prob.}(terminals \text{ in B}(r) \text{ do not transmit for } 2\tau_s + 1 \text{ slots on channel } i \mid r) \\ &= p' \left(1 - \frac{p'}{M}\right) p_c(r) p_b(r)^{2\tau_s + 1} \end{aligned} \quad (16)$$

where

$$p_c(r) = \text{Prob.}(terminals \text{ in C}(r) \text{ do not transmit in a slot in channel } i \mid r)$$

$$p_b(r) = \text{Prob.}(terminals \text{ in B}(r) \text{ do not transmit in a slot in channel } i \mid r)$$

$C(r)$  = the area of region  $N(x) \cap N(y)$

$B(r)$  = the area of region  $N(x) - N(y)$ .

According to the network model, the terminals in the network are distributed as a two-dimensional Poisson point process with density  $\lambda$ . Thus, the probability that there are  $i$  terminals in  $C(r)$  is given by

$$p(i) = \frac{(\lambda C(r))^i}{i!} e^{-\lambda C(r)}$$

Hence, the probability that no terminals in region  $C(r)$  transmit during one slot is

$$p_c(r) = \sum_{i=0}^{\infty} \left(1 - \frac{p'}{M}\right)^i \frac{(\lambda C(r))^i}{i!} e^{-\lambda C(r)} = e^{-\frac{p'}{M} \lambda C(r)} \quad (17)$$

Similarly, the probability that no terminals in region  $B(r)$  transmit during one slot is given by

$$p_b(r) = \sum_{i=0}^{\infty} \left(1 - \frac{p'}{M}\right)^i \frac{(\lambda B(r))^i}{i!} e^{-\lambda B(r)} = e^{-\frac{p'}{M} \lambda B(r)} \quad (18)$$

Hence,  $P_{IS}(r)$  can be expressed as

$$P_{IS}(r) = p' \left(1 - \frac{p'}{M}\right) e^{-\frac{p'}{M} \lambda C(r)} e^{-\frac{p'}{M} \lambda B(r)} (2\tau_s + 1) \quad (19)$$

As in [1], the probability density function of  $r$  is given by

$$f(r) = 2r, \quad 0 < r < 1.$$

Hence,  $P_{IS}$  is given by

$$\begin{aligned} P_{IS} &= \int_0^1 P_{IS}(r) f(r) dr = \\ &= 2p' \left(1 - \frac{p'}{M}\right) e^{-\frac{(2M\tau+1)p'N}{M}} \int_0^1 e^{-\frac{4p'N\tau}{\pi} q\left(\frac{r}{2}\right)} r dr \end{aligned} \quad (20)$$

From the Markov chain, we have  $P(S) = P(I)P_{IS}$ . Therefore, from the theorem given in Section 2, the limiting probability  $P_S$ , which is equal to TH, is given by

$$P_S = TH = \frac{P(I) P_{IS} D_S}{(1 - P(I)) D_S + D_I P(I)} = \frac{P_{IS} M}{a + p' M}$$

$$= \frac{M \left( 2p' \left( 1 - \frac{P'}{M} \right) e^{-\frac{(2M\tau+1)p'N}{M}} \int_0^1 e^{\frac{4p'N\tau}{\pi} q\left(\frac{r}{2}\right)} r dr \right)}{p'M + a} . \quad (21)$$

Numerical results are given in Section 6

#### 4.2 Throughput Performance of the M-CSMA-IC protocol

With the M-CSMA-IC protocol, a ready terminal senses all the  $M$  subchannels. Assume that there are  $k$  channels that are sensed to be idle,  $0 < k \leq M$ . Then this terminal chooses one of the idle channels to transmit its packet with probability equal to  $\frac{1}{k}$ . If no channel is sensed to be idle, it goes to the retransmission mode. The computation procedure for the M-CSMA-IC protocol is the same as the one for the M-CSMA-RC protocol. That is, assume that each subchannel is independent and that each terminal transmits in a slot with probability  $p'$ , where  $p'$  is defined to be

$$p' = p \cdot \text{Prob}(\text{at least one channel is sensed to be idle})$$

Let  $P_{cI}$  denote the limiting probability that a subchannel around terminal  $x$  is idle in a slot, we have,

$$p' = p \cdot (1 - (1 - P_{cI})^M) \quad (22)$$

Each subchannel can be modeled by the same two-state Markov chain as shown in Fig. 2. Now we derive the probability that a transmitting terminal in  $N(x)$  chooses channel  $CH_i(x)$  to transmit its packet, which is denoted as  $p_i$ . Assume that when channel  $CH_i(x)$  is sensed to be idle, there are  $j$  other channels that are idle at the same time. Then, a transmitting terminal will choose channel  $CH_i(x)$  to transmit its packet with probability  $\frac{1}{j+1}$ . Therefore,  $p_i$  can be expressed as

$$p_i = \text{Prob}(\text{channel } CH_i(x) \text{ is idle}) \cdot \sum_{j=0}^{M-1} \text{Prob}(\text{there are } j \text{ other idle channels}) \cdot$$

Prob(a transmitting terminal chooses channel  $CH_i(x)$  to transmit its packet)

$$\begin{aligned}
&= P_{cl} \sum_{j=0}^{M-1} \frac{1}{j+1} \binom{M-1}{j} (1-P_{cl})^{M-1-j} P_{cl}^j \\
&= \frac{1 - (1-P_{cl})^M}{M} = \frac{p'}{M}
\end{aligned} \tag{23}$$

Similar to the discussion of  $P_{cI}$  for the M-CSMA-RC protocol, from (2), we have

$$P_{cII} = e^{-\frac{p'}{M}N} \tag{24}$$

From (22), we have,

$$P_{cl} = \frac{a}{\left(1 - e^{-\frac{p'}{M}N}\right)M + a} \tag{25}$$

Substituting (25) in (22), we have

$$p' = p \left( 1 - \left( 1 - \frac{a}{\left(1 - e^{-\frac{p'}{M}N}\right)M + a} \right)^M \right) \tag{26}$$

Terminal  $x$  can be modeled by the same three-state Markov chain as shown in Fig. 3, the transition probability  $P_{II}$  is given by

$$P_{II} = 1 - p' = 1 - p(1 - (1 - P_{cl})^M)$$

Hence, from the Markov chain in Fig. 2, the steady state probability  $P(I)$  is given by

$$P(I) = \frac{1}{2 - P_{II}} = \frac{1}{1 + p'}$$

Also, in a manner similar to the M-CSMA-RC protocol, the transition probability  $P_{IS}(r)$  can be expressed as

$$P_{IS}(r) = \sum_{i=1}^M \text{Prob.}(x \text{ transmits over channel } i \text{ in a slot}) \cdot \text{Prob.}(y \text{ does not transmit})$$

on channel  $i$  in a slot) · Prob.(terminals in  $C(r)$  do not transmit in the same slot as  $x$  on channel  $i$  |  $r$ ) · Prob.(terminals in  $B(r)$  do not transmit for  $2\tau_s + 1$  slots on channel  $i$  |  $r$ )

$$\begin{aligned}
&= \sum_{i=1}^M p_i(1-p_i)p_c(r)p_b(r) \\
&= p'(1-\frac{p'}{M})p_c(r)p_b(r)^{2\tau_s+1}
\end{aligned} \tag{27}$$

The expressions  $p_c(r)$  and  $p_b(r)$  are the same as the ones for the M-CSMA-RC protocol. Hence,  $P_{IS}(r)$  can be expressed as

$$P_{IS}(r) = p'(1-\frac{p'}{M})e^{-\frac{p'}{M}\lambda_C(r)}e^{-\frac{p'}{M}\lambda_B(r)(2\tau_s+1)} \tag{28}$$

As in (20),  $P_{IS}$  is given by

$$P_{IS} = 2p'(1-\frac{p'}{M})e^{-(2M\tau+1)\frac{p'}{M}N} \int_0^1 e^{\frac{4p'N\tau}{\pi}q(\frac{r}{2})} r dr \tag{29}$$

Similarly, from the Markov chain model shown in Fig. 3, we have  $P(S) = P(I)P_{IS}$ . Therefore, the limiting probability  $P_S$ , which is equal to TH, is given by

$$\begin{aligned}
P_S = TH &= \frac{P(I)P_{IS}M}{(1-P(I))M+aP(I)} = \frac{P_{IS}M}{a+p'M} \\
&= \frac{2p'(1-\frac{p'}{M})e^{-(2M\tau+1)\frac{p'}{M}N} \int_0^1 e^{\frac{4p'N\tau}{\pi}q(\frac{r}{2})} r dr}{p'M+a}
\end{aligned} \tag{30}$$

Numerical results are given in Section 6.

We can further improve the throughput performance by using both the multiple channel and the busy tone schemes, which leads to a new protocol, the multiple channel BTMA protocol. Next we propose and analyze the performance of the multiple channel conservative BTMA (MC-BTMA) protocol.

## 5 Throughput Evaluation of the MC-BTMA-RC and MC-BTMA-IC protocols

In this section, we analyze the performance of the multiple channel conservative busy tone access protocol. The network model is the same as the one used in [1]. There are  $M$  subchannels available for each terminal in the network. Assume that the packet transmission time on each subchannel is  $T_s = M$ . As for the slotted BTMA protocols used in [1], the duration of a slot in time, which is denoted as  $a$ , is set equal to a propagation delay and the slotted MC-BTMA protocol only allows terminals to transmit at the beginning of even numbered slots. Let  $\tau_s$  be the number of slots in a packet transmission time or in a transmission period (TP), then we have  $\tau_s = \frac{M}{a}$ .  $\tau_s$  is assumed to be an even integer. Recall that  $\tau = \frac{1}{a}$ , thus  $\tau_s = M\tau$ .

The protocols are defined next.

### *MC-BTMA-RC protocol*

1. At the beginning of each even numbered slot, a silent terminal becomes ready with probability  $p_t$ . If it is ready, it chooses one of the  $M$  subchannels with probability  $\frac{1}{M}$ , and then senses that channel. Only when the chosen channel is sensed to be idle and no busy tone is heard, it transmits its packet. Otherwise, it goes to the retransmission mode.
2. When a silent terminal hears a transmission, it emits a busy tone to block all its neighboring terminals from transmitting over that subchannel until this transmission is over.

### *MC-BTMA-IC protocol*

1. At the beginning of each even numbered slot, a silent terminal may become ready with probability  $p_t$ . If it is ready, it senses all of the  $M$  subchannels. If  $i$  channels are sensed to be idle and no busy tone is heard,  $M \geq k > 0$ , it chooses one of the idle channels, with probability equal to  $\frac{1}{k}$ , to transmit its packet. If all the channels are sensed to be busy or busy tone is heard, it goes to the retransmission mode.

2. When a silent terminal hears a transmission from a subchannel, it emits a busy tone to block all its neighboring terminals from transmitting over that subchannel until this transmission is over.

The throughput computation procedure for the MC-BTMA-RC and MC-BTMA-IC protocols is the same as for the M-CSMA-RC and MC-CSMA-IC protocols except for three differences. The differences are:

1. The VP: The vulnerable period (VP) for the MC-BTMA protocol is the same as for the C-BTMA protocol. That is, the duration of the VP is only two slots, whereas the duration of the VP for the M-CSMA protocol was  $2\tau_s+1$  slots.
2. The blocking region: In the MC-BTMA protocol, when a terminal is transmitting, all its neighbors emit a busy tone. Hence, all its neighbors and the neighbors' neighbors of this terminal will be blocked. Therefore, the blocking region is  $2\pi(2R)^2 = \frac{4N}{\lambda}$ . In the M-CSMA protocol, when a terminal is transmitting, only its neighbors will sense this transmission, hence, the blocking region is  $2\pi R^2 = \frac{N}{\lambda}$ .
3. The duration of state I: Since BTMA protocols only allow terminals to transmit at the beginning of even numbered slots, the durations of state I in the Markov chains for CH(x) and terminal x is 2a.

We can easily obtain the expressions of the throughput and the intermediate results for the MC-BTMA protocol from the result for the M-CSMA protocol by substituting  $4N$  for  $N$  in the expression of  $P_{CI}$ , substituting  $1$  for  $2\tau_s+1$  in the expression of  $p_b$  and substituting  $2a$  for  $a$  in the expression of  $D_I$ . The results are listed below.

#### *MC-BTMA-RC protocol*

Substituting  $4N$  for  $N$  in (8), we have

$$P_{CII} = e^{-\frac{p'}{M}N^4} \quad (31)$$

From (10), we have



$$P_{cI} = \frac{2a}{\left(1 - e^{-\frac{p'}{M}N^4}\right)M + 2a} \quad (32)$$

and

$$p' = \frac{D_I P_c(I)}{(1 - P_c(I))D_S + D_I P_c(I)} = \frac{2ap_t}{\left(1 - e^{-\frac{p'}{M}N^4}\right)M + 2a} \quad (33)$$

Substituting 1 for  $2\tau_s+1$  in (19), we have

$$P_{IS}(r) = p' \left(1 - \frac{p'}{M}\right) e^{-\frac{p'N}{M}} \quad (34)$$

Therefore, the throughput TH can be given by

$$P_S = TH = \frac{P(I) P_{IS} D_I}{(1 - P(I)) D_I + D_I P(I)} = \frac{P_{IS} M}{2a + p' M} = \frac{p' \left(1 - \frac{p'}{M}\right) e^{-\frac{p'N}{M}} M}{2a + p' M} \quad (35)$$

*MC-BTMA-IC protocol*

Substituting  $4N$  for  $N$  in (24), we have

$$p_{cII} = e^{-\frac{p'}{M}N^4} \quad (36)$$

and

$$P_{cI} = \frac{2a}{\left(1 - e^{-\frac{p'}{M}N^4}\right)M + 2a} \quad (37)$$

Therefore,  $p'$  can be given by

$$p' = p_t \left( 1 - \left( \frac{2a}{\left(1 - e^{-\frac{p'}{M}N^4}\right)M + 2a} \right)^M \right) \quad (38)$$

Substituting 1 for  $2\tau_s+1$  in (28), we have

$$P_{IS} = p' \left(1 - \frac{p'}{M}\right) e^{-\frac{p'N}{M}} \quad (39)$$

Therefore, the throughput TH can be given by

$$P_s = TH = \frac{P(I) P_{IS} M}{(1 - P(I)) M + 2aP(I)} = \frac{P_{IS} M}{2a + p' M} = \frac{p' \left(1 - \frac{p'}{M}\right) e^{-\frac{p'N}{M}} M}{2a + p' M} \quad (40)$$

Numerical results are given in Section 6.

## 6. Numerical Results and Discussion

The performance of multihop networks is very difficult to analyze precisely because of the hidden terminal problem. The analysis of multiple channel multihop networks is even more difficult. But by using an approximation and a Markov chain model, we are able to evaluate the throughput performance of the CSMA based protocols approximately in multihop networks. In this section, we give numerical results.

Recall from [1] that in order to compare the performance of CSMA protocols and BTMA protocols, we assumed that  $p_t = 2p$ . Based on this assumption, we computed throughputs for different values of  $p$ ,  $a$ ,  $M$  and  $N$ . Fig. 3 shows the throughputs at  $a = 0.01$  and  $M = 2$ . Fig. 4 shows the throughputs at  $a = 0.1$  and  $M = 2$ . Fig.5 shows the throughputs at  $a = 0.1$ ,  $N = 6$  and various values of  $M$ .

The results show that the M-CSMA protocol has a better throughput performance than the single channel CSMA protocol. From the same figure, the M-CSMA-IC protocol has a higher throughput than the M-CSMA-RC protocol at low channel loads, but the M-CSMA-RC protocol has a better performance at higher channel loads. The reason for this is that the M-CSMA-IC protocol allows the terminals to sense the channels before they choose a channel. At low channel load, less terminals become ready at the same time and more channels are idle when the ready terminals sense them. Therefore, the probability that two or more terminals choose the same idle channel from the available idle channels is small. But when the channel load is high, the IC scheme becomes less effective. There are more terminals that become ready at the same time, and there are less idle channels when those ready terminals sense them. The probability that more than one terminal chooses the same idle channel is large. Hence, the M-CSMA-RC protocol has a better performance than the M-CSMA-IC protocol when the networks operate at higher channel loads.

The results also show that the MC-BTMA protocols consistently outperform the M-CSMA protocols. The reason for this is that the MC-BTMA protocols have a short VP consisting of only two slots, and because of the multiple channel scheme, it has more space reuse capability than the single channel BTMA protocol. Furthermore, the results also show that the MC-BTMA-IC has a consistently better performance than the MC-BTMA-RC protocol.

From the analysis in this paper, we have shown that the multiple channel scheme is another solution to the hidden terminal problem in addition to the busy tone scheme. If we use both the multiple channel and the busy tone techniques simultaneously, the resulting protocol, which is the MC-BTMA protocol, has a consistently better performance than both the single channel CSMA and C-BTMA protocols at all channel loads.

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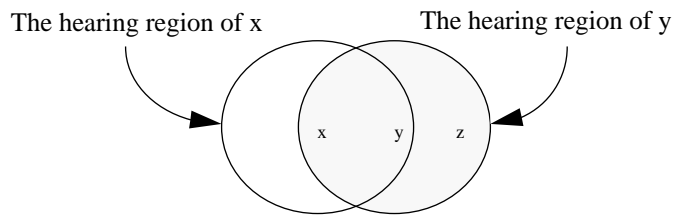


Fig. .1 Illustration of the hidden terminal problem in a multihop network.

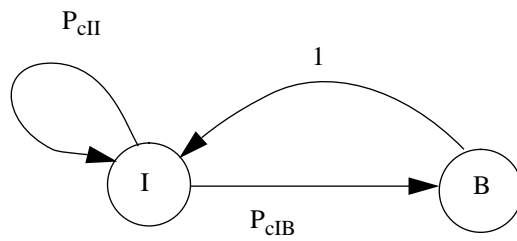


Fig.2 The Markov chain for the channel CH(x)

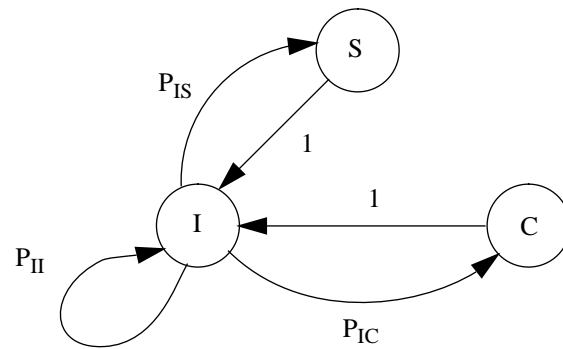


Fig3 The Markov Chain for terminal x

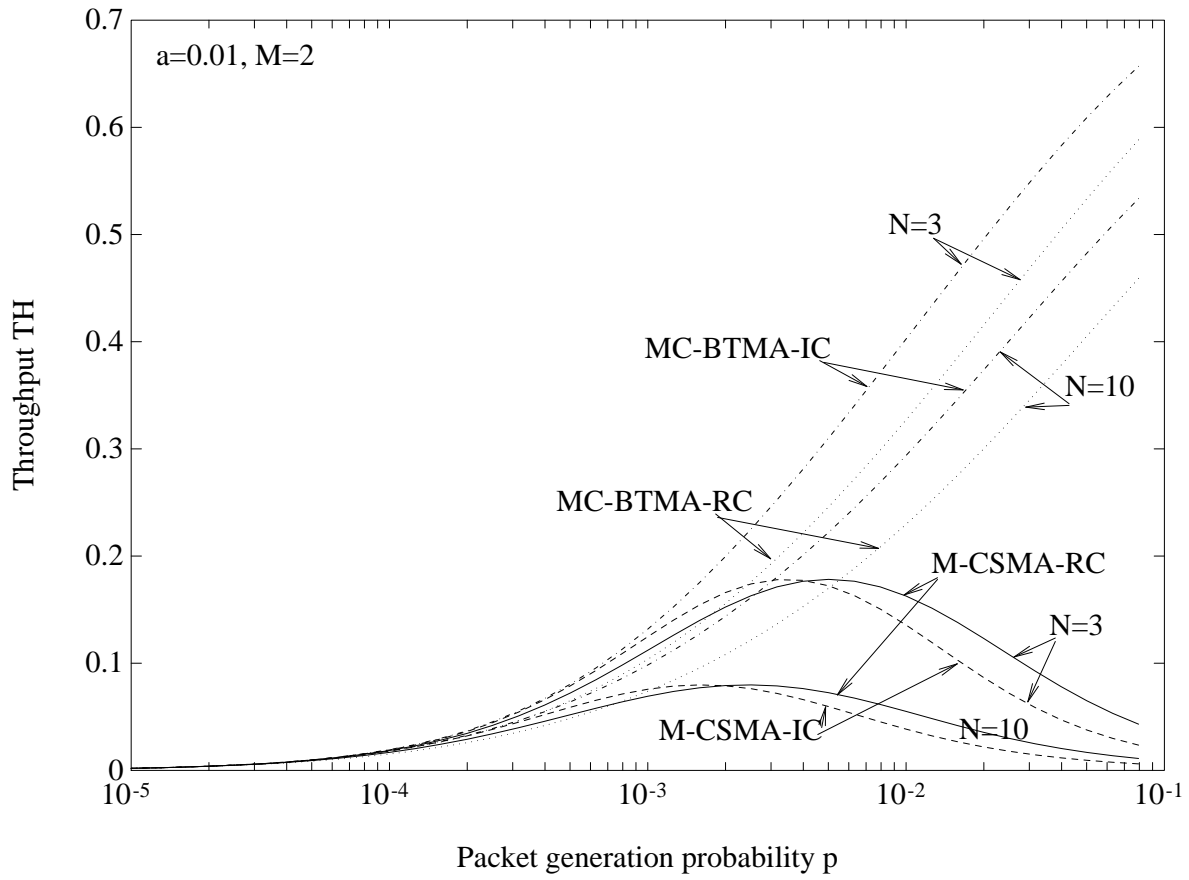


Fig.4 The throughput of the M-CSMA and MC-BTMA protocols for  $a = 0.01$



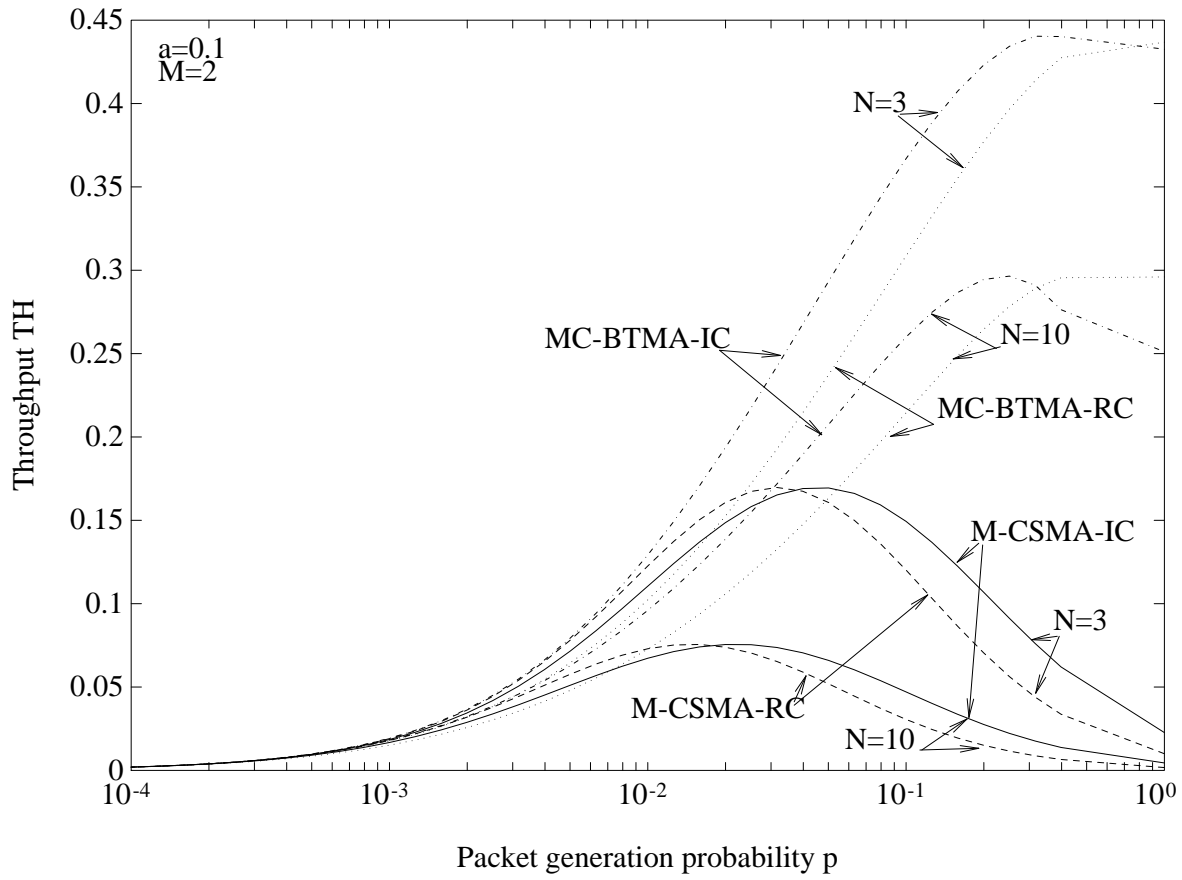


Fig.5 The throughput of the M-CSMA and MC-BTMA protocols for  $a = 0.1$ .

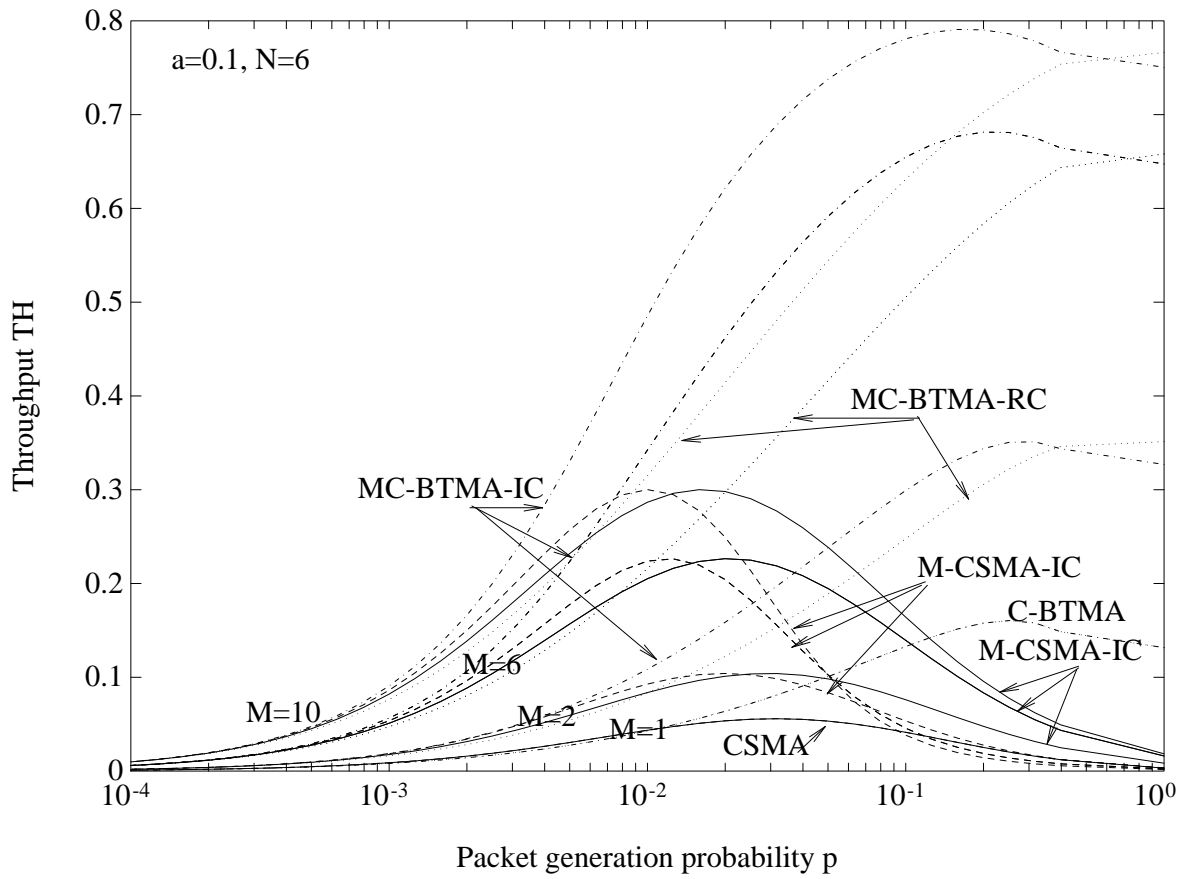


Fig.6 The throughput of the M-CSMA and MC-BTMA protocols for  $a = 0.1$  and  $N = 6$ .