

5-2013

Applications of Search Theory in Finance

Hongyu Song
Syracuse University

Follow this and additional works at: http://surface.syr.edu/fin_etd

 Part of the [Finance and Financial Management Commons](#)

Recommended Citation

Song, Hongyu, "Applications of Search Theory in Finance" (2013). *Finance - Dissertations*. Paper 12.

This Dissertation is brought to you for free and open access by the Whitman School of Management at SURFACE. It has been accepted for inclusion in Finance - Dissertations by an authorized administrator of SURFACE. For more information, please contact surface@syr.edu.

APPLICATIONS OF SEARCH THEORY IN FINANCE

Hongyu Song

ABSTRACT

In this dissertation, I apply two types of search based theoretical models to interpret two important issues in finance, respectively. **(1)** The first essay belongs to the field of corporate finance, in which a *random search model* is set up to describe the pre-IPO market searching and matching process between private firms with intent to sell equity in an IPO and investment banks (IB) that underwrite the issue. Our model tightly links many IPO-related phenomena such as IPO underpricing and long run underperformance under one unified searching framework. **(2)** The second essay is related to market microstructure, i.e. bid-ask spread, in which a *competitive search model* is proposed to re-interpret the existence of the market equilibrium bid-ask spread in a stylized security market, in which market dealers are in charge of posting an instantaneous bid price, investors choose whether to sell their share or not at this price. Different from the asymmetric information based explanation originated from two types of investors, our search based model emphasizes that since the market dealer provides necessary liquidity to the security market via playing such an intermediary role between actual buyers and sellers, the bid-ask spread charged thereafter should largely be justified as the compensation for the market dealer's endeavor in this process.

Key words: random search, pre-IPO market, IPO underpricing, IPO underperformance, competitive search, bid-ask spread, market microstructure, market dealer, stock liquidity, stock splits

APPLICATIONS OF SEARCH THEORY IN FINANCE

By

Hongyu Song

B.S., Tianjin University

M.S., Columbia University

M.A., SUNY at Albany

Dissertation

**Submitted in partial fulfillment of the requirements
for the degree of Doctor of Philosophy in Business Administration
in the Whitman School of Management at Syracuse University**

Syracuse, NY

May 2013

© 2013

Hongyu Song

All Rights Reserved

ACKNOWLEDGEMENTS

Many thanks go out to numerous people for helping me during my doctoral work at Syracuse University.

Firstly, I would like to express my highest gratitude to Dr. Chung Chen for being such a knowledgeable and supportive advisor. Without his generous support and enthusiastic guidance, I could not have finished my graduate study successfully. It has been my greatest luck to study under his excellent supervision. Thank you, Dr. Chen for everything you have done for me!

Secondly, I would like to sincerely thank my dissertation committee members: Dr. Donald Harter, Dr. Milena Petrova, Dr. Thomas Barkley, Dr. Pierre Yourougou, and my oral defense chair: Dr. Chihwa Kao for their valuable advice and suggestions for the revision of my dissertation. What I have learned from Dr. Harter's research methods course will benefit me forever in my life. Working as a TA for Dr. Barkley is my greatest pleasure and honor. He is always my role model and keeps reminding me of how to become a qualified finance teacher besides hard-working. More importantly, I am extremely grateful to Dr. Milena Petrova for her persistent help all these years.

Lastly, I would like to thank my wife Lijie Zhang and my parents for their faith in me and their unremitting love and support. This dissertation is also given as the most valuable gift to my cherished daughter, Alice Song, who was born at 7:23am, May 10, 2012 in Washington, DC.

TABLE OF CONTENTS

	PAGE
Abstract	i
Title page	ii
Copyright notice	iii
Acknowledgements	iv
List of Tables	vii
List of Figures	viii
Chapter One: Literature review on search theory	2
1.1 Introduction.....	2
1.2 Search based theoretical models.....	4
1.2.1 One-sided search.....	4
1.2.1.1 Discrete time search.....	4
1.2.1.2 Continuous time search.....	8
1.2.2 Diamond coconut model.....	11
1.2.3 Two-sided search.....	17
1.3 Relations between two-sided search models and Chapter two & Chapter three.....	21
References.....	22

Chapter Two: Searching in the pre-IPO market: interaction between private firms and investment banks	24
2.1 Introduction.....	25
2.2 A pre-IPO search model.....	33
2.3 Mathematical model and discussion.....	37
2.4 Empirical implications.....	53
2.5 Conclusion.....	66
Appendix A: Notation Table.....	68
Appendix B: Proofs of Propositions and Lemmas	70
References.....	80
Chapter Three: Revisit the bid-ask spread using competitive search	83
3.1 Introduction and background.....	84
3.2 Competitive search model.....	90
3.3 Discussions.....	94
3.4 Empirical implications.....	97
3.5 Calibration and simulation.....	106
3.6 Conclusion and future directions.....	110
Appendix C: Notation Table.....	112
Appendix D: Proofs of Propositions	114
References.....	119
Vita	121

LIST OF TABLES

	PAGE
Table2.1 key parameters and their typical values for the pre-IPO market	54
Table2.2 General equilibrium picture for the pre-IPO market.....	62
Table 3.1 Key parameters and their typical values for the bid-ask spread.....	106

LIST OF FIGURES

	Page
Figure1.1 Population flows in the steady state	14
Figure1.2 Unique equilibrium when λ is constant	15
Figure1.3 Multiple equilibriums when $\lambda = \lambda(n_T)$	16
Figure2.1 The schematic of the pre-IPO market.....	36
Figure2.2 The balanced steady state flow condition in the pre-IPO market.....	46
Figure2.3 (a)-(b) Effect of α_f or α_b on k/R	57
Figure2.4 Effect of θ on k/R	58
Figure2.5 (a)-(b) Effect of R on r and r/R when size effect > ratio effect	59
Figure2.6 (a)-(b) Effect of R on r and r/R when size effect < ratio effect.....	61
Figure2.7 Effect of r on R^*	64
Figure2.8 Effect of σ on R^*	65
Figure2.9 General equilibrium solution for R^* and MT	66
Figure 3.1 Effect of a on the relative bid-ask spread.....	107
Figure 3.2 Effect of b on the relative bid-ask spread.....	107
Figure 3.3 Effect of r on the relative bid-ask spread.....	108

Figure 3.4 Effect of λ on the relative bid-ask spread.....108

Figure 3.5(a)-(b) Effect of stock price level on the bid-ask spread.....109

Figure 3.6 Ratio between search and matching cost and inventory-holding cost.....110

This dissertation consists of three chapters: Chapter one introduces the fundamentals of search theory; Chapter two deals with the pre-IPO searching and matching between private firms and investment banks via a random search model; Chapter three revisits the bid-ask spread existing in the security markets via a competitive search model .

Chapter One: Literature Review on Search Theory

In this chapter, I provide a thorough review on search theory with two purposes in mind. The first purpose is that even though search theory is not unfamiliar to economists, few scholars at business schools have appreciated its importance very much. Thus introducing this important theory to management seems beneficial. More importantly, since the unified theme of my two essays is the application of search theory in the field of finance, search theory plays such a key role as the foundation of my model setup. Without a clear understanding of the origin and up-to-date academic progress of search theory, it is impossible to apply it efficiently and effectively.

1.1 Introduction

Classical demand and supply models in a frictionless market cannot explain many important market phenomena. For instance, how can we observe that there are unemployed workers and unfilled job positions simultaneously in a real job market? Moreover, how can apparently homogeneous workers in similar jobs end up earning totally different wages? While information friction, i.e. asymmetric information, could be one possible source which has already been discussed in many areas of economics and finance, here will introduce the concept of search friction and the related search theory. The importance of search theory can never be much more emphasized.

Generally speaking, search theory models the concept that it takes time (and/or any other resources) for one type of agent to locate the other type of agent when the cooperation of two types of agents is of necessity to accomplish a transaction. From the practical viewpoint, search

friction exists more or less in any exchange process since there is simply no such thing as a 100 percent centralized market where two types of agents can meet and trade instantly.

Since search theory initially studies the matching process between workers and job vacancies at equilibrium in labor economics, sometimes it has been called “matching theory” as well. I will make no difference between “search” and “matching” and always use the name of “search theory” thereafter. The summit academic recognition achieved by search theory is that three scholars, Peter A. Diamond, Dale Mortensen, and Christopher A. Pissarides, won the 2010 Nobel prize in economics "for their analysis of markets with search frictions". This is also the main reason why I will concentrate on the analysis of the job market when illustrating a variety of search theoretic models.

Although the applications of search theory are rather productive in many fields such as the marriage market(marriage and divorce), labor economics (the job market), monetary economics(the role of money as the medium of exchange) due to search theory’s rigorous but tractable framework, research papers devoted to the application of search theory in finance are very few. This low attention to search theory in finance may be partly because search theory is self-absorbed by using a set of mathematical-intensive notation system which is only narrowly communicated in a small circle of labor economists, and partly because the underlying meaning expressed by search theory (i.e. it always takes some time to identify the counterpart of a deal.) is so obvious that it seems trivial to formally model this relationship, in contrast to infinite versions of asymmetric information stories told in finance. Darrell Duffie (2002), Pierre-Olivier Weill (2003), Ricardo Lagos (2007), et.al. are all pioneers to introduce search theory to finance,

(specifically, to investments). While they have achieved many interesting and important results in this attempt, their work is largely ignored by the mainstream of finance.

1.2 Search based theoretical models

In this section, I begin with the basic discrete time job search model and then derive the continuous time version of job search model. Since those models only consider the optimal behavior of a worker when searching a job, they are classified as one-sided search models (McCall (1970), Mortensen (1970) and Gronau(1971)). Moreover, I will briefly introduce the Diamond coconut model proposed by 2010 Nobel laureate Peter Diamond (Diamond (1981, 1982)). The central message delivered by this model is “self-fulfilling”, i.e. “people's expectations as to the level of aggregate activity play a crucial role in actually determining this level of aggregate economic activity”. I conclude this section with a discussion of two types of two-sided search models (Pissarides(1985), Moen (1997) and Shimer(1996), Rogerson, Shimer and Wright(2005)) and their relationship to my following chapters.

1.2.1 One-sided search

One-sided search means that searching activities are concentrated on only one type of agent. With regard to the job search market, only workers are looking for a job while we assume that employers or firms are inactive. Thus, the job market condition faced by workers can be summarized as a random wage distribution. This is the way the search literature started and also a building block for the equilibrium analysis to follow.

1.2.1.1 Discrete time search

Assume that time is discrete and infinite. An agent is searching for a job in a fixed market situation. His objective function is the expected value of the sum of the discounted incomes from period 0 to infinity, i.e.:

$$E\sum_{t=0}^{\infty} \beta^t x_t \tag{1.1}$$

Here: β is the discount factor, $0 \leq \beta \leq 1$;

x_t is the income at period t ;

If the agent is not risk neutral, in the above objective function, x_t needs to be replaced by the utility of earning the income of x_t , $u(x_t)$. Although some interesting results could be derived from the risk-aversion, we always assume the risk-neutrality from now on.

The general picture is: each period, the agent is randomly meeting a job opportunity. The job opportunity is characterized by the offered wage, w , which is drawn from a known wage distribution $F(w)$. In addition, there is no density drift for $F(w)$ among different periods. If the agent accepts the job offer, w , he will keep it forever and his income for each period thereafter will equal to w . However, if the agent declines the offer, he will obtain the unemployment benefit, b , which is positive. Of course in so doing, the agent can still wait for the next period's job offer w' drawn from the independent and identical distribution (i.i.d.), $F(w)$ to test his luck.

The agent's optimal problem is to maximize his objective function (1.1) constrained by our model structure. Before going to the detail of the procedure, some economic intuition is provided here. We expect that the agent's optimal behavior follows the below decision tree:

$$\text{The agent} \left\{ \begin{array}{ll} \text{accepts the job offer,} & \text{if } w \geq w^* \\ \text{declines the job offer,} & \text{if } w < w^* \end{array} \right. \quad (1.2)$$

here w^* is the reservation wage

There exists a unique value, w^* , such that the agent will accept the offer if the wage offered by the job is larger than w^* (we assume that the agent will accept the offer if indifferent.), otherwise he will reject it. This is reasonable since the agent obtains the unemployment benefit (b) in this period and at the same time could wait for a better job offer in next period if he is patient and keeps unemployed now.

Then we define two value functions, U and V , corresponding to the payoff from unemployment and employment, respectively. The payoff from employment, V , is not a constant but depends on the offered wage, w , i.e. $V=V(w)$. The payoff from unemployment, U , is a constant. Apply dynamic programming rule, we have two Bellman equations:

$$V(w) = w + \beta V(w) \quad (1.3)$$

$$U = b + \beta \int_0^{\infty} \max\{U, V(w)\} dF(w) \quad (1.4)$$

The first Bellman equation says that if the agent accepts the job offer, w , he will keep the job forever without drop-out. The second Bellman equation says that if he keeps unemployed, he will obtain the unemployment benefit, b , and at the same time he has a chance to jump into a job as long as the corresponding value to employment $V(w)$ is larger than the value to unemployment U .

Let's use Equation (1.3) and (1.4) to solve for the reservation wage, w^* , according to the agent's decision tree.

Firstly, Equation (1.3) implies that: $V(w)=w/(1-\beta)$, i.e. the value to employment $V(w)$ is an increasing function of w .

Secondly, considering the integrand in Equation (1.4), $\max\{U, V(w)\}$, we need to compare the values of U and $V(w)$ in order to do integral. Combining with the agent's decision tree, there exists a unique w^* at which $V(w^*)=U$. Furthermore, we have:

$$\begin{aligned} V(w) < U & \text{ if } w < w^* \\ V(w) \geq U & \text{ if } w \geq w^* \end{aligned} \tag{1.5}$$

Finally, substitute $V(w)=w/(1-\beta)$ and $U=w^*/(1-\beta)$ into Equation (1.4) re-arrange it, we have:

$$w^* = (1-\beta)b + \beta \int_0^{\infty} \max\{w, w^*\} dF(w) \tag{1.6}$$

Equation (1.6) uniquely determines the reservation wage, w^* . Thus, the agent follows the optimal search strategy as described in (1.2).

Even this simple single agent discrete time search model has a super explanatory power to address two realistic job market phenomena, which, however, cannot be easily modeled in a conventional frictionless supply-demand model: one is that workers sometimes choose not to work even if they are offered a position and the other is that homogeneous workers earn different wages squarely due to their luck.

1.2.1.2 Continuous time search

We have already used the discrete time search model to illustrate the basic search model setup. Now we will switch to the continuous time search model. We will always apply this version of search model in Chapter two and Chapter three.

While the fundamental concept of search friction (i.e. it takes some time to find the other side of the deal.) is embedded in the discrete time search model via the fact that the agent can only meet one wage offer in any specific period, the continuous time counterpart explicitly defines the arrival of the job offer as a Poisson process with the parameter α .

Equation (1.3) and (1.4) are still our starting point. However, we need to make two convenient revisions:

(1) We assume the length of one period to be Δ . In this way the agent will get a job offer with probability $\alpha\Delta$ in each period and the job offer will provide the agent payoff of $w\Delta$, in proportion to time Δ .

(2) The discount factor β is expressed in the form of the discount rate r , i.e. $\beta=1/(1+r\Delta)$.

Now Equation (3) and (4) change into:

$$V(w) = w\Delta + \frac{1}{1+r\Delta}V(w) \quad (1.7)$$

$$U = b\Delta + \frac{\alpha\Delta}{1+r\Delta} \int_0^{\infty} \max\{U, V(w)\} dF(w) + \frac{1-\alpha\Delta}{1+r\Delta} U \quad (1.8)$$

Although Equation (1.8) seems very complicated, actually it addresses the same economic meaning as Equation (4): with probability $\alpha\Delta$, the unemployed agent meets a wage offer w and

decides whether to accept it or not; with probability $1 - \alpha\Delta$, he just keeps his unemployment status.

For Equation (1.7) and (1.8), multiply both sides by $(1+r\Delta)$, delete redundant items, and then let $\Delta \rightarrow 0$, we obtain the continuous time Bellman Equations (1.9) and (1.10):

$$rV(w) = w \tag{1.9}$$

$$rU = b + \alpha \int_0^{\infty} \max\{0, V(w) - U\} dF(w) \tag{1.10}$$

The above two equations are so important for search theory that it is imperative to discuss carefully their economic intuition here. Both equations have the similar structure: the left hand side is always the product of the discount rate and the value for specific status (this product has a formal name, the flow value.); the right hand side is the sum of the instantaneous payoff and the expected value of any changes in the value of the agent's current status.

To be specific, for Equation (1.9), the left hand side represents the flow value for employment; the right hand side has only one item, representing the instantaneous payoff of being employed w since we assume in our model that once the agent accept the job offer, he will continue to keep it forever (there is no mandatory laid-off or voluntary drop-out).

In the meantime, for Equation (1.10), the left hand side represents the flow value for unemployment; the right hand side has two items: the first one represents the instantaneous payoff of being unemployed, the second represents that the agent's expected value change from unemployment to employment.

Since the rule abstracted from Equation (1.9) and (1.10) will be repeatedly applied in Chapter two and Chapter three without from scratch when establishing the key equations for value functions, we iterate it here:

$$\begin{aligned}
 \text{Flow value} &= \text{Discount rate} \times \text{Value in a status} \\
 &= \text{Instantaneous payoff in this status} \\
 &\quad + \text{Expected value change from this status to another status} \quad (1.11)
 \end{aligned}$$

Following the same logic as the discrete time search model, the system can be characterized by one unique reservation wage w^* , i.e. the optimal strategy decision by an unemployed agent in the continuous time version of search model is to accept the job offer if $w \geq w^*$, otherwise to decline it.

Let's find the equation to identify the reservation wage w^* . Equation (1.9) implies that $V(w) = w/r$ (1.12). At w^* , $V(w^*) = U$, then $U = w^*/r$ (1.13). Substitute (1.12) and (1.13) into Equation (1.10), we obtain:

$$w^* = b + \alpha \int_0^\infty \max\left\{0, \frac{w}{r} - \frac{w^*}{r}\right\} dF(w) \quad (1.14)$$

Re-arrange Equation (1.14) and then divide the support of $F(w)$ into two intervals, $(0, w^*)$ and $[w^*, \infty)$ in order to reduce the integrand. Consider two cases for the integrand for Equation (1.14):

$$\max\left\{0, \frac{w}{r} - \frac{w^*}{r}\right\} \begin{cases} = 0 & \text{if } w \in (0, w^*) \\ = \frac{w}{r} - \frac{w^*}{r} & \text{if } w \in [w^*, \infty) \end{cases} \quad (1.15)$$

Thus, using (1.15), Equation (1.14) can be changed into:

$$w^* = b + \frac{\alpha}{r} \int_{w^*}^{\infty} (w - w^*) dF(w) \quad (1.16)$$

Equation (1.16) can be used to find the reservation wage w^* . Moreover, we can do comparative statics analysis according to (1.16) to explore the effects of the unemployment benefit b , the discount rate r and the arrival rate of job opportunity, on the reservation wage w^* .

1.2.2 Diamond coconut model

Previously discussed one-sided search belongs to a class of partial equilibrium models since it only considers the search behaviors from the workers' side and never takes the firm's search behaviors into consideration. Those models provide a first flavoring of the beauty of search model.

Let's discuss one famous search model-the Diamond coconut model, taken its name after Peter Diamond (http://en.wikipedia.org/wiki/Diamond_coconut_model). We will present the model first and then discuss its economic implication and possible application in finance.

Still we have a continuous and infinite time horizon. Now there are only two islands for the entire economy, one is the trading island (T) and the other is the production island (P). People can travel between those two islands freely and costlessly. The number of individuals living on the trading island and the production island is n_T and n_P , respectively ($n_T + n_P = 1$).

The preference of the island residents is quite peculiar: while consumption of anyone else's goods yields m unit of payoff, consumption of self-owned same goods yields zero unit of utility. This assumption makes "exchange" be necessary for individuals in order to obtain any utility.

Furthermore, only people on the production island can produce goods and only people on the trading island can exchange goods with each other. We assume that with the arrival rate α , an individual on the production island meets a production opportunity and the production cost is c , randomly drawn from distribution $F(c)$ ($c \in (c_L, c_H)$); with the encounter rate λ , an individual on the trading island meets with each other. Both α and λ follow a Poisson process.

Intuitively, in the original version of Diamond model, there is only one goods-coconut. Once the individual on the production island finds a palm tree with probability α , he cannot possess the coconut without climbing up the tree and picking it up. The height of the tree is denoted by c , corresponding to the production cost of our model. Hence, the taller the palm tree the individual meets, the harder for he to pick it up and then the higher the production cost of this coconut.

In this model any individual needs to make two important decisions. The first one is which island to stay, the production island or the trading island. Thus we have two value functions:

V_p : Value of being on the production island

V_T : Value of being on the trading island (1.17)

The second one is whether to produce goods if chance is available. In the language of coconut metaphor, whether to climb up the tree if he finds a palm tree on the production island.

Now we set up two key value functions according to the rule (1.11) derived in 1.2.1 without further explanation:

$$rV_T = \lambda(V_p - V_T + m) \tag{1.18}$$

$$rV_p = \alpha \int_{c_L}^{c_H} \max\{0, V_T - V_p - c\} dF(c) \tag{1.19}$$

There exists a reservation cost, c^* such that $V_T - V_P = c^*$, then the integrand in Equation (1.19) is reduced into:

$$\max\{0, V_T - V_P - c\} \begin{cases} =0 & \text{if } c \in (c^*, c_H] \\ =V_T - V_P - c & \text{if } c \in [c_L, c^*) \end{cases} \quad (1.20)$$

The reservation cost c^* characterizes the entire system. The counterpart in one-sided search model is the reservation wage w^* . When the individual on the production island finds a production opportunity with the cost less than c^* , he will choose to produce the goods; otherwise, he will choose not to do so.

Thus, to find an equation which can be used to solve for c^* is the most important objective of our analysis. Putting (1.20) to Equation (1.19), we obtain:

$$rV_P = \alpha \int_{c_L}^{c^*} (c^* - c) dF(c) \quad (1.21)$$

Subtract Equation (1.21) from Equation (1.18), we have:

$$(r + \lambda)c^* = \lambda m - \alpha \int_{c_L}^{c^*} (c^* - c) dF(c) \quad (1.22)$$

Equation (1.22) is what we need to find the reservation cost c^* . The counterpart in one-sided search model is the reservation wage w^* , characterized by Equation (1.16).

Next step, we set up a population flow equation in order to determine the number of individuals on both islands. In the steady state, with the flow rate λ , the individuals on the trading island exchange the goods with each other and then migrate onto the production island; with

flow rate $\alpha F(c^*)$, the individuals on the production island produce the goods and then migrate onto the trading island. This relationship can be illustrated by Figure 1.1:

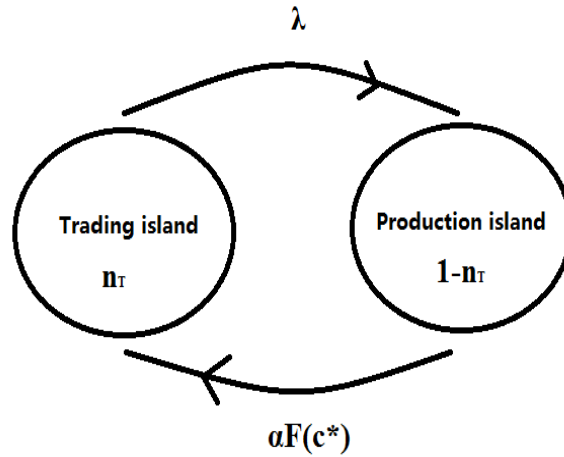


Figure 1.1 Population flows in the steady state

In order to keep the steady state, the flow-in and flow-out of each island has to be balanced, i.e.

$$n_T \lambda = \alpha F(c^*) (1 - n_T) \quad (1.23)$$

Combining Equation (1.22) and (1.23), our simplified one-goods (coconut) two-sector (production island and exchange island) economy is fully characterized by a pair of values (c^*, n_T^*) . Then the number of people living on the trading island is n_T^* and the number of people living on the production island is $1 - n_T^*$.

If we stop here, the significance of Diamond coconut model is not adequately expressed. Recall that the Diamond coconut model is used to show that people's expectation has an important impact on the pathway of the actual economy. To this purpose, review our assumption on λ , in which the matching technology on the trading island is embedded. Until now we assume

that λ is constant and independent of the number of people on the trading island, corresponding to constant returns to scale (CRS) for the matching technology. Under this condition, there exists unique steady state equilibrium shown in Figure 1.2.

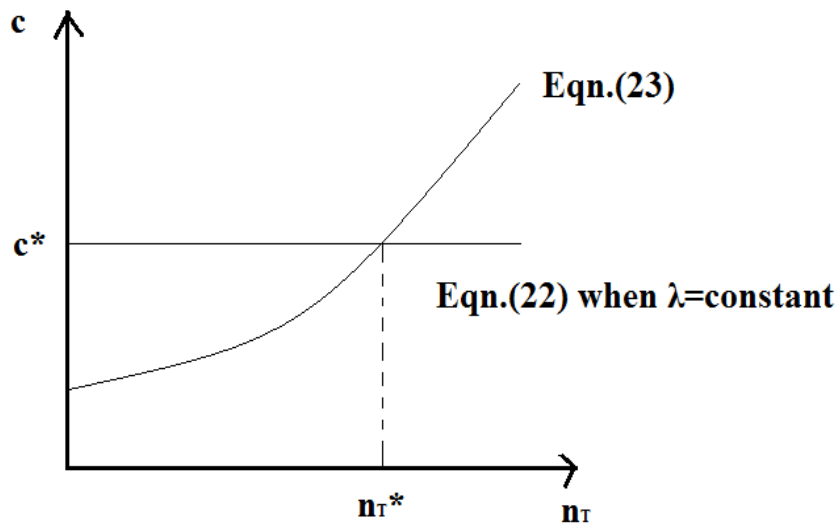


Figure 1.2 Unique equilibrium when λ is constant

However, if λ is not constant and positively dependent of the number of people on the trading island, corresponding to increasing returns to scale (IRS) for the matching technology, there could be multiple steady state equilibriums shown in Figure 1.3. We need to link Equation (1.22) and (1.23) and solve them simultaneously.

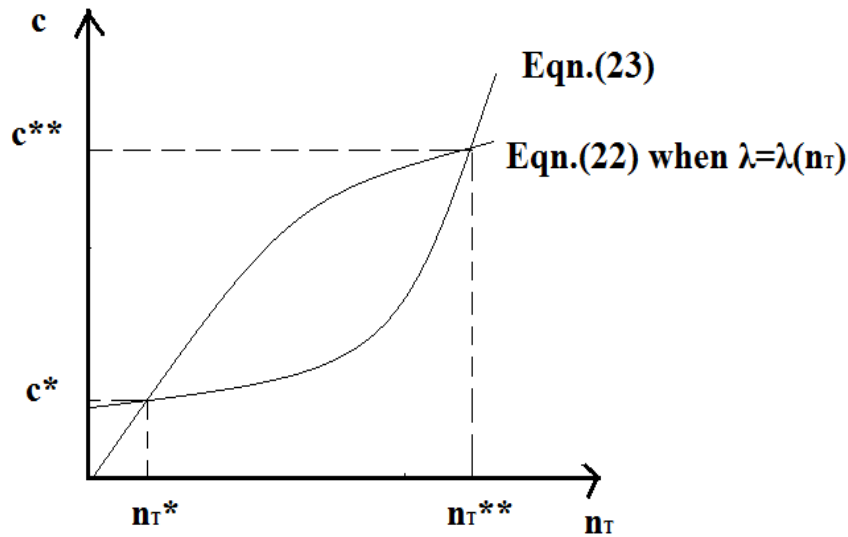


Figure 1.3 Multiple equilibriums when $\lambda = \lambda(n_T)$

Since it is more reasonable in reality that the matching function on the trading island is increasing returns to scale, i.e. the more the people live on the trading island, the quicker for the people to find a trading partner. Hence, our system could have two steady state equilibriums: the lower-level economy with fewer people on the trading island and lower reservation production cost, and the higher-level economy with more people on the trading island and higher reservation production cost. Without more information, we cannot predict which equilibrium the economy will fall into.

The diamond coconut model indicates that it is possible for two economies to differ in prosperousness even if their underlying fundamentals are identical, all depending on people's expectation. Emphasizing the role played by people's expectation distinguishes the study of natural sciences and social sciences. Financial markets are filled with this type of uncertainty resulting from expectation as well. If we appreciate its power, we are not going to be surprised

by such phenomena as unexplained differences in price for almost identical financial assets, excessive volatility in security markets even if there are no fundamental changes along way, which is also the original intent we introduce here the diamond coconut search model to finance researchers.

1.2.3 Two-sided search

Continuing our initial topic on job search, in this session, I introduce the more realistic two-sided search, i.e. we will consider the searching behavior of two sides of the market participants: workers and firms. Those models are mainly developed by Pissarides and Mortensen, the other two Nobel laureates in 2010.

(1) Four value functions

Since the two-sided search models are very complicated in structure and they are not just one model but a sequence of models, we only briefly discuss them here. Those models focus on the equilibrium macroeconomic variables such as the aggregated unemployment rate and the equilibrium wage. The general picture is that we both have a large number of homogenous unemployed workers and homogenous firms with unfilled openings at the same time. They are all searching and matching in one integrated job market. Since we have two types of agents: firms (f) and workers (w); and two states for each type of agents: occupied (V) and un-occupied (U), we have four value functions:

U_w : the unemployed worker's value function

V_w : the employed worker's value function

U_f : the unfilled firm's value function

V_f : the filled firm's value function

Two key modeling issues will determine final searching results: how workers and firms meet and how they decide the wage. Even small differences in institution arrangements could lead to large divergences in predictions.

(2) Matching methods

Two possible institution arrangements on how workers and firms meet have been analyzed in literature: one is the “random search” in which workers and firms just randomly meet each other; the other is the “directed search” in which workers' job search activities are directed to specific firms either because those firms post and advertise their offered wages or because those firms are natural focal points due to history or established reputation.

Although the two-sided search literature only concentrates on the random search and directed search, it has to be admitted that the actual search situation falls in between the above two extreme ends, i.e. in reality, some workers randomly search in the market; the others are attracted by the advertisements posted by firms, even it is still possible for the very same worker to do the two types of search activities instantaneously.

In literature, how workers and firms meet is characterized by the matching function technically. Suppose that the model has u workers and v openings (or firms), the number of contacts between workers and firms is mapped by the matching function $M=M(u,v)$, the same way as the Cobb- Douglas production function maps capital and labor into output.

Then the matching rate for workers α_w , and the matching rate for firms α_f can be expressed as:

$$\alpha_w=M(u,v)/u$$

$$\alpha_f = M(u,v)/v \quad (24)$$

Furthermore, if we assume that the matching function M is constant returns to scale, then we have:

$$\alpha_w = M(u,v)/u = M(u/u, v/u) = M(1, \theta) \equiv m(\theta)$$

$$\alpha_f = M(u,v)/v = M(u/u, v/u)/(v/u) = M(1, \theta)/\theta \equiv m(\theta)/\theta \quad (25)$$

Here $\theta \equiv v/u$, representing the “worker” market tightness. $1/\theta = u/v$, representing the “job” market tightness. In addition, according to the definition of θ , it is obvious that m is an increasing function of θ .

In the following chapters, we will apply intensely the matching function in our finance related model setup. However, it should be noted in advance that while the concept of the matching function is a convenient and easy-to-use abstract to model the search friction existing ubiquitously in the job market, it mimics the black box, thus short in details when describing the procedure from unemployment to employment.

Applying the matching function to the random search model is without difficulty. As to the directed search, the matching function needs to be revised a little bit.

The common image for the directed search model is that firms first post and commit to a wage and make the market at that wage, in doing so, they anticipate that workers will enter until workers are indifferent across all open markets. Thus within each open market(called submarket), there is a constant returns to scale matching function $M(u,v)$ in contrast to the matching function applied for the entire market in the random search model. In addition, each open market is distinguished by its posted wage.

At equilibrium, there is only one open market with one wage w^* posted, such that no firms have incentive to create a new market by posting a wage different from w^* and at the same time no workers want to enter that newly opened market.

(3) Wage determination methods

The second issue about the two-sided search is on how the worker's wage is determined, more generally how to divide the revenue between workers and firms. There are still two types of arrangements in literature.

The first approach is to assume that the wage is pinned down by the generalized Nash bargaining solution:

$$w^* = \operatorname{argmax} [V_f - U_f]^\Phi [V_w - U_w]^{1-\Phi} \quad \Phi \in (0,1) \quad (1.26)$$

Here, Φ represents the bargain power of the firm while $1-\Phi$ represents the bargain power of the worker.

With the same defect as the matching function, the generalized Nash bargaining solution does not provide the detailed bargaining process though it is simple and convenient to use for theoretical analysis.

The second approach to determine the worker's wage is quite straightforward and closely related to the second matching arrangement-the directed search. In this approach, the wage is not decided by the bargain process between the worker and the firm after matching, but pre-posted and published by the firm who has an opening. If we assume that the firm has to commit to its posted wage, i.e. the firm cannot break the deal once the worker is attracted to this firm, then the firm plays the role of a market dealer.

1.3 Relations between two-sided search and Chapter two & Chapter three

Combining the first arrangement for matching and the first arrangement for wage determination, we thus have the two-sided search model of “Random search and Bargaining”. In Chapter two, I will apply this version of search model to analyze the Pre-IPO interaction between private firms and investment banks, where many IPO-related phenomena such as IPO short-run underpricing and IPO long-run underperformance are tightly linked by one unified searching framework.

Combining the second arrangement for matching and the second arrangement for wage determination, we have the two-sided search model of “Directed search and Posting”, also called “Competitive search” in literature. In Chapter three, I will apply this version of search model to re-interpret the existence of the bid-ask spread for both centralized asset market and decentralized asset market. Different from the asymmetric information based explanation originated from two types of investors, our search based model emphasizes that since the market dealer provides necessary liquidity to the security market via playing such an intermediary role between actual buyers and sellers, the bid-ask spread charged thereafter should largely be justified as the compensation for the market dealer’s endeavor in this process.

References

Diamond, Peter A. [1981]. Mobility Costs, Frictional Unemployment, and Efficiency. *Journal of Political Economy*. 89(4), 798-812.

Diamond, Peter A. [1982]. Aggregate Demand Management in Search Equilibrium. *Journal of Political Economy*. 90(5), 881-894

Duffie, Darrell, et.al. [2002]. Securities lending, shorting, and pricing, *Journal of Financial Economics*. 66(2-3), 307-339.

Gronau, Reuben. [1971]. Information and frictional unemployment. *American Economic Review*. 61(3), 290-301.

Lagos, Ricardo et.al. [2007]. Search in asset markets: market structure, liquidity, and welfare. *American Economic Review*. 97(2), 198-202.

McCall, John J. [1970]. Economics of information and job search. *Quarterly Journal of Economics*. 84(1), 113-126.

Moen, Espen R. [1997]. Competitive search equilibrium. *Journal of Political Economy*. 105(2), 385-411.

Mortensen, Dale T. [1970]. A theory of wage and employment dynamics. *Microeconomic Foundations of Employment and Inflation Theory*.

Pissarides, Christopher A. [1985]. Short-run equilibrium dynamics of unemployment vacancies and real wages. *American Economic Review*. 75(4), 676-690.

Rogerson, R., Shimer, R., Wright, R., [2005], *Search-theoretic models of the labor market:*

A survey, *Journal of Economic Literature* XLIII, 959–988.

Shimer, Robert. [1996]. Contracts in frictional labor markets. MIT. Mimeo.

Weill, Pierre-Olivier. [2003]. Learning against the wind. Stanford University. Mimeo.

http://en.wikipedia.org/wiki/Diamond_coconut_model

Chapter Two: Searching in the Pre-IPO Market – Interaction between Private Firms and Investment Banks

ABSTRACT

In this chapter, we propose a search-based theoretical model to describe the pre-IPO market searching and matching process between private firms with intent to sell equity in an IPO and investment banks (IB) that underwrite the issue. Due to the wide existence of market search frictions, necessary time is required in order to form a strategic pair between a private firm and an investment bank for a successful IPO. Our model tightly links many IPO-related phenomena under one unified searching framework. We derive a closed-form formula for the investment bank's share of profit from an IPO transaction at the market equilibrium. The calibrated simulation result for this value is consistent with the "*seven percent solution*" initially identified by Chen and Ritter (2000). Our model suggests that IPO underpricing is not a deterministic but an empirical normality mainly originating from the market-wide co-movement between the total proceeds and the total number of successful IPOs. Our model also shows that the existence of the reservation initial offer price seems to be the sole driving force behind IPO underperformance, which also resolves the puzzle why quantity adjustment is more frequent than price adjustment in an IPO issuing process.

Key words: random search, pre-IPO market, IPO underpricing, IPO underperformance

2.1 Introduction

In corporate finance, the research effort devoted to initial public offerings (IPO) is unremitting not only because going public represents a critical stage for a privately owned firm's growth, namely broader financing channels along with more diversified control and ownership structures, but several intriguing phenomena observed in IPO processes, have not been fully explained. For example, Ritter and Welch's review paper (2002) and Carter et al. (1998) summarize three such stylized phenomena related to IPO: cyclicalities of the IPO market, IPO short-run underpricing and IPO long-run underperformance.

A typical IPO process consists of three stages: pre-IPO stage, IPO, and post-IPO stage. While enough literature puts great emphasis on IPO and post-IPO stages, *the pre-IPO stage is largely unexamined*. In another word, the interaction between private firms and investment banks in the pre-IPO market has not yet been studied in a systematic way. During the prolonged pre-IPO stage, there obviously exist intense searching and matching activities between private firms and investment banks. When any privately owned firm attempts to sell its equity for the first time at the primary security market, as its first move, it has to search a proper leading investment bank to underwrite the issue (Even for Google which uses Dutch auction as its IPO method, it still has Morgan Stanley and Credit Suisse First Boston as its underwriters). When choosing an underwriter, the firm needs to consider multiple factors, including its own characteristics such as the issue's size, the industry in which the firm operates, and the investment bank's characteristics such as the underwriter's reputation and expertise, etc. On the other hand, the investment bank has its own criteria to select which firm's issue to underwrite (and then market) under the possible constraint of limited human and financial resources in the house, not mentioning the aftermarket price stabilization role played by the investment bank.

When exploring the underlying causes of many intriguing IPO-related problems, we posit that concentrating on the pre-IPO stage could better reveal important findings as the order of events matters in this sequential game. A better understanding of the dynamic searching process of pre-IPO stage would enhance our analysis of IPO in the financial market. This thus motivates the current study of the modeling of the pre-IPO market searching behaviors of private firms and investment banks.

Furthermore, due to the complexity of IPO related activities, it is difficult, if not impossible, *to design one systematic framework to explain all IPO linked phenomena*. The challenge of designing a once- for- all IPO model mainly stems from the “three- player problem”, i.e. there are three key representative players who are getting involved in an IPO transaction, namely, the issuing private firm, the investment bank, and the outside general investors. However, if we only focus on the pre-IPO stage, the “three-player problem” can be transformed into the “two-player problem”, which will significantly reduce the degree of complexity of our analysis. This simplification makes it possible for IPO related phenomena be unified by the same underlying model without losing any significance. In addition, another advantage of the emphasis on the pre-IPO stage is that such questions as when private firms decides to go public and how intensively investment banks compete in the underwriting industry can be easily answered along the way as well.

While asymmetric information based models seem to dominate the mainstream of IPO modeling literature (actually most of corporate finance theoretical models can be traced back to asymmetric information models), our model established here is to address the interaction between private firms and investment banks in the pre-IPO stage from a totally different viewpoint, i.e. search-based angle. Asymmetric information models stereotypically assume that

in any relevant game structure, there always exists one type of player who knows more than the others (the information frictions). Depending on the comparative information advantage of the different types of players, researchers can thus easily borrow such basic frameworks from economics as the “principal-agent” model if the investment bank has more information than the issuing private firm, the “moral hazard” model if the strategic pair composed of the investment bank and the issuing private firm knows more than the outside general investors, and finally, the “winner’s curse” model when some investors (informed investors) are better informed than the other investors (uninformed or noise investors), to explain the IPO related phenomena. (Since our model solely focuses on the pre-IPO stage, the asymmetric information mentioned before specifically means the *information friction* only between private firms and investment banks.) However, our model puts sole stress on the *market search friction*, which doesn’t mean that the issue of the asymmetric information between private firms and investment banks is not important. Unfortunately we, however, avoid it on purpose here to emphasize more important searching and matching characteristics of the market equilibrium that has largely been ignored in traditional corporate finance literature. Using the searching and matching theory borrowed from macroeconomics, we have the ability to investigate the interaction of the many private firms and the many investment banks in the pre-IPO market, while a typical asymmetric information model is more targeted at the one- to- one bargaining process between issuing private firms and investment banks without a full consideration of the pressure from “peers”.

The fact that the success of IPO depends not only on the effort of the private firm but also on the professional support of the investment bank has a far-reaching effect on the modeling of the pre-IPO market behaviors of both agents and will further complicate the private firm’s IPO timing under uncertainty. In another word, even though the financial market demand condition

(e.g. investor sentiment is high) is permitted, it still takes time (and/or any other resources) for a private firm to locate a proper investment bank when the cooperation of both of them is of necessity to accomplish an IPO transaction. From the practical viewpoint, market search frictions exist more or less in any IPO process since there is simply no such thing as a 100 percent centralized market where two types of agents can meet and proceed instantly.

Thus, in this paper, we construct a *search-based theoretical model* to describe the *pre-IPO market* searching and matching process between private firms with intent to sell equity in an IPO and investment banks (IB) that underwrite the issue. We expect that our model can amend the lost chain in current IPO studies. Our main theoretical contribution to IPO literature is that we introduce the concept of market search frictions between private firms and investment banks to the modeling of the pre-IPO market optimal behaviors of both agents.

Firstly, our model solves the puzzle why quantity adjustment is more frequent than price adjustment in IPO issuing process. Ritter and Welch (2002) state that: *“Although offer prices are lowered, many firms withdraw their offering rather than proceed with their IPO. In other words, why is there quantity adjustment, rather than price adjustment? This is a puzzle not only for the IPO market, but for follow-on offerings as well.”* Our model indicates that the private firm has a *reservation initial offer price* in mind when going public. This reservation initial offer price is ultimately determined by the market searching structure represented by two conditions, the investment bank’s free-entry condition and the investment bank’s profit condition. Due to the existence of this reservation initial offer price, the initial offer price is downward-inelastic when touching on its reservation. Any suggested initial offer price which is less than its reservation initial offer price could only lead to no IPO.

Secondly, our model provides a “search-based” explanation why initial offer prices only partially adjust up in response to the positive public information during the IPO price formation stage (or book-keeping stage). Our model points at the investment bank as the main culprit of this incomplete adjustment of initial offer prices. Our model shows that the full adjustment of the initial offer price to the positive public information always better the welfare of the issuing private firm, but not always for the investment bank. It is probable that the investment bank’s share of proceeds will shrink along with the increase in the initial offer price. Intuitively, due to the observed market-wide co-movement between the initial offer price and the success arrival rate of IPO, while the increased initial offer price means that the size of the “IPO pie” becomes larger, it also reduces the investment bank’s proportion of the “IPO pie” much more seriously since going public for the private firm becomes less difficult (i.e. the faster success arrival rate of IPO), which will significantly weakens the professional role played by the investment bank in an IPO transaction. To one extreme, if the success arrival rate of IPO goes to infinity in the IPO market and thus is not a constraint for any IPO transaction, the investment bank will become totally useless, the effect of which on the investment bank’s share of proceeds from the “IPO pie” is equivalent to the decrease in the investment bank’s bargain power over the issuing private firm.

The phenomenon of the partial adjustment of initial offer prices to the positive public information is closely related to IPO short-run underpricing. IPO underpricing, the persistent positive significant average first day IPO returns has drawn a voluminous research attempting to explain this observation. Logue (1973) and Ibbotson (1975) are among the first to record that share prices tend to increase substantially in the first day of trading. While there is a significant variation in the first day returns over time and by country during the period of 1960 to 2010 on average, the closing price at the end of the first day of trading for IPO shares is 16.8 percent

above the offered price, which implies that a very large amount of “money left on the table” (Loughran and Ritter, 2002). Based on a shorter period of time underpricing is less prominent, but still economically significant. Loughran and Ritter (2004) report an average initial return during the period of 1980-2003 of 6.3%, with the highest underpricing of 32.3% observed in 1999-2000. To some extent, IPO underpricing can be simply measured by the difference between the first day closing price (which is largely affected by the current market sentiment of general investors) and the initial offer price (which is agreed upon by both the issuing private firm and the underwriter). Although our model never touches on the first day closing price of an IPO, we can reasonably assume that the positive public information will be fully reflected in the first day closing price. However, the same positive public information can only result into less amount of adjusting-up in the initial offer price under the influence of the investment bank, which will lead to IPO underpricing in the case that the first day close price is equal to the (imagined) initial offer price; or much severer magnitude of already-existing IPO underpricing in the case that the first day close price is in the first place higher than the (imagined) initial offer price.

Thirdly, our model explains why there exists an initial overpricing for IPO firms with respect to comparables non-IPO firms, which is tightly related to another important issue in the IPO research, the long-run post-issue underperformance. Ritter (1991) is the first to show that in the three years following the offering IPO firms underperformed significantly comparable firms from the same industry and with similar size. More recently Ritter and Welch (2002) examine 6249 IPOs during 1980-2001 and record that IPO issuers have a three-year return, which is 5.1% lower than comparable firms. During the period of 1980-2009, if a typical investor bought IPO shares at the first-day closing price and held them for three years, the average IPO would underperform the CRSP value-weighted market index by 19.7 percent and similar companies

matched by market capitalization and book-to-market ratio by 7.3 percent. Ritter (1991) explains this phenomenon with investors being overoptimistic about the future earnings of the IPO firms, and with firms taking advantage of market conditions. Subsequent studies (Brav and Gompers (1997), Brav, Geczy and Gompers (2000), Eckbo, Masulis and Norli (2000) and Eckbo and Norli (2005)) present evidence that long-term underperformance is consistent with the small growth firms exhibiting lower returns (Fama and French (1992)), or the failure of CAPM to explain returns for such firms.

Purnanandam and Swaminathan (2001) find that when initial offer prices are used, IPO firms are priced about 50 percent above comparables. Although they suggest that this initial overpricing with respect to comparables (this is a totally different concept from IPO underpricing.) helps predict long-run post-issue underperformance, they don't explain why there exists this initial overpricing in the first place. Our model points out that the existence of the reservation initial offer price for the private firm seems to be the sole driving force behind IPO underperformance. Only if the realized initial offer price is larger than the private firm's reservation price, the private firm will agree to go public. Therefore all observed initial offer prices in the IPO market will form a left-truncated distribution when compared to the original distribution consisting of both observed and potential initial offer prices together. Since the mean of the left-truncated distribution is always larger than the mean of the original distribution, the existence of initial overpricing for IPO firms is obvious.

Lastly, our search-based model offers an alternative innovative approach to studying the private firm's decision to go public and the dynamics of IPO activities without the assumption of asymmetric information, while the patterns of IPO activities are traditionally explained by the market-timing theory initially developed by Lucas and McDonald (1990), who study the firm's

decision to issue equity when information asymmetry regarding the value of assets in place exists. Based on the market-timing view issuers tend to postpone equity sales until good market conditions, when they are able to sell overpriced equity. This notion has been supported by Baker and Wurgler (2000) who document that equity issues are negatively correlated with future market returns.

This paper exemplifies one application of search theory in corporate finance. In macroeconomics and labor economics search theory is widely used to explore the matching behavior between workers and firms (Diamond (1984); Mortensen and Pissarides (1994); Jacquet and Tan (2007); Shimer (2007); Menzio (2007)). Although search theory is popular in economics represented by the fact that the 2010 Nobel prize in economics was jointly awarded to Peter A. Diamond, Dale Mortensen, and Christopher A. Pissarides won "for their analysis of markets with search frictions" and the 2012 Nobel prize in economics to Alvin E. Roth and Lloyd S. Shapley "for the theory of stable allocations and the practice of market design", it has been not used in the corporate finance literature, with very few exceptions. Silveira and Wright (2007) propose a search-based model to study the venture capital cycle. In their model, the capitalists (with funds) and entrepreneurs (with technical expertise) are searching in a decentralized venture capital market. They analyze the duration of each phase in the cycle and the flow of funds into the market.

Duffie, Garleanu and Pedersen (2002, 2005 and 2007), and Lagos (2007) are pioneers to introduce search theory to dynamic asset markets. Vayanos and Weill (2008) propose a search-based model to explain the on- the- run phenomenon in the over-the counter (OTC) fixed income markets. Their model shows that assets with identical cash flows can trade at different prices due to the existence of short-sellers and search frictions in the spot and the repo markets.

This paper is also related to but fundamentally different from Fernando et.al. (2005)'s matching model in which issuers and underwriters associate by mutual choice and matches are based on firms' and underwriters' relative characteristics at the time of issuance. But they never put the market search frictions into consideration when modeling the searching and matching process in the pre-IPO market, thus losing a track of the general picture of IPO processes.

The rest of this paper proceeds as follows: section 2.2 describes the pre-IPO search model; section 2.3 mathematically shows model structures of two cases and discusses major theoretical results derived from them; section 2.4 discusses empirical implications of our proposed model and simulation results; section 2.5 concludes the paper. Symbols and notations are summarized in Appendix A. Proofs of propositions and lemmas are provided in Appendix B.

2.2 A pre-IPO search model

In this section, we describe a stylized pre-IPO market including homogeneous private firms (denoted by f) whose final aim is always to go public at a good timing, and homogeneous investment banks (denoted by b), the support of which is of necessity for the success of an IPO. The initial number of private firms is normalized to 1 and the initial number of investment banks is n (the value of n is usually much smaller than 1). We assume that at a given time period each private firm (f) can only hire one investment bank (b) to underwrite its IPO and each investment bank can only serve one private firm customer.

Those two types of agents are continuously meeting with each other according to a standard Poisson process with meeting rates of α_f and α_b , respectively. Hence, on average, during each period each private firm will meet α_f number of investment banks and each investment bank will meet α_b number of private firms. The values of α_f and α_b cannot be infinite, which clearly

characterizes the presence of the search friction existing in the pre-IPO market. The values of α_f and α_b will ultimately depend on the relative number of private firms and investment banks in the market, i.e. the market tightness. The reciprocals of α_f and α_b ($1/\alpha_f$ and $1/\alpha_b$) thus represent the expected meeting time, accounting for not only the time spent on searching, but also the time consumed in the negotiation process by the two agents. It is optional to extend the model further by allowing for some type of hindrance originated from asymmetric information reflected in the magnitude of these two parameters.

The private firm and the investment bank simultaneously decide whether to form a strategic pair or not when meeting with each other. If either agent doesn't agree to form a pair, there will not be an IPO later. Some reasons for a private firm to decline to form this pair include: the private firm waits for another better offer from another investment bank; or the private firm waits for another good timing to go public. The same logic applies to the consideration of an investment bank. If the private firm and the investment bank both agree to form a strategic pair, the investment bank requires a profit of k , representing any underwriting related service fees such as the commission fee and other un-named benefits, and the private firm requires the residual part of the total proceeds, i.e. $R-k$, both due when the IPO with the total proceeds of R succeeds in the future. **In the proposed model, we assume that each firm can only issue one share of stock. So we can ignore the problem of how many shares will be outstanding for an IPO. In this way, from now on the total proceeds of R can be considered as the initial offer price of an IPO as well.**

The investment bank's profit k is the result of bargaining between the private firm and the investment bank when meeting with each other. We utilize the generalized Nash bargaining scheme to pin down the value of k , assuming that the investment bank's bargaining power is

characterized by a parameter θ . The value of θ falls in a range between 0 and 1. When θ approaches to 1 indicating that the investment bank has a higher bargaining power over the private firm and therefore, it can claim a larger amount of the profit of k from any fixed amount of the total IPO proceeds of R ; however, when θ approaches to 0, the opposite happens.

We assume that the occurrence of successful IPOs follows another standard Poisson process with a success arrival rate of σ , i.e. on average during each period there are σ number of successful IPOs among all proposed IPOs. The value of σ cannot be infinite either, which implies the concern that any IPO promoted by a strategic pair formed by a private firm and an investment bank is not guaranteed to be successful in the real world. According to a report of a consulting firm Dealogic, nearly 300 initial public offerings, valued at almost \$60 billion, were withdrawn in 2008. That's almost double the number from 2007. Generally speaking, the current macroeconomic environment, the financial market condition and even the advertising effort of investment banks can all influence the magnitude of σ , e.g. the looser the credit policy of the Federal Reserve System, the more optimistic the current stock market and the more intense the investment banks' underwriting activities, and hence the higher the value of σ . Once an IPO succeeds, the investment bank will return to the market and the private firm will exit the market. Moreover, a clone of the private firm will refill the market to keep the market equilibrium in the language of search theory.

In addition, we assume that both types of agents are risk neutral and the market on-going (risk-free) discount rate is denoted as r , which characterizes the time preference of private firms and investment banks.

In sum, the entire pre-IPO process can be illustrated by Figure 1. In the pre-IPO market, private firms and investment banks can stay in two distinguished states: the searching state where private firms and investment banks meet and negotiate with each other and the pair state where the strategic pair formed by one private firm and one investment bank waits for the success of the IPO.

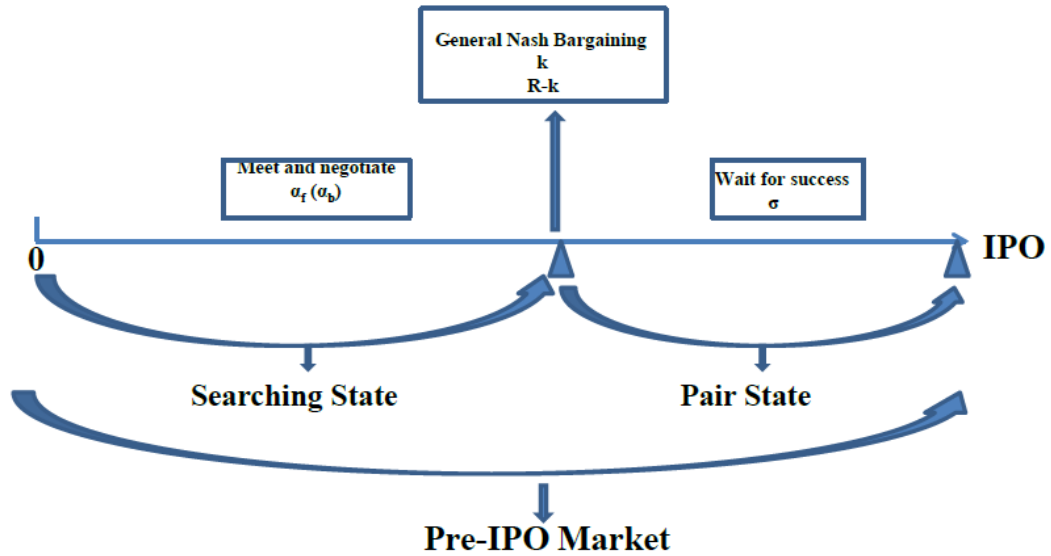


Figure 2.1 The schematic of the pre-IPO market

Since there are two types of agents (b denotes the investment bank and f denotes the private firm) and two states (0 indicates the searching state and 1 indicates the pair state), we thus define four state value functions:

V_f^0 : the value of a private firm who is searching an investment bank in the market;

V_f^1 : the value of a private firm who forms a strategic pair with an investment bank;

V_b^0 : the value of an investment bank who is searching a private firm in the market;

V_b^1 : the value of an investment bank who forms a strategic pair with a private bank.

These four value functions represent corresponding “utilities” or “welfares” obtained when staying in those two states for those two types of agents, respectively. We will compare the relative magnitude of value functions for each agent in two states so as to predict the optimal behavior of them.

2.3 Mathematical model and discussion

In this section, we apply the basic search equations to analyze the pre-IPO process between private firms and investment banks. Before going to the model in any details, we need to further clarify the role of the total proceeds or the initial offer price (R) supported by the current financial market. We assume that R is a random variable whose value is known only when private firms and investment banks meet with each other, but whose distribution is a common knowledge for both agents at the beginning of this game. (To be noted that R will be realized at the end of the game when the IPO succeeds.) Although private firms and investment banks are *ex ante* homogeneous, different meetings can lead to different values of R . Basing on the assumption of the distribution of R , the complication of our search-based model alters significantly. We will consider two cases in this paper. Firstly, we let the distribution of R be degenerated to a point, i.e. R is a known constant. In this case, we mainly derive a closed-form formula for the investment bank’s share of profit from an IPO transaction at the market equilibrium and discuss its theoretical implications. Secondly, we let the distribution of R be a general one with a cumulative distribution function of $F(R)$. In this case, we propose the concept of the reservation initial offer price and derive the two market equilibrium conditions to pin down its value. More complicated case can assume that a minimum value of R can be imposed on and thus any probability of realizing a value of R lower than that would in the first place

discourage the formation of a pre-IPO strategic pair between private firms and investment banks. In reality, this restraint may reflect legally minimum initial capital requirements listed in a stock exchange.

Case one: R is a known constant

(1) Basic model set-up

Since there are two types of agents, private firms and investment banks who are continuously searching in the Pre-IPO market, the interaction between them is modeled as a *two-sided search*, in marked contrast to a *one-sided search* where only one type of agents is actively searching in the market.

For this case, there doesn't exist any uncertainty about the total proceeds or the initial offer price(R). Both agents know the exact value of R before playing the game. Assuming the market prevalent value of the investment bank's share of profit is k^* , the four value functions defined in Section 1 satisfy the below four search equations:

$$r V_f^0 = \alpha_f (V_f^1 - V_f^0) \tag{2.1}$$

$$r V_f^1 = \sigma (R - k^* - V_f^1) \tag{2.2}$$

$$r V_b^0 = \alpha_b (V_b^1 - V_b^0) \tag{2.3}$$

$$r V_b^1 = \sigma (k^* + V_b^0 - V_b^1) \tag{2.4}$$

All four equations have the similar structure: the left hand side is called the flow value, which is always the product of the discount rate and the value for each specific state; the right

hand side is the expected value change from the agent's current state, which is the product of the state-changing rate (such as α_f , α_b and σ) and the change in the value of the agent's current state.

For instance, for Equation (2.1), the left hand side represents the flow value for a private firm who is searching an investment bank in the market; the right hand side is the private firm's expected value change jumping from the searching state to the pair state. In the same way, for Equation (2.2), the left hand side represents the flow value for a private firm who forms a strategic pair with an investment bank; the right hand side is the private firm's expected value change jumping from the pair state to the searching state.

We then define two surplus functions, S_f and S_b , for private firms and investment banks separately as below. Those two surplus functions are used in the private firm and investment bank's bargaining process to form an objective function under the framework of generalized Nash bargaining scheme.

$$S_f = V_f^1 - V_f^0 \quad (2.5)$$

$$S_b = V_b^1 - V_b^0 \quad (2.6)$$

Given the market prevalent value of the investment bank's share of profit k^* and the investment bank's bargaining power θ , we apply the generalized Nash bargaining scheme to divide the initial offer price (or the total proceeds) R between the private firm and the investment bank through solving a Cobb-Douglas like utility maximization problem with one choice variable of the investment bank's share of profit k . Although it does not provide the detailed bargaining process, the generalized Nash bargaining scheme is simple and convenient to use for theoretical analysis.

$$\text{Max } S_f^{1-\theta} S_b^\theta \text{ by choosing } k. \quad (2.7)$$

The market equilibrium requires that the market prevalent value of k^* be squarely consistent with the investment bank's share of profit k resulted from the general Nash bargaining scheme:

$$k=k^* \quad (2.8)$$

Linking the above eight equations from Equation (2.1) to (2.8), we can solve for eight variables ($V_f^0, V_f^1, V_b^0, V_b^1, S_f, S_b, k, k^*$) as a function of six model parameters ($\alpha_f, \alpha_b, r, \sigma, \theta, R$).

When we compare the relative value of V_f^0 and V_f^1 , the private firm's value functions in the two states, we can thus predict the private firm's decision on going public. The behavior rule is that if V_f^1 is larger than V_f^0 , i.e. the value of the private firm forming a strategic pair with the investment bank is larger than the value of it staying in the searching state, the private firm will go public; otherwise it will not. The private firm always prefers to stay in the state with the higher "utility" or "welfare".

According to Equation (2.1), we can find that $V_f^0 = \frac{\alpha_f}{\alpha_f + r} V_f^1$. As long as the discount rate r is positive, V_f^1 is always larger than V_f^0 , i.e. the private firm's value in the pair stage is always larger than that in the searching stage. Thus the private firm's best strategy is always to go public once the chance arrives. It is unnecessary for any private firm to wait since the initial offer price or the total proceeds R is a known fixed constant and there will no better offer in the market purely because of patience. This observation is summarized in Proposition 1.

Proposition 1: *If R is a known constant, it is meaningless for a private firm to delay going public. The private firm's best strategy is always to go public once the chance arrives.*

Further calculation provides a closed form formula for the investment bank's share of profit k from an IPO transaction at the market equilibrium (Please see Appendix B for the detailed derivation). As the market equilibrium condition, k is also equal to k^* , representing that the investment bank's share of profit (k) resulted from the general Nash bargaining scheme is in agreement with the market prevalent value of (k^*). Proposition 2 summarizes this important result.

Proposition 2: *If R is a known constant, the investment bank's share of profit k at the market equilibrium is expressed by Equation (2.9)*

$$k = k^* = \frac{\theta r(r + \sigma + \alpha_b)}{\theta r(r + \sigma + \alpha_b) + (1 - \theta)(r + \sigma)(r + \alpha_f)} R \quad (2.9)$$

Comparative statics analysis can be applied to Equation (2.9) to study the impact of different parameters on the investment bank's share of profit at the market equilibrium. We will focus on those five key parameters: α_b , α_f , θ , σ and R . When doing partial derivatives on Equation (2.9), we find that $\frac{\partial k}{\partial \alpha_f} < 0$ and $\frac{\partial k}{\partial \sigma} < 0$ while $\frac{\partial k}{\partial \alpha_b} > 0$, $\frac{\partial k}{\partial \theta} > 0$ and $\frac{\partial k}{\partial R} > 0$. Those results are included in Proposition 3. Intuitively, α_f and α_b characterize the market condition for the searching state. If an investment bank can meet more private firms during each time period, (corresponding to the case that a private firm can meet fewer investment banks during the same time period,) the market searching condition is benign to the side of investment banks who will obtain more share of profit for each successful IPO transaction. In the meantime, σ typifies the market condition for the strategic pair state. The faster the success arrival rate of IPO, the less important the role played by the investment bank in an IPO transaction. In the extreme case that the success arrival rate of IPO goes to infinity, the private firm even does not need the expertise

from the investment bank in order to go public. Regarding to this logic, the increase in the success arrival rate of IPO is equivalent to the decrease in the investment bank's bargaining power. Thus the investment bank's share of profit should decrease with the increase in the value of the success arrival rate of IPO (σ). In addition, if the initial offer price (R) becomes larger and the investment bank has more bargaining power (θ) over the private firm, that the investment bank will acquire more share of profit is obvious.

Proposition 3: *If R is a known constant, ceteris paribus, the investment bank's share of profit k at the market equilibrium is: (1) positively related to the meeting rate of the investment bank to the private firm (α_b); (2) negatively related to the meeting rate of the private firm to the investment bank (α_f); (3) positively related to the bargaining power of the investment bank (θ); (4) negatively related to the success arrival rate of IPO (σ); and (5) positively related to the initial offer price (R).*

One more important issue is associated with the *net effect* of the initial offer price (or the total proceeds) R on the investment bank's share of profit k at the market equilibrium. Proposition 3 only shows that, *all else equal*, the investment bank's share of profit is positively related to the initial offer price according to the positive sign of the *partial derivative* of k with respect to R (i.e. $\frac{\partial k}{\partial R} > 0$), which means that the higher the initial offer price of IPO, the more profit the investment bank will earn from its service; moreover, the ratio of that profit to the initial offer price does not change along with the initial offer price (i.e. $\frac{k}{R} = \text{fixed}$).

However, Proposition 4 is concerned with the *total derivative* of k with respect to R under the assumption that the success arrival rate (σ) of IPO is positively related to the initial offer price (R), which is reasonable since in reality we do observe a market wide co-movement

between the total proceeds of IPOs and the total number of successful IPOs for each period. For instance, during the boom of the IPO market, comparatively high initial offer prices are always accompanied by a large number of successful IPOs while relatively low initial offer prices follow a small number of IPOs during the bust period.

Specifically, let σ be an increasing function of R , i.e. $\sigma = \sigma(R)$ and $\frac{d\sigma}{dR} > 0$, the sign of $\frac{dk}{dR}$, i.e. the net effect of R on k cannot be determined without ambiguity since we have to consider two conflicting effects together now. The first effect is the “*size effect*”, also called the direct effect of R on k , which has already proved to be positive in Proposition 3. The second effect is the “*ratio effect*”, also called the indirect effect transmitted through σ when R changes, which is negative insofar as the faster success occurrence of IPOs impairs the role of “middle-man” played by investment banks and reduces the profit earned by investment banks from the underwriting service. If the negative ratio effect dominates the positive size effect, the investment bank’s share of profit will decrease when the initial offer price increases.

Although the similar procedure can be applied to the analysis of the net effect of the initial offer price R on the private firm’s share of profit (i.e. the residual part) $R-k$, the conclusion is explicit: the sign of $\frac{d(R-k)}{dR}$ is always positive since not only does the size of the “IPO pie” become larger, but the proportion of the private firm’s share also increases when confronting a higher initial offer price R .

Those results are summarized in Proposition 4:

Proposition 4: *If R is a known constant, under the assumption that the success arrival rate of IPO (σ) is positively related to the initial offer price(R), the relatively lower initial offer price(R) could be beneficial to the investment bank, but not to the private firm.*

Proposition 4 sheds light on IPO short-run underpricing. The crudest measure of IPO underpricing is the difference between the first day closing price (which is largely affected by the post-IPO investor sentiment) and the initial offer price (which is supported by the IPO market and agreed upon by the issuing firm and the underwriter). In other words, either the initial offer price is low or the first day closing price is high or both so as to have the phenomenon of IPO underpricing. While it is out of our reach to explain why the first day closing price is high, our search-based model can explain why the initial offer price could be relatively lower than it should have.

IPO underpricing is closely related to the observed phenomenon that initial offer prices only partially adjust to positive public information during IPO book keeping. In reality, the formation of the initial offer price is a rather complicated and dynamic process. Let's do a mental experiment in the pre-IPO stage to illustrate this process. As the starting point, the first day closing price (P_0) is imagined to be equal to the initial offer price (R_0). So there will squarely be no underpricing according to our simple definition of IPO underpricing above ($P_0 - R_0 = 0$). Suppose that some positive public information is released during the IPO book-keeping process. If the positive public information is expected to lead to the same amount of increase (ϵ) in the new first day closing price ($P_1 = P_0 + \epsilon$) and in the new initial offer price ($R_1 = R_0 + \epsilon$), then there will still be no underpricing since $P_1 - R_1 = (P_0 + \epsilon) - (R_0 + \epsilon) = P_0 - R_0 = 0$. This type of IPO price formation is called the fully adjustment of the initial offer price in response to the positive public information. However, if the new initial offer price is only partially adjusted up, there will be an

underpricing, which is what we most often observe in the IPO market. For instance, when the new first day closing price increases by ε and the new initial offer price only increases by 0.5ε , there will be an underpricing of 0.5ε since $P_1 - R_1 = (P_0 + \varepsilon) - (R_0 + 0.5 \varepsilon) = 0.5 \varepsilon > 0$.

Proposition 4 implies that the driving force of IPO underpricing mainly comes from investment banks' side since the relatively lower initial offer price (R) could be beneficial to the investment bank, but not to the private firm. Borrowing our previous analysis of the size effect and the ratio effect when the initial offer price R increases, the maximum achievable value of the initial offer price is not always an optimal choice from the viewpoint of the investment bank, while it is always better for the private firm to face a higher initial offer price if the ratio effect is dominating the size effect.

Another issue related to Proposition 4 is the uncertainty of IPO underpricing. As we have discussed before, we cannot deny the capacity of our model to interpret IPO underpricing. However, we have to admit that IPO underpricing is more as an empirical or statistical phenomenon than a theoretical one since our model also shows that when the size effect dominates the ratio effect, the full-adjustment of the initial offer price or even the over-adjustment of it in respond to positive public information is highly probable. In reality, we do observe that some IPOs show "over-pricing" (i.e. the first day closing price is lower than the initial offer price.) even though the frequency of those events is rare. As one of the most recent examples, Facebook (FB) priced its IPO at \$38 per share on May, 17, 2012. However, the average of the first five-day closing prices of FB is only \$33.66 per share, which represents a negative return of -11.42%.

(2) Endogenization of two meeting rates of α_f and α_b

The above content is basically a partial equilibrium search model with the two meeting rates of α_f and α_b as parameters of the model. More interestingly, the meeting rates of α_f and α_b can be endogenized once we apply the balanced steady state flow condition to the system shown in Figure 2.2.

Recall that initially the number of private firms is normalized to 1 and the number of investment banks is n . Let the number of the strategic pairs be m at the steady state, thus the number of private firms is $1-m$ and the number of investment banks is $n-m$ at the steady state then. The balanced steady state flow condition requires that:

$$(1-m) \alpha_f = \sigma m = (n-m) \alpha_b \tag{2.10}$$

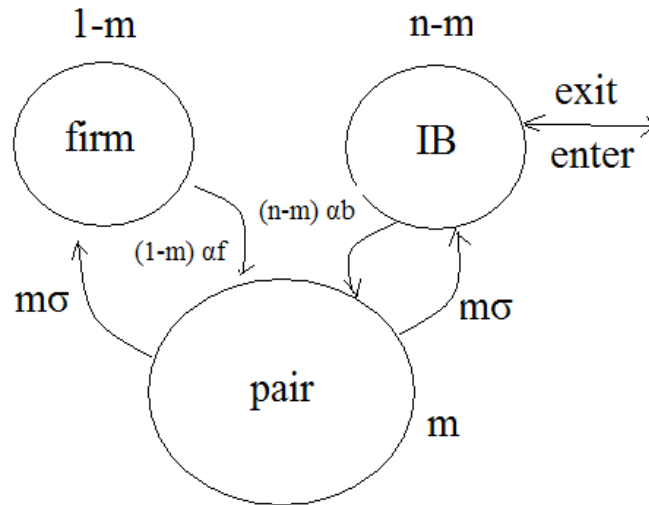


Figure 2.2 The balanced steady state flow condition in the pre-IPO market

Define the market tightness (MT) as the relative number of investment banks to private firms at the steady state:

$$MT = \frac{n-m}{1-m} \quad (2.11)$$

The matching technology between private firms and investment banks is abstracted in a matching function denoted as π that depends on the numbers of both types of agents in the pre-IPO market. Assuming that π has a constant rate of return with respect to those two numbers and has a functional form in Equation (2.12) (δ is a parameter in the function), the meeting rates of α_f and α_b can thus be expressed as a function of the market tightness MT in Equation (2.13) and (2.14):

$$\pi = \pi(1-m, n-m) = (1-m)^{1-\delta}(n-m)^\delta \quad (2.12)$$

$$\alpha_f = \pi/(1-m) = (1-m)^{-\delta}(n-m)^\delta = MT^\delta \quad (2.13)$$

$$\alpha_b = \pi/(n-m) = (1-m)^{1-\delta}(n-m)^{\delta-1} = MT^{\delta-1} \quad (2.14)$$

To close up our model, we need to assume the free entry for investment banks to the underwriting industry, which requires that the value of investment banks searching in the pre-IPO market be a fixed value of staying out of this market, L :

$$V_b^0 = L \quad (2.15)$$

In sum, this extended general equilibrium search model consists of Equation (2.1)-(2.8) and Equation (2.10)-(2.15). We can solve for thirteen variables ($V_f^0, V_f^1, V_b^0, V_b^1, S_f, S_b, k, k^*, \alpha_f, \alpha_b, n, m, MT$) as a function of six model parameters ($r, \sigma, \theta, \delta, R, L$).

Proposition 5: *if R is a known constant, the entire system is characterized by two equilibrium equations: one is the investment bank's profit condition (2.16) and the other is the investment bank's free entry condition (2.17)*

$$k = k^* = \frac{\theta r(r+\sigma+MT^{\delta-1})}{\theta r(r+\sigma+MT^{\delta-1})+(1-\theta)(r+\sigma)(r+MT^\delta)} R \quad (2.16)$$

$$L = \frac{\theta \sigma MT^{\delta-1} R}{\theta r(r+\sigma+MT^{\delta-1})+(1-\theta)(r+\sigma)(r+MT^\delta)} \quad (2.17)$$

Moreover, the number of IPO pairs at the market equilibrium is:

$$m = \frac{MT^\delta}{\sigma + MT^\delta} \quad (2.18)$$

the number of private firms at the market equilibrium is:

$$1 - m = \frac{\sigma}{\sigma + MT^\delta} \quad (2.19)$$

the number of investment banks at the market equilibrium is:

$$n - m = \frac{\sigma MT}{\sigma + MT^\delta} \quad (2.20)$$

In addition, the initial number of investment banks is:

$$n = \frac{\sigma MT + MT^\delta}{\sigma + MT^\delta} \quad (2.21)$$

Proposition 5 concludes our search based model. One of the key variables determined by this model is the market tightness MT, represented by the ratio of the number of investment banks and the number of private firms. The higher the value of the market tightness is, the more quickly for a private firm to find its underwriting investment bank. The market tightness MT can be solved directly from Equation (2.17). Once MT is resolved, the investment bank's share of profit k can be obtained from Equation (2.16). More importantly, the initial number of investment banks n can also be easily acquired based on Equation (2.21). Since the initial

number of private firms is normalized to 1, the magnitude of the initial number of investment banks will represent the competitive intensity in the underwriting industry. Therefore, Proposition 5 can be used to estimate how many investment banks can be supported in the underwriting industry by the IPO market.

Case two: R is a random variable

The search based model for the pre-IPO market with the initial offer price R being a known constant is systemically analyzed in the above section. Here we discuss the more general case with the initial offer price R being a random variable with a cumulative distribution function(CDF) of F(R). F(R) has a support $[0, \bar{R}]$. Here \bar{R} is the maximum possible value of R.

The basic structure of the four value functions is almost the same as before except that $V_f^1(R)$ and $V_b^1(R)$ are now functions of R. Here \tilde{R} is just a dummy variable for integration.

$$r V_f^0 = \alpha f \int_0^{\bar{R}} \max \{ V_f^1(\tilde{R}) - V_f^0, 0 \} dF(\tilde{R}) \quad (2.22)$$

$$r V_f^1(R) = \sigma [R - k^* - V_f^0(R)] \quad (2.23)$$

$$r V_b^0 = \alpha b \int_0^{\bar{R}} \max \{ V_b^1(\tilde{R}) - V_b^0, 0 \} dF(\tilde{R}) \quad (2.24)$$

$$r V_b^1(R) = \sigma (k^* + V_b^0 - V_b^1(R)) \quad (2.25)$$

Without solving the above complicated equation system, we can infer that $V_f^1(R)$ and $V_b^1(R)$ are both non-decreasing functions of R, the economic meaning of which is that the values(or utilities) to form a strategic pair for both agents increase with the observed initial offer price. More importantly, there should exist a reservation initial offer price R^* at which

$$V_f^1(R^*) = V_f^0 \quad V_b^1(R^*) = V_b^0 \quad (2.26)$$

If the initial offer price observed when meeting with each other is smaller than the reservation initial offer price (i.e. $R < R^*$), then for any agent, the value to be in the strategic pair state will always be less than the value to be in the searching state (i.e. $V_f^1(R) < V_f^0$) since $V_f^1(R) < V_f^1(R^*) = V_f^0$, here the sign “<” comes from the non-decreasing property of $V_f^1(R)$ and the sign “=” comes from the definition of the reservation initial offer price. Thus it is wise to wait a little longer time for a better deal to arrive. Proposition 6 resolves a private firm’s decision on going public:

Proposition 6: *If R is a random variable with a CDF of $F(R)$ (the support of R is $[0, \bar{R}]$), there exists a reservation initial offer price R^* for a private firm, whose optimal behavior rule is that when the realized value of R is larger than R^* , the private firm will go public, when the realized value of R is less than R^* , it will stay private. Moreover, the mean of the left-truncated distribution is always larger than the mean of the original distribution, i.e.*

$$E(R/R > R^*) = \frac{\int_{R^*}^{\bar{R}} R dF(R)}{1 - F(R^*)} > E(R) = \int_0^{\bar{R}} R dF(R).$$

The importance of Proposition 6 cannot be exaggerated. We need to understand its economic implications from two aspects:

(1) Proposition 6 resolves the puzzle why quantity adjustment is more frequent than price adjustment for the IPO market.

Ritter and Welch (2002) state: “Although offer prices are lowered, many firms withdraw their offering rather than proceed with their IPO. In other words, why is there quantity adjustment, rather than price adjustment? This is a puzzle not only for the IPO market, but for

follow-on offerings as well.” Proposition 6’s answer to this puzzle is that: due to the existence of the private firm’s reservation initial offer price R^* , the initial offer price R is downward-inelastic when touching on R^* ; any suggested value of R which is less than R^* only leads to no IPO.

Specifically, according to Proposition 6, the private firm’s optimal behavior rule is that: as long as the initial offer price is higher than its reservation initial offer price, the firm is willing to proceed with its IPO (but still needs the underwriting expertise from the investment bank and the demand support from the general investors in order to finally have a successful IPO), otherwise, there will be no IPO. Suppose that there is an initial offer price which is only slightly higher than the private firm’s reservation initial offer price, but which is not supported by the current IPO market condition. While an alternative lower initial offer price could be suggested by the investment bank, the private firm’s optimal behavior rule indicates that there will be no IPO since this new initial offer price is probably higher than the private firm’s reservation initial offer price.

(2) Proposition 6 also indicates that existence of the reservation initial offer price is the sole driving force behind IPO long run underperformance.

Purnanandam and Swaminathan (2001) find that when initial offer prices are used, IPO firms are priced about 50 percent above comparables. They suggest that this initial overpricing with respect to comparables (this is a different concept from IPO underpricing.) helps predict long-run underperformance. According to Proposition 6, we can interpret the mean of the left-truncated distribution $E(R|R > R^*)$ as the average initial offer price for only private firms who go public and the mean of the original distribution $E(R)$ as the average initial offer price for all private firms, either going public or not. Since the mean of the left-truncated distribution is always larger than the mean of the original distribution, i.e. $E(R|R > R^*) > E(R)$, the initial

overpricing of IPO firms with respect to comparables is rather obvious. Furthermore, in some sense $E(R)$ can also represent a long-run market intrinsic value of a homogeneous private firm in our model in a stock market without growth. As long as we believe that an IPO firm's market value reverts from the average initial offer price $E(R|R > R^*)$ to its long-run intrinsic value $E(R)$, the occurrence of IPO underperformance is clearly understandable.

When applying the similar general Nash bargain scheme (Equation (2.7)), the balanced steady state flow condition (Equation (2.10)), the investment bank's value of staying out of the underwriting industry (Equation (2.15)), and the market equilibrium condition (Equation (2.8)) as those when R is a known constant, the entire system can be reduced into two conditions in Propositions 7.

Proposition 7: *if R is a random variable with a CDF of $F(R)$ (the support of R is $[0, \bar{R}]$), the entire system is characterized by two equations (the investment bank's profit condition and the investment bank's free entry condition) with two key variables (the reservation initial offer price R^* and the market tightness MT).*

$$R^* = \left[\frac{(1-\theta)MT^\delta}{r} + \frac{\theta MT^{\delta-1}}{r+\sigma} \right] \int_{R^*}^{\bar{R}} [1 - F(R)] dR \quad (2.27)$$

$$\theta \sigma R^* = [\theta r + (1 - \theta)(r + \sigma)MT]L \quad (2.28)$$

We will use Equation (2.27) and (2.28) to pin down the reservation initial offer price R^* with calibrated parameters in section 2.4. Given that the market tightness MT is fixed, based on Equation (2.27) only, we can find the effects of r and σ on R^* , which is represented in Lemma 1:

Lemma 1: *If R is a random variable with a CDF of $F(R)$ (the support of R is $[0, \bar{R}]$), the higher the discount rate r and the higher the success arrival rate of IPO σ , the lower the reservation initial offer price R^* for a given market tightness MT .*

Furthermore, recall that the expected length of each agent staying in the searching state can be expressed by $1/\alpha_f$ for private firms and $1/\alpha_b$ for investment banks when R is a known constant. When R is a random variable with a CDF of $F(R)$, the corresponding expected length of each agent staying in the searching state has to be extended from $1/\alpha_f$ to $\frac{1}{\alpha_f[1-F(R^*)]}$ for private firms and from $1/\alpha_b$ to $\frac{1}{\alpha_b[1-F(R^*)]}$ for investment banks, insofar as the agents are willing to form a strategic pair only if the realized initial offer price R is larger than the reservation initial offer price R^* , which is represented in Lemma 2:

Lemma 2: *With the same model structure, the expected lengths of both agents staying in the searching state when R is a random variable are longer than those when R is a known constant. Uncertainty of R reduces the market efficiency.*

2.4 Empirical implications

In this section, we first calibrate the key parameters of our models according to the typical data from IPO markets. Then we combine the theoretical predictions of our model with the simulation results to illustrate the empirical implications of our model. Our results show that our search based model fits well into the real market.

(1) Parameter calibration

Table 1 summarizes the key parameters and their typical values used in our model simulation.

Parameter	Notation	Typical Value
The success arrival rate of IPO	σ	15/month
The parameter in the matching technology function π	δ	0.5
The (risk-free)discount rate	r	0.5%/month
The investment bank's bargaining power	θ	0.5
The meeting rate of an investment bank to a private firm	α_b	10/month
The meeting rate of a private firm to an investment bank	α_f	0.1/month

Table 2.1 key parameters and their typical values for the pre-IPO market

We use the median number of IPOs per month from 1980 to 2011 as the success arrival rate of IPO. So we choose 15 times per month for σ . The reason why we don't use the mean is because the median excludes the extreme effects of the stock market crisis such as 1998-1999 and 2008-2009. Without additional information, we always assume that the matching parameter δ and the investment bank's bargaining power θ are 0.5. While we can use the current risk-free interest rate as the discount rate r in our model, since the current risk-free interest rate is almost zero, we choose the median of monthly 10-year Treasury constant maturities nominal yields from January, 1980 to December, 2011, which is approximately 0.5%/month, as the discount rate applied in our model. Moreover, estimating the values of the two meeting rates α_b and α_f is not an easy task. We apply two approaches here:

In the first approach, we apply Equation (2.11) and (2.13) to estimate those two meeting rates. Assuming that there are initially 1000 private firms and 20 investment banks in a typical pre-IPO market, 10 IPO strategic pairs will be formed during each month. Translated into our model's language, the initial number of private firms is normalized to 1, then the initial number of investment banks n is $20/1000=0.02$ and the number of strategic pairs m is $10/1000=0.01$.

Using Equation (2.11), the market tightness will be $(0.02-0.01)/(1-0.01) = 0.0101$; using Equation (2.13), the meeting rate of a private firm to an investment bank α_f will be $0.0101^{0.5} = 0.1005 \approx 0.1/\text{month}$ and the meeting rate of an investment bank to a private firm α_b will be $0.0101^{-0.5} = 9.95 \approx 10/\text{month}$. As we assume that the matching parameter (δ) is 0.5, α_b will also be the reciprocal of α_f according to Equation (2.13) and (2.14).

In the second approach, we use Equation (2.10) to estimate the meeting rate of an investment bank to a private firm α_b . The basic pre-IPO market structure is that the number of private firms is much larger than that of investment banks. So it is more likely the meeting rate of a private firm to an investment bank is the controlling step for the searching state (please compare the values of α_f with α_b to see this point). In addition, for most IPO cases, when a private firm and an investment bank form a strategic pair, the success of this IPO is almost expected as long as the strategic pair is patient to wait for its turn. Thus we can reasonably assume that the meeting rate of an investment to a private firm is approximately equal to the success arrival rate of IPO when $n-m \approx m$ from Equation (2.9). In sum, the three important rates have such a relationship as: $15 = \sigma \approx \alpha_b \gg \alpha_f = 1/\alpha_b$.

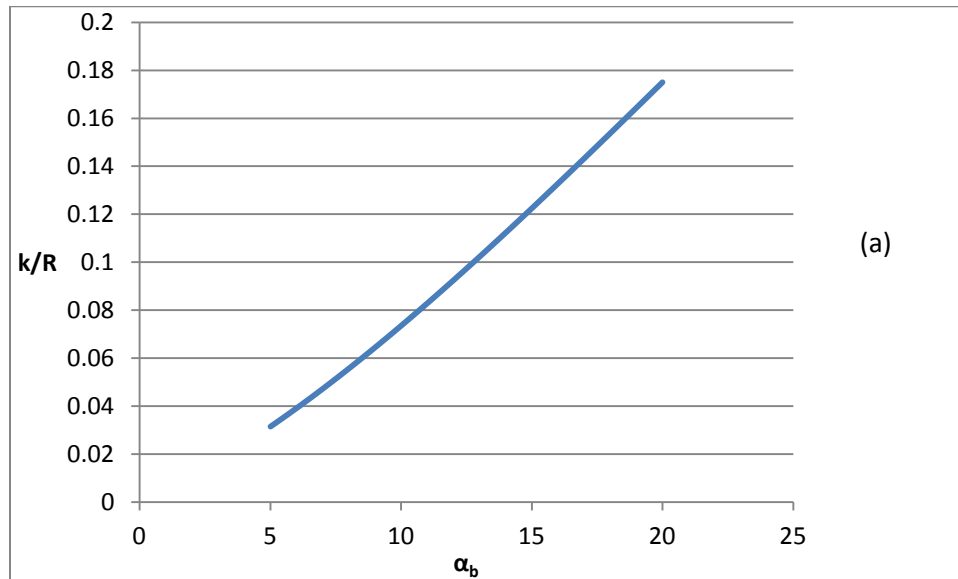
Combining those two approaches, we thus choose 10 times per month as the meeting rate of an investment bank to a private firm. The meeting rate of a private firm to an investment bank will be $1/10 = 0.1$ times per month.

(2) Simulation results when R is a known constant

Before going to the detailed simulation results, our model shows that the investment bank's relative share of profit from an IPO transaction at the market equilibrium (k/R) is 7.35% when simply plugging the calibrated values of model parameters in Table 2.1 into Equation (2.9).

This result is well consistent with the “seven percent solution” initially identified by Chen and Ritter (2000).

Figure 2.3 (a) and (b) show the effects of the market search efficiency (α_f or α_b) on the investment bank’s relative share of profit (k/R) when the values of the other parameters in Table 2.1 are fixed. In Figure 2.3(a), the meeting rate of an investment bank to a private firm changes from 5 times per month to 20 times per month and in Figure 2.3(b) the meeting rate of a private firm changes from 0.2 times per month to 0.05 per month. Consistent with Proposition 3, Figure 2.3 illustrates that the faster the meeting rate of an investment bank to a private firm and the slower the meeting rate of a private firm to an investment bank, the higher the investment bank’s relative share of profit from an IPO at the market equilibrium.



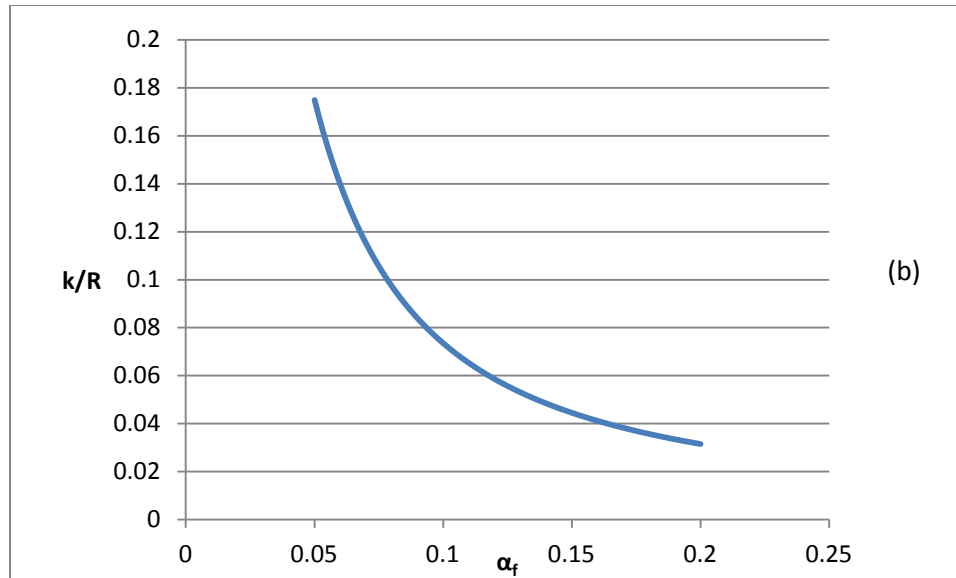


Figure 2.3 (a) and (b) Effect of α_f or α_b on k/R

Figure 2.4 shows the effect of the investment bank's bargaining power on the investment bank's relative share of profit when the values of the other parameters in Table 2.1 are fixed. When the investment bank's bargaining power changes from 0.1 to 0.9, we can see that the investment bank's relative share of profit increases significantly. In a real pre-IPO market, the investment bank's bargaining power will be a crucial factor affecting its profit earning ability. But this value can fluctuate case by case and industry by industry,

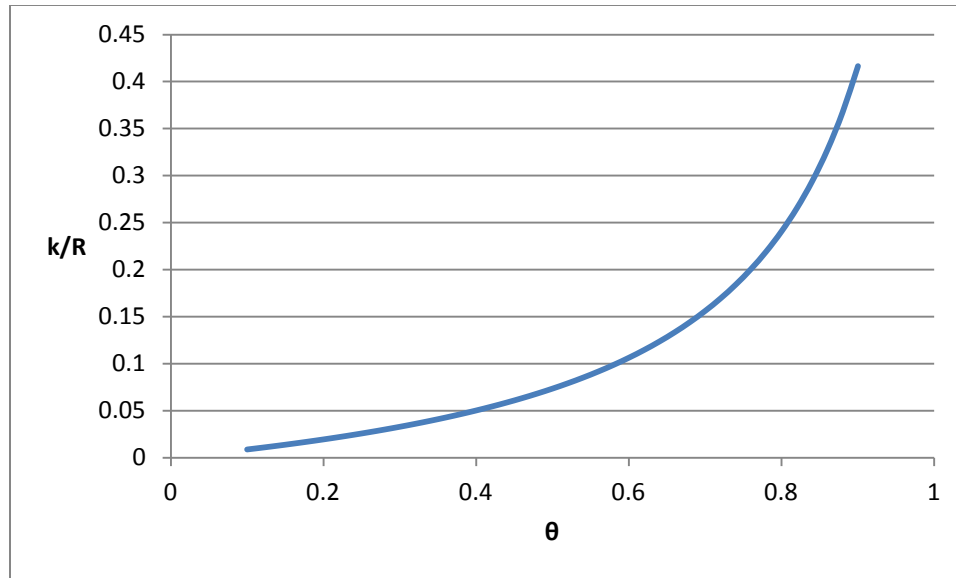


Figure 2.4 Effect of θ on k/R

Figure 2.5 and Figure 2.6 provide two opposite scenarios to illustrate the effect of the potentially positive initial offer price adjustment (R) on the investment bank's share of profit (k) under the condition that the success arrival rate of IPO is positively related to the initial offer price. (To be noted again that in our model each firm can only issue one share of stock. So R means both the total proceeds from the IPO and the initial offer price of the IPO.)

In Figure 2.5, since the positive size effect of the increase in the initial offer price R dominates the negative ratio effect of the increase in the success arrival rate σ , we observe that the investment bank's share of profit k increases with the initial offer price. Figure 2.5(a) shows that this negative ratio effect and Figure 2.5(b) shows the net effect (both size effect and ratio effect) of R on k . The initial offer price ranges from \$0.05 to \$0.5 billion, surrounding \$0.2 billion per IPO transaction. The value of \$0.2 billion is chosen due to the fact that from 2000 to 2011 there are 1519 offerings with \$352.616 billion of gross proceeds in total and so the average value of an IPO transaction is \$0.232 billion \approx \$0.2 billion.

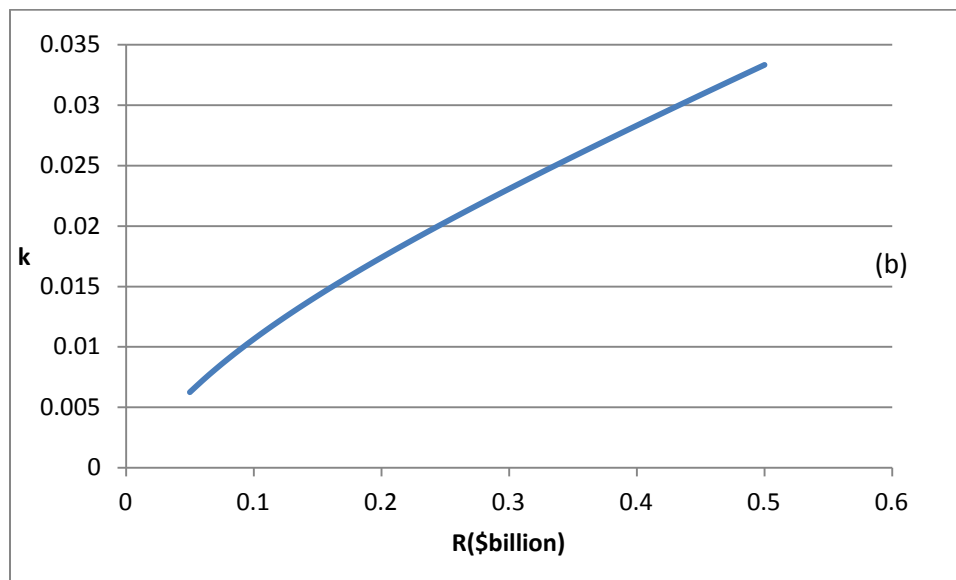
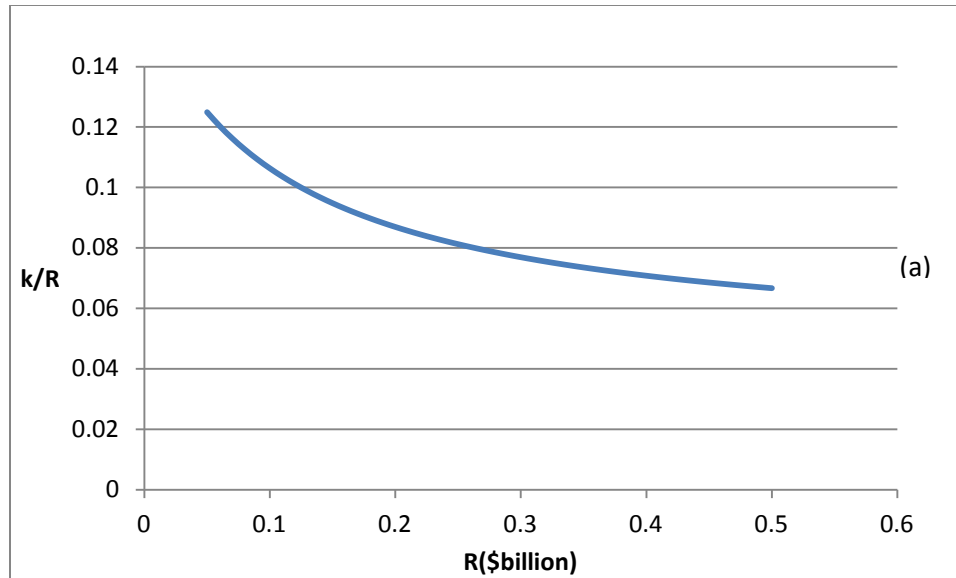


Figure 2.5 (a) and (b) Effect of R on r and r/R when size effect > ratio effect

In Figure 2.6, since the positive side effect of the increase in the initial offer price R is dominated by the negative ratio effect of the increase in the success arrival rate σ , we observe that the investment bank's share of profit k decreases with the initial offer price. Figure 2.6(a)

shows that this negative ratio effect and Figure 2.6(b) shows the net effect (both size effect and ratio effect) of R on k.

Besides little difference in the range of R and σ , the main difference between Figure 2.5 and Figure 2.6 comes from the slope linking the co-movement of R and σ . Although the values are both positive, the slope used for Figure 2.6 is about 9 times as large as that used for Figure 2.5, leading to the totally different net effect of R on k. We cannot overrate the importance of Figure 2.6(b), which indicates that under our reasonable ranges of parameters, the potentially positive adjustment in the initial offer price may decrease the investment bank's share of profit. For instance, if R increases from \$0.21billion to \$0.23billion, k will decrease from \$0.0163billion to \$0.0156billion in Figure 2.6(b). This scenario provides a straight-forward search based explanation why initial offer prices only partially adjust to the positive public information during the IPO price formation stage. In addition, we also find that \$0.237 billion is a turning point for R in our case. If the current value of R is larger than \$0.237billion, k will increase along with the increase in R, which indicates that the over-adjustment (overpricing) is also possible in reality. In sum, our model suggests that IPO underpricing is not a deterministic but an empirical normality.

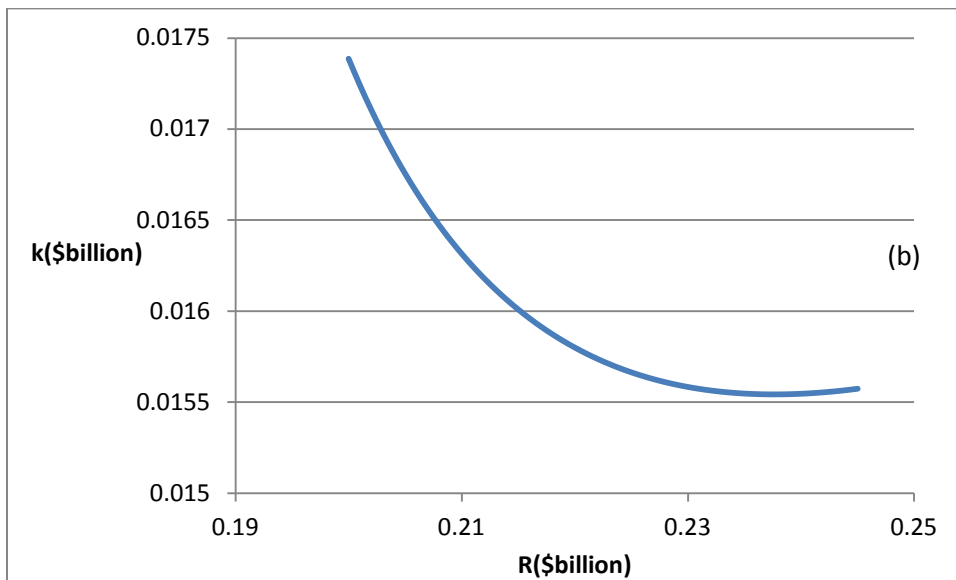
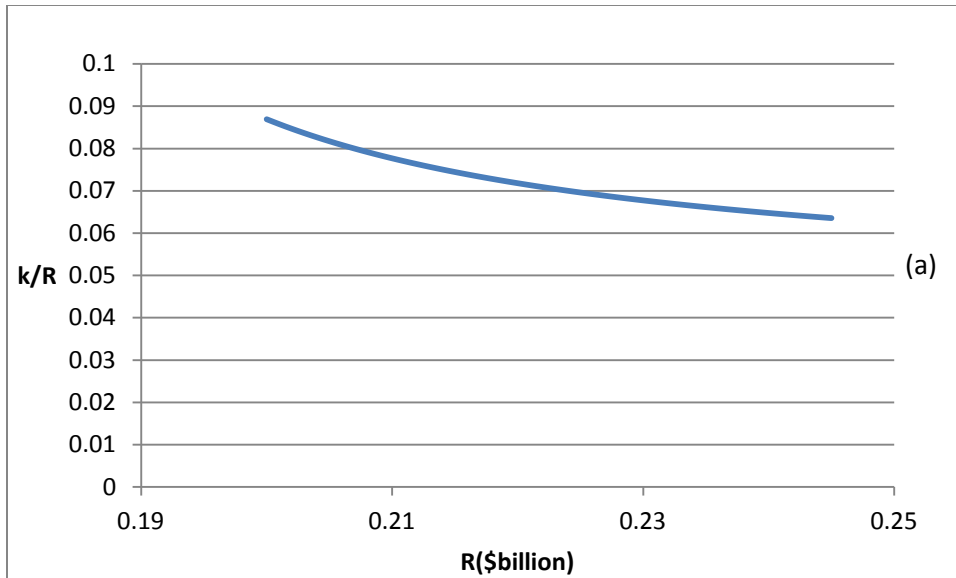


Figure 2.6(a) and (b) Effect of R on r and r/R when size effect < ratio effect

Basing on Proposition 5, Table 2.2 provides a general equilibrium picture for the pre-IPO market when the two meeting rates of α_f and α_b are endogenized. Recall that one salient feature of our model is that we focus on the pre-IPO stage, thus making it possible to determine the number of investment banks supported by the underwriting industry endogenously. To our knowledge, few models explicitly consider the market capacity of the underwriting industry. For

this purpose, we need to estimate the value of investment banks staying out of the market (L). Here we use the average market value of investment banks, roughly \$20billion as our input. Again we set the initial offer price(R) be \$0.2 billion here.

Table 2.2 illustrates that if there are initially 1000 private firms in the market, this market has the capacity of holding 15 investment banks. At the market equilibrium, during each time period 994 private firms and 9 investment banks are continuously searching and 6 strategic pairs have been formed to wait for success. (We assume that one investment bank serves one private firm in our model), For each successful IPO transaction, the investment bank can earn \$0.0161billion, i.e. 7.91% of the total IPO proceeds.

Variable name	Notation	Result
The initial number of investment bank	n	0.015
The market equilibrium number of IPO pairs	m	0.006
The market equilibrium number of investment banks	n-m	0.009
The market equilibrium number of private firms	1-m	0.994
The investment bank's share of profit	k(\$ billion)	0.0161
The investment bank's relative share of profit	k/R	7.91%
The market tightness	MT	0.0089
The meeting rate of an investment bank to a private firm	α_b	10.6/month
The meeting rate of a private firm to an investment bank	α_f	0.094/month

Table 2.2 General equilibrium picture for the pre-IPO market

(3) Simulation results when R is a random variable

If R is a random variable, besides the typical values in Table 2.2, we need to assume a functional form and a support for the cumulative distribution function of $F(R)$. Let's use the simplest one for $F(R)$:

$$F(R) = R/0.3, \quad \text{here } R \in [0, 0.3]$$

The upper limit of R is \$0.3billion. This means that the initial offer price is uniformly distributed on the interval of $[0, 0.3]$, i.e. it is equally possible for R to be any value of $[0, 0.3]$. (To be noted that the assumption on the functional form of the probability distribution of R can have a great impact on the simulation results.)

The key contribution of our random version of search model (the model when R is a random variable) is to establish the concept of the reservation initial price R^* under the framework of market searching. Moreover, our theoretical model provides a unique and tractable structure to quantify this value and investigate the factors which may affect its magnitude with as few assumptions as possible. Therefore, the main task left in this part is to illustrate how to determine the reservation initial offer price R^* and to study which factors may affect it quantitatively.

As a starting point, we first solve for the reservation initial offer price R^* only using Equation (2.16). We find that this reservation initial offer price R^* is \$0.19billion, at which the left hand side of Equation (2.16) is equal to the right hand side of it. This result is considered as the partial equilibrium solution for R^* since we don't combine Equation (2.16) and (2.17) together and we just assume that the market tightness MT is fixed and known. Don't confuse the value of \$0.19billion, the reservation initial offer price R^* resulted from in this random model, with the value of \$0.2billion, the average of the total proceeds or initial offer price R from an

IPO transaction used previously. The reservation initial offer price R^* is an unobservable variable implicitly determined by the model structure and is embedded in private firms' mind while the average of the initial offer price R is observable from the market.

Now consider the effects of two factors on the reservation initial offer price R^* still under the framework of the partial equilibrium, the results of which are illustrated in Figure 2.7 and 2.8, respectively.

In Figure 2.7, when the monthly discount rate increases from 0.1% to 1.1%, the reservation initial offer price decreases from \$0.246billion to \$0.160billion correspondingly.

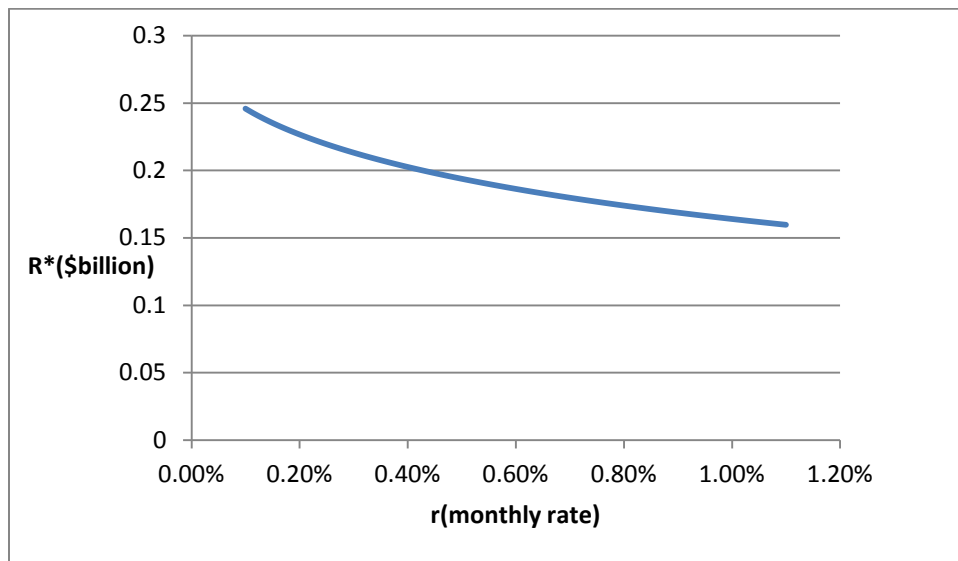


Figure 2.7 Effect of r on R^*

In Figure 2.8, when the success arrival rate of IPO increases from 5 to 20 times per month, the reservation initial offer price decreases from \$0.196billion to 0.194billion correspondingly.

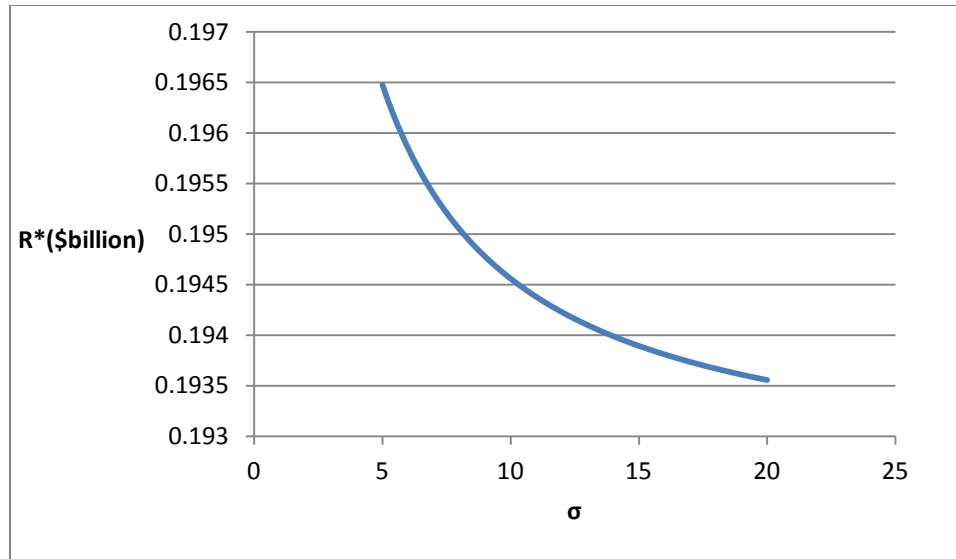


Figure 2.8 Effect of σ on R^*

More importantly, Figure 2.9 shows the general equilibrium solution for the reservation initial offer price R^* at the point where the red line comes across the blue line. The corresponding reservation initial offer price R^* is \$0.192billion and the market tightness MT is 0.009246. Here the red line denotes the investment bank's free entry condition and the blue line represents the investment bank's profit condition. The reservation initial offer price at the market equilibrium has to satisfy both conditions simultaneously. Once we know the value of R^* , the

mean of the truncated distribution can be calculated as $E(R|R>0.192) = \frac{\int_{0.192}^{0.3} R dF(R)}{1-F(0.192)} = 0.246$, which

is about 64 percent ($\frac{0.246-0.15}{0.246} = 64\%$) higher than the unconditional mean $E(R)$

$= \int_0^{0.3} R dF(R) = 0.15$. In terms of IPO language, when initial offer prices are used, IPO firms are

priced about 64 percent above all the private firms in the pre-IPO market.

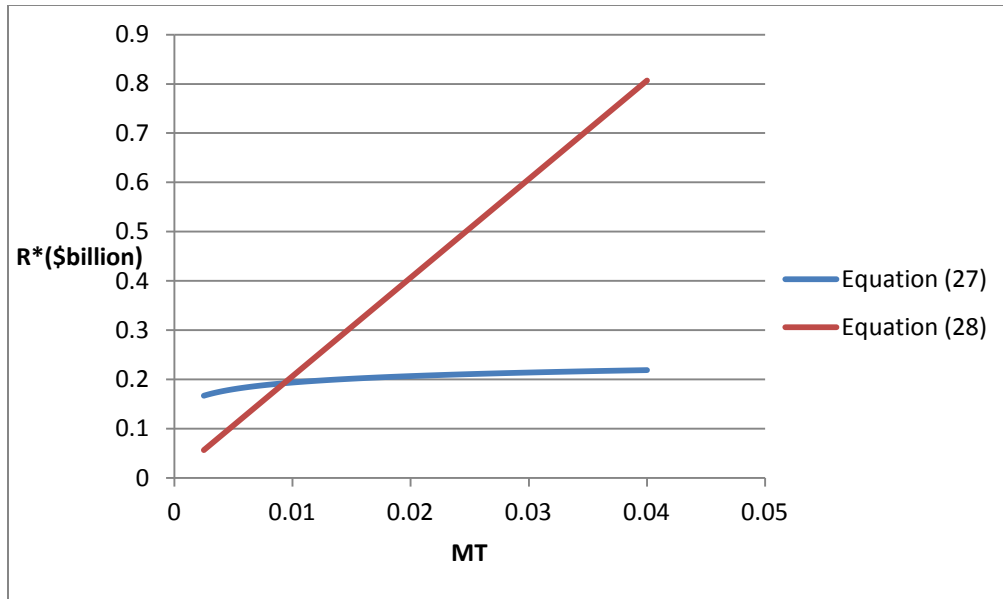


Figure 2.9 General equilibrium solution for R^* and MT

2.5 Conclusion

In this paper, the optimal strategies of private firms who are eager to go public and investment banks that are assumed to be necessary to serve the IPO process are simultaneously investigated under the framework of two-sided search theory. Four useful value functions for both types of agents are established to represent the corresponding utilities obtained when staying in two distinct states, the searching state and the pair state. One important characteristic of our model is that the intent of every private firm who always wants to go to public is compared with the revealed preference of only the partial number of private firms who indeed form a strategic pair with investment banks to realize an IPO.

Aided by this model, the complex IPO process can be reduced into a system with a finite number of equations and a finite number of variables, making the research exploration in IPO areas more tractable. Such IPO related phenomena and puzzles as IPO underpricing and IPO underperformance can be creatively explained from the “search” angle. Our model suggests that

IPO underpricing is not a deterministic but an empirical normality mainly originating from the market-wide co-movement between the total proceeds and the total number of successful IPOs, while the existence of the reservation initial offer price seems to be the sole driving force behind IPO underperformance, which also explains the puzzle why quantity adjustment is more frequent than price adjustment in an IPO issuing process. Empirically, our model is pioneering to pin down the reservation initial offer price under the framework of market searching. Moreover, our derived closed-form of the investment bank's share of profit from an IPO transaction at the market equilibrium reproduces the "seven percent solution" observed in the underwriting industry, which indicates the validity of our search based model.

As the predicted collapse of the used car market due to the existence of lemons by Akerlof never happened in the U.S., we claim that the market searching structure may be more fundamental than the secondary consideration of asymmetric information. Our perspective is also consistent with Ritter and Welch's (2002) observation that "asymmetric information is not the primary driver of many IPO phenomena".

Appendix A: Notation Table

f	private firm
b	investment bank
l	the initial number of private firms normalized to 1
n	the initial number of investment banks
m	the equilibrium number of IPO pairs
α_f	the meeting rate of a private firm to an investment bank, i.e. how many investment banks a private firm can meet during each time period.
α_b	the meeting rate of an investment to a private firm, i.e. how many private firms an investment bank can meet during each time period.
σ	the success arrival rate of IPO pairs
R	the total proceeds of an IPO or the initial offer price of an IPO
k	investment bank's share of profit from an IPO
k^*	investment bank's share of profit from an IPO at the market equilibrium
State"0"	the searching state
State"1"	the IPO pair state
V_f^0	the value of a private firm who is searching an investment bank in the market
V_f^1	the value of a private firm who forms a strategic pair with an investment bank
V_b^0	the value of an investment bank who is searching a private firm in the market
V_b^1	the value of an investment bank who forms a strategic pair with a private bank
r	the (risk-free) discount rate
S_f	the surplus function for a private firm, equals $V_f^1 - V_f^0$
S_b	the surplus function for a private firm, equals $V_b^1 - V_b^0$
θ	investment bank's bargaining power, the parameter in the Nash bargaining scheme
MT	the market tightness, equals $\frac{n-m}{1-m}$, denotes the relative number of investment bank to

	private firm in market equilibrium
π	the matching technology function between f and b
δ	the parameter in the matching technology function π
L	the value of investment bank staying out of the market
R^*	the reservation initial offer price of an IPO
$F(R)$	the cumulative distribution function(CDF) of the initial offer price of R

Appendix B: Proofs of Propositions and Lemmas

Proposition 1

From (2.1) and (2.2), we can find the expressions of V_f^0 and V_f^1 :

$$(2.2) \rightarrow V_f^1 = \frac{\sigma(R-k^*)}{r+\sigma} \quad (\text{B-1})$$

$$(\text{B-1}) \text{ and } (2.1) \rightarrow V_f^0 = \frac{\alpha_f}{\alpha_f+r} V_f^1 = \frac{\alpha_f \sigma (R-k^*)}{(\alpha_f+r)(r+\sigma)} \quad (\text{B-2})$$

Then, $V_f^0 < V_f^1$ as long as r is positive. Since V_f^1 is always larger than V_f^0 i.e. the firm's value in a pair state is always larger than in a searching state, the firm's best strategy is always to go public once chance arrives.

Proposition 2

Assuming the outside options V_f^0 and V_b^0 as given, find the expressions of the two surplus functions (S_f and S_b) according to their definitions (2.5) and (2.6). The values of those surplus functions depend on k which is chosen by the bargain process of two parties. Then apply the first order condition to solve the Generalized Nash bargain problem (2.7) and put the market equilibrium condition (2.8) into the F.O.C., we can derive the formula for k^* .

Specifically, let the outside options V_f^0 and V_b^0 be given:

Consider (2.2) when the investment bank's share is k ,

$$V_f^1(k) = \frac{\sigma(R-k)}{r+\sigma} \quad (\text{B-3})$$

$$\text{Then, } S_f(k) = V_f^1(k) - V_f^0 = \frac{\sigma(R-k)}{r+\sigma} - V_f^0 = \frac{\sigma(R-k) - (r+\sigma)V_f^0}{r+\sigma} \quad (\text{B-4})$$

Consider (2.4) when the investment bank's share is k ,

$$V_b^1(k) = \frac{\sigma(k+V_b^0)}{r+\sigma} \quad (\text{B-5})$$

$$\text{Then, } S_b(k) = V_b^1(k) - V_b^0 = \frac{\sigma(k+V_b^0)}{r+\sigma} - V_b^0 = \frac{\sigma k - rV_b^0}{r+\sigma} \quad (\text{B-6})$$

Replace S_f and S_b by (B-4) and (B-6), (2.7) is changed into:

$$\text{Max } S_f^{1-\theta} S_b^\theta = \text{Max} \left[\frac{\sigma(R-k) - (r+\sigma)V_f^0}{r+\sigma} \right]^{1-\theta} \left(\frac{\sigma k - rV_b^0}{r+\sigma} \right)^\theta \quad (\text{B-7})$$

Since $(r+\sigma)$ are parameters, the above optimal problem is equivalent to:

$$\text{Max } [\sigma(R-k) - (r+\sigma)V_f^0]^{1-\theta} (\sigma k - rV_b^0)^\theta \quad (\text{B-8})$$

Apply the first order condition to (B-8) for k :

$$(1-\theta)[\sigma(R-k) - (r+\sigma)V_f^0]^{-\theta} (-\sigma)(\sigma k - rV_b^0)^\theta + [\sigma(R-k) - (r+\sigma)V_f^0]^{1-\theta} \theta (\sigma k - rV_b^0)^{\theta-1} \sigma = 0 \quad (\text{B-9})$$

(B-9) is simplified into:

$$\sigma k = \theta \sigma R + (1-\theta)rV_b^0 - \theta(r+\sigma)V_f^0 \quad (\text{B-10})$$

We need to the formulas for V_b^0 and V_f^0 , which can be derived from (2.1)-(2.4):

$$(2.1)-(2.4) \rightarrow V_b^1 - V_b^0 = \frac{\sigma k^*}{r+\alpha_b+\sigma} \quad (\text{B-11})$$

Put (B-11) back into (2.3):

$$V_b^0 = \frac{\alpha_b \sigma k^*}{r(r+\alpha_b+\sigma)} \quad (\text{B-12})$$

Recall that $V_f^o = \frac{\alpha_f \sigma (R - k^*)}{(\alpha_f + r)(r + \sigma)}$ (B-2). Putting (B-2) and (B-12) into (B-10), we get:

$$\begin{aligned} \sigma k &= \theta \sigma R + (1 - \theta) r \frac{\alpha_b \sigma k^*}{r(r + \alpha_b + \sigma)} - \theta (r + \sigma) \frac{\alpha_f \sigma (R - k^*)}{(\alpha_f + r)(r + \sigma)} \\ &= \theta \sigma R + (1 - \theta) \frac{\alpha_b \sigma k^*}{r + \alpha_b + \sigma} - \theta \frac{\alpha_f \sigma (R - k^*)}{\alpha_f + r} \end{aligned} \quad (\text{B-13})$$

Divide both sides of (B-13) by σ :

$$k = \theta R + (1 - \theta) \frac{\alpha_b k^*}{r + \alpha_b + \sigma} - \theta \frac{\alpha_f (R - k^*)}{\alpha_f + r} \quad (\text{B-14})$$

Apply the market equilibrium condition (2.8) to (B-14):

$$k^* = \theta R + (1 - \theta) \frac{\alpha_b k^*}{r + \alpha_b + \sigma} - \theta \frac{\alpha_f (R - k^*)}{\alpha_f + r} \quad (\text{B-15})$$

$$\text{Then } k = k^* = \frac{\theta r(r + \sigma + \alpha_b)}{\theta r(r + \sigma + \alpha_b) + (1 - \theta)(r + \sigma)(r + \alpha_f)} R \quad (2.9)$$

$$\frac{\partial k}{\partial R} = \frac{\theta r(r + \sigma + \alpha_b)}{\theta r(r + \sigma + \alpha_b) + (1 - \theta)(r + \sigma)(r + \alpha_f)} > 0$$

Proposition 3

From (2.9), we can do comparative static analysis and find the signs of the following first derivations:

The sign of $\frac{\partial k}{\partial \alpha_f}$ is negative which is straightforward since α_f only shows up in the denominator of

(2.9)

The sign of $\frac{\partial k}{\partial \alpha_b}$ is positive which can be seen if we re-arrange (2.9) as below:

$$k = k^* = \left[1 - \frac{(1-\theta)(r+\sigma)(r+\alpha_f)}{\theta r(r+\sigma+\alpha_b) + (1-\theta)(r+\sigma)(r+\alpha_f)} \right] R \quad (\text{B-16})$$

where α_b only shows up in the denominator of (B-16), but this part has a negative sign in front.

We need to use the formula that: $\left(\frac{x}{y}\right)' = \frac{x'y - xy'}{y^2}$ to show that the sign of $\frac{\partial k}{\partial \theta}$ is positive:

$$\frac{\partial k}{\partial \theta} = \frac{r(r+\sigma+\alpha_b)(r+\sigma)(r+\alpha_f)}{[\theta r(r+\sigma+\alpha_b) + (1-\theta)(r+\sigma)(r+\alpha_f)]^2} R > 0 \quad (\text{B-17})$$

Proposition 4

Let R be the total initial offer price of an IPO and assume $\sigma = \sigma(R)$ and $\frac{d\sigma}{dR} > 0$.

Define the coefficient in Equation (2.9) as β , then,

$$k = k^* = \frac{\theta r(r+\sigma+\alpha_b)}{\theta r(r+\sigma+\alpha_b) + (1-\theta)(r+\sigma)(r+\alpha_f)} R = \beta R \quad (\text{B-18})$$

Note that β is a function of σ which is also a function of R , i.e. $\beta = \beta(\sigma) = \beta[\sigma(R)]$.

$$\text{So } \frac{\partial k}{\partial R} = \beta + \frac{\partial \beta}{\partial \sigma} \frac{d\sigma}{dR} R \quad (\text{B-19})$$

Let's check out the sign of $\frac{\partial \beta}{\partial \sigma}$:

$$\frac{\partial \beta}{\partial \sigma} = \frac{\theta(1-\theta)r(r+\alpha_f)(-\alpha_b)}{[\theta r(r+\sigma+\alpha_b) + (1-\theta)(r+\sigma)(r+\alpha_f)]^2} \quad (\text{B-20})$$

Then $\frac{\partial \beta}{\partial \sigma} < 0$.

Combining our assumption that $\frac{d\sigma}{dR} > 0$, the sign of $\frac{\partial k}{\partial R}$ in (B-19) cannot be decided since the first

item is positive and the second item is negative.

Proposition 5

Define the market tightness as:

$$MT = \frac{n-m}{1-m} \quad (2.11)$$

The arrival rates of α_f and α_b can be derived as:

$$\alpha_f = \pi/(1-m) = (1-m)^{-\delta}(n-m)^\delta = MT^\delta \quad (2.13)$$

$$\alpha_b = \pi/(n-m) = (1-m)^{1-\delta}(n-m)^{\delta-1} = MT^{\delta-1} \quad (2.14)$$

Put (2.13) and (2.14) into (2.9), we can get (2.16):

$$k = k^* = \frac{\theta r(r + \sigma + MT^{\delta-1})}{\theta r(r + \sigma + MT^{\delta-1}) + (1 - \theta)(r + \sigma)(r + MT^\delta)} R \quad (2.16)$$

Put (B-12) into (2.15):

$$V_b^o = \frac{\alpha_b \sigma k^*}{r(r + \alpha_b + \sigma)} = L \quad (B-21)$$

Replace k^* by (2.16), α_f by (2.13) and α_b by (2.14) in (B-21), we can get (2.17):

$$\begin{aligned} V_b^o = \frac{\alpha_b \sigma k^*}{r(r + \alpha_b + \sigma)} = L &= \frac{MT^{\delta-1} \sigma \left[\frac{\theta r(r + \sigma + MT^{\delta-1})}{\theta r(r + \sigma + MT^{\delta-1}) + (1 - \theta)(r + \sigma)(r + MT^\delta)} R \right]}{r(r + \sigma + MT^{\delta-1})} \\ &= \frac{MT^{\delta-1} \sigma \theta R}{\theta r(r + \sigma + MT^{\delta-1}) + (1 - \theta)(r + \sigma)(r + MT^\delta)} \end{aligned} \quad (2.17)$$

Let's derive (2.18)-(2.21):

Put (2.13) and (2.14) into (2.10), we get:

$$(1-m) MT^\delta = \sigma m = (n-m) MT^{\delta-1} \quad (\text{B-22})$$

$$\text{Consider the first equal sign, } m = \frac{MT^\delta}{\sigma+MT^\delta} \quad (2.18)$$

$$\text{Then, } 1 - m = \frac{\sigma}{\sigma+MT^\delta} \quad (2.19)$$

$$\text{Consider the second equal sign, } n - m = \frac{\sigma}{MT^{\delta-1}} m = \frac{\sigma}{MT^{\delta-1}} \frac{MT^\delta}{\sigma+MT^\delta} = \frac{\sigma MT}{\sigma+MT^\delta} \quad (2.20)$$

$$\text{Use (2.18) and (2.20), then, } n = \frac{\sigma MT}{\sigma+MT^\delta} + m = \frac{\sigma MT}{\sigma+MT^\delta} + \frac{MT^\delta}{\sigma+MT^\delta} = \frac{\sigma MT+MT^\delta}{\sigma+MT^\delta} \quad (2.21)$$

Proposition 6

$V_f^1(R)$ and $V_b^1(R)$ are both non-decreasing functions of R , suppose there exists a reservation offer price R^* at which

$$V_f^1(R^*) = V_f^0 \quad V_b^1(R^*) = V_b^0 \quad (\text{B-23})$$

We can prove that If $R < R^*$, then $V_f^1(R) < V_f^0$ because $V_f^1(R) < V_f^1(R^*) = V_f^0$, the sign “<” comes from the non-decreasing property of $V_f^1(R)$ and the sign “=” comes from the definition of the reservation return (B-23).

Proposition 7

The two equilibrium equations in this proposition have the same economic meaning with those in proposition 2. These two equations can thus be derived with the same basic steps except that the latter is more complicated due to the randomness of R . Before deriving (2.27) and (2.28), let's first find several useful equations. The total surplus S is a function of R , i.e. $S=S(R)=S_b(R)+S_f(R)$. Recall (B-4) and (B-6):

$$S(R) = \frac{\sigma k - rV_b^0}{r + \sigma} + \frac{\sigma(R - k) - (r + \sigma)V_f^0}{r + \sigma} = \frac{\sigma R - (r + \sigma)V_f^0 - rV_b^0}{r + \sigma} \quad (\text{B-24})$$

In the above equation, the division of R between f and b is unrelated to the total surplus S , i.e. k does not show in (B-24). When $R = R^*$, according to the definition of R^* , (B-23), $S(R^*) = 0$. So, the numerator of (B-24) equals to 0 at $R = R^*$, i.e.

$$\sigma R^* = (r + \sigma)V_f^0 + rV_b^0 \quad (\text{B-25})$$

In addition, according to the F.O.C. for the generalized Nash bargain problem (B-10):

$$\sigma k^* = \theta \sigma R + (1 - \theta)rV_b^0 - \theta(r + \sigma)V_f^0 \quad (\text{B-26})$$

Now we use (B-25) and (B-26) to derive (2.27):

$$(2.23) \rightarrow V_f^1(R) = \frac{\sigma(R - k^*)}{r + \sigma} \quad (\text{B-27})$$

Insert (B-27) into (2.22) and re-format (2.22) according to Proposition 6:

$$\begin{aligned} rV_f^0 &= \alpha_f \int_0^{\bar{R}} \max \{V_f^1(\tilde{R}) - V_f^0, 0\} dF(\tilde{R}) = \alpha_f \int_{R^*}^{\bar{R}} \left[\frac{\sigma(R - k^*)}{r + \sigma} - V_f^0 \right] dF(R) \\ &= \frac{\alpha_f}{r + \sigma} \int_{R^*}^{\bar{R}} [\sigma(R - k^*) - (r + \sigma)V_f^0] dF(R) \end{aligned} \quad (\text{B-28})$$

Insert (B-26) into (B-28):

$$rV_f^0 = \frac{\alpha_f}{r + \sigma} \int_{R^*}^{\bar{R}} [\sigma R - \theta \sigma R - (1 - \theta)rV_b^0 + \theta(r + \sigma)V_f^0 - (r + \sigma)V_f^0] dF(R) \quad (\text{B-29})$$

Simplify (B-29) via (B-25):

$$rV_f^0 = \frac{\alpha_f}{r + \sigma} \int_{R^*}^{\bar{R}} [(1 - \theta)\sigma R - (1 - \theta)rV_b^0 - (1 - \theta)(r + \sigma)V_f^0] dF(R)$$

$$\begin{aligned}
&= \frac{\alpha_f}{r+\sigma} \int_{R^*}^{\bar{R}} \{(1-\theta)\sigma R - (1-\theta)[rV_b^0 + (r+\sigma)V_f^0]\} dF(R) \\
&= \frac{\alpha_f}{r+\sigma} \int_{R^*}^{\bar{R}} [(1-\theta)\sigma R - (1-\theta)\sigma R^*] dF(R) \\
&= \frac{\alpha_f(1-\theta)\sigma}{r+\sigma} \int_{R^*}^{\bar{R}} (R - R^*) dF(R)
\end{aligned} \tag{B-30}$$

Integrate (B-30) by parts:

$$rV_f^0 = \frac{\alpha_f(1-\theta)\sigma}{r+\sigma} \int_{R^*}^{\bar{R}} [1 - F(R)] dR \tag{B-31}$$

In the same way, we can derive a formula for rV_b^0 :

$$(2.25) \rightarrow V_b^1(R) = \frac{\sigma(k^* + V_b^0)}{r+\sigma} \tag{B-32}$$

Insert (B-32) into (2.24) and re-format (2.24) according to Proposition 6:

$$\begin{aligned}
rV_b^0 &= \alpha_b \int_0^{\bar{R}} \max \{ V_b^1(\tilde{R}) - V_b^0, 0 \} dF(\tilde{R}) = \alpha_b \int_{R^*}^{\bar{R}} \left[\frac{\sigma(k^* + V_b^0)}{r+\sigma} - V_b^0 \right] dF(R) \\
&= \frac{\alpha_b}{r+\sigma} \int_{R^*}^{\bar{R}} [\sigma(k^* + V_b^0) - (r+\sigma)V_b^0] dF(R) \\
&= \frac{\alpha_b}{r+\sigma} \int_{R^*}^{\bar{R}} [\sigma k^* - rV_b^0] dF(R)
\end{aligned} \tag{B-33}$$

Insert (B-26) into (B-33):

$$rV_b^0 = \frac{\alpha_b}{r+\sigma} \int_{R^*}^{\bar{R}} [\theta\sigma R + (1-\theta)rV_b^0 - \theta(r+\sigma)V_f^0 - rV_b^0] dF(R) \tag{B-34}$$

Simplify (B-34) via (B-25):

$$rV_b^0 = \frac{\alpha_b}{r+\sigma} \int_{R^*}^{\bar{R}} [\theta\sigma R + (1-\theta)rV_b^0 - \theta(r+\sigma)V_f^0 - rV_b^0] dF(R)$$

$$\begin{aligned}
&= \frac{\alpha_b}{r+\sigma} \int_{R^*}^{\bar{R}} [\theta\sigma R - \theta(r+\sigma)V_f^0 - \theta rV_b^0] dF(R) \\
&= \frac{\alpha_b}{r+\sigma} \int_{R^*}^{\bar{R}} [\theta\sigma R - \theta\sigma R^*] dF(R) \\
&= \frac{\alpha_b\theta\sigma}{r+\sigma} \int_{R^*}^{\bar{R}} (R - R^*) dF(R) \tag{B-35}
\end{aligned}$$

Integrate (B-35) by parts:

$$rV_b^0 = \frac{\alpha_b\theta\sigma}{r+\sigma} \int_{R^*}^{\bar{R}} [1 - F(R)] dR \tag{B-36}$$

Multiply (B-31) by $\frac{r+\sigma}{r}$:

$$\begin{aligned}
rV_f^0 \left(\frac{r+\sigma}{r}\right) &= \left(\frac{r+\sigma}{r}\right) \frac{\alpha_f(1-\theta)\sigma}{r+\sigma} \int_{R^*}^{\bar{R}} [1 - F(R)] dR \\
(r+\sigma)V_f^0 &= \frac{\alpha_f(1-\theta)\sigma}{r} \int_{R^*}^{\bar{R}} [1 - F(R)] dR \tag{B-37}
\end{aligned}$$

Add (B-37) and (B-36) together, we get:

$$(r+\sigma)V_f^0 + rV_b^0 = \frac{\alpha_f(1-\theta)\sigma}{r} \int_{R^*}^{\bar{R}} [1 - F(R)] dR + \frac{\alpha_b\theta\sigma}{r+\sigma} \int_{R^*}^{\bar{R}} [1 - F(R)] dR \tag{B-38}$$

Recall (B-25), the left hand side equals σR^* , then,

$$\begin{aligned}
\sigma R^* &= \frac{\alpha_f(1-\theta)\sigma}{r} \int_{R^*}^{\bar{R}} [1 - F(R)] dR + \frac{\alpha_b\theta\sigma}{r+\sigma} \int_{R^*}^{\bar{R}} [1 - F(R)] dR \\
R^* &= \left[\frac{\alpha_f(1-\theta)}{r} + \frac{\alpha_b\theta}{r+\sigma} \right] \int_{R^*}^{\bar{R}} [1 - F(R)] dR \tag{B-39}
\end{aligned}$$

Replace α_f by MT^{δ} and α_b by $MT^{\delta-1}$ in (B-39), we get (2.27):

$$R^* = \left[\frac{(1-\theta)MT^{\delta}}{r} + \frac{\theta MT^{\delta-1}}{r+\sigma} \right] \int_{R^*}^{\bar{R}} [1 - F(R)] dR \tag{2.27}$$

Now we use (B-36), (2.27) and (2.15) to derive (2.28):

$$\begin{aligned}
rV_b^0 = rL &= \frac{\alpha_b \theta \sigma}{r + \sigma} \int_{R^*}^{\bar{R}} [1 - F(R)] dR = \frac{MT^{\delta-1} \theta \sigma}{r + \sigma} \int_{R^*}^{\bar{R}} [1 - F(R)] dR \\
&= \frac{MT^{\delta-1} \theta \sigma}{(r + \sigma)} \frac{R^*}{\left[\frac{(1-\theta)MT^\delta}{r} + \frac{\theta MT^{\delta-1}}{r + \sigma} \right]} = \frac{\theta \sigma}{(r + \sigma)} \frac{R^*}{\left[\frac{(1-\theta)MT}{r} + \frac{\theta}{r + \sigma} \right]} = \frac{\theta \sigma R^*}{\frac{(r + \sigma)(1-\theta)MT}{r} + \theta}
\end{aligned} \tag{B-40}$$

Rearrange (B-40), we get:

$$\theta \sigma R^* = [\theta r + (1 - \theta)(r + \sigma)MT]L \tag{2.28}$$

Lemma 1

This is derived from (2.27) when assuming MT is fixed, which is a partial equilibrium analysis.

Lemma 2

This is from the definition of the meeting rates and steady state flow balances. The inverse of arrival rates has the time unit. When R is a constant known, the expected length of each agent staying in the searching state is $1/\alpha_f$ for private firms and $1/\alpha_b$ for investment banks; When R is a random variable, the corresponding expected length of each agent staying in the searching state is $\frac{1}{\alpha_f[1-F(R^*)]}$ for private firms, $\frac{1}{\alpha_b[1-F(R^*)]}$ for investment banks.

Moreover, $1/\alpha_f < \frac{1}{\alpha_f[1-F(R^*)]}$ and $1/\alpha_b < \frac{1}{\alpha_b[1-F(R^*)]}$.

References

- Baker, M. and J. Wurgler. [2000]. The Equity Share in New Issues and Aggregate Stock Returns. *Journal of Finance* 55, 2219-2257.
- Brav, A., Geczy, C. and P.A. Gompers. [2000]. Is the Abnormal Return Following Equity Issuances Anomalous? *Journal of Financial Economics* 56, 209-249.
- Brav, A. and P.A. Gompers. [2003]. The Role of Lock-ups in Initial Public Offerings. *Review of Financial Studies* 16, 1-29.
- Carter, R.B., Dark, F.H. and A.K. Singh. [1998]. Underwriter Reputation, Initial Returns, and the Long-Run Performance IPO Stocks. *Journal of Finance* 53, 285-311.
- Chambers, D. and E. Dimson. [2009]. IPO Underpricing over the Very Long Run. *Journal of Finance* 64, 1407-1443.
- Chen, H.C. and Ritter, J.R. [2000]. The Seven Per Cent Solution. *Journal of Finance*, 55, 1105-1131
- Diamond, P. [1984]. *A Search-Equilibrium Approach to the Micro Foundations of Macroeconomics*. MIT press.
- Duffie, Darrell, Nicolae Gârleanu, and Lasse H. Pedersen, 2002, Securities Lending, Shorting, and Pricing, *Journal of Financial Economics* 66, 307–339.
- Duffie, Darrell, Nicolae Gârleanu, and Lasse H. Pedersen, 2005, Over-the-counter markets, *Econometrica*, 73, 1815–1847.
- Duffie, Darrell, Nicolae Gârleanu, and Lasse H. Pedersen, 2007, Valuation in over-the-counter markets, *Review of Financial Studies* 20, 1865–1900.
- Eckbo, B.E., Masulis, R.W. and O. Norli. [2000]. Seasoned Public Offerings: Resolution of the New Issues Puzzle. *Journal of Financial Economics* 56, 251-291.

- Eckbo, B.E., O. Norli. [2005]. Liquidity Risk, Leverage and Long-run IPO Returns. *Journal of Corporate Finance* 11, 1-35.
- Fama, E.F. and K.R. French. [1992]. The Cross-section of Expected Stock Returns. *Journal of Finance* 47, 427-465.
- Fernando, S. Chitru, Gatchev, A. Vladimir and Spindt, A. Paul. [2005]. Wanna Dance? How Firms and Underwriters Choose Each Other. *Journal of Finance*, 60(5), 2437 - 2469
- Ibbotson, R.G.. [1975]. "Hot Issue" Markets. *Journal of Finance* 30, 1027-1042.
- Jacquet, N. and S. Tan. [2007]. On the Segmentation of Markets. *Journal of Political Economy* 115, 639-64.
- Lagos, Ricardo et.al. [2007]. Search in asset markets: market structure, liquidity, and welfare. *American Economic Review*. 97(2), 198-202.
- Logue, D. [1973]. Premia on Unseasoned Equity Issues, 1965-69. *Journal of Economics and Business* 25, 133-141.
- Loughran, T. and J.R. Ritter. [2002]. Why Don't Issuers Get Upset About Leaving Money on the Table in IPOs?. *Review of Financial Studies* 15, 413-444.
- Loughran, T. and J.R. Ritter. [2004]. Why Has IPO Underpricing Changed Over Time?. *Financial Management* 33, 5-37.
- Lowry, M., Officer, M.S. and G.W. Schwert. [2010]. The Variability of IPO Initial Returns. *Journal of Finance* 65, 425-465.
- Lucas, D.J., R.L. McDonald. [1990]. Equity Issues and Stock Price Dynamics. *Journal of Finance* 45, 1019-1043.
- Menzio, G. [2007]. A Theory of Partially Directed Search. *Journal of Political Economy* 115, 748-76.

- Mortensen, D.T. and Pissarides, C.A. [1994]. Job Creation and Job Destruction in the Theory of Unemployment. *The Review of Economic Studies* 61, 397-415.
- Purnanandam, A.K. and B. Swaminathan. [2001]. Are IPOs really underpriced? *The Reviews of Financial Studies* 17, 811-848.
- Ritter, J.R. [1991]. The Long-run Performance of Initial Public Offerings. *Journal of Finance* 46, 3-27.
- Ritter, J.R. and I. Welch [2002]. A Review of IPO Activity, Pricing, and Allocations. *Journal of Finance* 57, 1795-1828.
- Shimer, R. [2007]. Mismatch. *American Economic Review* 97, 1074-1101.
- Shouyong, S. [2009]. Directed Search for Equilibrium Wage-Tenure Contracts. *Econometrica* 77, 561-584.
- Silveira, R. and R. Wright [2007]. The Venture Capital Cycle. Working paper, University of Pennsylvania.
- Vayanos, D. and P. Weill [2008]. A Search-Based Theory of the On-the-Run Phenomenon. *Journal of Finance*, 63, 1361-1398.
- Yung, C., Colak, G. and W. Wang. [2008]. Cycles in the IPO market. *Journal of Financial Economics*, 89, 192–208.

Chapter Three: Revisiting the Bid-ask Spread Using Competitive Search

ABSTRACT

In this chapter, we set up a competitive search model to re-interpret the existence of the market equilibrium bid-ask spread in a stylized security market, in which market dealers are in charge of posting an instantaneous bid price, investors choose whether to sell their share or not at this price, and the fluctuation of market sentiment is mimicked by an arrival rate of arbitrage opportunities with a Poisson process. Different from the asymmetric information based explanation originated from two types of investors, our search based model emphasizes that since the market dealer provides necessary liquidity to the security market via playing such an intermediary role between actual buyers and sellers, the bid-ask spread charged thereafter should largely be justified as the compensation for the market dealer's endeavor in this process. Our model provides a closed-form bid-ask spread formula which has a capacity to reproduce many empirical observations with respect to the effects of the market dealer's maintenance cost, the discount rate prevalent in the market, the overall market uncertainty, and the dividends payoff on the magnitude of the bid-ask spread. Our model further indicates that the absolute bid-ask spread is positively related to the stock price level while the percentage bid-ask spread is negatively related to the stock price level, which solves the puzzle on the impact of stock splits on stock liquidity without the assumption of asymmetric information.

Key words: competitive search, bid-ask spread, market microstructure, market dealer, stock liquidity, stock splits

3.1 Introduction and background

As traditional capital asset pricing models (CAPM) only consider the market systematic risk that is compensated by the equity risk premium (ERP), they cannot explain a variety of pricing puzzles and anomalies associated with the cross-sectional expected stock returns. Many researchers found that liquidity premium is one of the important explanatory factors but ignored in CAPM(Pastor (2003)). While we all can feel that financial assets differ in their liquidity such as some of stocks are much easier to trade and the others are not if without a longer waiting time or a larger price impact, the exact meaning of liquidity is rather elusive. The origin of liquidity difference of different assets is not what we are going to investigate here. Insofar as the liquidity of a stock is typically measured by its bid-ask spread in literature(Amihud (1986)), rather, in this paper, we will focus on one more easily tackled but correlated question: why does there exist a bid-ask spread for a traded asset and how to quantify it theoretically?

Although there are many papers focusing on the market microstructure trying to interpret why there exists a bid-ask spread in a security market, including one popular model of bid-ask spread proposed by Kyle(Kyle (1985)), which is mainly based on the asymmetric information, showing that dealers have to widen the bid-ask spread in order to trade against informed investors, our paper here attempts to treat the bid-ask spread from a different viewpoint, i.e. *search-based angle*, which doesn't mean that the issue of asymmetric information between dealers and investors is not important. Unfortunately we, however, avoid the trace of asymmetric information deliberately in our model in order to emphasize more insightful and more fundamental search and matching characteristics of the market equilibrium of the bid-ask spread phenomenon.

Introduced here as the background information, in macroeconomics, search theory is widely

used to explore the matching behavior between workers and firms. The typical framework is in this way: there are two types of rational agents, workers and firms; they meet with each other depending on the current market tightness; the pure meeting of each other doesn't necessarily lead to a permanent job contract since the worker would expect that he or she might come across a better job offer if he or she just waits a little longer, in the meantime the firm has the same thought to sign a higher productive worker as long as it is more patient(Diamond (1984)).

With regard to the search mechanism between workers and firms, there are two key modeling issues which can significantly influence the final jointly searching results: (1) how workers and firms meet with each other and (2) how they decide the wage (i.e. how they split the total profit from a successful hiring). Minor differences in institution arrangements may lead to diverse predictions. Two possible arrangements have been thoroughly analyzed in search literature: one is called "*random search*" in which workers and firms just randomly meet with each other and the final wage for workers is determined via the generalized Nash bargaining scheme once they meet; the other is called "*competitive search*" in which workers' job search activities are not random but directed to specific firms either because those firms post and advertise their offered wages or because those firms are natural focal points due to history or established reputation (Rogerson, Shimer and Wright 2005).

To the best of our knowledge, We are the first to resort to *competitive search*, also known as *directed search with price posting*, to investigate the interaction between market dealers and investors in a stylized security market in order to re-interpret the existence of the bid-ask spread at the market equilibrium. The common image for our competitive search is: assuming that only one type of asset is being traded between market dealers and investors, each market dealer first posts its own bid price and will commit to it without collusion with any other market dealer;

since there are many market dealers, there will be many possible sub-markets and each sub-market is distinguished by its own posted bid price; in doing so, the market dealers anticipate that investors will enter until investors are indifferent across all sub-markets. The market equilibrium of our competitive search is: there will, however, be only one sub-market with one bid price, such that no market dealer has any incentive to open a new sub-market by posting a bid price different from the uniform one, meanwhile no investors want to enter that newly opened sub-market.

The center theme delivered by search theory is that it always takes time for one agent to locate another one if the success of a transaction requires the collaboration of both agents. Thus, the difficulty of identifying the counter-party of the transaction (either buying or selling a security) in a security market justifies a role of a market intermediary who is introduced to facilitate the easiness of buying and selling through shortening the time wasted in matching and dampening the price impact of a larger order.

The above represents the concept of market search friction that exists in any security market, but more saliently exists in over the counter (OTC) market(Vayanos (2008)) and(Lagos (2009)), government or corporate bonds market, foreign exchange market and NASDAQ stock market than in NYSE market where the security transaction is more centralized.

It is admitted that with the rapid progress of advanced IT technology and more centralized market transactions, searching for the counter-party has become more efficient. While the market search friction plays a less important role for the operations of security markets than previously, we, however, can still appreciate the merits of search based market microstructure modeling from two aspects:

Firstly, centralized markets historically all evolve from initial de-centralized markets. When

considering the compensation for a role of a market dealer playing in a centralized market as the form of the bid-ask spread, we need to compare two states, one is the real centralized market, and the other is the imaginary de-centralized market, even if the de-centralized market is not the current status of the market transaction. To be more specific, only if we explore how difficult it is to meet the other side of the transaction in the imaginary de-centralized market, can we then evaluate accurately how well the liquidity that the real centralized market provides to investors, and justify the amount of the bid-ask spread required by a market dealer in the current centralized market. Hence, the market search friction modeled in this manuscript should be embedded implicitly in the bid-ask spread for any centralized market.

Secondly, our competitive search based model well characterizes the matching process between market dealers and investors. As we have mentioned before, our competitive search based model is distinguished from random search based models by how the search process proceeds. While random search based models assume 1) the random matching between the types of agents such as workers and firms, 2) the determination of price by bargaining once they meet, which are not fit into our market microstructure environment, our competitive search based model allows market dealers to post a widely known bid price in public ex ante in order to direct or attract the arrival of investors. Furthermore, our symmetric equilibrium lets all market dealers play the same strategy at the market equilibrium, i.e. post the same bid price in the entire market. In this way, our search based model well mimics the operation of NASDAQ stock market.

The existence of market dealers for any security is so indispensable and so natural that it is hard to image what would happen if there were not such a role there. As we never take a pause to challenge why the New York Stock Exchange charges a service fee due to the secondary market liquidity provided to the entire society by it, so in the same logic, any reasonable explanation of

why the bid-ask spread is charged by market dealers should be traced back to the service offered and the role played by market dealers, and the function of asymmetric information matters only to modulate the above fundamentals, which is the basic belief of the authors and the motivation of this paper as well.

Different from traditional bid-ask spread theories which pay much attention to the inventory-holding cost and the asymmetric information cost associated with two types of investors in the market, the key contribution of our paper is that we emphasize that the bid-ask spread can mainly be justified as the compensation for market dealers' endeavor in providing necessary liquidity to the security market via playing an intermediary role between actual buyers and sellers. In another word, our model stresses the importance of searching and matching cost. Our model further shows that "peer pressure" resulted from competition from other market dealers can downplay their role as a market intermediary while they are always willing to charge investors with the highest possible bid-ask spread.

Our model provides a closed-form bid-ask spread formula based on competitive search in a stylized security market, the simulation results of which are well consistent with the real security markets, thus proving the validity of our model.

Specifically, our model shows that the maintenance cost has two opposite effects on the bid-ask spread. Its direct effect on the bid-ask spread is positive, which is in agreement with the predication of inventory holding cost theory of bid-ask spread. Our model also indicates that the indirect effect of the maintenance cost is negative since higher maintenance cost could reduce the total number of market dealers, which also represents a more competitive structure for market dealers, leading to a less amount of bid-ask spread. Overall, the positive direct effect of the maintenance cost on the bid-ask spread dominates the negative indirect effect according to our

simulation results.

If we treat the dividends payoff as the “negative” maintenance cost, it is natural for our competitive search based model to acquire the positive effect of the dividends payoff on the bid-ask spread. While asymmetric information based models reach the same conclusion, the underlying mechanism is totally different as those information based models presume that there exists a positive relation between the level of information asymmetry and the magnitude of the bid-ask spread. Thus larger amount of dividends payoff signals the market the less information asymmetry, causing the bid-ask spread to shrink.

Our model also explicitly studies the influences of the discount rate and the instantaneous market opportunity on the bid-ask spread. Our model indicates that both the discount rate and the instantaneous market opportunity have a positive effect on the bid-ask spread. The above results are obvious if we consider the discount rate as the opportunity cost of holding stocks without earning the interest rate of a market dealer’s own fund meanwhile if we treat the instantaneous market opportunity as the measure of the overall market uncertainty.

More importantly, our model studies the impact of the price level on the bid-ask spread, which is closely related to a very important and prevalent financial market phenomenon both in the U.S. and worldwide-*stock splits*. People show great interest in the relationship between stock splits and stock liquidity. Contrary to the confusing explanations and observations originated from varieties of asymmetric information based models, our competitive search model clearly shows that the (absolute) bid-ask spread is positively related to the stock price level, but the percentage bid-ask spread is negatively related the stock price level.

In addition, better than conventional market microstructure models, our model has a capacity to determine the total number of market dealers at the market equilibrium since we don’t let the

total number of market dealers fixed beforehand when modeling the dynamic interaction between market dealers and investors in a security market. This number can be pinned down by the system via a free entry and exit condition for market dealers, which is also the reason why our model is named as the “competitive” search model.

The rest of this paper proceeds as follows: section 3.2 sets up our competitive search model; section 3.3 solves the model and derives the main results; section 3.4 discusses the empirical implications of the model; section 3.5 calibrates the key parameters of the model and illustrates the effects of many factors on the bid-ask spread, and section 3.6 concludes and points the future directions. Symbols and notations are summarized in Appendix C. Proofs of propositions are provided in Appendix D.

3.2 Competitive search model

(1) Assumptions

In a simplified world of a security market with only one type of asset or stock to be traded, we have many market dealers (denoted by f) who publicly post an instantaneous bid price (W), at which many potential investors (denoted by w) would choose whether to sell their share or not at this price. We assume that each investor initially has only one share of the stock inherited from endowment; each market dealer can only serve one transaction at a time and each transaction consists of only one share of the stock.

Furthermore, assume that time is continuous and goes from zero to infinity, both agents (f and w) are risk neutral with the (risk-free) discount rate of r . In addition, even though heterogeneity of agents is more realistic, both agents are assumed to be homogenous in this model respectively.

Let the number of investors be normalized to 1. During each time period, $(1-u)$ of them sell

their shares to market dealers and the left of them (u) don't sell their shares to market dealers. Thus, the number of market dealers who are occupied by stock transactions has to be $(1-u)$. If we assume that the number of market dealers who are idle is assumed to be v , then the total number of market dealers for this stock will be $(1-u) + v = 1 - u + v$. We define the market tightness as $\theta = v/u$, one key system variable which characterizes the tightness of the market condition, namely, the higher this ratio is, the more the number of idle market dealers and the fewer the number of investors who still have shares of the stock to be sold out, hence it is easier for any investor to sell his or her share if he or she is willing to.

Another note to be emphasized is that since the basic structure of selling and buying is similar in nature, the following model will only be concentrated on one half of the market activities, i.e. the selling part of the market. Namely, we only consider the relation between market dealers and investors in which market dealers post an instantaneous bid price and investors decide whether to sell the share or not. The modeling of the buying part of the market in which market dealers post an instantaneous ask price and investors decide whether to buy the share or not will not become too insurmountable if the selling part of the market is clearly understood. Therefore, from this viewpoint, our model can be classified into a class of partial equilibrium models.

(2) General picture

In sum, the general picture is: there are two types of agents continuously interacting with one another in the selling part of the market. They need to match with each other to realize their respective optimal profits.

Investors, whose final aim is to sell one share of the stock they obtain from the initial endowment. If an investor holds the share, each period he or she will extract b units of utility (or

money) forever. We can think of one share of the stock as one “Lucas tree” which can produce b units of dividends during each time period and b has the same unit as the bid price W . However, with an arrival rate of m that is an increasing and concave function of the market tightness θ , the investor may change his or her mind and sell the share to a market dealer at any bid price W currently posted in the market. To be stressed here, in the strictest term, our investors’ search behavior is not random but directed by the bid price W publicly known, which is a key difference between our model and other search based models in Finance pioneered by Duffie (Duffie (2005))and Weill(Weill (2008)).

Market dealers play an intermediary role by posting a bid price W signaling that they are always available to buy investors’ shares at W . Market dealers need to compete with each other for providing the “liquidity service” to all investors, which implicitly determines the number of market dealers this security market can finally support at the market equilibrium. A market dealer incurs a maintenance cost of a by posting a bid price W in the market. When the transaction is settled, the market dealer will pay the bid price W during each time period to the investor in exchange for the share which can be sold later at the price P by the market dealer then. (Since we only consider the selling part of the market, from now on, the price P that the market dealer can realize in the other side of the market will be treated as a parameter in our model while the bid price W is still a choice variable.) The price difference between P and W , titled with the bid-ask spread, compensates the market dealer for working as a counter party in any stock selling transaction. In the language of economics, $(P-W)$ can also be considered as the normal profit of the market dealer.

With an arrival rate of λ that follows a standard Poisson process, the market dealer who holds one share from the previous transaction cancels this transaction, i.e. stop paying W afterwards.

Different from traditional market microstructure models where the overall market sentiment is represented by the arbitrage transaction opportunities that investors are facing, our model, however, stresses the market sentiment embedded in the market dealers' transaction decisions. Through this delicate mechanism design, our model is endowed with the power to mimic the instantaneous arbitrage opportunities for the market dealer due to the fluctuation of the entire market sentiment.

To be clarified here, the price P at which the market dealer can sell out the share, the bid price W posited by the market dealer, the maintenance cost a occurring to the market dealer when posting a bid price in the market, and the dividends b produced by one share of the stock are not "stock variables" but "flow variables" in response to the time factor in our model. Both market dealers and investors are constrained by the basic market structure (the market tightness, θ and the functional form of the matching function, m) and the instantaneous market opportunities (λ). Whether each selling transaction can be realized mainly depends on whether it is profitable or not for both parties.

(3) Mathematical model

In this sub-section, we will establish the basic mathematical equations to model the interaction between investors (w) and market dealers (f) in the selling part of the security market.

We first define four key value functions since there are two types of agents (f and w) and each agent can be in two states (U means that the agent is in the idle state, V means the agent is in the occupied state). The exact meanings of the four value functions are explained below:

U_f : the value of a market dealer who posts a bid price and waits for a business;

V_f : the value of a market dealer who buys one share from an investor ;

U_w : the value of an investor who keeps one share in hand and waits for a chance to sell the

share to a market dealer;

V_w : the value of an investor who sells one share to a market dealer.

For any posted bid price W , the above four value functions satisfy the following four competitive search equations:

$$r U_w = m(\theta) (V_w - U_w) + b \quad (3.1)$$

$$r V_w = W + \lambda(U_w - V_w) \quad (3.2)$$

$$r V_f = P - W + \lambda(U_f - V_f) \quad (3.3)$$

$$r U_f = [m(\theta) / \theta] (V_f - U_f) - a \quad (3.4)$$

Assume the free exit and entry for any market dealer, we have the below free entry condition for the market dealer:

$$U_f = 0 \quad (3.5)$$

The market equilibrium is characterized by a solution to the following optimal problem:

$$\begin{aligned} (W^*, \theta^*) = \operatorname{argmax} U_f(W, \theta) \\ \text{s.t. (1) } U_w(W, \theta) = U_w(W^*, \theta^*) \\ (2) U_f(W^*, \theta^*) = 0 \end{aligned} \quad (3.6)$$

3.3 Discussions

Definition1 (Symmetric Equilibrium): *If (W^*, θ^*) solve the optimal problem (3.6), then (W^*, θ^*) define a market symmetric equilibrium.*

The underlying meaning of the above optimal problem is that at a predetermined market condition (θ^*) , given that all other market dealers post W^* , any agents (either market dealers or investors) have no incentive to create an alternative market with a W which is different from W^* . This market equilibrium definition is well consistent with the concept of Nash equilibrium.

Proposition 1: *The market symmetric equilibrium can also be characterized by the below*

optimal problem of (3.7), i.e. (3.6) and (3.7) are equivalent with each other and will give the same (W^*, θ^*) .

$$\begin{aligned} (W^*, \theta^*) &= \operatorname{argmax} U_w(W, \theta) \\ \text{s.t. } U_f(W^*, \theta^*) &= 0 \end{aligned} \quad (3.7)$$

Linking Equation (3.1) - (3.6) together, we can solve for the four value functions (U_f^* , V_f^* , U_w^* , V_w^*) and the two variables (W^* , θ^*) when assuming (r , λ , P , a , b , the functional form of m) are all exogenous

According to our equilibrium definition, the two values of (W^*, θ^*) exactly characterize the entire system. They can be resolved from two reduced equations summarized in Proposition 2.

Proposition 2: *The original six-equation system (Equation (3.1)-(3.6)) can be reduced into two fundamental equations: Equation (3.8) is the Free entry equation and Equation (3.9) is the Nash equilibrium equation, which can then be used to solve for (W^*, θ^*) .*

Free entry equation:

$$m(\theta^*)(P-W^*) - a(r+\lambda)\theta^*=0 \quad (3.8)$$

Nash equilibrium equation:

$$m'(\theta^*)(P-b) - a[r+\lambda + m(\theta^*) - \theta^* m'(\theta^*)] = 0 \quad (3.9)$$

Deriving a closed form formula for the bid-ask spread at the market equilibrium for our stylized security market under the framework of competitive search is one of the most important objectives of our paper. Once (W^*, θ^*) are solved via Equation (3.8) and (3.9), the corresponding bid-ask spread in our model will be equal to $(P-W^*)$.

Without further assumption about the functional form of the matching function m , we cannot derive a closed form for the bid-ask spread from Equation (3.8) and (3.9). However, our model

clearly indicates that such system parameters as (r, λ, P, a, b) all have a significant influence on the magnitude of the bid-ask spread at the market equilibrium. Comparative statics analysis can still be applied on those two equations via implicit function theorem (IFT) to draw many important economic implications.

In particular, if we assume a specific functional form for the matching function as $m(\theta) = \theta^{1/2}$, which is consistent with the increasing and concave properties of m , the entire system can be easily solved since Equation (3.9) is now independent of W^* :

$$\text{Equation (8)} \implies P - W^* = a(r + \lambda)\theta^{*1/2} \quad (3.10)$$

$$\text{Equation (9)} \implies \theta^{*1/2} = [(r + \lambda)^2 + (P - b)/a]^{1/2} - (r + \lambda) \quad (3.11)$$

Thus we have Proposition 3.

Proposition 3: *If we assume that the matching function $m(\theta)$ between market dealers and investors has a functional form of $m(\theta) = \theta^{1/2}$ (so m is an increasing and concave function of θ), the prevailing bid-ask spread at the market equilibrium can be expressed by Equation (3.12):*

$$P - W^* = a(r + \lambda) \{ [(r + \lambda)^2 + (P - b)/a]^{1/2} - (r + \lambda) \} \quad (3.12)$$

In a real security market, there are two closely related but different concepts about the bid-ask spread, one is the quoted bid-ask spread, the other is the effective bid-ask spread. While the quoted bid-ask spread is defined as the difference between the lowest market ask price for a security and the highest market bid price for the same security, the effective bid-ask spread is calculated as twice the difference between the actual execution price and the midquote (the midquote is the average of the market bid and ask price) for a buy order, and twice the difference between the midquote and the actual execution price for a sell order. Here the meaning of “buy” or “sell” is considered from the viewpoint of investors. For instance, a sell order is flowed in if the current quoted market bid and ask prices are \$5.00 and \$5.30 respectively. (The midquote is

then $(5.00+5.30)/2=\$5.15$.) Suppose that the market deal steps in front of the previously quoted bid price and the sell order is actually executed at $\$5.10$, the effective bid-ask spread is $2(5.15 - 5.10) = \$0.10$ while the quoted bid-ask spread is $5.30-5.00=\$0.30$. Since the effective bid-ask spread better captures the cost of a round-trip order for investors by including the actual execution price in the bid-ask spread calculation, we are going to define the effective bid-ask spread below with the purpose to fit into our theoretical model which only considers the selling part of the security market. When we treat the market equilibrium bid price W^* as the actual execution price and treat the price P as the midquote, the effective bid-ask spread used in our model is defined below:

Definition 2 (Effect bid-ask spread): *The effective bid-ask spread can be expressed as twice the difference between P and W^* . According to Equation (3.12),*

$$\text{The effective bid-ask spread} = 2(P - W^*) = 2a(r + \lambda) \{ [(r + \lambda)^2 + (P - b)/a]^{1/2} - (r + \lambda) \}. \quad (3.13)$$

3.4 Empirical implications

In this section, we discuss the results of comparative statics analysis of our model, draw important empirical implications and comment the significance and contributions of our search based model.

Basing on the derived effective bid-ask spread formula Equation (3.13), we can clearly see that there are five key parameters (a , b , r , λ , P) which can significantly influence the bid-ask spread $2(P - W^*)$. In the following, we will discuss in detail the effect of each system parameter on the bid-ask spread systemically. Although we assume a simple functional form for the matching function m when deriving a closed form bid-ask spread formula, in fact the functional form of m should have a substantial impact on the bid-ask spread, the discussion of which will deserve the work of one full paper and thus has to be omitted here. The only point to be noted is

that there may exist multiple equilibriums (Lagos (2007))if the matching function of m has the property of increasing returns with respect to its two arguments, v and u , leading to two possible levels of liquidity cost (corresponding to the high bid-ask spread equilibrium and the low bid-ask spread equilibrium) for assets with almost identical cash flows(Mandal (2011)) and (Blanchard (1989)).

(1) The effect of the maintenance cost occurring to a market dealer (a) on the bid-ask spread

It has long been known that the maintenance cost occurring to a market dealer is one of the most important factors which can affect the magnitude of the bid-ask spread in the security market. Traditional inventory holding cost theories (Bollena (2004))claim that as the cost of maintaining the role of a market dealer increases, the gap between the bid price and the ask price (i.e. the bid-ask spread) will widen in order to compensate the market dealer for this unavoidable cost. According to Equation (2.13), the maintenance cost a has two opposite effects: the direct effect of a on the bid-ask spread is obviously positive, the indirect effect of a on the bid-ask spread is negative. Intuitively speaking, with the increase in a , the total number of market dealers $(1-u+v)$ in the market should be reduced, either u increases or v decreases, or both, then the market tightness $(\theta=v/u)$, will decrease, which may lead to the possible decrease in the bid-ask spread (Please check Equation (2.10)). If we assume that the positive direct effect of the maintenance cost dominates the negative indirect one, which is more likely to happen in reality, our model successfully predicts the same result as conventional inventory holding cost theories do.

(2) The effect of the dividends payoff (b) on the bid-ask spread

The effect of the dividends payoff on the bid-ask spread is more straightforward in our

model when compared with asymmetric information based models, i.e. there exists a negative relation between the dividends payoff (b) and the bid-ask spread. While the prediction of our competitive search based model with respect to the effect of the dividends payoff on the bid-ask spread is consistent with the conclusion of asymmetric information based theories(Howe (1992)), both of which are supported by empirical evidence, the underlying story is totally different.

Our model can explain this negative relation without difficulty if we think of the dividends payoff as the “negative” maintenance cost to a market dealer, i.e. from the viewpoint of the market dealer, the dividends from holding one share of stock represent some positive carrying benefit. Since the main part of the market dealer’s own fund is occupied by the stock inventory holding, the more the amount of dividends paid out to the market dealer, the lower the bid-ask spread required by him.

As to the asymmetric information based theories, their underlying hypothesis is that a positive relation exists between the level of information asymmetry and the magnitude of the bid-ask spread. Insofar as the payment of dividends signals material relevant information to the market, thus reducing information asymmetry, dividends policy may influence the bid-ask spread. Moreover, based on the above logic, an inverse relation between the dividend yield and the bid-ask spread should exist, "ceteris paribus."

Although we believe that our competitive search based explanation is more persuasive than those asymmetric information based stories, whether the negative effect of the dividends payoff on the bid-ask spread originates from our search based market friction or from the asymmetric information friction is more an empirical issue than a theoretical one, the answer of which will be left for further exploration.

(3) The effect of the stock price level (P) on the bid-ask spread

Purely looking at the formula of the bid-ask spread in Equation (2.13), we may draw a conclusion that there exists a monotone positive relation between the stock price level P and the bid-ask spread without hesitation. This observation is fully in agreement with Copeland and Galai's model of information effects on the bid-ask spread (Copeland (1983)), i.e. the bid-ask spread is a positive function of the price level. The only difference is that we derive the same result from the perspective of search and matching without the assumption of asymmetric information.

One critical reason why we are greatly interested in the effect of the stock price level on the bid-ask spread is that we are attempting to apply our model to touch on a rather important and prevalent U.S. financial market phenomenon—stock splits, and its effect on stock liquidity. As we all know, stock splits are one of the intriguing anomalies in the financial market. Since they only lead to nominal changes in stock prices and there is no any real impact on the equity ownership of shareholders, stock splits are not supposed to have any material effect on the stock price behavior and the measure of liquidity subsequent to the splits though the opposite is true in reality.

Referring to the effect of stock splits on the bid-ask spread, two strands of theories are competing with each other. The liquidity and trading range hypothesis claims that the motivation for stock splits is to bring stock prices down to a preferred trading range in order to improve liquidity. This hypothesis is strongly supported by management in practice because most managers who are in charge of stock splits do believe that the above consideration is indeed the dominating concern of their decisions on stock splits. When the bid-ask spread is utilized as our measure of liquidity, the accompanying economic implication is that the bid-ask spread should decrease after stock splits, i.e. *improved liquidity follows stock splits*.

Alternative asymmetric information based theory proposed by Conroy, Harris, and Benet (Conroy (1990)) suggests that stock splits with the feature or function of worsening liquidity can serve as a costly but valid signal of “favorable future prospects of the firm”. The corresponding implication is that the bid-ask spread should increase after stock splits, i.e. *worsen liquidity follows stock splits*.

When resorting to empirical evidence to appraise the validity of those two rival theories, the existing empirical results about the impact of stock splits on liquidity are mixed as well. The inconclusive evidence partly reflects the challenge in selecting and interpreting the proper proxy for the measure of liquidity. While the liquidity and trading range hypothesis selects the absolute bid-ask spread (i.e. $2(P-W^*)$) as its measure of liquidity, the asymmetric information based theory prefers to applying the percentage bid-ask spread or relative bid-ask spread ((i.e. $2(P-W^*)/P$). Each theory uses the corresponding empirical results to support its own claim on the relation between stock splits and liquidity. For instance, Conroy, Harris, and Benet in the same paper find that “percentage spreads increase after splits, representing a liquidity cost to investors” for NYSE listed companies.

Summarizing the conflicting empirical results, on the one side, the bid-ask spread is positively related to the stock price level, on the other side, the percentage bid-ask spread is negatively related to the stock price. Our search based model can resolve this apparently controversial issue elegantly:

Firstly, consider Equation (2.13), let all the other parameters fixed, when P decreases (b also decreases in the same proportion during this process), the absolute bid-ask spread will decrease as well, showing that the absolute bid-ask spread is positively related to the stock price level P ;

Secondly, divide both sides of Equation (2.13) by P in order to obtain the formula for the percentage bid-ask spread. Roughly speaking, the numerator of the right-hand side of Equation (2.13) has the power of $\frac{1}{2}$ for P , the denominator has the power of 1 for P , then the percentage bid-ask spread has the power of $-\frac{1}{2}$ for P . Thus, the percentage bid-ask spread is approximately a decreasing function of the stock price level P .

(4) The effect of the discount rate (r) on the bid-ask spread

The effect of the discount rate on the bid-ask spread is roughly positive since in most of times, the positive effect of the discount rate on the bid-ask spread dominates its negative effect. It is not difficult to understand this positive effect if we think of the discount rate as the opportunity cost of holding the stock without earning the interest for the market dealer's own fund. The higher the discount rate is, the more the interest given up by the market dealer when he keeps an inventory of stocks, thus the higher the bid-ask spread will be required for his compensation.

(5) The effect of the instantaneous market opportunity (λ) on the bid-ask spread

λ is unique to our competitive search based model. While the effect of λ on the bid-ask spread is very similar to that of the discount rate r since both parameters show up in the same position in our derived bid-ask spread formula, the underlying economic meaning of λ is rather subtle and thus sensitive to explanation.

Formally, λ is defined as the market dealer's cancellation rate for an existing deal, i.e. when the market dealer expects that the posted bid price W^* may not be appropriate for the ongoing market sentiment or economic situation, the market dealer will cancel it. From this viewpoint, we image λ as the measure of the instantaneous market opportunity.

Alternatively, since too low or too high W^* is equally likely to cause the market dealer to

cancel the existing deal, λ can also be treated as the measure of the overall market uncertainty (because in our concise model, we only have one asset or stock and thus one security market). Higher λ means that a market dealer feels that there exists more uncertainty in the market and thus it is more likely for the posted bid price W^* unfit for the current market condition, leading to a deal cancelled more often.

In sum, we conclude that:

Proposition 4(Determinants of the bid-ask spread):

- (1) *The bid-ask spread is positively related to the market dealer's maintenance cost a , the discount rate r and the instantaneous market opportunity λ ;*
- (2) *The bid-ask spread is negatively related to the dividends from holding one share b .*
- (3) *The bid-ask spread is positively related to the stock price level P ; but the percentage bid-ask spread is negatively related to the stock price level P .*

Moreover, there are several important caveats which need to be further discussed and clarified below.

(A) De-centralized market vs. centralized market

Basically speaking, if there is a centralized market for the transaction of an asset, most of the market search friction considered here will become trivial. Thus, from this narrowest viewpoint, our model applies to over the counter market, the foreign exchange market, and the other de-centralized markets such as the real estate market.

However, from the perspective of the normal market operation, the market dealer (or any other agent with the same responsibility but the different title in an asset market) still needs to be credited with providing the necessary matching service to both actual buyers and sellers. Just because of the existence of this type of agent, the market search friction in a centralized market

can be reduced to the current minimum level. Thus, our model can also be utilized to assess the magnitude of liquidity cost in terms of the bid-ask spread in a centralized market.

Furthermore, in history, centralized markets all evolve from the initial de-centralized markets. Our search based model can earn its merits from this viewpoint. When considering the compensation for a role of a market dealer playing in a centralized market as the form of the bid-ask spread, we need to compare two states, one is the real centralized market, and the other is the imaginary de-centralized market, even if the de-centralized market is not the current status of the market. To be more specific, only if we explore how difficult it is to meet the other side of the transaction in the imaginary de-centralized market, we can then evaluate accurately how well the current centralized market provides to investors and justify the amount of the bid-ask spread required by a market dealer in the real centralized market. Hence, the market search and matching friction modeled in this manuscript, should be embedded implicitly in the bid-ask spread for any centralized market.

More importantly, our search based model is distinguished from traditional search models by how the search process works. While traditional search models assume 1) the random matching between the types of agents such as workers and firms, 2) the determination of W^* by bargaining once they meet, our search model lets market dealers to post a widely known bid price W^* in public ex ante in order to direct or attract the arrival of investors. Due to the above feature, our model can be classified as “directed search and posting” or “competitive search”(Moen (1997)) and (Shimer (1996)). Furthermore, our symmetric equilibrium lets all market dealers play the same strategy at equilibrium, i.e. post the same bid price W^* in the entire market. In this way, our search model well mimics the operation of NASDAQ stock market.

(B) Search and matching friction cost vs. Inventory-holding cost and asymmetric

information cost

The earlier version of theories of the bid-ask spread focused so much on the concept of inventory-holding cost for a market dealer that it ignored the information content represented by the bid-ask spread. Although modern theories of the bid-ask spread (Bollena (2004)) pay enough attention to asymmetric information cost associated with two types of investors in the market, the random or noise investors and the informed investors, to our knowledge, there is no such a theory or model except ours that puts the important search and matching friction cost into consideration. The relative weight of the inventory-holding cost and the search and matching cost is basically an empirical issue, on which our model has the potential to shed light. Basing on our bid-ask spread formula in Equation (2.13), though the two terms are coupled with each other in our model, if the search and matching cost is roughly estimated as by W^* and the inventory-holding cost is proxied by the maintenance cost a . So the ratio of the two types of costs will be W^*/a .

(C) The number of market dealers: Competition vs. monopoly

Another interesting result is related to the number of market dealers which can be supported by the market. Since the market tightness $\theta^* = v/u$, if u is known, we can pin down the total number of market dealers at the market equilibrium, which is equal to $(1-u + v)$, here, $1-u$ is the number of market dealers who have a business, v is the number of market dealers who are idle.

One salient feature of our model is that we don't assume the total number of market dealers be fixed, ex ante. This number is endogenously decided by the system via a free entry and exit condition for market dealers, which is the reason why our model is called the "competitive" search model. Intuitively, some researchers give a pre-emptive monopoly position to a market dealer. Therefore their models may lead to a comparatively higher bid-ask spread owing to the

monopoly profit.

3.5 Calibration and simulation

In this section, we calibrate the key parameters of our bid-ask spread formula according to the typical values of the stock market in order to show the impacts of those parameters on the bid-ask spread quantitatively. Our simulation results illustrate that the bid-ask spread indicated by our competitive search based model well fits into empirical observations. However, it should be noted that the choices of values of model parameters may have a significant effect on the magnitude of the bid-ask spread.

Table 3.1 summarizes the parameter values which will be used in our model.

Parameter	Notation	Typical Value
Maintenance cost occurring to a market dealer	a	\$15 (1.5P)
Dividends produced each period by one share of stock	b	\$9.70 (0.97P)
Stock price level	P	\$10
Discount rate	r	0.04
Instantaneous market opportunity	λ	0.05

Table 3.1: Key parameters and their typical values for the bid-ask spread

Figure 3.1 shows the positive effect of the maintenance cost (a/P) on the relative bid-ask spread ($2(P-W^*)/P$) when the values of the other parameters from Table 3.1 are fixed. We let the maintenance cost change from one time to five times of the stock price level P.

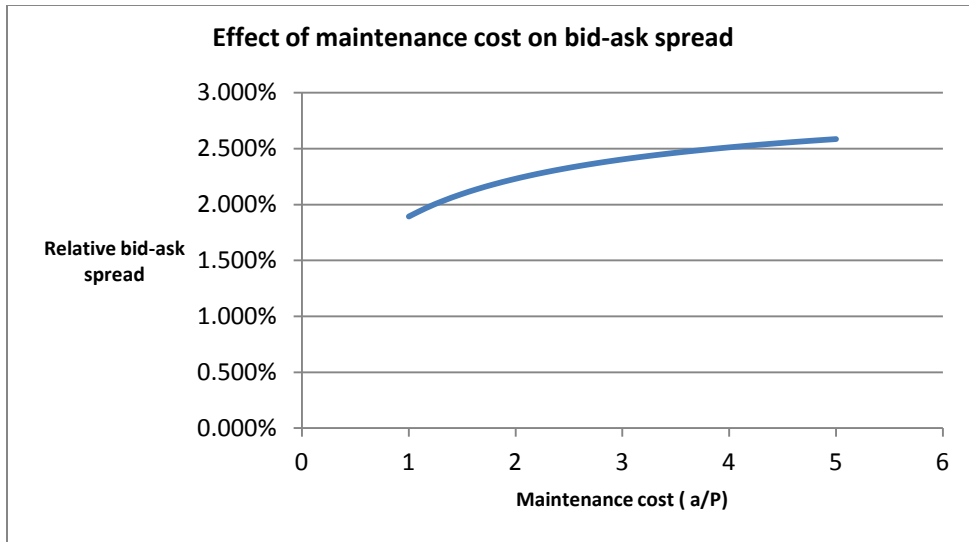


Figure 3.1 Effect of a on the relative bid-ask spread

Figure 3.2 shows the negative effect of the dividend yield (b/P) on the relative bid-ask spread $(2(P-W^*)/P)$ when the typical values of the other parameters are still used from Table 3.1. The possible values of the dividend rate range from 0.95 P to 0.99P. Here P is the stock price level.

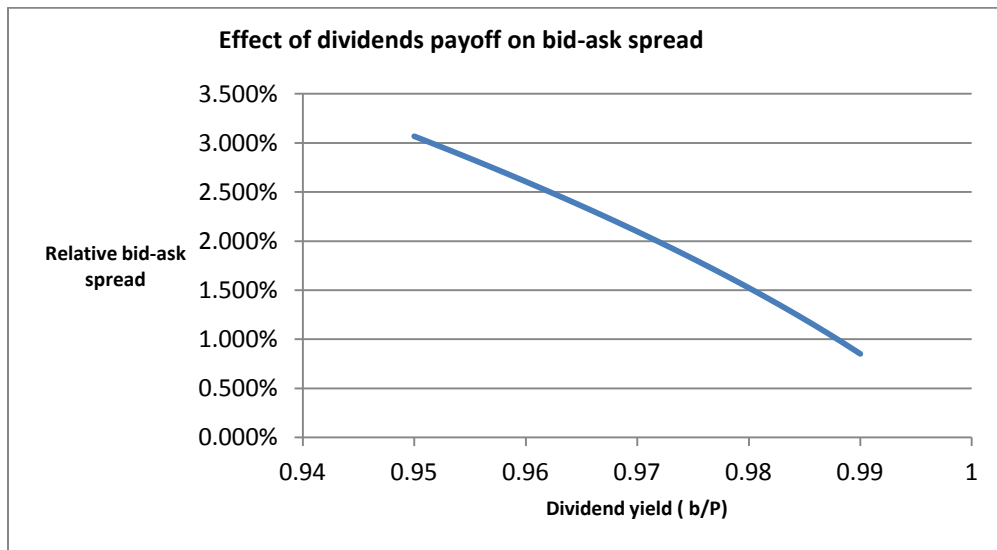


Figure 3.2 Effect of b on the relative bid-ask spread

Figure 3.3 shows the positive effect of the discount rate (r) on the relative bid-ask spread $(2(P-W^*)/P)$ when the typical values of the other parameters are used from Table 3.1. The

possible values of the discount rate range from 0.02 to 0.06.

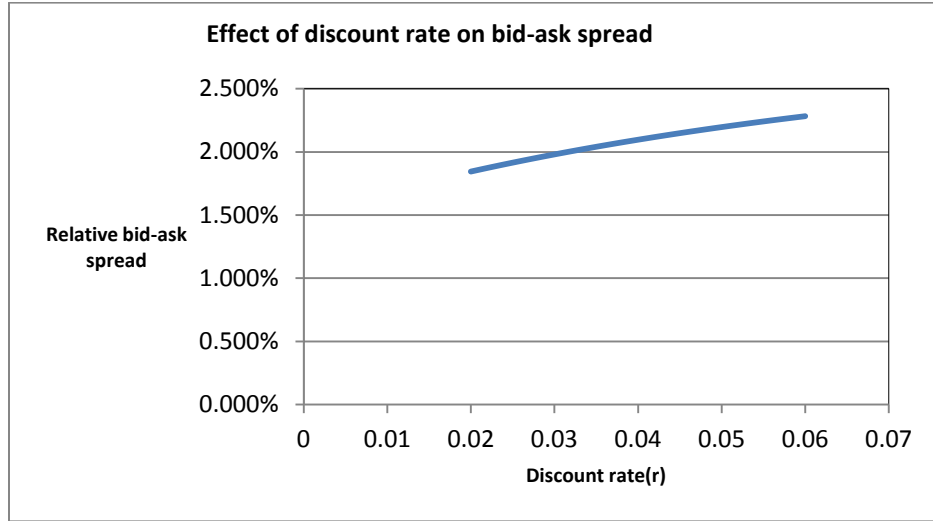


Figure 3.3 Effect of r on the relative bid-ask spread

Figure 3.4 shows the positive effect of the market instantaneous opportunity (λ) on the bid-ask spread $(2(P-W^*)/P)$ when the typical values of the other parameters are used from Table 3.1. The possible values of the market instantaneous opportunity range from 0.03 to 0.07.

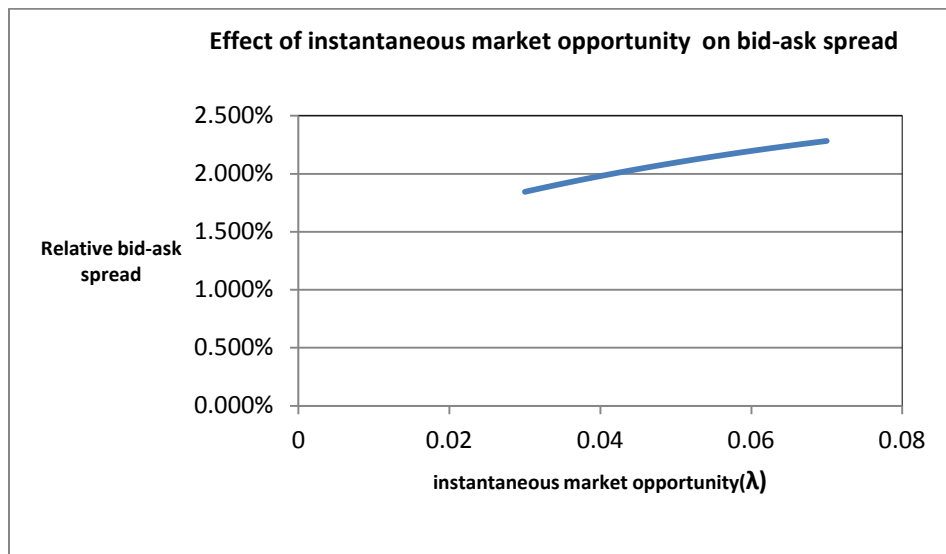


Figure 3.4 Effect of λ on the relative bid-ask spread

Figure 3.5 (a) and (b) show the effects of the stock price level (P) on both the relative (percentage bid-ask spread) (Figure 3.5(a)) and the absolute (dollar bid-ask spread) (Figure

3.5(b)) when the typical values of the other parameters are used from Table 3.1 except that we keep the dividends payoff b as a constant ratio of the stock price level P , i.e. $b=0.97P$. The possible values of the stock price level range from \$5 to \$15. We can see clearly that the absolute bid-ask spread is positively related to the stock price level; while the percentage bid-ask spread is negatively related to the stock price level.

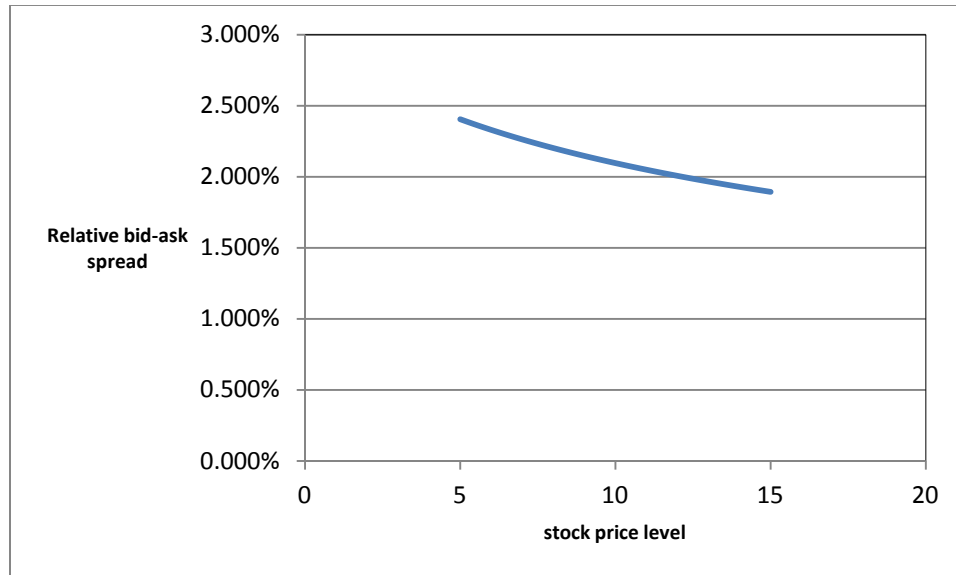


Figure 3.5(a) Effect of stock price level on the relative bid-ask spread

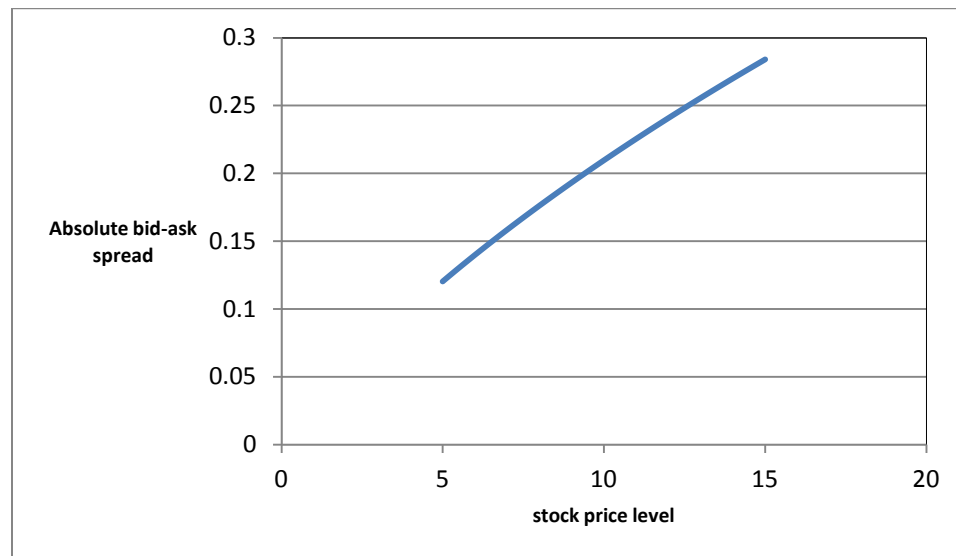


Figure 3.5(b) Effect of stock price level on the absolute bid-ask spread

In addition, we can also evaluate the ratio between the search and matching cost and the inventory-holding cost by W^*/a , showed in Figure 3.6 where the search and matching cost becomes less important when the maintenance cost increases.

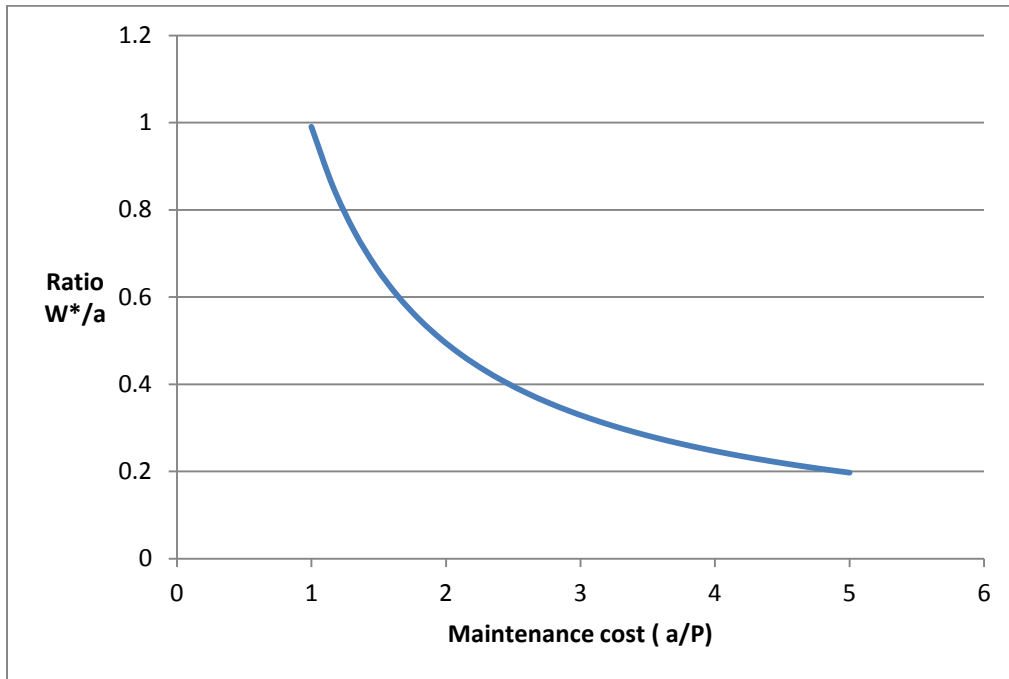


Figure 3.6 Ratio between search and matching cost and inventory-holding cost

3.6 Conclusion and future directions

In this paper, the optimal behaviors of both market dealers and investors are simultaneously investigated under the framework of competitive search theory. Four useful value functions for both agents are established to represent the corresponding utilities obtained when staying in two distinct states, the idle state and the occupied state.

Our competitive search based model shows that the bid-ask spread charged by market dealers is legitimated by their liquidity service provided to actual buyers and sellers in the security market. The more difficult for an investor to locate a counter party to trade in a de-centralized market, the higher the bid-ask spread required by market dealers in a centralized market.

Moreover, our model stress that the ultimate bid-ask spread prevalent in the centralized market will also be cut down by the competition among market dealers. Our derived bid-ask spread formula indicates that the magnitude of the bid-ask spread is affected by many factors such as the cost for maintaining a market dealer's position, the (risk-free) discount rate, the overall market uncertainty, the dividends payoff and the stock price level, etc.

Our model's theoretical prediction on the effect of dividends policy on the bid-ask spread is consistent with asymmetric information based theories and empirical observations. Our model successfully addresses the apparent controversial issue on the effect of the stock price level on the absolute bid-ask spread and on the percentage bid-ask spread without the assumption of asymmetric information.

Several directions for the future work are suggested, including introducing more detailed economic analysis of the influencing parameter λ ; considering the more complicated case of an integrated two-side search covering the price formation mechanism for both bid price and ask price; and combining the theoretical predictions with the empirical results to further testify the validity of the proposed model.

Appendix C Notation Table

f: market dealers.

w: investors.

P: the instantaneous price at which a market dealer can dispose of one share of stock. (Since we only model the selling part of the market, P can be treated as parameter.)

W: the instantaneous bid price posted by a market dealer.

W*: the instantaneous bid price at the market equilibrium.

2(P-W*): twice the difference between P and W , denoting the effective bid-ask spread at the market equilibrium.

1: the initial number of investors, normalized to 1.

u: the number of investors who don't sell their shares.

1-u: the number of investors who sell their shares, which is also the number of market dealers who have business.

v: the number of market dealers who are idle.

1-u+v: the total number of market dealers including occupied and idle.

a: the maintenance cost occurring to a market dealer when posting a bid price in the market.

b: the dividends produced each period by one share as one Lucas tree.

θ : the market tightness, equals $\frac{v}{u}$.

θ^* : the market tightness at the market equilibrium.

m (θ): the matching technology function between f and w , which is an

increasing and concave function of θ . Thus, the meeting rate for investors is $m(\theta)$, for market dealers is $m(\theta)/\theta$, respectively.

λ : the instantaneous market opportunity or the overall market uncertainty.

r : the (risk-free) discount rate.

State "U": means that it is in the idle state.

State "V": means that it is in the occupied state.

U_f : the value of a market dealer who posts a bid price and waits for a business.

V_f : the value of a market dealer who buys one share from an investor.

U_w : the value of an investor who keeps one share in hand and waits for an chance to sell.

V_w : the value of an investor who sells one share to a market dealer.

Appendix D Proofs of Propositions

Proposition 1: Market equilibrium (Equivalence of two optimal problems: (3.6) and (3.7))

From the optimal problem (3.6),

$$L_1 = U_f(W, \theta) + \mu [U_w(W, \theta) - U_w(W^*, \theta^*)]$$

$$\text{F.O.C. for } W: \quad \frac{\partial L_1}{\partial W} = \frac{\partial U_f}{\partial W} + \mu \frac{\partial U_w}{\partial W} = 0$$

$$\text{for } \theta: \quad \frac{\partial L_1}{\partial \theta} = \frac{\partial U_f}{\partial \theta} + \mu \frac{\partial U_w}{\partial \theta} = 0$$

From the optimal problem (3.7),

$$L_2 = U_w(W, \theta) + \eta U_f(W, \theta)$$

$$\text{F.O.C. for } W: \quad \frac{\partial L_2}{\partial W} = \frac{\partial U_w}{\partial W} + \eta \frac{\partial U_f}{\partial W} = 0$$

$$\text{for } \theta: \quad \frac{\partial L_2}{\partial \theta} = \frac{\partial U_w}{\partial \theta} + \eta \frac{\partial U_f}{\partial \theta} = 0$$

When set $\mu = 1/\eta$, those two sets of conditions are equivalent with each other.

Proposition 2: Two equilibrium equations

(1) Derive the free entry equation:

Step one: Solve for U_f .

For a market dealer, (3.3)-(3.4), we get:

$$r(V_f - U_f) = P - W + a - (V_f - U_f)(\lambda + m/\theta)$$

$$(V_f - U_f) = (P - W + a) / (r + \lambda + m/\theta)$$

Put the above relation back into (3.4),

$$\begin{aligned} rU_f &= -a + (m/\theta) * (V_f - U_f) = -a + (m/\theta) * [(P - W + a) / (r + \lambda + m/\theta)] \\ &= -a + [m(P - W + a)] / [\theta(r + \lambda) + m] \end{aligned}$$

$$=[m(P-W)-a \theta(r+\lambda)] / [\theta(r+\lambda)+m]$$

$$\text{So } U_f = [m(P-W)-a \theta(r+\lambda)] / [r\theta(r+\lambda)+rm]$$

Step two: Use (3.5), since $U_f=0$, the numerator has to be zero,

$$\text{i.e. } m(P-W)-a \theta(r+\lambda)=0$$

At market equilibrium, W^* and θ^* , we get:

$$\boxed{m(\theta^*)(P-W^*) - a (r+\lambda)\theta^*=0}$$

(2) Derive the Nash equilibrium equation:

Step one: Solve for U_f .

According to the result derived from the free entry equation, we have known that: $U_f = [m(P-W)-a \theta(r+\lambda)] / [r\theta(r+\lambda)+rm]$

Step two: Solve for U_w .

For an investor, (3.2)-(3.1), we get:

$$r(V_w-U_w)=W-b-(V_w-U_w)(\lambda+m)$$

$$(V_w-U_w)=(W-b)/(r+\lambda+m)$$

Put the above relation back into (3.1),

$$\begin{aligned} rU_w &= b+m(V_w-U_w) = b+ m^*[(W-b)/(r+\lambda+m)] \\ &= [mW+(r+\lambda)b] / (r+\lambda+m) \end{aligned}$$

$$\text{So } U_w = [mW+(r+\lambda)b] / [r(r+\lambda+m)]$$

Step three: Solve the optimal problem (3.6)

Use the LaGrange method, set up two Lagrange multipliers μ and η since there are two constraints (actually the second constraint has no effect on the first order conditions).

$$L = U_f(W, \theta) - \mu [U_w(W, \theta) - U_w(W^*, \theta^*)] - \eta [U_f(W^*, \theta^*)]$$

Put $U_f(W, \theta)$ and $U_w(W, \theta)$ derived from Step one and Step two into L,

we get: $L = [m(P-W) - a\theta(r+\lambda)] / [r\theta(r+\lambda) + rm]$

$$- \mu[rU_w^*(r+\lambda+m) - mW - (r+\lambda)b]$$

$$- \eta[U_f(W^*, \theta^*)]$$

Here, U_w^* is the abbreviated form of $U_w(W^*, \theta^*)$.

Now consider:

First order condition for W :

$$-m/[r\theta(r+\lambda) + rm] + \mu m = 0$$

$$\text{So, } \mu = 1/[r\theta(r+\lambda) + rm] = 1/\{r[\theta(r+\lambda) + m]\}$$

First order condition for θ :

(Note that m is also a function of θ)

$$(1/r) \{ [m'(P-W) - a(r+\lambda)] [\theta(r+\lambda) + m] - [m(P-W) - a\theta(r+\lambda)] [(r+\lambda) + m'] \} / [\theta(r+\lambda) + m]^2 - \mu(rU_w^* m' - m'W) = 0$$

Then put $\mu = 1/\{r[\theta(r+\lambda) + m]\}$ into the above equation, delete the factor of $1/r$ and $1/[\theta(r+\lambda) + m]$ on both items,

$$[m'(P-W) - a(r+\lambda)] - \{ [m(P-W) - a\theta(r+\lambda)] [(r+\lambda) + m'] \} / [\theta(r+\lambda) + m] - (rU_w^* m' - m'W) = 0$$

Apply the free entry condition, at market equilibrium (W^*, θ^*) , we know that: $m(\theta^*)(P-W^*) - a\theta^*(r+\lambda) = 0$, So the middle item is equal to zero.

Finally only the first and last items are left:

$$[m'(P-W) - a(r+\lambda)] - (rU_w^* m' - m'W) = 0$$

$$m'(P-W) - a(r+\lambda) - m'(rU_w^* - W) = 0$$

$$m'(P - rU_w^*) - a(r+\lambda) = 0$$

Moreover, replace rU_w^* by $rU_w(W^*, \theta^*) = [mW^* + (r+\lambda)b] / (r+\lambda+m)$,

$$m'[P(r+\lambda+m) - mW^* - (r+\lambda)b] - a(r+\lambda)(r+\lambda+m) = 0$$

$$m'[P(r+\lambda)+mP-mW^*-(r+\lambda)b]-a(r+\lambda)(r+\lambda+m)=0$$

According to the free entry condition:

$$m(\theta^*)(P-W^*)-a\theta^*(r+\lambda)=0$$

$$\text{so } mP-mW^*=a\theta^*(r+\lambda),$$

Thus, at market equilibrium, the F.O.C. for θ changes into:

$$m'[P(r+\lambda)+a\theta^*(r+\lambda)-(r+\lambda)b]-a(r+\lambda)(r+\lambda+m)=0$$

$$m'(P+a\theta^*-b)-a(r+\lambda+m)=0$$

$$\text{i.e. } m'(\theta^*)(P-b)-a[r+\lambda+m(\theta^*)-\theta^*m'(\theta^*)]=0$$

Proposition 3: Bid-ask spread formula

This proposition has already been proved in the paper. The brief proof is reproduced here for your reference.

$$\text{If } m(\theta)=\theta^{1/2},$$

$$\text{The free entry equation changes into: } \theta^{*1/2}(P-W^*)-a(r+\lambda)\theta^*=0$$

$$\text{i.e. } P-W^*=a(r+\lambda)\theta^{*1/2}$$

The Nash equilibrium equation changes into:

$$0.5\theta^{*-1/2}(P-b)-a(r+\lambda+0.5\theta^{*1/2})=0$$

$$0.5\theta^{*-1/2}(P-b)-a(r+\lambda+0.5\theta^{*1/2})=0$$

Let $\theta^{*1/2}=X$, then, $\theta^{*-1/2}=1/X$, the above equation can be changed into:

$$X^2+2(r+\lambda)X-(P-b)/a=0$$

Solve the above quadratic equation, get X ,

$$\theta^{*1/2}=X=[(r+\lambda)^2+(P-b)/a]^{1/2}-(r+\lambda)$$

Thus, the bid-ask spread is:

$$P-W^*=a(r+\lambda)\theta^{*1/2}=a(r+\lambda)\{[(r+\lambda)^2+(P-b)/a]^{1/2}-(r+\lambda)\}$$

Proposition 4: Determinants of bid-ask spread

It is easy to check out the signs of the first derivations of our bid-ask spread with respect to those determinants.

References

- Amihud, Y., Mendelson, H., [1986]. Asset pricing and the bid-ask spread, *Journal of Financial Economics* 17 223-249
- Blanchard, O., Diamond, P., [1989]. The beveridge curve, *Brookings Papers on Economic Activity* 1, 1-60.
- Bollena, N.P.B., Smith, T., Whaley, R. E. , [2004]. Modeling the bid/askspread: Measuring the inventory-holding premium, *Journal of Financial Economics* 72, 97-141.
- Conroy, R. M., Harris, R. S., Benet, B. A., [1990]. The effects of stock splits on bid-ask spreads, *Journal of Finance* 45 1285-1295
- Copeland, T. E., Galai, D., [1983]. Information effects on the bid-ask spread, *Journal of Finance* 38 1457-1469.
- Diamond, P., [1984]. A search-equilibrium approach to the micro foundations of macroeconomics (MIT press).
- Duffie, D., Garleanu, N., Pedersen, L.H., [2005]. Over-the-counter markets, *Econometrica* 73, 1815-1847.
- Howe, J.S., Lin, J.C., [1992]. Dividend policy and the bid-ask spread: An empirical analysis, *Journal of Financial Research* 15, 1-10.
- Kyle, A.S., [1985]. Continuous auctions and insider trading, *Econometrica* 53, 1335-1355.
- Lagos, R., Rocheteau, G., [2007]. Search in asset markets: Market structure, liquidity, and welfare, *American Economic Review* 97, 198-202.
- Lagos, R., Rocheteau, G., [2009]. Liquidity in asset markets with search frictions, *Econometrica* 77, 403-426.
- Mandal, A. , [2011]. Matching function: Estimations using jolts, *The International Journal of*

- Applied Economics and Finance 5, 157-166.
- Moen, E. R., [1997]. Competitive search equilibrium, *Journal of Political Economy* 105, 385-411.
- Pastor, L., Stambaugh, R.F. , [2003]. Liquidity risk and expected stock returns, *Journal of Political Economy* 111, 642-685.
- Rogerson, R., Shimer,R., Wright, R., [2005]. Search-theoretic models of the labor market: A survey, *Journal of Economic Literature* XLIII, 959–988.
- Shimer, R., [1996]. Contracts in frictional labor markets MIT. Mimeo.
- Vayanos, D., Weill, P., [2008]. A search-based theory of the on-the-run phenomenon, *Journal of Finance* 63, 1361-1398.
- Weill, P., [2008]. Liquidity premia in dynamic bargaining markets, *Journal of Economic Theory* 140, 66-96.

Hongyu (Garrison) Song's Vita

Education

Ph.D., Business Administration, Syracuse University, May, 2013

M.A., Economics (Ph.D. program), State University of New York at Albany, 2008

M.S., Chemical Engineering, Columbia University, 2005

B.S., Engineering Economics & Chemical Engineering, Tianjin University, 1999

Research Interests

IPO, Market Microstructure, Searching and Matching Theory

Teaching Interests

Corporate Finance, Investments, Financial Modeling, Derivatives & Risk Management, International Finance, Public Finance, Financial Statement Analysis, Microeconomics, Macroeconomics, Econometrics

Conferences

Presenter: (1) "Searching in the pre-IPO market: relation between private firms and investment banks"
FMA, 2012, Atlanta, GA

Adelphi University, Robert B. Willumstad School of Business, February, 2013

(2) "Revisit bid-ask spread using competitive search"

Syracuse University, Whitman School of Management, Brown Bag Series, February, 2012

University of New Orleans, College of Business Administration, January, 2012

Discussant: (1) "Does the Duration and Accuracy of Asymmetric Trading Matter To Market Makers?"
FMA, 2012, Atlanta, GA

(2) "Capital Constraints and Systemic Risk"

FMA, 2010, New York, NY

Teaching Experience

Instructor

(1) Essentials of Finance (**Fin 301**) (2) Introductory Statistics to Management (**MAS 261**)

Teaching Assistant

Corporate Financial Policy and Strategy (**FIN 751**) Decision Tools for Managers (**MAS 362**)

Time Series Modeling and Analysis (**MAS 777**) Linear Regression Models (**MAS 766**)

Data Analysis for Managers (**MBC 638**) Principles of Real Estate (**FIN 400**)

Selected Honors and Awards

Full Tuition Scholarship and Graduate Assistantship Syracuse University 2008-2013

Prestigious Presidential Fellowship State University of New York at Albany 2006-2008

University Distinguished Undergraduate Student Tianjin University 1999

Samsung Inc. Scholarship Tianjin University 1998

Professional Affiliations

Member of FMA, AFA, AEA

Candidate of CFA institute (**Passed all three levels of CFA exams**)

Member of The Golden Key International Honor Society and The Honor Society of Phi Kappa Phi

Computer Skills

Microsoft Certified Professional Systems Engineer (**MCSE**)

Microsoft Certified Professional Database Administrator (**MCDBA**)

SAS, STATA, financial modeling with Excel, and HTML design