December 2016

SENSOR MANAGEMENT FOR LOCALIZATION AND TRACKING IN WIRELESS SENSOR NETWORKS

Nianxia Cao
Syracuse University

Follow this and additional works at: http://surface.syr.edu/etd

Part of the Engineering Commons

Recommended Citation
Cao, Nianxia, "SENSOR MANAGEMENT FOR LOCALIZATION AND TRACKING IN WIRELESS SENSOR NETWORKS" (2016). Dissertations - ALL. 563.
http://surface.syr.edu/etd/563

This Dissertation is brought to you for free and open access by the SURFACE at SURFACE. It has been accepted for inclusion in Dissertations - ALL by an authorized administrator of SURFACE. For more information, please contact surface@syr.edu.
Abstract

Wireless sensor networks (WSNs) are very useful in many application areas including battlefield surveillance, environment monitoring and target tracking, industrial processes and health monitoring and control. The classical WSNs are composed of large number of densely deployed sensors, where sensors are battery-powered devices with limited signal processing capabilities. In the crowdsourcing based WSNs, users who carry devices with built-in sensors are recruited as sensors. In both WSNs, the sensors send their observations regarding the target to a central node called the fusion center for final inference. With limited resources, such as limited communication bandwidth among the WSNs and limited sensor battery power, it is important to investigate algorithms which consider the trade-off between system performance and energy cost in the WSNs. The goal of this thesis is to study the sensor management problems in resource limited WSNs while performing target localization or tracking tasks.

Most research on sensor management problems in classical WSNs assumes that the number of sensors to be selected is given a priori, which is often not true in practice. Moreover, sensor network design usually involves consideration of multiple conflicting objectives, such as maximization of the lifetime of the network or the inference performance, while minimizing the cost of resources such as energy, communication or deployment costs. Thus, in this thesis, we formulate the sensor management problem in a classical resource limited WSN as a multi-objective optimization problem (MOP), whose goal is to find a set of sensor selection strategies which reveal the trade-off between the target tracking performance and the number of selected sensors to perform the task. In this part of the thesis, we propose a novel mutual information upper bound (MIUB) based sensor selection scheme, which has low computational complexity, same as the Fisher information (FI) based sensor selection scheme, and gives estimation performance similar to the mutual information (MI) based sensor selection scheme. Without knowing the number of sensors to be selected a priori, the MOP gives a set of sensor selection strategies that reveal differ-
ent trade-offs between two conflicting objectives: minimization of the number of selected sensors and minimization of the gap between the performance metric (MIUB and FI) when all the sensors transmit measurements and when only the selected sensors transmit their measurements based on the sensor selection strategy.

Crowdsourcing has been applied to sensing applications recently where users carrying devices with built-in sensors are allowed or even encouraged to contribute toward the inference tasks. Crowdsourcing based WSNs provide cost effectiveness since a dedicated sensing infrastructure is no longer needed for different inference tasks, also, such architectures allow ubiquitous coverage. Most sensing applications and systems assume voluntary participation of users. However, users consume their resources while participating in a sensing task, and they may also have concerns regarding their privacy. At the same time, the limitation on communication bandwidth requires proper management of the participating users. Thus, there is a need to design optimal mechanisms which perform selection of the sensors in an efficient manner as well as providing appropriate incentives to the users to motivate their participation. In this thesis, optimal mechanisms are designed for sensor management problems in crowdsourcing based WSNs where the fusion center (FC) conducts auctions by soliciting bids from the selfish sensors, which reflect how much they value their energy cost. Furthermore, the rationality and truthfulness of the sensors are guaranteed in our model. Moreover, different considerations are included in the mechanism design approaches: 1) the sensors send analog bids to the FC, 2) the sensors are only allowed to send quantized bids to the FC because of communication limitations or some privacy issues, 3) the state of charge (SOC) of the sensors affects the energy consumption of the sensors in the mechanism, and, 4) the FC and the sensors communicate in a two-sided market.
SENSOR MANAGEMENT FOR LOCALIZATION AND TRACKING IN WIRELESS SENSOR NETWORKS

By

Nianxia Cao
B.S., Northwestern Polytechnical University (NPU), China, 2009
M.S., Northwestern Polytechnical University (NPU), China, 2012

DISSERTATION

Submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Electrical and Computer Engineering

Syracuse University
December 2016
ACKNOWLEDGMENTS

First and foremost, I would like to express my deepest gratitude and sincere thanks to my advisor, Prof. Pramod K. Varshney, for his guidance, support and encouragement during my doctoral study. I have been amazingly fortunate to have an advisor who gave me the freedom to explore on my own, and at the same time guidance to recover when my steps faltered. He has been influencing me with his enthusiasm towards research, as well as immense knowledge, and will keep guiding me in the future.

Besides my advisor, I would like to thank the rest of my thesis committee: Prof. Chiwa Kao, Prof. Biao Chen and Prof. Yinbing Liang, Prof. Yanzhi Wang, and Prof. Kishan Mehrotra for their insightful comments and suggestions.

Dr. Swastik Brahma, Dr. Engin Masazade, Dr. Thakshila Wimalajeewa, and Dr. Sora Choi have provided technical support for part of the work in this thesis.

I would like to thank all my fellow labmates in the Sensor Fusion Laboratory for their constant support in my research and life: Yujiao Zheng, Swarnendu Kar, Arun Subramanian, Hao He, Aditya Vempaty, Sid Nadendla, Raghed Bardan, Sijia Liu, Bhavya Kailkhura, Shan Zhang, Qunwei Li, Prashant Khanduri, Swatantra Kafle and Pranay Sharma.

My PhD research has been supported by U.S. Air Force Office of Scientific Research (AFOSR) Grants No. FA9550-10-1-0458, FA9550-10-1-0263 and National Science Foundation (NSF) Grant No. 1307775, for which I am very grateful.

Most importantly, none of this would have been possible without the love and constant encouragement of my parents, my brother, and my husband and my newborn baby.
I would like to thank them for always being there cheering me up and standing by me through the good times and bad.
# Table of Contents

Acknowledgments v

List of Tables xii

List of Figures xiii

1 Introduction 1

1.1 Sensor Management ........................................ 1
  1.1.1 Sensor Management in Classical WSNs .................. 2
  1.1.2 Sensor Management in WSNs with Unreliable Sensor Observations .. 3
  1.1.3 Sensor Management in Crowdsourcing based WSNs .......... 4

1.2 Summary of Contributions and Outline of Thesis ............. 6

1.3 Bibliographic Notes ....................................... 9

2 Background and Preliminaries 11

2.1 Introduction ............................................ 11

2.2 Preliminaries for Target Tracking ............................ 11
  2.2.1 System Model ......................................... 11
  2.2.2 Fisher Information ...................................... 13
  2.2.3 Mutual Information .................................... 15
  2.2.4 Particle Filter for Target Tracking ..................... 15

2.3 Preliminaries for Target Localization ....................... 16
  2.3.1 Monte Carlo Method for Target Localization .......... 17
2.4 Multiobjective Optimization based Sensor Selection ........................................ 18
  2.4.1 NSGA-II ................................................................. 19

3 Multiobjective Optimization method based Sensor Selection in Wireless Sensor Networks 21
  3.1 Introduction ................................................................. 21
  3.2 A Multiobjective Optimization based Sensor Selection Method for Target Tracking in Wireless Sensor Networks ................................................................. 23
    3.2.1 Solution Selection from the Pareto-optimal Front ...................... 25
    3.2.2 Simulation Results ....................................................... 26
  3.3 Sensor Selection for Target Tracking in Wireless Sensor Networks with Uncertainty 30
    3.3.1 Uncertainty Model of Sensor Observations .............................. 31
    3.3.2 Sensor Selection Criteria for Uncertain WSNs .......................... 33
    3.3.3 Multiobjective Optimization based Sensor Selection .................. 42
  3.4 Portfolio Theory based Sensor Selection in Wireless Sensor Networks with Unreliable Observations ................................................................. 49
    3.4.1 System Model ............................................................. 49
    3.4.2 Portfolio Theory based Sensor Selection .................................. 52
    3.4.3 NBI method for Multiobjective Optimization ............................. 55
    3.4.4 Simulation Experiments .................................................. 56
  3.5 Summary ................................................................. 58

4 Sensor Management in Crowdsourcing based Sensor Networks: A Mechanism Design Approach 72
  4.1 Introduction ................................................................. 72
  4.2 An Incentive-based Mechanism for Location Estimation in Wireless Sensor Networks 74
    4.2.1 System Model and Problem Formulation ............................... 75
    4.2.2 Analysis of the Problem .................................................. 78
4.2.3 Simulation Results .............................................. 81
4.3 Target Tracking via Crowdsourcing: A Mechanism Design Approach ............. 85
  4.3.1 Target Tracking in Wireless Sensor Networks .................................. 86
  4.3.2 Formulation of the Auction Design Problem .................................... 87
  4.3.3 Analysis of the Auction Design Problem ........................................ 92
  4.3.4 Implementation of the Proposed Mechanism .................................... 97
  4.3.5 Simulation Experiments .................................................. 103
4.4 Summary ........................................................................... 106

5 Optimal Auction Design with Quantized Bids in Crowdsourcing based Sensor Networks ............................................. 111
  5.1 Introduction ......................................................................... 111
  5.2 Optimal Auction Design with Binary Bids ..................................... 113
    5.2.1 Auction Model .......................................................... 113
    5.2.2 Auction Design Problem Formulation ............................... 114
    5.2.3 Optimal Auction Design for Given Quantization Thresholds .......... 117
    5.2.4 Optimal Quantization Thresholds .................................... 122
  5.3 Optimal Auction Design with Quantized Bids for Target Tracking ............. 126
    5.3.1 System Model .......................................................... 126
    5.3.2 Sensor Selection for Measurement Transmission .................... 129
    5.3.3 Analysis of the Optimal Auction Design Problem for Measurement Selection 131
    5.3.4 Optimization Problem Analysis ..................................... 133
    5.3.5 Sensor Selection for Bidding ......................................... 138
    5.3.6 Simulation Results .................................................. 138
  5.4 Summary ........................................................................... 140

6 Charging State Aware Optimal Auction Design for Sensor Selection in Crowdsourcing based Sensor Networks 145
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.1</td>
<td>Introduction</td>
<td>146</td>
</tr>
<tr>
<td>6.2</td>
<td>Formulation of the Auction Design Problem</td>
<td>147</td>
</tr>
<tr>
<td>6.2.1</td>
<td>System Model</td>
<td>147</td>
</tr>
<tr>
<td>6.2.2</td>
<td>Probabilistic Bid Information</td>
<td>147</td>
</tr>
<tr>
<td>6.2.3</td>
<td>Problem Formulation</td>
<td>148</td>
</tr>
<tr>
<td>6.3</td>
<td>Problem Analysis</td>
<td>151</td>
</tr>
<tr>
<td>6.4</td>
<td>Simulation Experiments</td>
<td>157</td>
</tr>
<tr>
<td>6.5</td>
<td>Summary</td>
<td>162</td>
</tr>
</tbody>
</table>

**7 Cloud Sensing Enabled Target Localization** 166

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.1</td>
<td>Introduction</td>
<td>166</td>
</tr>
<tr>
<td>7.2</td>
<td>Problem Formulation</td>
<td>169</td>
</tr>
<tr>
<td>7.2.1</td>
<td>Basic Assumptions</td>
<td>169</td>
</tr>
<tr>
<td>7.2.2</td>
<td>Motivation for Incentive Compatibility Constraint</td>
<td>170</td>
</tr>
<tr>
<td>7.2.3</td>
<td>Expected Utility Functions</td>
<td>171</td>
</tr>
<tr>
<td>7.3</td>
<td>Constraints of Individual Rationality, Incentive Compatibility, and Ex Post Efficiency 172</td>
<td></td>
</tr>
<tr>
<td>7.3.1</td>
<td>Individual Rationality and Incentive Compatibility</td>
<td>172</td>
</tr>
<tr>
<td>7.3.2</td>
<td>Ex Post Efficiency</td>
<td>176</td>
</tr>
<tr>
<td>7.4</td>
<td>Sensor Selection in Optimal Mechanism</td>
<td>177</td>
</tr>
<tr>
<td>7.4.1</td>
<td>Optimal Trading Rule and Corresponding Payment</td>
<td>178</td>
</tr>
<tr>
<td>7.4.2</td>
<td>Sensor Selection Problem</td>
<td>181</td>
</tr>
<tr>
<td>7.5</td>
<td>Simulation Experiments</td>
<td>181</td>
</tr>
<tr>
<td>7.6</td>
<td>Summary</td>
<td>184</td>
</tr>
</tbody>
</table>

**8 Market Based Sensor Mobility Management for Target Localization** 188

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.1</td>
<td>Introduction</td>
<td>189</td>
</tr>
<tr>
<td>8.2</td>
<td>Market Based Mobile Sensor Scheduling</td>
<td>191</td>
</tr>
<tr>
<td>8.2.1</td>
<td>System Model</td>
<td>191</td>
</tr>
</tbody>
</table>
LIST OF TABLES

1.1 Connection between publications & chapters ............................... 10

3.1 Sensors with the most significant MI or FI at different time steps .......... 40

3.2 Mean values of the probabilities $p_{si}$, the expected FI gain $r_i$, and the variance of $r_i$. 53

3.3 MSE Performance of the solutions found by the NBI method. .................. 57

5.1 Optimal quantization thresholds with one buyer. .......................... 124

5.2 Optimal thresholds with one buyer ............................................. 125
LIST OF FIGURES

3.1 An example target trajectory and WSN with $N = 16$ sensors. 27
3.2 Pareto optimal front obtained by using NSGA-II at time step (a) $t = 9$ and (b) $t = 12$. y-axis represents the objective function $f_1$ and x-axis represents the objective function $f_2$. 28
3.3 MSE performance and average number of sensors selected at each time step of tracking when using knee-point solutions and the compromise solution. 29
3.4 MSE of sensor selection based on multiobjective optimization and convex relaxation based sensor selection method which selects $A$ sensors at each time step of tracking. 30
3.5 MSE and the average number of selected sensors when the sensor measurements are quantized into $M = 5$ and $M = 3$ bits for the compromise solution. 31
3.6 WSN with 36 unreliable sensors. Numbers above the stars indicate sensor index (left) and its sensing probability (right). 38
3.7 Target tracking performance with analog data, 5-bit quantized data, and 2-bit quantized data, (a) MSE performance; (b) average percentage of reliable sensors selected. 60
3.8 Target tracking performance for MI and MIUB, $A = 2$. 61
3.9 Pareto optimal front obtained by using NSGA-II at time step $t = 3$ and $t = 6$, (a) FI gap shown in (3.31); (b) MIUB gap shown in (3.33). 62
3.10 Tracking performance at each time step with different solution selection methods. 63
3.11 Tracking performance for MOP with NSGA-II, convex relaxation, and weighted sum methods (a) MSE for MIUB; (b) MSE for FI. 64
3.12 Turn off sensors with relatively low sensing probabilities. ....................... 65
3.13 Comparison the performance for WSNs with and without uncertainty. .......... 65
3.14 Tracking performance under varying network densities (a) \( N = 9 \); (b) \( N = 16 \); (c) \( N = 25 \), (d) \( N = 49 \). ............................... 66
3.15 Tracking performance of MIUB and FI, sensors’ sensing probabilities are reversely deployed. ................................................................. 67
3.16 Tracking performance of MIUB and FI in a WSN with obstacle in center. .... 67
3.17 Sensors’ sensing probabilities are all uniformly distributed between 0 and 1 for every Monte Carlo trial. ............................................. 68
3.18 Sensor measurements have higher noise \( \sigma = 1 \). ...................................... 68
3.19 Target moves with a relatively large process noise parameter \( q = 0.01 \). ........ 69
3.20 Sensor measurements are quantized to 3 bits. ........................................... 69
3.21 An example scenario. ................................................................. 70
3.22 Efficient frontier obtained through NBI. The numbers around the frontier are the solution numbers corresponding to Table 3.3. ............................... 70
3.23 Expected return and variance as a function of the number of sensors. ........ 71
3.24 MSE of target localization as a function of the number selected sensors. ........ 71

4.1 The utility of the FC and the corresponding MSE as a function of the total number of sensors in the WSN .................................................. 83
4.2 The utility of the FC and the corresponding FIM as a function of the total number of sensors in the WSN. Red-Circle: \( v_{FC} = 0.2 \), Green-Diamond: \( v_{FC} = 0.6 \), Blue-Triangle: \( v_{FC} = 0.9 \), Magenta-Plus: FIM based Sensor Selection. ........ 84
4.3 The lifetime of the network under our mechanism and the FIM based sensor selection (SS) method. .......................................................... 85
4.4 Flowchart of the crowdsourcing-based target tracking mechanism. \( T_s \) denotes the total number of tracking steps. ................................. 86
4.5 Trellis of the dynamic programming algorithm for time step \( t \) ................. 100
4.6 Bit allocation with $M = 5$ and $M = 8$ (a) The number of selected sensors. (b) The utility of the FC. (c) MSE at each time step. (d) The utility of the FC with prior information excluded. ................................. 102

4.7 Impact of different number of sensors in the ROI (a) The utility of the FC. (b) The MSE at each time step. ................................................................. 108

4.8 The remaining number of active sensors in the WSN ........................................ 109

4.9 Impact of residual energy consideration (a) The utility of the FC. (b) The MSE at each time step. ................................................................. 110

5.1 Utility of the seller as function of the nonidentical thresholds with $N = 2$, $v_0 = 10$, $a_1 = 2$, $b_1 = 8$, and $a_2 = 12$, $b_2 = 20$. .................................................. 125

5.2 Utility of the seller as function of the nonidentical thresholds with $N = 2$, $v_0 = 10$, $a_1 = 5$, $b_1 = 15$, and $a_2 = 8$, $b_2 = 20$. .................................................. 126

5.3 Flowchart of the crowdsourcing-based target tracking mechanism. The FC selects sensors for bidding every $T_s$ time steps, and $T$ denotes the total number of tracking steps. ................................................................. 142

5.4 Illustration of WSN layout. ................................................................................... 143

5.5 MSE and utility of FC when different number of sensors are available in each bidding window. ................................................................. 143

5.6 MSE and utility of the FC with different bidding window $T_s$. .................. 144

6.1 Sensor network example. ................................................................................... 158

6.2 Performance with relatively large $v_{FC}$: (a) utility of the FC, (b) total payment made by the FC to the selected sensors. ................................. 163

6.3 MSE of localization: (a) $v_{FC} = 100$, (b) $v_{FC} = 1$. ........................................ 164

6.4 Utility of the FC as the total number of sensors in the ROI increases. .......... 165

7.1 Cloud Sensing Framework ................................................................. 168
7.2 WSN with $N = 9$ randomly distributed sensors: Case 1 (a) Sensors and the target, the sensor with a square is selected. (b) The optimal gain, cost, and tradeoff of the WSN as a function of the number of selected sensors. ........................................ 186

7.3 WSN with $N = 9$ randomly distributed sensors: Case 2 (a) Sensors and the target, the sensors with squares are selected. (b) The optimal gain, cost, and tradeoff of the WSN as a function of the number of selected sensors. ........................................ 187

8.1 Example of market based mobile sensor network ............................................. 190

8.2 Supply and Demand curve of the FC and the sensors ........................................... 200

8.3 The demanded distance and the corresponding FIM of the FC as a function of the budget .......................................................... 200

8.4 The equilibrium price and distance and the estimating MSE as a function of the initial location of the sensors ............................................. 201
CHAPTER 1
INTRODUCTION

Wireless sensor networks (WSNs) are composed of a large number of densely deployed sensors, where sensors are battery-powered devices with limited signal processing capabilities. When programmed and networked properly, WSNs are very useful in many application areas including battlefield surveillance [1], environment monitoring and target tracking [2], industrial processes [3] and health monitoring and control [4]. In this thesis, we consider two different types of sensor networks: a) classical WSNs, where sensors are used in the applications in a dedicated manner; and b) crowdsourcing based WSNs, where users who carry devices with built-in sensors are recruited as sensors for inference tasks.

1.1 Sensor Management

Tasks associated with WSNs often require coverage of broad areas and a large number of sensors that can be densely deployed over the Region of Interest (ROI). However, in many WSNs, energy is a scarce resource that needs to be conserved to prolong the operational lifetime of the network [5], and the bandwidth for communication is often quite limited. Thus, it is inefficient to utilize all the sensors in the ROI including the uninformative ones, which hardly contribute to the inference task but consume resources. This issue has been investigated and addressed via the development of
sensor selection schemes, whose goal is to select the best non-redundant set of sensors for inference tasks while satisfying some performance and/or resource constraints [6]. The sensor selection problem for target localization and target tracking has been considered in [7–17] among others, where the sensor sets are selected to get the desired information gain or reduction in estimation error about the target state. Transmission of quantized measurements is required in typical WSNs that have limited resources (energy and bandwidth). This gives rise to the more general problem of bit allocation. Given the total bandwidth constraint, the Fusion Center (FC) in this case determines the optimal bandwidth distribution for the channels between the sensors and the FC.

Sensor selection schemes often require \textit{a priori} information about the number of sensors to be selected at each time, denoted as \(A\), and computationally efficient algorithms are developed in order to find the optimal \(A\) sensors that achieve the maximum performance gain. Realistically, in many applications like target tracking, it is unlikely that the number of sensors that need to be selected at each time step of tracking is known to the system designer before operation begins. Therefore, it is quite necessary and important to investigate sensor selection strategies that determine the optimal number of sensors to be selected as well as which sensors to select based on the WSN conditions.

1.1.1 Sensor Management in Classical WSNs

The sensor selection problem for target localization and target tracking has been considered in [7–17] among others, where the sensor sets are selected to get the desired information gain or reduction in estimation error about the target state. In [7–10], the mutual information (MI) or entropy is considered as the performance metric, and in [11, 12], the sensors that have the lowest posterior Cramer-Rao lower bound (PCRLB), which is the inverse of the Fisher information (FI), are selected. In [13], the authors compared the two sensor selection criteria namely MI and PCRLB for the sensor selection problem based on quantized data, and showed that the PCRLB based sensor selection scheme achieves similar mean square error (MSE) with significantly less computational effort. In [14], the sensor selection problem was formulated as an integer programming problem, which was relaxed and solved through convex optimization. In [15], a multi-step sensor selection
strategy by reformulating the Kalman filter was proposed, which was able to address different performance metrics and constraints on available resources. In [16], the authors aimed to find the optimal sparse collaboration topologies subject to a certain information or energy constraint in the context of distributed estimation. Transmission of quantized measurements is required in typical WSNs that have limited resources (energy and bandwidth). This gives rise to the more general problem of bit allocation. Given the total bandwidth constraint, the Fusion Center (FC) determines the optimal bandwidth distribution for the channels between the sensors and the FC. In [18], a myopic bandwidth allocation problem is considered and the algorithms to solve the problem, namely, convex relaxation, approximate dynamic programming (A-DP), generalized Breiman, Friedman, Olshen, and Stone (GBFOS) and greedy search, are compared.

1.1.2 Sensor Management in WSNs with Unreliable Sensor Observations

The previous research on sensor selection assumes that the WSNs operate reliably during the target tracking process without any interruptions. The fact is that, in some situations, the sensor observations are quite uncertain [19–23]. For example, sensors may have temporary failure, there may be abrupt changes in the operating environment [21, 22], or other interference such as traffic or birds/animals that may change the power received by the sensors. Moreover, some random interruptions may appear over the communication channels in the system, and adversaries may jam wireless communications using different attack strategies [23]. These types of uncertainties would result in the set of sensor observations with insufficient information about the target at the fusion center. In other words, in such an uncertain WSN, sensor observations may contain useful information regarding the target only with a certain probability. It is important to investigate the sensor selection problem in such an uncertain environment. In our work here, we study the uncertainty caused by occlusions, i.e., the sensors may not be able to observe the target when blocked by some obstacles. Regarding the representation of this type of uncertainty, the authors in [19] and [20] introduced a stochastic model for sensor measurements. Furthermore, the work in [21]
and [22] generalized the model in [19, 20] to multiple sensors by considering a more realistic viewpoint in that the sensors have different uncertainty at different time instants. For the problems involving uncertain WSNs, even though there are studies about the Kalman filter for target tracking [20, 24–26], and about the target localization problem with non-ideal channels [27, 28], the sensor selection problem in WSNs with uncertain sensor observations has not been considered in the literature and is addressed in this thesis.

1.1.3 Sensor Management in Crowdsourcing based WSNs

Crowdsourcing is the practice of obtaining needed services, ideas, or content by soliciting contributions from a large group of people, rather than from traditional employees or suppliers [29]. Many of today’s sensing applications allow users carrying devices with built-in sensors, such as sensors built in smart phones, and automobiles, to contribute towards an inference task with their sensing measurements, which is exactly an application of crowdsourcing. For instance, today’s smart phones are embedded with various sensors, such as camera, microphone, accelerometer, and GPS, which can be used to acquire information regarding a phenomenon of interest. An advantage of such architectures is that they do not need a dedicated sensing infrastructure for different inference tasks, thereby providing cost effectiveness. Another advantage of such architectures is that they allow ubiquitous coverage.

Systems and applications that rely on utilizing an infrastructure where crowdsourced sensing measurements of participating users are used are poised to revolutionize many sectors of our life. Some example application domains include social networks, environmental monitoring [30, 31], green computing [32], target localization and tracking [33–37], healthcare [38] (such as predicting and tracking disease patterns/outbreaks), and tracking traffic patterns [39, 40]. For instance, the OpenSense project [30] involves the design of a sensing infrastructure for real-time air quality monitoring using heterogeneous sensors owned by the general public, while [31] involves the design of a similar system to monitor noise levels. GreenGPS [32] uses data from sensors installed in automobiles to map fuel consumption on city streets and construct fuel efficient routes between
arbitrary end-points. Various systems to estimate object locations and to track them using smartphone sensors have also been proposed. For instance, work reported in [34, 36] utilizes built-in sensors in smartphones such as camera, digital compass and GPS, to estimate a target location as well as to monitor the velocity of moving objects. In [33, 35, 37], proximity sensors in built-in smartphones are used to track objects (such as lost/stolen devices) installed with electronic tags (such as Bluetooth or RFID tags). Such systems have important commercial applications (such as tracking lost/stolen objects or accurately estimating arrival time of buses) as well as defense related applications (estimating the enemy’s vehicle position prior to an attack).

Existing sensing applications and systems assume voluntary participation of users, for example, [33–37, 39, 41]. While participating in a sensing task, users consume their own resources such as energy and processing power. Moreover, users may also have concerns regarding their privacy. As a result, existing applications and systems may suffer from insufficient number of participants because it may not be rational for the users to participate. Thus, there is a need to design sensing architectures that can provide appropriate incentives to the users to motivate their participation. Furthermore, users, being selfish in nature, may manipulate protocols of the sensing architectures for their own benefits. Thus, a critical property that any mechanism involving selfish entities should exhibit is strategy-proofness or truthfulness. As has been shown in [42], mechanisms that are not truthful are prone to market manipulations and can have inefficient outcomes.

Market-based mechanisms for sensor management have started to gain attention only recently [43–45]. In [43], the authors explored the possibility of using economic concepts for sensor management without explicitly formulating a specific problem. The authors in [44] used the concept of Walrasian equilibrium [46] to model market-based sensor management. In [45], the authors also proposed a Walrasian equilibrium-based dynamic bit allocation scheme for target tracking in energy-constrained wireless sensor networks (WSNs) using quantized data. However, as shown in [47], Walrasian markets can be unstable and can fail to converge to the equilibrium. Furthermore, computing the equilibrium prices and allocations can be computationally prohibitive. Accordingly, the authors ( [44] and references therein) propose algorithms to compute an approximate equilib-
rium. Moreover, the mechanisms proposed in [44, 45] are not truthful and are, therefore, prone to market manipulations.

1.2 Summary of Contributions and Outline of Thesis

This thesis is organized as follows: In Chapter 2, a preliminary introduction to some background material relevant to this thesis is included. Chapter 3 investigates the multi-objective optimization method based sensor management problem in classical WSNs, and two different scenarios when 1) all the sensors in the network have reliable performance, and, 2) some sensors operate in an unreliable manner are considered. In Chapter 4 to Chapter 6, optimal mechanisms are designed for sensor management problems in crowdsourcing based WSNs. Chapter 4 assumes that the sensors send analog bids to the fusion center to compete for participation while Chapter 5 considers a more practical situation where the sensors are only allowed to send quantized bids to the fusion center. Chapter 6 includes further consideration that the sensors have different energy consumptions with different charging status. Further, Chapter 7 studies a two-sided auction mechanism for sensor management problems in the target localization problem. In Chapter 8, the demand-supply model is applied for the management of the sensors in WSNs from a financial point of view. We then conclude this thesis in Chapter 9. The main contributions of each chapter are as follows.

Chapter 3 proposes a multiobjective optimization method for the sensor selection problem in a resource limited classical WSN for target tracking. Three performance metrics, Fisher information (FI), mutual information (MI), and mutual information upper bound (MIUB) are considered as objective functions for characterizing the estimation performance for the multiobjective optimization problem (MOP). At each time step of tracking, the sensor selection strategy is obtained from the Pareto-optimal solutions which reflect different trade-offs between the total number of selected sensors and estimation accuracy. Numerical results are presented to show that the MIUB based selection scheme (MIUB-SS) selects more reliable sensors compared with the FI based selection scheme (FI-SS) while saving computational cost compared with the MI based selection
scheme (MI-SS). Furthermore, for the MOP framework, we show that the compromise solution on the Pareto front of the MOP achieves good estimation performance while obtaining savings in terms of the number of selected sensors. In this chapter, we further propose a portfolio theory based sensor selection framework in uncertain WSNs for target localization. Our task is to select sensors that consider both the expected FI gain and the FI variability, i.e., risk. Thus, we apply the portfolio theory from economics and finance to our problem, and formulate our sensor selection problem as a multiobjective optimization problem (MOP), which is solved by the normal boundary intersection (NBI) method.

In Chapter 4, we first limit our focus to the design of an incentive-based mechanism for the sensor selection problem in target localization. Then, we study the more general problem of designing an incentive-based mechanism for dynamic bit allocation in the target tracking process. In this chapter, to accomplish the sensor management task in the target localization and tracking problems, the FC conducts an auction by soliciting bids from the selfish sensors, where the bids reflect the information available at the sensor and the energy cost of the sensors. Furthermore, the rationality and truthfulness of the sensors are guaranteed in our model.

Chapter 5 considers the design of an auction mechanism when the bidders quantize their private value estimates regarding the object/target prior to communicating them to the auctioneer. The designed auction mechanism maximizes the utility of the auctioneer (i.e., the auction is optimal), prevents bidders from communicating falsified quantized bids (i.e., the auction is incentive-compatible), and ensures that bidders will participate in the auction (i.e., the auction is individually-rational). The chapter also investigates the design of the optimal quantization thresholds using which buyers quantize their private value estimates, and the bandwidth allocation problem for the bidding process.

In Chapter 6, crowdsourcing based WSNs with rechargeable sensors are used for target localization. For rechargeable sensors, the state of charge (SOC) is one of the key factors that decides the sensors’ energy cost for the localization task. To conserve limited resources, the FC employs an optimal sensor selection scheme obtained through an auction design approach. The sensors com-
pete to participate in the target localization task by sending bids based on their energy efficiency (analog data) and SOC (quantized data) to the FC. Aiming at maximizing the expected utility, the FC designs an optimal auction mechanism incorporating both analog information about sensors’ energy efficiency and quantized information of the SOC and decides on the winning sensor(s) as well as the payment to the winner(s).

In Chapter 7, we introduce “cloud sensing” as a paradigm for enabling sensing-as-a-service in the context of target localization in WSNs. We present a bilateral trading mechanism consisting of a sensing service provider (fusion center) that “sells” information regarding the target through sensor management, and a user who seeks to “buy” information regarding the target. Our mechanism, aware of resource costs involved in service provisioning, maximizes the expected total gain from the trade while assuring individual rationality and incentive compatibility. The impossibility of achieving ex post efficiency is also shown in the paper. Design of the mechanism enables the study of the tradeoff between information gain and the costs of the WSN for sensor management.

In Chapter 8, we propose a framework for the mobile sensor scheduling problem in target location estimation by designing an equilibrium based two-sided market model where the FC is modeled as the consumer and the mobile sensors are modeled as the producers. To accomplish the task, the FC provides incentives to the sensors to motivate them to optimally relocate themselves in a manner that maximizes the information gain for estimating the location of the target. On the other hand, the sensors calculate their own best moving distances that maximize their profits. Price adjustment rules are designed to compute the equilibrium prices and moving distances, so that a stable solution is reached.

Finally, in Chapter 9, we summarize the findings and results of this thesis. Several directions and ideas for future research are also presented.
1.3 Bibliographic Notes

Most of the research work appearing in this thesis has either already been published or is in several stages of publication at various venues. The relationship between the chapters and the publications is shown in Table 1.1, while the list of publications is provided as follows.

Journal papers


Conference papers


<table>
<thead>
<tr>
<th></th>
<th>Chapter 3</th>
<th>Chapter 4</th>
<th>Chapter 5</th>
<th>Chapter 6</th>
<th>Chapter 7</th>
<th>Chapter 8</th>
</tr>
</thead>
<tbody>
<tr>
<td>J1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>J2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>J3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>J4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 1.1: Connection between publications & chapters
CHAPTER 2

BACKGROUND AND PRELIMINARIES

2.1 Introduction

The goal of sensor management problems in target localization and tracking problems is to, 1) determine the number of sensors to be employed in inference tasks; 2) select a subset of sensors from the sensor network; and 3) allocate bandwidth among the sensors in the WSN, while guaranteeing the system performance. In this chapter, we present some preliminaries on the signal processing framework employed in this thesis and review some relevant concepts and algorithms in target localization and tracking and auction theory.

2.2 Preliminaries for Target Tracking

2.2.1 System Model

In a target tracking problem, a moving target emitting (or reflecting) a signal over an area of interest is tracked by a WSN consisting of $N$ sensors. The target state is assumed to be a 4-dimensional vector $\mathbf{x}_t = [x_t, y_t, \dot{x}_t, \dot{y}_t]^T$ where $x_t$ and $y_t$ are the target positions, and $\dot{x}_t$ and $\dot{y}_t$ are the target velocities in the horizontal and vertical directions. Even though the approaches developed in this
thesis are applicable to more complex dynamic models, here we assume a linear dynamic model

\[ x_{t+1} = Fx_t + w_t, \]  

(2.1)

where \( F \) is the state transition matrix and \( w_t \) is the Gaussian process noise with zero mean and covariance matrix \( Q \):

\[
F = \begin{bmatrix}
1 & 0 & D & 0 \\
0 & 1 & 0 & D \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix},
\quad Q = q \begin{bmatrix}
\frac{D^3}{3} & 0 & \frac{D^2}{2} & 0 \\
0 & \frac{D^3}{3} & 0 & \frac{D^2}{2} \\
\frac{D^2}{2} & 0 & D & 0 \\
0 & \frac{D^2}{2} & 0 & D
\end{bmatrix},
\]

(2.2)

where \( D \) is the sampling interval and \( q \) is the process noise parameter. It is assumed that the signal emitted by the target follows a power attenuation model [27]. Thus, the signal power received by sensor \( i \) which is located at \((x_i, y_i)\) is

\[ P_{i,t}(x_t) = \frac{P_0}{1 + d_{i,t}^2} \]  

(2.3)

where \( P_0 \) is the emitted signal power from the target at distance zero, \( n \) is the signal decay exponent and \( \alpha \) is a scaling parameter. In (2.3), \( d_{i,t} \) is the distance between the target and the \( i^{th} \) sensor at time step \( t \), i.e., \( d_{i,t} = \sqrt{(x_t - x_i)^2 + (y_t - y_i)^2} \).

At time step \( t \), the received signal at sensor \( i \) is given by

\[ z_{i,t} = h_{i,t} + n_{i,t} \]  

(2.4)

where \( h_{i,t} = \sqrt{P_{i,t}(x_t)} \). The measurement noise samples \( n_{i,t} \) are assumed to be independent across time steps and across sensors and they follow Gaussian distribution with parameters \( \mathcal{N}(0, \sigma^2) \). In order to reduce the cost of communication, the sensor measurements, \( z_{i,t} \)'s, are quantized into \( M \)-bits before transmission to the fusion center. The quantized measurement of sensor \( i \) at time step
$D_{i,t}$ is defined as:

$$D_{i,t} = \begin{cases} 
0 & -\infty < z_{i,t} < \eta_1 \\
1 & \eta_1 < z_{i,t} < \eta_2 \\
\vdots \\
L-1 & \eta_{(L-1)} < z_{i,t} < \infty 
\end{cases}$$

(2.5)

where $\eta = [\eta_0, \eta_1, \ldots, \eta_L]^T$ is the set of quantization thresholds with $\eta_0 = -\infty$ and $\eta_L = \infty$ and $L = 2^M$ is the number of quantization levels. For simplicity, the quantization thresholds are assumed to be identical at each sensor and are designed according to the Fisher Information based heuristic quantization as in [48]. Then, given target state at time step $t$, the probability that $D_{i,t}$ takes value $l$ is,

$$p(D_{i,t} = l|x_t) = Q\left(\frac{\eta_l - h_{i,t}}{\sigma}\right) - Q\left(\frac{\eta_{l+1} - h_{i,t}}{\sigma}\right)$$

(2.6)

Given $x_t$, the sensor measurements become conditionally independent, so the likelihood function of $D_t = [D_{1,t}, D_{2,t}, \ldots, D_{N,t}]^T$ can be written as,

$$p(D_t|x_t) = \prod_{i=1}^{N} p(D_{i,t}|x_t)$$

(2.7)

### 2.2.2 Fisher Information

Posterior Cramer-Rao Lower Bound (PCRLB) provides the theoretical performance limit for a Bayesian estimator [49]. Let $p(z_t, x_t)$ denote the joint probability density function of the sensor measurements and the target state, and let $\hat{x}_t$ denote the estimate of $x_t$. The PCRLB on the estimation error is represented as [49],

$$E \left\{ [\hat{x}_t - x_t][\hat{x}_t - x_t]^T \right\} \geq J_t^{-1},$$

(2.8)

where $J_t$ is the Fisher information (FI) matrix. It has been shown in [18] that, the FI matrix for Bayesian estimation is composed of two parts: the FI obtained from the sensor measurements and the FI corresponding to a priori information. Furthermore, under the assumption that the sensor
measurements are conditionally independent given the target state $x_t$, the FI obtained from the measurements of multiple sensors can be written as the summation of each sensor’s FI plus the FI from the prior information,

$$J_t = E[-Δ_{x_t}^t \log p(z_t, x_t)]$$

$$= E[-Δ_{x_t}^t \log p(z_t | x_t)] + E[-Δ_{x_t}^t \log p(x_t)]$$

$$\triangleq \sum_{i=1}^N \int_{x_t} J_{S,i,t}^t(x_t)p(x_t)dx_t + J_t^P,$$

where $J_t^P$ is the FI matrix of the a priori information, and $J_{S,i,t}^t(x_t)$ represents the standard FI of each sensor as a function of the target state $x_t$.

$$J_{S,i,t}^t(x_t) = E[-Δ_{x_t}^t \log p(D_i | x_t)]$$

$$= \int_{z_{i,t}} \frac{1}{p(z_{i,t} | x_t)} \left( \frac{\partial p(z_{i,t} | x_t)}{\partial x_t} \right) \left( \frac{\partial p(z_{i,t} | x_t)}{\partial x_t} \right)^T d_{z_{i,t}}$$

$$= \frac{4\kappa_{i,t} h_{i,t}^2}{(1 + d_i^2)^2} \times \begin{bmatrix} (x_i - x_t)^2 & (x_i - x_t)(y_i - y_t) & 0 & 0 \\ (x_i - x_t)(y_i - y_t) & (y_i - y_t)^2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

where

$$\kappa_{i,t} = \frac{1}{8\pi} \sum_{l=0}^{L-1} \frac{\gamma_{i,l}}{p(D_i = l | x_t)}$$

and

$$\gamma_{i,l} = e^{-\frac{(\eta_l - h_{i,t})^2}{2}} - e^{-\frac{(\eta_{l+1} - h_{i,t})^2}{2}}$$

A detailed derivation of $J_{S,i,t}^t(x_t)$ can be found in [18].
2.2.3 Mutual Information

Information-theoretic sensor management for target tracking seeks to minimize the uncertainty in the estimate of the target state conditioned on the sensor measurements \([50]\). Entropy, which is defined by Shannon \([51]\), represents the uncertainty or randomness in the estimate of the target state \(x_t\). Moreover, because of the relationship between the entropy and the MI \([52]\), the sensor selection problem for target tracking can be solved by maximizing the MI between the target state and the sensor measurements.

Given the distribution of the target state and the likelihood function of the sensor measurements, the MI for the analog data can be written as \([9, 13]\)

\[
I(x_t, z_t) = H(z_t) - H(z_t|x_t)
\]

\[
= -\int_{z_t} \left\{ \int_{x_t} p(z_t|x_t)p(x_t)dx_t \right\} \left\{ \log_2 \left[ \int_{x_t} p(z_t|x_t)p(x_t)dx_t \right] \right\} dz_t
\]

\[
+ \sum_{i=1}^{N} \int_{x_t} \left[ \int_{z_{i,t}} p(z_{i,t}|x_t) \log_2 p(z_{i,t}|x_t)dz_{i,t} \right] p(x_t)dx_t,
\]

where \(H(z_t)\) is the entropy of the sensor measurements \(z_t\), and \(H(z_t|x_t)\) is the conditional entropy of the sensor measurements \(z_t\) given the target state \(x_t\).

2.2.4 Particle Filter for Target Tracking

The target tracking problem requires the estimation of the target state using a sequence of sensor measurements. For nonlinear systems, the extended Kalman filter (EKF) provides suboptimal solutions. However, when the sensor measurements are quantized, even for linear and Gaussian systems, the EKF fails to provide an acceptable performance especially when the number of quantization levels is small \([53]\). Thus, we employ a sequential importance resampling (SIR) particle filter to solve our nonlinear target tracking problem with analog and quantized sensor measurements \([54, 55]\). The SIR algorithm is based on the Monte Carlo method, and can be used for recursive Bayesian filtering problems under very weak assumptions \([55]\). The main idea of the
Algorithm 2.1 SIR Particle Filter for target tracking

1: Set $t = 1$. Generate initial particles $x_s^0 \sim p(x_0)$ with $\forall s \cdot w_s^0 = N_s^{-1}$.
2: while $t \leq T_s$ do
3: $x_s^t = F x_{s,t-1} + v_t$ (Propagating particles)
4: $p(x_t|z_{1:t}) = \frac{1}{N_s} \sum_{s=1}^{N_s} \delta(x_t - x_s^t)$
5: Obtain sensor data $z_t$
6: $w_s^t \propto p(z_t|x_s^t)$ (Updating weights through obtained data)
7: $w_s^t = \frac{w_s^t}{\sum_{s=1}^{N_s} w_s^t}$ (Normalizing weights)
8: $\hat{x}_t = \sum_{s=1}^{N_s} w_s^t x_s^t$
9: $\{x_s^t, N_s^{-1}\} = \text{Resampling}(x_s^t, w_s^t)$
10: $t = t + 1$
11: end while

The SIR particle filter is to find a discrete representation of the posterior distribution $p(x_t|z_{1:t})$ ($x_t$ is the target state and $z_{1:t}$ are the sequential measurements from the sensors from time step 1 to $t$) by using a set of particles $x_s^t$ with associated weights $w_s^t$, 

$$p(x_t|z_{1:t}) \approx \sum_{s=1}^{N_s} w_s^t \delta(x_t - x_s^t),$$

(2.13)

where, $\delta(\cdot)$ is the Dirac delta measure, and $N_s$ denotes the total number of particles. When the number of particles is large enough, the weighted sum of the particles based on the Monte Carlo characterization will be an equivalent representation of the posterior distribution. The resampling step in the SIR particle filter avoids the situation that all but one of the importance weights are close to zero after a few iterations, which is known as the degeneracy phenomenon in the particle filter. Algorithm 2.1 provides a summary of the SIR particle filtering algorithm for the target tracking problem, where $T_s$ denotes the number of time steps over which the target is tracked.

### 2.3 Preliminaries for Target Localization

We consider a network consisting of $N$ selfish users which are deployed in a square ROI of size $b^2$. The task of the users is to localize a target which is assumed to emit an isotropic signal from its location. We assume that the target and all the sensors are based on flat ground and have the
same height, so that we can formulate the problem using a 2-D model. It is assumed that the signal emitted by the target follows a power attenuation model [27]. Thus, the signal amplitude received by the sensing device of user $i$ which is located at $(x_i, y_i)$ is

$$z_i = \sqrt{\frac{P_0}{1 + d_i^2}} + n_i$$  \hspace{1cm} (2.14)

where $P_0$ is the signal power of the source at distance zero and $d_i$ is the distance between the target $(x, y)$ and sensor $i$. The noises $n_i$ are independent across sensors and modeled as standard Gaussian distribution $\mathcal{N}(0, 1)$. The sensor measurements $z_i$ are quantized locally as in (2.5). The overall FI, as shown in Section 2.2.2, can be written as the sum of the standard FI of the individual sensors and the FI due to the prior information, $J = \sum_{i=1}^{N} J_i^D + J^P$.

### 2.3.1 Monte Carlo Method for Target Localization

Based on the sensor measurements, the FC estimates the location of the target through the importance sampling based Monte Carlo method [13]. The posterior probability density function (pdf) of the target location given the sensor measurements is approximated by a set of particles,

$$p(x|D) = \sum_{s=1}^{N_s} w^s \delta(x - x^s)$$

where $x^s; s = 1, \ldots, N_s$ are the particles with associated weights $w^s; s = 1, \ldots, N_s$ and $N_s$ denotes the number of particles. The particles are initially generated from the prior distribution of the target $p(x_0)$ with equal weights $1/N_s$. The weights are updated according to the conditional distribution of the selected sensor measurements and the normalized weights $\tilde{w}^s$. The particles yield the final estimate of the target location, $x = \sum_{s=1}^{N_s} \tilde{w}^s x^s$. 
2.4 Multiobjective Optimization based Sensor Selection

The mathematical description of an $n$-objective optimization problem is given as

\[
\begin{align*}
\min_{\alpha} & \quad \{f_1(\alpha), f_2(\alpha), \ldots, f_n(\alpha)\} \\
\text{subject to} & \quad a \leq \alpha_i \leq b, h(\alpha) = 0, g(\alpha) \leq 0,
\end{align*}
\]

(2.15)

where $\alpha$ is the vector of decision variables with elements $\alpha_i$, $a$ and $b$ define the bounds on decision variables, functions $h(.)$ and $g(.)$ represent the equality and inequality constraints of the problem respectively. For the MOP, the solutions satisfying the constraints of (2.15) form the feasible set $C$. In an optimization problem involving the minimization of all the objectives, the solution $\alpha^1$ dominates the solution $\alpha^2$ ($\alpha^1 \succ \alpha^2$) if and only if

\[
\begin{align*}
&f_u(\alpha^1) \leq f_u(\alpha^2) \quad \forall u \in \{1, 2, \ldots n\} \\
&f_v(\alpha^1) < f_v(\alpha^2) \quad \exists v \in \{1, 2, \ldots n\},
\end{align*}
\]

(2.16)

$\alpha^*$ is called a Pareto optimal solution if and only if there is no $\alpha$ in $C$ that dominates $\alpha^*$, and the set of Pareto optimal outcomes is called the Pareto front. Pareto optimality criterion has been commonly used in approaches to solve MOPs [56, 57], though a novel class of optimality criteria, namely p-optimality criteria, is used in [58]. Approaches using Pareto optimality criterion preserve only Pareto optimal solutions in each solution updating step. The authors in [58] state that a non-Pareto optimal solution may be improved at a later stage in such a way that outperforms one or more Pareto optimal solutions in every objective. In this thesis, we apply Pareto optimality criterion in each iteration of the solution update process. Another well-known technique for solving MOPs is to minimize a weighted sum of the objectives, which yields a single solution corresponding to the weights used. With this approach, if a uniform spread of weights is employed to obtain different solutions, it rarely produces a uniform spread of points on the Pareto front. Some of the optimal solutions may become closely spaced and hence reducing the number of design alternatives [59].
2.4.1 NSGA-II

NSGA-II [60] first generates an initial population of size $P$ where each solution in the population is a feasible solution of the MOP. In our problem, a solution in the population is represented as a vector of $N$ elements where each element is a binary variable. NSGA-II is an elitist algorithm where good solutions are always preserved in the population. The values of the objective functions for each solution in the population form the fitness values of the solution. Then all the solutions in the population are sorted based on their non-domination. As an example, solutions with Rank 1 consist of all non-dominated solutions, then solutions with Rank 2 consist of all the solutions which are dominated by only one of the solutions in the population and so on. If two solutions in the population have the same fitness value, then they are sorted based on their crowding distance, which is a closure measure of each solution.

NSGA-II uses the rank of a solution to create the mating population. The offspring solutions are generated by using binary tournament selection [60]. If both of the selected solutions have the same fitness value, then the solution with larger crowding distance is selected. In our problem where we have binary decision variables, we use a real-parameter recombination operator called uniform crossover (UX), where offspring solutions $c_1$ and $c_2$ are obtained from parent solutions $p_1$ and $p_2$ according to,

$$
\begin{align*}
  c_1 &= \xi p_1 + (1 - \xi)p_2 \\
  c_2 &= (1 - \xi)p_1 + \xi p_2
\end{align*}
$$

(2.17)

where $\xi$ is defined by a random number $q$ between $[0, 1]$ [61]

$$
\begin{align*}
  \xi &= 1 \quad q \leq 0.5 \\
  \xi &= 0 \quad q > 0.5
\end{align*}
$$

(2.18)

Along with the UX, the uniform mutation procedure is performed. In uniform mutation, an off-
spring solution \( c_l \) is obtained from the parent solution \( p_l \) according to

\[
c_l = \delta (1 - p_l) + (1 - \delta) p_l
\]

(2.19)

where \( \delta \) is also determined according to (2.18). Then the new population with all the parents and offsprings are sorted again based on their non-dominance and the population size is decreased to the original population size \( P \) by eliminating all the lower rank solutions. Remaining solutions are then fed to a binary tournament selection operator and so on. After several generations \( G \), the population will preserve solutions near or on the Pareto optimal front.
CHAPTER 3
MULTIOBJECTIVE OPTIMIZATION METHOD BASED SENSOR SELECTION IN WIRELESS SENSOR NETWORKS

3.1 Introduction

Sensor network design usually involves consideration of multiple conflicting objectives, such as maximization of the lifetime of the network or the inference performance, while minimizing the cost of resources such as energy, communication or deployment costs [62–65]. The problems that investigate the trade-offs among such conflicting objective functions are called Multiobjective Optimization Problems (MOPs). In this chapter, we first study the sensor selection method under the assumption that the sensors in the WSN are all reliable by utilizing FI as the performance metric in an MOP framework. Two objectives are optimized simultaneously: minimization of the total number of sensors selected at each time, and minimization of the information gap between the FI when all the sensors transmit their measurements and the FI when only the selected sensors transmit their measurements. We use the determinant of the Fisher Information Matrix (FIM) to quantify the Fisher Information as a scalar where maximization of the determinant of the FIM
corresponds to minimization of the area of the uncertainty ellipsoid [66]. We solve the MOP and generate the Pareto-optimal solutions between the two conflicting objectives using a state-of-the-art multiobjective evolutionary algorithm, Nondominating Sorting Genetic Algorithm-II (NSGA-II) [60]. By using the multiobjective optimization approach, we seek sensor selection strategies that deliver significant savings in terms of number of selected sensors without sacrificing much from the estimation performance which is achieved by selecting all the sensors in the network.

We then investigate the sensor selection problem in an uncertain WSN, and generalize the approach by addressing the issues that arise due to uncertainty. As we will see in this chapter, the FI based selection scheme (FI-SS) tends to select sensors which are relatively close to the target, while the MI based selection scheme (MI-SS) selects sensors that have high sensing probabilities, and achieves better performance. The better performance of MI-SS comes along with high computational complexity. Thus, we propose to use a mutual information upper bound (MIUB) as the performance metric for the sensor selection problem. The complexity of computing MIUB is similar to that of evaluating FI, and is much lower than that of computing MI. We also show through simulation experiments that the MIUB based selection scheme (MIUB-SS) hardly degrades the tracking performance. Furthermore, we consider our sensor selection problem with uncertainty under the MOP framework, where the Nondominating Sorting Genetic Algorithm-II (NSGA-II) is applied to dynamically select an optimal set of sensors at each time step. Numerical results show that MIUB-SS selects more sensors than FI-SS under the MOP framework. We also compare our framework with some other sensor selection methods, e.g., weighted sum method and convex optimization method, and show that NSGA-II with the compromise solution (to be discussed later in this chapter) adaptively decides the optimal number of sensors at each time step of tracking and achieves satisfactory estimation performance while obtaining savings in terms of number of sensors.

Regarding the uncertain WSNs with unreliable sensor observations, we also study the sensor selection problem for target localization when there is uncertainty associated with sensor observations in the sense that sensor observations may probabilistically contain only noise. Such a
consideration complicates the sensor selection problem since it introduces the risk of selecting sensors that may provide only noise. In other words, while selecting sensors, it becomes necessary to consider not only the maximization of information gain from the sensors, but to also consider the minimization of the risk (the reliability of the selected sensors) involved. Moreover, the dependence among the sensors also affects the sensor selection result. Therefore, the main objective is to find a sensor selection scheme that considers: 1) the expected information gain of each sensor, 2) the reliability of the sensor observations, and 3) the dependence among the sensors. The contributions are as follows: 1) We model our sensor selection problem using portfolio selection theory [67, 68], which studies decision making under uncertainty and risk; 2) We formulate our sensor selection problem as a multiobjective optimization problem (MOP), where the expected Fisher information (FI) gain is maximized while minimizing the risk due to unreliable sensor observations. The MOP is then solved with the normal boundary intersection (NBI) method [69]. Simulation results show the efficiency of our proposed model. Specifically, they show that the risk of the sensor portfolio can be reduced through diversification, and that our approach is more efficient than the method that maximizes only the expected FI gain.

The rest of this chapter is organized as follows: In Section 3.2, we study the sensor selection method where the sensors in the WSN are assumed to be all reliable, and the scenario when the sensors may be unreliable are studied in Section 3.3. In Section 3.4, we investigate the portfolio selection theory based sensor selection problem for target localization. A brief summary is presented in Section 3.5.

3.2 A Multiobjective Optimization based Sensor Selection Method for Target Tracking in Wireless Sensor Networks

In this section, we propose a sensor selection strategy for target tracking in Wireless Sensor Networks by formulating it as a multiobjective optimization problem (MOP). At each time step of tracking, we obtain tradeoff solutions between two conflicting objectives: minimization of the
number of selected sensors and minimization of the information gap between the Fisher Information when all the sensors transmit measurements and the Fisher Information when only the selected sensors transmit their measurements based on the sensor selection strategy. The model of the tracking system and the Fisher information are as introduced in Section 2.2.

Let $\alpha_t = [\alpha_{1,t}, \ldots, \alpha_{N,t}]$ be the sensor selection strategy at time step $t$. The elements of $\alpha_t$ are binary variables, i.e., $\alpha_{i,t} = 1$, if sensor $i$ is selected and $\alpha_{i,t} = 0$ otherwise. Then, $A = \sum_{i=1}^{N} \alpha_{i,t}$, is the number of sensors selected at time step $t$. Based on the sensor selection strategy $\alpha_t$, the FIM at time step $t$ can be written as,

$$J_t(\alpha_t) = \sum_{i=1}^{N} \alpha_{i,t} J^D_{i,t} + J^P_t$$

Since $J_t(\alpha_t)$ is a matrix, we maximize FIM at time step $t$ by maximizing its determinant, which corresponds to minimizing the area of the uncertainty ellipsoid [66]. At each time step of tracking, we determine the sensor selection strategy as a result of the following multiobjective optimization problem where the objective functions are: minimization of the information gap between the Fisher Information when all the sensors transmit measurements and the Fisher Information obtained when the selected sensors transmit measurements based on the sensor selection strategy $\alpha_t$,

$$f_1(\alpha_t) = \frac{\log \det \left( \sum_{i=1}^{N} J^D_{i,t} + J^P_t \right)}{\log \det \left( \sum_{i=1}^{N} \alpha_{i,t} J^D_{i,t} + J^P_t \right)} - \log \det \left( \sum_{i=1}^{N} \alpha_{i,t} J^D_{i,t} + J^P_t \right)$$

and minimization of the number of selected sensors,

$$f_2(\alpha_t) = \frac{1}{N} \sum_{i=1}^{N} \alpha_{i,t}$$

In this section, the sensor selection strategies reflecting different trade-offs between objective
functions (3.2) and (3.2) are obtained as solutions to the following MOP,

\[
\min_{\alpha'} \left[ \frac{\log \det (J_t) - \log \det (J_t(\alpha_t))}{\log \det (J_t)}, \frac{1}{N} \sum_{i=1}^{N} \alpha_{i,t} \right],
\]

\[\text{s.t. } \alpha_{i,t} \in \{0, 1\} \text{ for all } i = 1, \ldots, N \tag{3.3}\]

In this section, we solve the above MOP which has binary decision variables using a state-of-the-art multiobjective evolutionary algorithm, Nondominated sorting genetic algorithm (NSGA)-II [60] (refer to Section 2.4 for details of NSGA-II). This algorithm yields all the solutions on the Pareto front that explore all the possible tradeoffs between conflicting objectives.

### 3.2.1 Solution Selection from the Pareto-optimal Front

Since NSGA-II provides \( P \) non-dominated solutions, it is necessary to select one particular solution from the Pareto-front which can yield the desired trade-off between the conflicting objectives. In [70], the knee of the trade-off curve is introduced as the solution where a small decrease in one objective is achieved by a large increase in the other. Let \( \alpha^a \) and \( \alpha^b \) be two adjacent (neighboring) solutions on the Pareto-optimal front where \( f_1(\alpha^a) > f_1(\alpha^b) \) and \( f_2(\alpha^a) < f_2(\alpha^b) \). Then we can compute the slope of the curve between solutions \( \alpha^a \) and \( \alpha^b \) from,

\[
\text{slope}\{\alpha^b\} = 180 - \left[ \arctan \left( \frac{f_1(\alpha^a) - f_1(\alpha^b)}{f_2(\alpha^a) - f_2(\alpha^b)} \right) \frac{180}{\pi} \right] \tag{3.4}
\]

For our problem, we define \( \alpha^1 \) as the all zero solution where none of the sensors are selected with \( f_2(\alpha^1) = 0 \) and maximizes the information gap, \( f_1(\alpha^1) \). Similarly, we define \( \alpha^P \) as the all one solution which yields \( f_1(\alpha^P) = 0 \) and \( f_2(\alpha^P) = 1 \). We call the Pareto-optimal solution which maximizes (3.34) as the knee point solution given by,

\[
\alpha_t = \arg \max_{\alpha^2, \ldots, \alpha^P} \text{slope}\{\alpha^\rho\} \tag{3.5}
\]

where \( \alpha^\rho (\rho \in \{2, 3, \ldots P\}) \) represents the solutions on or near the Pareto-optimal front.
Alternatively, the utopia point $F^*$ of a MOP is defined as \[59\],
\[
F^* = [f_1^*, \ldots, f_n^*]^T
\]
(3.6)
where $f_j^*$ is the individual minima of objective $f_j \ (j \in \{1, \ldots, n\})$ defined as,
\[
f_j^* = \min_{\alpha} \{f_j(\alpha) \mid \alpha \in C\}
\]
(3.7)
and let $F(\alpha^\rho) = [f_1(\alpha^\rho), \ldots, f_n(\alpha^\rho)]^T$ where $\rho \in \{1, 2, \ldots, P\}$. In \[59\], the point which is closest to the utopia point has been defined as the compromise solution (CS). In this thesis, we use the Euclidean distance to find CS as,
\[
\alpha_t = \arg \min_{\alpha_1, \ldots, \alpha_P} \sqrt{\sum_{j=1}^n (f_j^* - f_j(\alpha^\rho))^2}
\]
(3.8)
In the next section, we present an illustrative example.

### 3.2.2 Simulation Results

In our simulations, we consider the WSN shown in Fig. 3.1, which has $N = 4 \times 4 = 16$ sensors and the size of the ROI is selected as $b^2 = 50 \times 50 \text{ m}^2$. The source power is $P_0 = 1000$ and the target motion follows the white noise acceleration model with parameters $\tau = 2.5 \times 10^{-3}$ and $D = 1.25$ seconds. The variance of the measurement noise is selected as $\sigma = 0.2$. The prior distribution about the state of the target, $p(x_0)$, is assumed to be Gaussian with mean $\mu_0 = [-23 -23 2 2]^T$ and the covariance $\Sigma_0 = \text{diag}[2 2 0.01 0.01]$ so that initially the target remains in the ROI with high probability. We quantize the sensor measurements using $M = 5$ bits and the initial $N_s = 1000$ particles are drawn from $p(x_0)$. For NSGA-II, we set the population size as $P = 100$ and the number of generations $G = 100$. Before running NSGA-II, we include the two extreme solutions, i.e., all zero and all one solutions to the initial population. The mean square error (MSE) for estimation at each time step of tracking is averaged over $T_{\text{trial}} = 100$ trials as,
where $\hat{x}_t^{tr}$ and $x_t$ are the estimated and the actual target states at time $t$ of the $t_{th}$ trial.

In Fig. 3.2, we present the Pareto optimal front for our optimization problem obtained using NSGA-II. It is interesting to note that at the end of $G$ generations, NSGA-II yields $N + 1$ different solutions on the Pareto-optimal front where each solution corresponds to the optimal selection of $A$ sensors out of $N$ sensors where $A \in \{0, 1, \ldots, N\}$. At time step $t = 9$, the target is not relatively close to any of the sensors in the network and the fusion center has relatively large uncertainty about the target location. Therefore, using the compromise solution defined in (3.38), multiple sensors are selected to achieve an acceptable information gain. However, at time step $t = 12$, the target is relatively close to the sensor located at $((x_i, y_i) = (-8.3, -8.3))$, then this sensor is able to achieve significant information gain. Therefore, the Pareto-optimal front at $t = 12$ is steeper than the one obtained at $t = 9$.

The sensor selection strategy from the Pareto-optimal front that we employ determines the
Fig. 3.2: Pareto optimal front obtained by using NSGA-II at time step (a) $t = 9$ and (b) $t = 12$. y-axis represents the objective function $f_1$ and x-axis represents the objective function $f_2$.

overall tracking performance. In Fig. 3.3, we compare the MSE performance and the average number of selected sensors at each time step of tracking using (3.5) and (3.8). In order to find the knee point solution, we rename $P$ as the number of non-identical solutions on the Pareto-optimal front rather than the population size of NSGA-II, so we get $P \triangleq N + 1$. Simulation results show that the knee point solution in (3.5) always selects the sensor which provides the largest gain in terms of Fisher information. However, the sensor selection strategy using (3.8), selects the sensors as a function of target location. When the target is close to one particular sensor, only this sensor is selected. Otherwise, if the target is equally distant from multiple sensors, multiple sensors are selected based on (3.8). Finally, we consider a variant of the knee-point solution and choose the solution where the slope between two adjacent Pareto-optimal solutions falls below $5^\circ$. In this case, tracking performance is similar to the solution obtained by (3.8). Such a variant of the knee point definition may not be practical, since determining the best value of the slope for sensor selection may require a search over different slope values. Therefore, in the rest of our simulations, we use the compromise solution obtained according to (3.8).
In Fig. 3.4, we compare the tracking performance based on NSGA-II and (3.8) with the convex relaxation based sensor selection method similar to [14, 18] which always chooses $A$ sensors out of $N$ sensors at each time step of tracking. Note that in Fig. 3.3, we presented the number of sensors selected at each time step of tracking using NSGA-II algorithm and CS criteria as in (3.8). In our simulations, we choose $A = 1$ and $A = 3$ which correspond to minimum and maximum number of sensors selected by NSGA-II with CS. With $A = 1$, the convex relaxation based sensor selection method gives poor tracking performance. On the other hand, with $A = 3$ and $A = 16$ the performance improvement in tracking as compared to the MOP approach is negligible. Thus, compared to the convex relaxation method, the multiobjective optimization method gives satisfactory tracking performance while saving in terms of the number of selected sensors. In Fig. 3.4, we also compare the MSE performance of the MOP framework with the weighted sum approach where the sensor selection scheme chooses those sensors which minimize the summation of both objectives, i.e. $w_1 f_1(\alpha_t) + (1 - w_1) f_1(\alpha_t)$ with $w_1 = 0.5$. Simulation results also show that the MSE performance of the MOP based sensor selection is better than the sensor selection.
scheme which minimizes the weighted sum of the objectives.

Fig. 3.4: MSE of sensor selection based on multiobjective optimization and convex relaxation based sensor selection method which selects $A$ sensors at each time step of tracking.

Finally, in Fig. 3.5, we compare the MSE and the average number of selected sensors when the sensor measurements are quantized into $M = 5$ and $M = 3$ bits respectively. Simulation results show that when $M = 5$, a sensor can transmit very accurate information for the target location, and Fisher information of a few sensors can dominate the total Fisher information. However, with $M = 3$, each sensor provides coarse information about the target location and more sensors need to be selected as a compromise solution since none of the sensors dominate the overall FIM.

### 3.3 Sensor Selection for Target Tracking in Wireless Sensor Networks with Uncertainty

In this section, we propose a multiobjective optimization framework for the sensor selection problem in uncertain Wireless Sensor Networks (WSNs). The uncertainties of the WSNs result in a set of sensor observations with insufficient information about the target. We propose a novel mutual
information upper bound (MIUB) based sensor selection scheme, which has low computational complexity, same as the Fisher information (FI) based sensor selection scheme, and gives estimation performance similar to the mutual information (MI) based sensor selection scheme. Without knowing the number of sensors to be selected \textit{a priori}, the multiobjective optimization problem (MOP) gives a set of sensor selection strategies that reveal different trade-offs between two conflicting objectives: minimization of the number of selected sensors and minimization of the gap between the performance metric (MIUB and FI) when all the sensors transmit measurements and when only the selected sensors transmit their measurements based on the sensor selection strategy.

### 3.3.1 Uncertainty Model of Sensor Observations

In this section, we apply the dynamic model of target motion shown in Section 2.2. As discussed earlier, sensor observations may be uncertain due to sensor failures, natural interference or some random interruptions. Regarding different uncertainties, there are different probabilistic models [71]. In this chapter, we consider the scenario that the sensor observation uncertainty is caused
by some obstacles, and assume the following probabilistic measurement model, which has been proposed in [19] and generalized in [21] and [22]: the sensor observation is assumed to contain only noise if the sensor cannot sense the target due to obstacles, and since such uncertainty may happen at any time for any sensor, the sensing probability may not be identical across the sensors in the WSN, i.e.,

\[
    z_{i,t} = \begin{cases} 
        h_{i,t}(x_t) + v_{i,t}, & \text{with probability } p_{s}^{(i)} \\
        v_{i,t}, & \text{with probability } 1 - p_{s}^{(i)}
    \end{cases},
\]

where \( p_{s}^{(i)} \) is the sensing probability of sensor \( i \), \( h_{i,t}(x_t) = \sqrt{P_{i,t}(x_t)} \) represents the signal amplitude received by sensor \( i \) at time step \( t \), and \( v_{i,t} \) is the measurement noise, which is assumed to be independent across time steps and across sensors, follows a Gaussian distribution with parameters \( \mathcal{N}(0, \sigma^2) \). The likelihood function for sensor measurements \( z_t = [z_{i,t}, \ldots, z_{N,t}]^T \) given the target state \( x_t \) is simply the product of each sensor \( i \)'s likelihood function. Given \( x_t, z_{i,t} \) follows the Gaussian distribution \( \mathcal{N}(h_{i,t}(x_t), \sigma^2) \) with probability \( p_{s}^{(i)} \), and follows the Gaussian distribution \( \mathcal{N}(0, \sigma^2) \) with probability \( 1 - p_{s}^{(i)} \), i.e.,

\[
    p(z_{i,t}|x_t) = p_{s}^{(i)} \mathcal{N}(h_{i,t}(x_t), \sigma^2) + (1 - p_{s}^{(i)}) \mathcal{N}(0, \sigma^2). \tag{3.11}
\]

For communication between the fusion center and the sensors, we consider the following two practical scenarios:

1. the sensors directly send their analog measurements \( z_t \) to the fusion center; and,

2. the sensors quantize their analog measurements to \( M \) bits, and then transmit the quantized data to the fusion center for tracking.

Analog sensor measurements contain complete information about the observation, at the expense of high communication cost; on the other hand, quantized measurements save communication burden, but lose some information about the target.
The quantized measurement of sensor \( i \) at time step \( t \), \( D_{i,t} \), is defined as:

\[
D_{i,t} = \begin{cases} 
0 & \eta_0 \leq z_{i,t} \leq \eta_1 \\
1 & \eta_1 \leq z_{i,t} \leq \eta_2 \\
\vdots & \\
L - 1 & \eta_{(L-1)} \leq z_{i,t} \leq \eta_L 
\end{cases},
\]

(3.12)

where \( \eta = [\eta_0, \eta_1, \ldots, \eta_L]^T \) is the set of quantization thresholds with \( \eta_0 = -\infty \) and \( \eta_L = \infty \) and \( L = 2^M \) is the number of quantization levels. The probability that \( D_{i,t} \) takes the value \( l \) is

\[
p(D_{i,t} = l | x_t) = \Pr(\eta_l \leq z_{i,t} \leq \eta_{l+1} | x_t) \\
= p_s^{(i)} \Pr(\eta_l \leq z_{i,t} \leq \eta_{l+1} | z_{i,t} \sim N(h_{i,t}(x_t), \sigma^2)) \\
+ (1 - p_s^{(i)}) \Pr(\eta_l \leq z_{i,t} \leq \eta_{l+1} | z_{i,t} \sim N(0, \sigma^2)) \\
= p_s^{(i)} \left[ Q \left( \frac{\eta_l - h_{i,t}(x_t)}{\sigma} \right) - Q \left( \frac{\eta_{l+1} - h_{i,t}(x_t)}{\sigma} \right) \right] \\
+ (1 - p_s^{(i)}) \left[ Q \left( \frac{\eta_l}{\sigma} \right) - Q \left( \frac{\eta_{l+1}}{\sigma} \right) \right],
\]

(3.13)

where \( Q(\cdot) \) denotes the complementary distribution of the standard Gaussian distribution with zero mean and unit variance

\[
Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} \exp\{-\frac{y^2}{2}\} \, dy.
\]

(3.14)

Since the sensor measurements are conditionally independent, the likelihood function of \( D_t = [D_{1,t}, D_{2,t}, \ldots, D_{N,t}]^T \) can be written as the product of each sensor \( i \)'s likelihood function.

### 3.3.2 Sensor Selection Criteria for Uncertain WSNs

In this section, we present and investigate three performance metrics, FI, MI, and MIUB, for the sensor selection problem in an uncertain WSN. After formulating the three performance metrics mathematically for the analog data and quantized data respectively, we compare them with respect to the resulting tracking performance.
Fisher Information

Posterior Cramer-Rao Lower Bound (PCRLB) provides the theoretical performance limit for a Bayesian estimator [49]. Let \( p(z_t, x_t) \) denote the joint probability density function of the sensor measurements and the target state, and let \( \hat{x}_t \) denote the estimate of \( x_t \). The PCRLB on the estimation error is represented as [49],

\[
E \left\{ [\hat{x}_t - x_t][\hat{x}_t - x_t]^T \right\} \geq J_t^{-1},
\]

where \( J_t \) is the Fisher information (FI) matrix. It has been shown in [18] that, the FI matrix for Bayesian estimation is composed of two parts: the FI obtained from the sensor measurements and the FI corresponding to a priori information. Furthermore, under the assumption that the sensor measurements are conditionally independent given the target state \( x_t \), the FI obtained from the measurements of multiple sensors can be written as the summation of each sensor’s FI plus the FI from the prior information,

\[
J_t \triangleq \sum_{i=1}^{N} \int_{x_t} J_{i,t}^S(x_t)p(x_t)dx_t + J_t^P,
\]

where \( J_t^P \) is the FI matrix of the a priori information, and \( J_{i,t}^S(x_t) \) represents the standard FI of each sensor as a function of the target state \( x_t \),

\[
J_{i,t}^S(x_t) = \int_{z_{i,t}} \frac{1}{p(z_{i,t}|x_t)} \left( \frac{\partial p(z_{i,t}|x_t)}{\partial x_t} \right) \left( \frac{\partial p(z_{i,t}|x_t)}{\partial x_t} \right)^T d z_{i,t}.
\]

Fisher information for the analog sensor measurement model

The FI for analog data is obtained by substituting the likelihood function \( p(z_{i,t}|x_t) \) given in (3.11) into (3.17). The derivative of \( p(z_{i,t}|x_t) \) is

\[
\frac{\partial p(z_{i,t}|x_t)}{\partial x_t} = p_s^{(i)} \frac{z_{i,t} - h_{i,t}(x_t)}{\sigma^2 \sqrt{2\pi\sigma^2}} \exp \left\{ -\frac{(z_{i,t} - h_{i,t}(x_t))^2}{2\sigma^2} \right\} \frac{\partial h_{i,t}(x_t)}{\partial x_t},
\]
where
\[
\frac{\partial h_{i,t}(x_t)}{\partial x_t} = -\frac{\alpha n h_{i,t}(x_t) d_{i,t}^{n-2}}{2} \begin{bmatrix} x_i - x_t \\ y_i - y_t \\ 0 \\ 0 \end{bmatrix}.
\] (3.19)

Substituting (3.11), (3.18), and (3.19) into (3.17) and letting \(J_{i,t}^{SA}(x_t)\) denote the standard FI matrix for analog data, \(J_{i,t}^{SA}(x_t)\) is obtained as follows:

\[
J_{i,t}^{SA}(x_t) = (p_s^{(i)})^2 \kappa_{i,t}^{A}(x_t) \left(\frac{\partial h_{i,t}(x_t)}{\partial x_t}\right) \left(\frac{\partial h_{i,t}(x_t)}{\partial x_t}\right)^T \\
= \kappa_{i,t}^{A}(x_t) \frac{(p_s^{(i)})^2 h_{i,t}^2(x_t)}{(1 + d_{i,t}^2)^2} \times \begin{bmatrix} (x_i - x_t)^2 & (x_i - x_t)(y_i - y_t) & 0 & 0 \\ (x_i - x_t)(y_i - y_t) & (y_i - y_t)^2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},
\] (3.20)

where
\[
\kappa_{i,t}^{A}(x_t) = \int_{z_{i,t}} \frac{1}{p(z_{i,t}|x_t)} \left[ \frac{z_{i,t} - h_{i,t}(x_t)}{\sigma^2 \sqrt{2\pi}\sigma^2} \exp \left\{ -\frac{(z_{i,t} - h(x_t))^2}{2\sigma^2} \right\} \right]^2 dz_{i,t}.
\] (3.21)

**Fisher information for the quantized sensor measurement model**

The FI of quantized data is calculated by replacing the likelihood function \(p(z_{i,t}|x_t)\) given in (3.17) with \(p(D_{i,t}|x_t)\) in (3.13). Since the derivative of the likelihood function of the quantized observations is

\[
\frac{\partial p(D_{i,t}|x_t)}{\partial x_t} = \frac{p_s^{(i)}}{\sigma \sqrt{2\pi}} \left[ \exp \left\{ -\frac{(\eta_t - h_{i,t}(x_t))^2}{2\sigma^2} \right\} - \exp \left\{ -\frac{(\eta_{t+1} - h_{i,t}(x_t))^2}{2\sigma^2} \right\} \right] \frac{\partial h_{i,t}(x_t)}{\partial x_t},
\] (3.22)
we derive the FI for quantized data by substituting (3.19) into (3.22) as follows:

\[
J_{SQ}^{i,t}(x_t) = \sum_{D_{i,t}} \frac{1}{p(D_{i,t}|x_t)} \left( \frac{\partial p(D_{i,t}|x_t)}{\partial x_t} \right)^2
\]

\[
= (p_s(i))^2 \kappa_{Q,i,t}(x_t) \left( \frac{\partial h_{i,t}(x_t)}{\partial x_t} \right) \left( \frac{\partial h_{i,t}(x_t)}{\partial x_t} \right)^T
\]

\[
= \kappa_{Q,i,t}(x_t) \frac{(p_s)^2 h_{i,t}^2(x_t)}{(1 + d_{i,t}^2)^2} \times
\left[
\begin{array}{cccc}
(x_i - x_t)^2 & (x_i - x_t)(y_i - y_t) & 0 & 0 \\
(x_i - x_t)(y_i - y_t) & (y_i - y_t)^2 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{array}
\right],
\]

(3.23)

where

\[
\kappa_{Q,i,t}(x_t) = \sum_{D_{i,t}} \frac{1}{2\pi\sigma^2 p(D_{i,t}|x_t)} \left[ \exp \left\{ -\frac{(\eta_t - h_{i,t}(x_t))^2}{2\sigma^2} \right\} - \exp \left\{ -\frac{(\eta_{t+1} - h_{i,t}(x_t))^2}{2\sigma^2} \right\} \right]^2.
\]

(3.24)

Thus, we get the FI for the analog observation model in (3.20), and for the quantized observation model in (3.23).

**Mutual Information**

Information-theoretic sensor management for target tracking seeks to minimize the uncertainty in the estimate of the target state conditioned on the sensor measurements [50]. Entropy, which is defined by Shannon [51], represents the uncertainty or randomness in the estimate of the target state \(x_t\). Moreover, because of the relationship between the entropy and the MI [52], the sensor selection problem for target tracking can be solved by maximizing the MI between the target state and the sensor measurements.

Given the distribution of the target state and the likelihood function of the sensor measurements,
the MI for the analog data can be written as [9, 13]

\[
I(x_t, z_t) = H(z_t) - H(z_t|x_t)
\]

\[
= - \int_{z_t} \left\{ \int_{x_t} p(z_t|x_t)p(x_t)dx_t \right\} \left\{ \log_2 \left[ \int_{x_t} p(z_t|x_t)p(x_t)dx_t \right] \right\} dz_t
\]

\[
+ \sum_{i=1}^{N} \int_{x_t} \left[ \int_{z_{i,t}} p(z_{i,t}|x_t) \log_2 p(z_{i,t}|x_t)dz_{i,t} \right] p(x_t)dx_t,
\]

(3.25)

where \(H(z_t)\) is the entropy of the sensor measurements \(z_t\), and \(H(z_t|x_t)\) is the conditional entropy of the sensor measurements \(z_t\) given the target state \(x_t\). Similarly, the MI for the quantized sensor measurements can be written as

\[
I(x_t, D_t) = H(D_t) - H(D_t|x_t)
\]

\[
= - \sum_{D_t} \left\{ \int_{x_t} p(D_t|x_t)p(x_t)dx_t \right\} \left\{ \log_2 \left[ \int_{x_t} p(D_t|x_t)p(x_t)dx_t \right] \right\}
\]

\[
+ \sum_{i=1}^{N} \int_{x_t} \left[ \sum_{D_{i,t}} p(D_{i,t}|x_t) \log_2 p(D_{i,t}|x_t) \right] p(x_t)dx_t,
\]

(3.26)

where the summation over \(D_t\) is taken over all possible combinations of the quantized measurements of the set of sensors.

**Mutual Information Upper Bound (MIUB)**

The computational complexity of evaluating the MI for a set of \(A\) sensors increases exponentially with the number of sensors \(A\), so that it becomes impractical to compute the MI in (3.25) and (3.26) when the number of sensors to be selected is large [9] [72]. The chain rule for the MI is described as follows (we only show the MI for analog data, results for quantized data are similar):

\[
I(z_t; x_t) = \sum_{i=1}^{N} I(z_{i,t}; x_t|z_{i-1,t}, \ldots, z_{1,t}).
\]

(3.27)
Fig. 3.6: WSN with 36 unreliable sensors. Numbers above the stars indicate sensor index (left) and its sensing probability (right).

Since $z_{1,t}, \ldots, z_{N,t}$ are conditionally independent given the target state $\mathbf{x}_t$, $z_{i,t} \rightarrow x_t \rightarrow z_{j,t}$ ($i, j \in \{1, \ldots, N\}$) form a Markov chain, and we have the following data processing inequality [52]:

$$I(z_{i,t}; x_t | z_{i-1,t}, \ldots, z_{1,t}) \leq I(z_{i,t}; x_t | z_{i-1,t}, \ldots, z_{2,t}) \leq \cdots \leq I(z_{i,t}; x_t).$$

(3.28)

Thus, $\sum_{i=1}^{N} I(z_{i,t}; x_t)$ is an upper bound on $I(z_t; x_t)$. We use this mutual information upper bound (MIUB) as the performance metric for our sensor selection problem. It can be easily shown that the computational complexity of evaluating MIUB for selecting $A$ out of $N$ sensors increases linearly with $A$, which is the same with that of computing FI.

Comparison of Performance Metrics for Sensor Selection by Numerical Experiments

In this subsection, we compare the performance of the above three performance metrics, FI, MI, and MIUB, for the sensor selection problem through some numerical experiments.

a) Simulation setting: In our simulations, we consider the WSN shown in Fig. 3.6, which has
\( N = 6 \times 6 = 36 \) sensors deployed in the ROI of area \( b^2 = 50 \times 50 \ m^2 \). In practical situations, sensing probability \( p_s \) depends on many factors such as sensor characteristics, obstacle features and locations, weather, and other environmental elements [73, 74]. Since these probabilities are context and scenario dependent, we do not study their estimation in this chapter and leave it as a future research topic. Thus, in the current work, we assume that the sensing probabilities of the sensors are already known to the fusion center. Generally, if the sensors around the target track have higher sensing probabilities compared to other sensors in the WSN, it is highly likely that the algorithm will select those sensors owing to both higher signal power and sensing probability. Our interest is in considering more challenging cases to test the performance of our algorithm. Thus, we assume that the sensors around the target track have relatively low sensing probabilities as shown in the figure. Moreover, the sensing probabilities may be identical for some sensors if they are in the same environment, however, if the sensors have the same sensing probability, the selection results would be similar to Section 3.2. Thus, we consider the scenario in which the sensors in the WSN all have different sensing probabilities.

For the linear dynamical model of the target given in (2.1), the time interval is \( D = 1.25 \) seconds and the process noise parameter \( q = 2.5 \times 10^{-3} \). The source power is \( P_0 = 1000 \) and the variance of the measurement noise is selected as \( \sigma = 0.2 \). The sensors quantize their observations to \( M \) bits for quantized data, and the quantization thresholds \([\eta_1, \ldots, \eta_{L-1}]\) are selected to be the values which evenly partition the interval \([-\sigma, \sigma + \sqrt{P_0}]\). The prior distribution about the state of the target, \( p(x_0) \), is assumed to be Gaussian with mean \( \mu_0 = [-23 - 24 2 2]^T \) and covariance \( \Sigma_0 = \text{diag}[\sigma_x^2, \sigma_x^2, 0.01, 0.01] \) where we select \( \sigma_x = 6 \). The initial \( N_s = 5000 \) particles are drawn from \( p(x_0) \). The mean square error (MSE) is used to measure errors between the ground truth and the estimates, and the MSE of the estimation at each time step of tracking is averaged over \( T_{\text{total}} \) trials as,

\[
\text{MSE}_t = \frac{1}{T_{\text{total}}} \sum_{t=1}^{T_{\text{total}}} (\hat{x}_{tr}^t(1) - x_t(1))^2 + (\hat{x}_{tr}^t(2) - x_t(2))^2,
\]

where \( \hat{x}_{tr}^t \) and \( x_t \) are the estimated and the actual target states at time \( t \) of the \( tr^{th} \) trial.

\textit{b) Sensors with highest MI or FI at different time steps:} We first consider analog and two
quantization communication schemes ($M = 5$ and 2) for one Monte Carlo run. The sensors with highest MI or FI are listed in Table 3.1. Note that, 1) since the FI in our work is a matrix, we consider the determinant of the FI matrix, which corresponds to the area of the uncertainty ellipsoid [66]; 2) we are interested in the effect of the sensors’ distances from the target and the sensing probabilities on the performance metrics, thus we compute the performance metric for each sensor instead of focusing on different sets of multiple sensors; 3) for individual sensors, the MI and MIUB are identical.

Generally, quantized data contains less information compared with the analog data. We first discuss the results for analog data and 5-bit quantized data. We observe from Table 3.1 that the sensors with highest MI or FI are identical for Analog data and 5-bit quantized data, which means that 5-bit quantization preserves most information of the analog data as far as sensor selection is concerned. Additionally, we investigate three distinct time steps to compare the results:

- At time step 1, the target is relatively close to sensors 2 and 7 with a similar distance from the target, so that sensors 2 and 7 have the most significant MI and FI. Sensors 1 and 8 have very low sensing probabilities though they have a similar distance to the target as sensors 2 and 7, and therefore have low MI and FI.

- At time step 3, the target is much closer to sensor 8 than the other sensors, so that sensor 8 has the highest MI and FI even though it has a low sensing probability.

- At time step 8, sensor 15 is the closest one to the target with a very low sensing probability,
and sensor 16 is the second closest with a higher sensing probability. In this case, sensor 15 has the highest FI while sensor 16 has the highest MI.

The 2-bit quantized data contains much less information about the target compared to the analog data and the 5-bit quantized data, so that the sensing probability of sensors affects the FI and MI more with the 2-bit quantized data. Thus, the sensors with relatively higher sensing probabilities have higher FI and MI than the other sensors for the 2-bit quantized data case as shown in Table 3.1.

Therefore, we conclude that for analog data or quantized data with a large number of quantization levels, MI is more affected by the sensing probabilities of the sensors than FI; for quantized data with small number of quantization levels, both MI and FI are considerably affected by the sensing probabilities. Moreover, FI-SS tends to select sensors which are closer to the target compared to MI-SS, which can be explained from Equation (3.45) and (3.23) with the corresponding parameters, i.e., the distance between the target and the sensors dominates FI. However, such an explanation cannot be found for MI. In other words, the sensor’s distance from the target, sensing probability, and the number of quantization levels are all important factors that determine the tracking performance of the WSNs.

c) Tracking performance: In Fig. 3.7, we show the performance of the WSN given in Fig. 3.6 when only one sensor is selected at each time step over $T_{total} = 500$ Monte Carlo runs. Fig. 3.7(a) shows that MI-SS has better MSE performance than FI-SS with both analog data and 5-bit quantized data. We explain the result by investigating the percentage of reliable sensors (the fusion center treats a sensor as unreliable if its amplitude is quite close to noise) among the selected ones over 500 Monte Carlo trials in Fig. 3.7(b). We observe that, in 500 Monte Carlo trials, around 60% of the sensors selected by MI-SS are reliable, and only around 40% of the sensors selected by FI-SS are reliable, which explains the better estimation performance of MI-SS. Although the sensor selection scheme with 2-bit quantized data selects even more reliable sensors, there is no improvement with respect to the MSE performance because of the significant information loss.

---

1 In the experiments, we check if it is within the region $[-3\sigma, 3\sigma]$. 
in the quantization process. As is shown in Fig. 3.7(a), the sensor selection scheme based on analog data has the best tracking performance; 5-bit quantized data based sensor selection scheme achieves performance that is close to that with the analog data; and 2-bit quantized data based sensor selection scheme performs much worse. Thus, we only show simulation results for the 5-bit quantized data in the following simulation experiments.

\textit{d) Performance of MIUB-SS:} The complexity of computing MIUB for selecting \( A \) out of \( N \) sensors is the same as that of computing FI (both increase linearly with \( A \)), and is much less than that of evaluating the MI (increases exponentially with \( A \)). Fig. 3.8 shows the results of MI-SS and MIUB-SS when \( A = 2 \) sensors are selected, and we observe similar performance for MI-SS and MIUB-SS in terms of both the percentage of reliable sensors selected by the schemes and the MSE performance. In other words, MIUB-SS obtains performance similar to MI-SS but with much lower computational complexity. Thus, in the next section, we utilize MIUB-SS, instead of MI-SS, in the multiobjective optimization framework, and compare it with FI-SS.

### 3.3.3 Multiobjective Optimization based Sensor Selection

In this section, we utilize the MOP framework to find the sensor selection strategy that can determine the optimal sensor set. In our work, the sensor selection strategies reflect different trade-offs between two objective functions: the estimation performance and the number of selected sensors, which are dependent on the binary decision variables.

**Objective Functions based on Fisher Information (FI) and Mutual Information Upper Bound (MIUB)**

\textit{a) FI based objective functions:} Let \( \alpha_t = [\alpha_{1,t}, \ldots, \alpha_{N,t}] \) be the sensor selection strategy at time step \( t \). The elements of \( \alpha_t \) are binary variables, i.e, \( \alpha_{i,t} = 1 \), if sensor \( i \) is selected and \( \alpha_{i,t} = 0 \) otherwise. Then, \( A = \sum_{i=1}^{N} \alpha_{i,t} \) is the number of sensors selected at time step \( t \). Based on the
sensor selection strategy \( \alpha_t \), the FI matrix at time step \( t \) can be written as,

\[
J_t(\alpha_t) = \sum_{i=1}^{N} \alpha_{i,t} J_{i,t}^{D} + J_{t}^{P},
\]

(3.30)

where \( J_{i,t}^{D} \triangleq \int_{x_t} J_{i,t}^{S}(x_t)p(x_t)dx_t \) from (3.16). We determine the sensor selection strategy from the solution of the MOP where the objective functions are: minimization of the information gap between the FI based on the measurements of all the sensors and the FI based on the sensor set selected by strategy \( \alpha_t \),

\[
f_1(\alpha_t) = \frac{\log \det \left( \sum_{i=1}^{N} J_{i,t}^{D} + J_{t}^{P} \right) - \log \det \left( \sum_{i=1}^{N} \alpha_{i,t} J_{i,t}^{D} + J_{t}^{P} \right)}{\log \det \left( \sum_{i=1}^{N} J_{i,t}^{D} + J_{t}^{P} \right)}
\]

(3.31)

and minimization of the normalized number of selected sensors,

\[
f_2(\alpha_t) = \frac{1}{N} \sum_{i=1}^{N} \alpha_{i,t}.
\]

(3.32)

b) MIUB based objective functions: The objective functions based on MIUB are very similar to that for FI: minimization of the normalized information gap between the MIUB based on the measurements from all the sensors and the MIUB based on the measurements from the sensor set based on the sensor selection strategy \( \alpha_t \),

\[
f_1(\alpha_t) = \frac{\sum_{i=1}^{N} I^{(i)} - \sum_{i=1}^{N} \alpha_{i,t} I^{(i)}}{\sum_{i=1}^{N} I^{(i)}},
\]

(3.33)

where \( I^{(i)} \) denotes \( I(z_{i,t}; x_t) \) (or \( I(D_{i,t}; x_t) \)), and minimization of the normalized number of selected sensors (the same as (3.32)).

**Solution Selection from the Pareto-optimal Front**

We solve the above MOP which has binary decision variables using the NSGA-II algorithm which is presented in Section 2.2. Since NSGA-II provides \( P \) non-dominated solutions, it is necessary to
select one particular solution from the Pareto-front which can yield the desired trade-off between
the conflicting objectives. There are many approaches that one can employ in selecting a single
solution from the Pareto-optimal front. As discussed in [56–58], the selection is guided by the pref-
erence of the decision maker. For example, the decision maker may suggest a reference direction
or reference points or other clues while solving a multiobjective optimization problem to enable the
selection of a single solution or a preferred set of solutions on the Pareto-optimal front [75]. In this
chapter, we select either the knee point solution or the compromise solution as the single solution
from the optimal Pareto front. In [70], the knee of the trade-off curve is introduced as the solution
where a small decrease in one objective is associated with a large increase in the other. Let \( \alpha^a \) and
\( \alpha^b \) be two adjacent (neighboring) solutions on the Pareto-optimal front where
\( f_1(\alpha^a) > f_1(\alpha^b) \)
and
\( f_2(\alpha^a) < f_2(\alpha^b) \).
Then we can compute the slope of the curve between solutions \( \alpha^a \) and \( \alpha^b \) from,
\[
\text{slope}\{\alpha^b\} = 180 - \left[ \arctan\left( \frac{f_1(\alpha^a) - f_1(\alpha^b)}{f_2(\alpha^a) - f_2(\alpha^b)} \right) \frac{180}{\pi} \right].
\] (3.34)
For our problem, we define \( \alpha^1 \) as the all zero solution where none of the sensors are selected, so
that \( f_1(\alpha^1) = 1 \) and \( f_2(\alpha^1) = 0 \). Similarly, we define \( \alpha^P \) is the all one solution which yields
\( f_1(\alpha^P) = 0 \) and \( f_2(\alpha^P) = 1 \). We call the Pareto-optimal solution which maximizes (3.34) as the
knee point solution given by,
\[
\alpha_t = \arg \max_{\alpha^2, \ldots, \alpha^P} \text{slope}\{\alpha^\rho\},
\] (3.35)
where \( \alpha^\rho \) (\( \rho \in \{2, 3, \ldots P\} \)) represents the solutions on or near the Pareto-optimal front.

Alternatively, the utopia point \( F^* \) of a MOP is defined as [59],
\[
F^* = [f_1^*, \ldots, f_n^*]^T,
\] (3.36)
where \( f_j^* \) is the individual minima of objective \( f_j \) \( (j \in \{1, \ldots, n\}) \) defined as
\[
f_j^* = \min_{\alpha} \{f_j(\alpha) \mid \alpha \in \mathcal{C}\},
\] (3.37)
and let $F(\alpha^\rho) = [f_1(\alpha^\rho), \ldots, f_n(\alpha^\rho)]^T$ where $\rho \in \{1, 2, \ldots, P\}$. In [59], the point which is closest to the utopia point has been defined as the compromise solution. In this chapter, we use the Euclidean distance to find the compromise solution as,

$$
\alpha_t = \arg\min_{\alpha^1, \ldots, \alpha^P} \sqrt{\sum_{j=1}^n (f_j^* - f_j(\alpha^\rho))^2}.
$$

(3.38)

In the next section, we present some numerical results.

**Numerical Experiments for the MOP Framework**

In this section, we conduct some simulation experiments to investigate the performance of the multiobjective optimization method. The WSN considered in this subsection is the same as shown in Fig. 3.6 in Section 3.3.2, and the system parameters are also the same as Section 3.3.2. Note that, for NSGA-II, the population size is chosen as $P = 100$. We choose the number of generations according to the diversity metric introduced in [60]. The diversity metric measures the extent of spread achieved among the obtained solutions, which is defined as

$$
\Delta = \frac{d_f^{(E)} + d_i^{(E)} + \sum_{i=1}^{N-1} |d_i^{(E)} - \bar{d}^{(E)}|}{d_f^{(E)} + d_i^{(E)} + (N - 1)d^{(E)}},
$$

(3.39)

where $d_f^{(E)}$ and $d_i^{(E)}$ are the Euclidean distances between the extreme solutions and the boundary solutions of the obtained nondominated set, $d_i^{(E)}$ is the Euclidean distance between consecutive solutions in the obtained nondominated set of solutions, and $\bar{d}^{(E)}$ is the average of $d_i^{(E)}$ for $i \in \{1, \ldots, N\}$. We observe that for both FI-SS and MIUB-SS, the diversity metric converges after 100 generations for all the 20 time steps. Thus, in our simulation experiments, we set the number of generations as $G = 100$. Also, before running NSGA-II, we include the two extreme solutions, i.e, all zero and all one solutions to the initial population. Note that for all the simulation results in this subsection, we only consider the 5-bit quantized data.
Pareto optimal front  In Fig. 3.9, we present the Pareto optimal front for our MOP obtained using NSGA-II, where Fig. 3.9(a) is for FI-SS and Fig. 3.9(b) shows the result for MIUB-SS. It is interesting to note that at the end of $G$ generations, NSGA-II yields $N + 1$ different solutions on the Pareto-optimal front where each solution corresponds to the optimal selection of $A$ sensors out of $N$ sensors where $A \in \{0, 1, \ldots, N\}$. We know from (3.20), (3.23) and Table 3.1 that the distance between the target and the sensor plays a more important role than the sensing probability for FI-SS. At time step $t = 3$, the target is relatively close to sensor 8, and sensor 8 itself is able to achieve significant FI gain. At time step $t = 6$ the target is not relatively close to any of the sensors in the network and the fusion center has relatively large uncertainty about the target location. Thus, the Pareto front for FI-SS at $t = 3$ is steeper than that at $t = 6$. However, compared with FI-SS, MIUB-SS prefers the sensors with high sensing probability and selects more sensors, so that the Pareto front of MIUB-SS at $t = 3$ or $t = 6$ is not as steep as that for FI-SS. Moreover, we observe that the compromise solution and the knee point solution are identical when the Pareto front is relatively steep.

Solution selection method  The solution, i.e., the sensor selection strategy, that we choose from the Pareto optimal front determines the overall tracking performance. In Fig. 3.10, we compare the average number of active sensors\(^2\) at each time step of tracking and the MSE performance using the knee point solution (3.35) and the compromise solution (3.38) with MIUB-SS and FI-SS under the MOP framework. We observe similar results for MIUB and FI-SS that the knee point solution always selects one sensor for target tracking, and thus gives poorer MSE performance. However, the sensor selection strategy using the compromise solution in (3.38) selects the sensors which balance the tradeoff between the performance gain (MIUB and FI) and the total number of selected sensors. Thus, in the rest of our simulations, we use the compromise solution to choose the sensor selection strategy from the Pareto optimal front.

Recall the results shown in Fig. 3.7 and Fig. 3.8 that MIUB-SS selects more reliable sensors

\(^2\)We show the number of active sensors (the selected sensors) to investigate the energy cost of each solution selection method, because selecting more sensors for data transmission incurs more energy cost.
when the number of sensors to be selected is given. Furthermore, Fig. 3.10 shows that when the number of sensors to be selected is not known, MIUB-SS tends to select more sensors than FI-SS under the MOP framework, such that the MSE performance of MIUB-SS is better than FI-SS.

**NSGA-II, convex optimization, and weighted sum methods** In Fig. 3.11, we compare the tracking performance based on NSGA-II and (3.38) with the convex relaxation based sensor selection method similar to [14,18] which always chooses $A$ sensors out of $N$ sensors at each time step of tracking. Note that our MOP based sensor selection scheme selects different number of sensors at each time step as shown in Fig. 3.10, while convex optimization method needs to fix the number of sensors to be selected before performing sensor selection. Thus, we apply the convex relaxation method to select the minimum and maximum number of sensors selected by NSGA-II with compromise solution in Fig. 3.10. With the minimum number of sensors, the convex relaxation based sensor selection method gives poor tracking performance. On the other hand, selecting the maximum number of sensors or all the sensors through convex relaxation method negligibly improves the MSE performance compared to the MOP approach. Thus, compared to the convex relaxation method, the multiobjective optimization method gives reasonably good tracking performance while saving in terms of the number of selected sensors with both MIUB-SS and FI-SS. We also compare the MSE performance of the MOP framework with the weighted sum approach where the sensor selection scheme chooses those sensors which minimize the summation of both objectives, i.e. $w_1 f_1(\alpha_t) + (1 - w_1) f_2(\alpha_t)$ with $w_1 = 0.5$. Simulation results show that for MIUB-SS (Fig. 3.11(a)), the NSGA-II method obtains similar MSE performance as the weighted sum method, while for FI-SS (Fig. 3.11(b)), the weighted sum method achieves much worse MSE performance. It should be noted that, the weighted sum method requires the selection of the weight $w_1$. Moreover, it rarely produces a uniform spread of points on the Pareto front with a uniform spread of weights, and some of the optimal solutions may become closely spaced and hence reducing the number of design alternatives [59].
A Naive strategy We consider a naive sensor selection method in which the fusion center turns off the sensors with relatively low sensing probabilities before sensor selection. In Fig. 3.12, we present the results when the fusion center turns off the sensors whose sensing probabilities are lower than some threshold $p^{th}_s$, where $p^{th}_s = 0.5$ and $p^{th}_s = 0.15$ are considered. Note that, for the WSN in Fig. 3.6, the sensors that are relatively close to the target will be turned off because they have low sensing probabilities. As shown in the previous results, MIUB-SS prefers to select more reliable sensors, and FI-SS selects the sensors that are close to the target. Turning off sensors before selection performs worse for MIUB-SS because it reduces the selection alternatives, and it performs better for FI-SS because more reliable sensors are selected when the closest sensors with low sensing probabilities are no longer available.

Comparison with the performance when there is no uncertainty In Fig. 3.13, we present the target tracking performance when the sensors are all reliable, e.g., $p_s = 1$ for all the sensors, and compare with the results with uncertain observations. We observe that, with uncertain observations, both FI-SS and MIUB-SS achieve worse MSE performance though they both tend to select more sensors. Moreover, compared with FI-SS, MIUB-SS selects many more sensors with uncertain observations, and therefore achieves better MSE performance.

WSNs with different densities In Fig. 3.14, we show the performance of MIUB-SS and FI-SS by considering WSNs with different densities, where $N = 9$, $N = 16$, $N = 25$, and $N = 49$ sensors are considered. Note that we randomly generate the sensing probabilities for the sensors from the uniform distribution between $[0, 1]$. We observe similar conclusion from Fig. 3.14 that MIUB-SS selects more reliable sensors, thus provides better MSE performance than that of FI-SS.

WSN with other instances of sensing probabilities In Fig. 3.15, the sensors’ sensing probabilities are distributed in a reverse manner as compared with Fig. 3.6, i.e., the sensors that are around the target track have relatively high sensing probabilities. In this condition, MIUB-SS and FI-SS select similar number of sensors with similar MSE performance. The reason is
that, under this scenario, MIUB-SS and FI-SS both select the sensors around the target track with high sensing probabilities. In Fig. 3.16, we assume that there is a static obstacle at the center of the ROI, so that the sensors which are closer to the center of ROI have lower sensing probabilities. Similar performance is observed that MIUB-SS selects more reliable sensors compared with FI-SS, thus provides better tracking performance. We have also conducted experiments for the following scenarios: 1) the sensors’ sensing probabilities are all uniformly distributed between 0 and 1 for every Monte Carlo trial (Fig. 3.17); 2) the sensor measurements have higher noise (Fig. 3.18); 3) target moves with a relatively large process noise parameter $q = 0.01$ (Fig. 3.19); 4) the sensor measurements are quantized to 3 bits (Fig. 3.20), and the results provides similar insights as the previous figures.

3.4 Portfolio Theory based Sensor Selection in Wireless Sensor Networks with Unreliable Observations

In this section, we propose a portfolio theory based sensor selection framework in Wireless Sensor Networks (WSNs) with unreliable sensor observations for target localization. Fisher information (FI) is used as the sensor selection metric in our work. Our objective is to find a sensor selection scheme that considers both the expected FI gain and the reliability of the sensors, where we observe that the FI variability captures the reliability of the sensors. Based on portfolio theory, we formulate our sensor selection problem as a multiobjective optimization problem (MOP), which is solved by the normal boundary intersection (NBI) method. Simulation results show the advantages of performing portfolio theory based sensor selection.

3.4.1 System Model

The target model considered in this section is shown in Section 2.3. Sensor observations may be unreliable due to sensor failures, natural interference or some random interruptions. Regarding different uncertainties, there are different probabilistic models [71]. In this section, we assume
the following probabilistic measurement model, which has been proposed in [19] and generalized in [21] and [22]: the sensor observation is assumed to contain only noise with some probability, i.e.,

\[
    z_i = \begin{cases} 
        h_i(x) + v_i, & \text{with probability } p_{s_i} \\
        v_i, & \text{with probability } 1 - p_{s_i}
    \end{cases} \quad (3.40)
\]

where, \( h_i(x) = \sqrt{P_i(x)} \) represents the signal amplitude received by sensor \( i \), and \( v_i \) is the measurement noise, which is assumed to be independent across sensors, follows a Gaussian distribution with parameters \( \mathcal{N}(0, \sigma^2) \). Thus, given \( x \) and \( p_{s_i}, z_i \) follows the Gaussian distribution \( \mathcal{N}(h_i(x), \sigma^2) \) with probability \( p_{s_i} \), and follows the Gaussian distribution \( \mathcal{N}(0, \sigma^2) \) with probability \( 1 - p_{s_i} \), i.e.,

\[
    p(z_i| x, p_{s_i}) = p_{s_i} \mathcal{N}(h_i(x), \sigma^2) + (1 - p_{s_i}) \mathcal{N}(0, \sigma^2) \quad (3.41)
\]

Since it is difficult to know the exact value of the probabilities \( p_{s_i}, i = 1, \cdots, N \), we assume that \( p_{s_i} \) is a random variable which follows a Beta distribution \( \text{Beta}(\lambda_i, \beta_i) \) with shape parameters \( \lambda_i \) and \( \beta_i \). Further, we consider sensor observations to have a correlation structure with the correlation coefficient between sensor \( i \) and \( j \) defined as

\[
    \rho_{i,j} = \exp(-d^{S}_{i,j}/d^0) \quad (3.42)
\]

where \( d^{S}_{i,j} \) is the distance between sensor \( i \) and sensor \( j \), and \( d^0 \) is a constant parameter [76].

**Fisher Information**

For the system considered in this paper, the FI matrix conditioned on the probability \( p_s \) is

\[
    J = \mathbb{E}[-\Delta^x \log p(x, z|p_s)] \quad (3.43)
\]
where the expectation is with respect to $p(x, z|p_s)$, and

$$
\mathbf{J} = \sum_{i=1}^{N} \mathbb{E}_{x}\{\mathbb{E}_{z_i}[-\Delta_x^{x} \log p(z_i|x, p_s)]\} + \mathbb{E}_{x}[\log p(x)]
$$

$$
\triangleq \sum_{i=1}^{N} \mathbb{E}_{x}[\mathbf{J}_i^{S}(x, p_{s_i})] + \mathbf{J}^{P}
$$

(3.44)

where, $\mathbf{J}^{P}$ is the FI matrix of the a priori information, and $\mathbf{J}_i^{S}(x, p_{s_i})$ represents the standard FI matrix of sensor $i$ as a function of the target state $x$ and probability $p_{s_i}$,

$$
\mathbf{J}_i^{S}(x, p_{s_i}) = \kappa_i(x) p_{s_i}^2 P_0 n^2 d_{i}^{2n-4} 4(1 + d_i^n)^3 
$$

$$
\times \frac{1}{\sigma^2 \sqrt{2\pi}} \left[ \begin{array}{ccc}
(x_i - x)^2 & (x_i - x)(y_i - y) & 0 \\
(x_i - x)(y_i - y) & (y_i - y)^2 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
\end{array} \right] 
$$

(3.45)

where

$$
\kappa_i(x) = \int_{z_i} \frac{1}{p(z_i|x, p_{s,i})} \left[ \frac{1}{\sigma^2 \sqrt{2\pi}} \exp \left\{ - \frac{(z_i - h_i(x))^2}{2\sigma^2} \right\} \right] \frac{d z_i}{d z_i}
$$

(3.46)

We use the trace of the FI matrix as the performance metric [77, 78],

$$
\text{tr}(\mathbf{J}) = \sum_{i=1}^{N} \mathbb{E}_{x}\{\text{tr}(\mathbf{J}_i^{S}(x, p_{s_i}))\} + \text{tr}(\mathbf{J}^{P})
$$

(3.47)

where $\mathbb{E}_{x}\{\text{tr}(\mathbf{J}_i^{S}(x, p_{s_i}))\}$ is a function of the probability $p_{s_i}$ and is denoted as $r_i$,

$$
r_i = p_{s_i}^2 \mathbb{E}_{x}\left\{ \frac{\kappa_i(x) P_0 n^2 d_{i}^{2n-2}}{4(1 + d_i^n)^3} \right\}
$$

(3.48)

In (3.48), $r_i$ is a random variable, and represents the amount of FI gain obtained from sensor $i$’s observation. Moreover, $r_i$ ($i \in \{1, \cdots, N\}$) are correlated with each other since the probabilities $p_{s_i}$ ($i \in \{1, \cdots, N\}$) of the sensors are correlated.
**Monte Carlo Method based Target Localization**

Based on the sensor measurements, the fusion center estimates the location of the target through the importance sampling based Monte Carlo method as shown in Section 2.3.1, where the posterior pdf of the target location given the sensor measurements is approximated by a set of particles,

\[ p(x|z, p_s) = \sum_{s=1}^{N_s} w^s \delta(x - x^s). \]

### 3.4.2 Portfolio Theory based Sensor Selection

The objective of this section is to employ portfolio theory to develop a sensor selection scheme in a WSN with unreliable sensor observations. The probabilistic nature of sensor observations as modeled in (3.40) makes the sensor selection problem challenging.

In Markowitz’s portfolio selection theory [67, 68], an investor forms a portfolio by selecting a set of assets, where an asset is an investment instrument that can be bought and sold [67]. Each asset has its return value, which is a random variable, where the expected value of the return of each asset corresponds to the mean value of the asset and measures the growth of the investment, and the risk of the asset corresponds to the variance in the value of the asset and measures the degree of variation in the investment’s growth [79].

The sensor selection scheme we are seeking aims at maximizing the expected FI and minimizing the FI variability. We analyze our sensor selection problem by mapping it to the portfolio selection problem with Markowitz’s model. We observe an analogy between the sensor selection problem and the selection of portfolios in finance. We relate FI to the return of the assets, then the expected FI corresponds to the expected return, and the covariance matrix \( \Sigma \), which characterizes the reliability of the sensors, corresponds to the risk.

In the following subsections, we first introduce the motivation for our portfolio selection theory based sensor selection scheme, then we formulate the optimization problem and introduce the NBI method to solve the problem.
Table 3.2: Mean values of the probabilities $p_s$, the expected FI gain $r_i$, and the variance of $r_i$.

<table>
<thead>
<tr>
<th>Sensor $i$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mathbb{E}(p_s)$</td>
<td>0.8</td>
<td>0.57</td>
<td>0.50</td>
<td>0.33</td>
<td>0.33</td>
<td>0.33</td>
</tr>
<tr>
<td>$\mathbb{E}(r_i)$</td>
<td>20.98</td>
<td>15.38</td>
<td>12.05</td>
<td>8.68</td>
<td>23.6</td>
<td>28.08</td>
</tr>
<tr>
<td>$\sqrt{\text{var}(r_i)}$</td>
<td>5.95</td>
<td>6.3</td>
<td>5.46</td>
<td>5.03</td>
<td>13.36</td>
<td>15.76</td>
</tr>
</tbody>
</table>

Motivation for Portfolio Theory based Sensor Selection

Traditional sensor selection schemes only consider the maximization of the information gain. However, in our system, the sensor observations may not always contain useful information about the target, and the probabilities $p_s$ bring uncertainty to the FI in (3.48).

In Fig. 3.21, we present a specific WSN example to illustrate our problem. The WSN contains $N = 6$ sensors, where sensors 1, 2, 3, and 4 have similar distances from the expected location of the target, and sensors 5, 6 are relatively close to the target. Also, sensor 5 and sensor 6 are relatively close to each other compared to the other sensors, so that their correlation coefficient $\rho_{5,6}$ is larger. For the Beta distribution of $p_s$, the parameters $\lambda_i$ for all the sensors are assumed to be identical as $\lambda_i = 2$ for $i = \{1, \cdots, 6\}$, and the parameters $\beta_i$ are shown in Fig. 3.21, where larger $\beta_i$ indicates lower mean value of $p_s$ ($\mathbb{E}(p_s) = \frac{\lambda_i}{\lambda_i + \beta_i}$).

In Table 3.2, we list the mean values of the probabilities $p_s$, the expected FI gain $r_i$ of (3.48), and the variance of $r_i$. The expected FI gains of sensor 6 and sensor 5 are relatively high (because they are relatively close to the expected location of the target). However, the mean value of the probabilities $p_{s_6}$ and $p_{s_5}$ are quite low, which means that the average probabilities for sensor 5 and sensor 6 to give observations that contain useful information about the target is only around 33%. In other words, selecting sensor 5 and sensor 6 indicates greater risk in terms of reliability of the system. Moreover, sensor 5 and sensor 6 are highly correlated (the correlation coefficient $\rho_{5,6}$ is 0.95), which means that if one of them has an unreliable observation, the observation of the other one is very likely to be unreliable, so that it would be more reasonable not to select sensor 5 and sensor 6 at the same time. In the problem setup, if we select sensors according to the traditional sensor selection schemes, which select sensors with higher expected FI gain, the effect
of the reliability of the sensors and the dependence among the sensors to the performance of the network would be ignored.

We notice from Table 3.2 that the variance of the FI gain for sensor 5 and sensor 6 are relatively high compared to the other sensors, which indicates greater risk of selecting sensors with low reliability. In other words, the variance of the FI indicates the reliability of the sensor observations. The covariance matrix of the FI gain \( r_i \) shows not only the variance of each sensor’s FI gain, but also the dependence among the sensors. Therefore, we are interested in seeking the sensor selection scheme that considers not only the expected FI gain, but also the covariance matrix of the FI of the sensors while designing sensor selection strategies that optimize information gain while minimizing risk with dependence among sensors.

**Optimization Problem for Portfolio Theory based Sensor Selection**

Markowitz’s portfolio selection model directs the analysis to risk-averse investors who seek to minimize risk for a given mean, so that the mean-variance criterion is used to select assets [68], where the expected return of the portfolio is maximized while the variance of the portfolio is minimized. In our problem, we have two conflicting objectives, i.e., maximizing the expected FI gain and minimizing the FI variability. Let \( \alpha \) be a sensor selection vector, where \( 0 \leq \alpha_i \leq 1 \) represents the probability that sensor \( i \) is selected by the fusion center. Based on the sensor selection strategy \( \alpha \), the FI obtained from the sensor observations is \( \bar{r} = \alpha^T r \), which has the mean

\[
\mathbb{E}(\bar{r}) = \alpha^T \mathbb{E}(r) = \sum_{i=1}^{N} \alpha_i \mathbb{E}(r_i)
\]

(3.49)

and variance

\[
\sigma^2 = \alpha^T \Sigma \alpha = \sum_{i=1}^{N} \sum_{j=1}^{N} \alpha_i \alpha_j \text{cov}(r_i, r_j)
\]

(3.50)

for \( i, j \in \{1, 2, \cdots N\} \).

With two conflicting objectives, i.e., maximizing the expected FI gain \( \mathbb{E}(\bar{r}) \) and minimizing the FI variability \( \sigma^2 \), we are interested in finding the tradeoff between the two objectives. Thus, we for-
mulate the sensor selection problem as a multiobjective optimization problem. The multiobjective optimization problem for sensor selection is formulated as follows:

\[
\begin{align*}
\text{minimize} & \quad \{ -\alpha^T \mathbb{E}(r), \alpha^T \Sigma \alpha \} \\
\text{subject to} & \quad 0 \leq \alpha \leq 1, 1^T \alpha \leq R^N
\end{align*}
\]

(3.51)

where \( R^N \) is the maximum number of sensors that can be selected.

### 3.4.3 NBI method for Multiobjective Optimization

Our objective is to maximize the expected FI gain while minimizing the FI variability, i.e., risk, and we have formulated it as a MOP as shown in (3.51). A well-known technique for solving MOPs is to minimize a weighted sum of the objectives, which yields a single solution corresponding to the weights used. With this approach, if a uniform spread of weights is employed to obtain different solutions, it rarely produces a uniform spread of points on the Pareto front. Some of the optimal solutions may become closely spaced and hence reducing the number of design alternatives [59, 80]. The NBI method finds the trade-off among the various conflicting objectives, and is successful in producing an evenly distributed set of points in the Pareto set given an evenly distributed set of parameters [69]. Since the two objective functions in (3.51) are both convex, the gradient-based approaches used in the NBI method yield a global optimum. Moreover, NBI is computationally efficient in locating Pareto optimal points [62]. Thus, we apply the NBI method to solve our problem in (3.51).

We denote \([-\alpha^T \mathbb{E}(r), \alpha^T \Sigma \alpha]^T\) in (3.51) as \( F(\alpha) \), and the set of attainable objective vectors is denoted by \( \mathcal{F} \). Let \( \alpha^*_i \) be the minimizer of the \( i^{th} \) objective and \( F^*_i = F(\alpha^*_i) \), then the convex hull of individual minima (CHIM) is defined as the convex combinations of \( F^*_i - F^* \), where \( F^* \) is the utopia point (the vector containing the individual global minima of each objective function). The central idea of the NBI method is motivated by the fact that the intersection point between the boundary of \( \mathcal{F} \) and the normal pointing toward the origin emanating from any point in the CHIM is
a Pareto optimal point as long as the trade-off surface in the objective space is convex [69]. Thus, we find the optimal sensor set by solving the following subproblems:

\[
\begin{align*}
\max_{\alpha, \tau} & \quad \tau \\
\text{subject to} & \quad \Phi \beta + \tau \hat{n} = F(\alpha) \\
& \quad \alpha \geq 0, 1^T \alpha \leq R^N
\end{align*}
\]

(3.52)

where \( \Phi \beta \) represents a point in the CHIM, \( \hat{n} \) denotes the unit normal to the CHIM simplex pointing toward the origin, and \( \Phi \beta + \tau \hat{n} \) represents the set of points on the normal [69]. And each NBI subproblem can be solved by any optimization method that is appropriate for the problem.

### 3.4.4 Simulation Experiments

In this section, we show the performance of our sensor selection approach through simulation experiments. We consider the WSN as shown in Fig.3.21. The signal power at distance zero is \( P_0 = 1000 \), and the signal decay exponent is set to be \( n = 2 \). We assume that the prior pdf of the target location \( x \) is \( \mathcal{N}(\mu_0, \Sigma_0) \) with \( \mu_0 = [0, 0]^T \) and \( \Sigma_0 = \text{diag}(2^2, 2^2) \). The performance of the location estimator is determined in terms of the mean square error (MSE) via 500 simulation runs. For the correlation structure of the sensor observations in (3.42), \( d^0 \) is set to be 5 in our experiments [76].

**a) Efficient Frontier:** In Fig. 3.22, we present the efficient frontier\(^3\) of our portfolio selection theory based sensor selection problem, where the NBI method is employed to solve the MOP in (3.51). For NBI, we set the resolution of the Pareto front as 10, and MATLAB’s “fmincon” function is used for finding the minimizers of each objective function and each NBI subproblem. In “fmincon”, \( \alpha_i \) is initialized to 1 for each sensor \( i \), and the algorithm tolerances of “fmincon” is set to \( 10^{-7} \). Table 3.3 lists all the solutions of the multiobjective optimization problem in (3.51) and the MSE with the corresponding solution \( \alpha \). Here we employ two solution selection methods to

---

\(^3\)In portfolio theory, efficient frontier is defined as the portfolio set which has the highest expected return for its level of risk. Note that in Fig. 3.22, both the \( \mathbb{E}(\bar{r}) \) and \( \sigma^2 \) have been normalized.
Table 3.3: MSE Performance of the solutions found by the NBI method.

<table>
<thead>
<tr>
<th>Method</th>
<th>MSE</th>
<th>$\alpha_1$</th>
<th>$\alpha_2$</th>
<th>$\alpha_3$</th>
<th>$\alpha_4$</th>
<th>$\alpha_5$</th>
<th>$\alpha_6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sol. 1</td>
<td>0.42</td>
<td>0.38</td>
<td>0.08</td>
<td>0.05</td>
<td>0.01</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Sol. 2</td>
<td>0.33</td>
<td>0.72</td>
<td>0.16</td>
<td>0.09</td>
<td>0.02</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Sol. 3</td>
<td>0.3</td>
<td>1</td>
<td>0.27</td>
<td>0.17</td>
<td>0.03</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Sol. 4</td>
<td>0.23</td>
<td>1</td>
<td>0.63</td>
<td>0.37</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Sol. 5</td>
<td>0.32</td>
<td>1</td>
<td>0.65</td>
<td>0</td>
<td>0</td>
<td>0.35</td>
<td>0</td>
</tr>
<tr>
<td>Sol. 6</td>
<td>0.37</td>
<td>1</td>
<td>0.31</td>
<td>0</td>
<td>0</td>
<td>0.69</td>
<td>0</td>
</tr>
<tr>
<td>Sol. 7</td>
<td>0.42</td>
<td>1</td>
<td>0.04</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.96</td>
</tr>
<tr>
<td>Sol. 8</td>
<td>0.57</td>
<td>0.65</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.35</td>
<td>1</td>
</tr>
<tr>
<td>Sol. 9</td>
<td>0.68</td>
<td>0.3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.7</td>
<td>1</td>
</tr>
<tr>
<td>Sol.10</td>
<td>0.82</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

select one particular solution from the Pareto-front which can yield the desired trade-off between the conflicting objectives: knee point solution and compromise solution [70]. The knee of the trade-off curve represents the solution where a small decrease in one objective is associated with a large increase in the other, and the compromise solution is the one that is closest to the utopia point. As shown in Fig. 3.22, the knee point solution is Sol. 1, and the compromise solution is Sol. 5. We observe from Table 3.3 that both the compromise solution and the knee point solution select sensors with better reliability, and the compromise solution provides lower MSE than the knee point solution.

b) Comparison with the method that maximizes only the expected FI gain: We now show the significance of our portfolio theory based sensor selection scheme by comparing it with the scheme that maximizes only the expected FI gain. From Table. 3.2, we know that sensor 5 and sensor 6 have relatively large expected FI gain, so that they will be selected by the sensor selection scheme that maximizes only the expected FI gain, i.e., Sol. 10 in Fig. 3.22 is the solution that only maximizes the expected FI gain. However, sensor 5 and sensor 6 have poor reliability, and they are highly correlated with each other. Moreover, we can observe from Table. 3.3 that Sol. 10 gives the poorest MSE performance among all the solutions on the efficient frontier. Note that, Sol. 4, where sensor 1 has probability 1 of being selected, and sensor 5 and sensor 6 have probability 0 to be selected, gives the best MSE performance compared to the other solutions. In other words, the sensors that give the best MSE performance do not necessarily have high expected FI gain, and
the reliability of sensor observations also plays a very important role. Our portfolio theory based sensor selection scheme finds all the efficient solutions, and the compromise solution achieves a reasonable tradeoff between the expected FI gain and the reliability of the sensors to optimize the overall performance.

\(\text{c) Diversification:}\) In portfolio selection theory, the variance of the return of the portfolio, which represents risk, can be reduced by including additional assets in the portfolio, which is referred to as diversification [67]. In Fig. 3.23, we assume that every sensor has equal probability to be selected and each element of the sensor selection vector is equal, i.e., \(\alpha_i = 1/N\) for each sensor \(i\). We observe that the expected FI gain of the sensors does not change much when the number of sensors increases, and the variance decreases because of the diversification process. Note that, although the FI in (3.44) increases as the number of sensors increases, the vector \(\alpha\) averages the expected return. As shown in [67], the risk can be reduced through diversification, but there is likely to be a lower limit to the variance, since the sensors are positively correlated.

In Fig. 3.24, we plot the MSE of our sensor selection scheme when the number of selected sensors increases. We assume that 25 sensors are randomly deployed in the network, and the sensors are selected through the portfolio selection theory based sensor selection scheme. Specifically, on the efficient frontier obtained through the NBI method, we select the compromise solution to select sensors for our target localization task. Fig. 3.24 shows that the MSE decreases as the number of selected sensors increases.

### 3.5 Summary

In this chapter, we have proposed multiobjective optimization methods for the sensor selection problem in a resource limited wireless sensor network (WSN) for target tracking and localization. In the target tracking problem, two scenarios were considered: 1) all the sensors are assumed to be reliable, and, 2) a fraction of the sensors in the WSN perform in an unreliable manner. At each time step of tracking, the sensor selection strategy has been obtained from the Pareto-optimal
solutions which reflect different trade-offs between the total number of the selected sensors and the estimation accuracy. We have considered three performance metrics, Fisher information (FI), mutual information (MI), and mutual information upper bound (MIUB), as objective functions for characterizing the estimation performance for the multiobjective optimization problem (MOP). Numerical results showed that the MIUB based selection scheme (MIUB-SS) selects more reliable sensors compared with the FI based selection scheme (FI-SS) while saving computational cost compared with the MI based selection scheme (MI-SS). Furthermore, for the MOP framework, we have shown that the compromise solution on the Pareto front of the MOP achieves good estimation performance while obtaining savings in terms of the number of selected sensors.

For the target localization problem, we have proposed a portfolio theory based sensor selection approach for the target localization problem in an uncertain wireless sensor network (WSN). We have mapped our sensor selection problem to portfolio selection theory in finance and shown that it is important and necessary to select sensors that consider not only the expected FI gain, but also the reliability of the sensors, i.e., risk. Since FI variability represents the reliability of each sensor, our portfolio theory based sensor selection problem has been formulated as a MOP to achieve the trade-off between the expected FI gain and the FI variability. The normal boundary intersection (NBI) method has been used to solve the MOP. In the simulation part, we have shown the efficiency of our approach, and that the risk of the sensor portfolio can be reduced through diversification.
Fig. 3.7: Target tracking performance with analog data, 5-bit quantized data, and 2-bit quantized data, (a) MSE performance; (b) average percentage of reliable sensors selected.
Fig. 3.8: Target tracking performance for MI and MIUB, $A = 2$. 
Fig. 3.9: Pareto optimal front obtained by using NSGA-II at time step $t = 3$ and $t = 6$, (a) FI gap shown in (3.31); (b) MIUB gap shown in (3.33).
Fig. 3.10: Tracking performance at each time step with different solution selection methods.
Fig. 3.11: Tracking performance for MOP with NSGA-II, convex relaxation, and weighted sum methods (a) MSE for MIUB; (b) MSE for FI.
Fig. 3.12: Turn off sensors with relatively low sensing probabilities.

Fig. 3.13: Comparison the performance for WSNs with and without uncertainty.
Fig. 3.14: Tracking performance under varying network densities (a) $N = 9$; (b) $N = 16$; (c) $N = 25$, (d) $N = 49$. 
Fig. 3.15: Tracking performance of MIUB and FI, sensors’ sensing probabilities are reversely deployed.

Fig. 3.16: Tracking performance of MIUB and FI in a WSN with obstacle in center.
Fig. 3.17: Sensors’ sensing probabilities are all uniformly distributed between 0 and 1 for every Monte Carlo trial.

Fig. 3.18: Sensor measurements have higher noise $\sigma = 1$. 
Fig. 3.19: Target moves with a relatively large process noise parameter $q = 0.01$.

Fig. 3.20: Sensor measurements are quantized to 3 bits.
Fig. 3.21: An example scenario.

Fig. 3.22: Efficient frontier obtained through NBI. The numbers around the frontier are the solution numbers corresponding to Table 3.3.
Fig. 3.23: Expected return and variance as a function of the number of sensors.

Fig. 3.24: MSE of target localization as a function of the number selected sensors.
CHAPTER 4
SENSOR MANAGEMENT IN CROWDSOURCING BASED SENSOR NETWORKS: A MECHANISM DESIGN APPROACH

4.1 Introduction

The main objective of this chapter is to design a market-based mechanism [81] to trade information for localizing/tracking a target, with the mechanism being computationally efficient, individually rational (to rationalize user participation), incentive-compatible (to ensure strategy-proofness), and profitable (to ensure feasibility). However, as opposed to conventional market scenarios, the problem at hand portrays two unique characteristics— a) Here, the traded commodity in the market is information. At what prices would information trade, given that the prices users would want to sell their information is dependent on their participatory costs?, and, b) The information acquisition process is in a resource-constrained environment with participants having limited energy, and bandwidth availability for communication. How do we allocate resources efficiently in such a
resource-constrained environment? To answer both questions, we propose to use auctions [81–83]. One of the chief virtues of auctions is their ability to determine appropriate prices of traded commodities [84]. Further, there is also substantial agreement among economists that auctions are the best way to allocate resources in a resource-constrained environment [85]. Essentially, auctions seek an answer to the basic question ‘Who should get the resources and at what prices?’

In this chapter, we first limit our focus on the design of an incentive-based mechanism for sensor selection problem in target localization. Then we study the more general problem of designing an incentive-based mechanism for dynamic bit allocation\(^1\) for target tracking.

In the Section 4.2, we focus on the design of an incentive-compatible mechanism for estimating the location of a target by considering the users (sensors) who provide information about the target to be selfish in nature. Specifically, we propose a reverse auction\(^2\) based mechanism in which an auctioneer (Fusion Center (FC)) conducts an auction to estimate a target location by soliciting bids from the selfish users. The bid of each sensor corresponds to its valuation per unit information, which depends on the energy resources available at the sensor. We consider the network to be bandwidth constrained so that the FC needs to select an optimal subset of sensors as winners of the auction with the objective of maximizing its own utility and make appropriate payments to the selected sensors. To address the participatory concerns of the selfish users, we design the mechanism so that it is always in the best interest of the selfish users to participate in the auction. Further, our proposed auction mechanism is truthful so that it is not prone to market manipulations. We also provide computationally efficient algorithms to implement the proposed mechanism.

In Section 4.3, we propose a reverse auction based mechanism in which an auctioneer (FC) conducts an auction to estimate the target location at each tracking step by soliciting bids from the selfish users (sensors\(^3\)). The bids of the sensors reflect how much they value their energy costs. Moreover, the sensors’ value estimates of their energy costs may also increase as the residual

\(^1\)It should be noted that, for a given total number of bits per time step that can be transmitted from sensors to the fusion center (FC), dynamic bit allocation distributes the resources more efficiently, and thus provides better estimation performance as compared to the sensor selection problem [18].

\(^2\)A reverse auction is one in which the roles of buyers and seller are reversed.

\(^3\)In this chapter, we refer to users as sensors, unless mentioned explicitly.
energy depletes, which we also consider in our model. Our auction mechanism is comprised of two components— a) \textit{bandwidth allocation function}, which determines how to distribute the limited bandwidth (bits) between the sensors and the FC, and, b) \textit{pricing function}, which determines the payment to be made to each sensor. The focus of this chapter is to design these two functions.

To implement the proposed auction model in a computationally efficient manner, we use dynamic programming by formulating the proposed mechanism as a multiple-choice knapsack problem (MCKP) [86, 87]. As shown in Section 4.3, the dynamic programming approach finds the exact equilibrium of our model. Formally, the key contributions of the section are as follows.

- We propose an incentive-based mechanism to trade information for tracking a target. The proposed mechanism is computationally efficient, individually rational, incentive-compatible (truthful), and profitable. To the best of our knowledge, we are the first to propose a market mechanism for tracking a target using selfish sensors that exhibits the aforementioned properties.

- We propose a pseudo-polynomial time procedure to implement the proposed auction mechanism using dynamic programming. The dynamic programming approach can provably sustain the market at the exact equilibrium. Our solution is thus stable.

- Via extensive simulations, we show the effectiveness of our proposed mechanism, study its characteristics, and also show the benefits of the “energy-awareness” of the mechanism when the participatory costs (value estimates) of the users are dependent on their residual energy.

4.2 An Incentive-based Mechanism for Location Estimation in Wireless Sensor Networks

In this section, we propose a framework for target location estimation by designing an incentive-compatible mechanism in a wireless sensor network containing sensors that are selfish and profit-motivated. To accomplish the task, the fusion center (FC) conducts an auction by soliciting bids
from the selfish sensors, where the bids reflect the information available at the sensor and the remaining energy of the sensors. Furthermore, the truthfulness of the sensors is guaranteed in our model. Computationally efficient algorithms to implement our mechanism are provided. Simulation results show the effectiveness of our proposed approach.

4.2.1 System Model and Problem Formulation

System Model

The system model and Monte Carlo method for target localization considered in this section is as shown in Section 2.3.

We assume that the FC is at the center of the ROI and we employ an energy-efficient on-off keying scheme, where only the sensors that are selected by the FC transmit their quantized measurements. We assume that there are no errors in data transmission. We consider a simple model of energy consumption at sensor \( i \) for transmitting \( M_i \) bits over distance \( d_i^F \) between sensor \( i \) and the FC which is defined as

\[
E_{ti}(M_i, d_i) = E_{elec} \times M_i + \epsilon_{amp} \times M_i \times d_i^{F^2} \tag{4.1}
\]

Thus, the remaining energy of sensors \( i \) is \( E_{ri} = E_0 - E_{ti} \), where \( E_0 \) is the initial energy of every sensor in the network. In the model given in (4.1), the sensor dissipates \( E_{elec} = 50 \text{ nJ/bit} \) to run the transmitter and \( \epsilon_{amp} = 100 \text{ pJ/bit/m}^2 \) for the transmitter amplifier.

Information Valuation

The sensors that are competing to sell their measurements are the bidders or potential sellers in the sensor network. We assume that each bidder \( i \) has a valuation \( v_i \) per unit of information and \( v_i \) is the true valuation of \( i \). The FC is assumed to be unaware of the true valuations of the sensors so that the sensors have to advertise their valuations to the FC, giving the sensors an opportunity to
lie about their valuations for potentially an extra benefit. We also assume that the FC’s uncertainty about the value estimate of bidder \( i \) can be described by a continuous probability distribution \( f_i : [a_i, b_i] \to \mathbb{R}_+ \) (positive real number field) over a finite interval \((a_i, b_i)\), where \( a_i \) is the lowest possible value which \( i \) might assign to its data and \( b_i \) is the highest possible value which \( i \) might assign to its data and \(-\infty \leq a_i \leq b_i \leq \infty\). \( F_i : [a_i, b_i] \to [0, 1] \) denotes the cumulative distribution function, where \( F_i(v_i) = \int_{a_i}^{v_i} f_i(u_i)du_i \). We let \( T \) denote the set of all possible combinations of sensors’ value estimates:

\[
T = [a_1, b_1] \times \ldots \times [a_n, b_n].
\]

Also, for any sensor \( i \), the set of all the combinations of the other sensors’ value estimates is

\[
T_{-i} = [a_1, b_1] \times \ldots \times [a_{i-1}, b_{i-1}] \times [a_{i+1}, b_{i+1}] \times \ldots \times [a_n, b_n].
\]

The value estimates of the \( N \) sensors are assumed to be statistically independent random variables. Thus, the joint pdf of the vector \( \mathbf{v} = (v_1, \ldots, v_N) \) is

\[
f(\mathbf{v}) = \prod_{j \in \{1, 2, \ldots, N\}} f_j(v_j) \quad (4.2)
\]

We assume that bidder \( i \) treats the other sensors’ value estimates in a similar way as the FC. Thus, both the FC and the bidder \( i \) consider the joint pdf of the vector of values for all the sensors other than \( i \) \((v_1, \ldots, v_{i-1}, v_{i+1}, \ldots, v_N)\) to be

\[
f_{-i}(\mathbf{v}_{-i}) = \prod_{j \in \{1, \ldots, i-1, i+1, \ldots, N\}} f_j(v_j) \quad (4.3)
\]

Further, we assume the FC to derive a benefit from performing location estimation and consider that the valuation of the FC per unit of information of the selected sensors is \( v_{FC} \).
Problem Formulation

Based on the above definitions and assumptions, we consider an auction design problem in the WSNs. We consider a direct revelation mechanism, where the bidders simultaneously and confidentially announce their value estimates to the FC. The FC then determines from whom it should buy the data and how much it should pay to each sensor. Thus, our objective is to maximize the FC’s utility as a function of the user selection scheme variable and the payment vector. Since the utility function and the information being traded need to be additive and separable [89], the trace of the FIM is considered as the measure of information in this chapter. We define the expected utility $U_{FC}$ for the FC from the auction mechanism as

$$U_{FC}(p, q) = \int_{T} v_{FC} \left( \sum_{i=1}^{N} q_i(v) J_i^D + J^P \right) - \sum_{i=1}^{N} p_i(v) \right] f(v) dv$$ (4.4)

where $p = [p_1, \ldots, p_N]$ is the payment vector and $p_i$ is the expected payment that the FC makes to sensor $i$ which is a function of the vector of announced value estimates $v = [v_1, \ldots, v_N]$. $q = [q_1, \ldots, q_N]$ is a Boolean vector which represents the selection state of the sensors, i.e., $q_i = 1$ when sensor $i$ is selected and $q_i = 0$ when it is not. $f(v)$ is the pdf of the value estimates and $dv = dv_1 \ldots dv_n$. Since sensor $i$ knows that its value estimate is $v_i$, its expected utility $U_i(p_i, q_i, v_i)$ is described as

$$U_i(p_i, q_i, v_i) = \int_{T-i} \left[ p_i(v) - \text{tr} \left( J^P_i \right) \frac{v_i}{E_{ri}/E_0} q_i(v) \right] f_{-i}(v_{-i}) dv_{-i}$$ (4.5)

As described earlier, $J^D_i$ is the expected FIM and $E_{ri}$ is the remaining energy for sensor $i$; and $dv_{-i} = dv_1 \ldots dv_{i-1} dv_{i+1} \ldots dv_n$. If sensor $i$ claimed that $w_i$ was its value estimate when $v_i$ was its true value estimate, its expected utility $\tilde{U}_i$ would be

$$\tilde{U}_i = \int_{T-i} \left[ p_i(w_i, v_{-i}) - \text{tr} \left( J^P_i \right) \frac{v_i}{E_{ri}/E_0} q_i(w_i, v_{-i}) \right] f_{-i}(v_{-i}) dv_{-i}$$
where \((w_i, v_{-i}) = (v_1, \ldots v_{i-1}, w_i, v_{i+1} \ldots v_n)\). The FC estimates the FIM and is assumed to know the residual energy of all the sensors, so that the sensors do not need to transmit their valuations every time a target location needs to be determined. Thus, the auction mechanism based sensor selection problem can be explicitly formulated as follows:

\[
\begin{align*}
\text{max} \quad & \mathcal{U}_{\text{FC}}(p, q) \\
\text{s.t.} \quad & \sum_{i=1}^{N} M_i q_i(v) \leq R \quad i \in \{1, \ldots N\}, \quad \forall v \in \mathcal{V} \\
& U_i(p_i, q_i, v_i) \geq 0, \quad i \in \{1, \ldots N\}, \quad \forall v_i \in [a_i, b_i] \\
& U_i \geq \tilde{U}_i
\end{align*}
\]

(4.6)

The first constraint in (4.6) guarantees that the FC can buy no more than \(R\) bits from all the sensors; we call it the bandwidth limitation (BL) constraint. We assume that the FC cannot force a sensor to participate in an auction. If it did not participate in the auction, the sensor will not get paid, but also would not have any energy cost, so its utility would be zero. Thus, to guarantee that the sensors will participate in the auction, the individual-rationality (IR) condition, which is shown in the second constraint, must be satisfied. Finally, we assume that the FC can not prevent any sensor from lying about its value estimate if the sensor is expected to gain from lying. Thus, to prevent sensors from lying, honest responses must form a Nash equilibrium in the auction game. This is addressed in the last constraint, which is called the incentive-compatibility (IC) constraint.

### 4.2.2 Analysis of the Problem

In this subsection, we analyze the optimization problem proposed in Section 4.2.1. We define

\[
B_i(q_i, v_i) = \int_{T_{-i}} \text{tr} \left( J_i^P \right) q_i(v_i, v_{-i}) f_{-i}(v_{-i}) dv_{-i}
\]

(4.7)

for any sensor \(i\) and any value estimate \(v_i\). So \(B_i(q_i, v_i)\) denotes the expected amount of information that sensor \(i\) is going to sell to the FC conditioned on the valuations of the other sensors \(v_{-i}\).
Our first result is a simplified characterization of the IC constraint of the feasible auction mechanism.

**Lemma 4.2.1.** The IC constraint holds if and only if the following conditions hold:

1. If \( v_i \leq w_i \) then \( B_i(q_i, v_i) \geq B_i(q_i, w_i) \) (4.8)

2. \( U_i(p_i, q_i, v_i) = U_i(p_i, q_i, b_i) + \int_{v_i}^{b_i} \frac{B_i(q_i, v_i)}{E_{ri}/E_0} dv_i \) (4.9)

**Proof.** If \( v_i \leq w_i \), we first consider the case in which bidder \( i \) claims that \( w_i \) is its value estimate when \( v_i \) is its true value estimate.

\[
U_i(p_i, q_i, v_i) \\
\geq \int_{T-i} \left[ p_i(w_i, v_i) - \text{tr} \left( J_i^D \right) \frac{v_i}{E_{ri}/E_0} q_i(w_i, v_i) \right] f_{-i}(v_{-i}) dv_{-i} \\
= U_i(p_i, q_i, w_i) + \left( \frac{v_i - w_i}{E_{ri}/E_0} \right) \int_{T-i} \text{tr} \left( J_i^D \right) q_i(w_i, v_i) f_{-i}(v_{-i}) dv_{-i}
\]

So we can get,

\[
U_i(p_i, q_i, v_i) \geq U_i(p_i, q_i, w_i) + \left( \frac{v_i - w_i}{E_{ri}/E_0} \right) B_i(q_i, w_i)
\] (4.10)

Thus, the IC constraint is equivalent to (4.10). We will show that (4.10) implies (4.8) and (4.9).

By considering the other case that bidder \( i \) claimed that \( v_i \) is its value estimate when \( w_i \) is its true value estimate, similar results are obtained,

\[
U_i(p_i, q_i, w_i) \geq U_i(p_i, q_i, v_i) + \left( \frac{v_i - w_i}{E_{ri}/E_0} \right) B_i(q_i, v_i)
\]

Denote \( A_i = \frac{(w_i - v_i)}{E_{ri}/E_0} \), and combining the two cases, we get

\[
A_i B_i(q_i, w_i) \leq U_i(p_i, q_i, v_i) - U_i(p_i, q_i, w_i) \leq A_i B_i(q_i, v_i)
\]

\( B_i(q_i, v_i) \) is decreasing in \( v_i \), so it is Riemann integrable. Thus, (4.8) and (4.9) hold.
We can also easily show that the conditions in the Lemma also imply (4.10). The details are skipped for brevity.

Based on Lemma 4.2.1, problem (4.6) can be simplified as follows.

**Theorem 4.2.1.** The optimal auction of (4.6) is equivalent to

\[
\max_q \int_T \left\{ \sum_{i=1}^{N} q_i(v) \left[ v_{FC} \text{tr} \left( J_i^D \right) - \frac{\text{tr} \left( J_i^D \right)}{E_{ri}/E_0} \left( v_i + \frac{F_i(v_i)}{f_i(v_i)} \right) \right] \right\} f(v) dv
\]

s.t. \( \sum_{i=1}^{N} M_i q_i(v) \leq R \) \( i \in \{1, \ldots N\} \), \( \forall v \in T \)

and the payment to sensor \( i \) is given by

\[
p_i(v) = v_i \frac{\text{tr} \left( J_i^D \right)}{E_{ri}/E_0} q_i(v) + \int_{v_i}^{b_i} \frac{\text{tr} \left( J_i^D \right)}{E_{ri}/E_0} q_i(w_i, v_{-i}) dw_i
\]

**Proof.** By Lemma 4.2.1, we know that for a feasible auction,

\[
\int_T \left( p_i(x, v) - v_i \frac{\text{tr} \left( J_i^D \right)}{E_{ri}/E_0} q_i(v) \right) f(v) dv
\]

\[= \int_{a_i}^{b_i} U_i(p_i, q_i, v_i) f_i(v_i) dv_i
\]

\[= U_i(p_i, q_i, b_i) + \int_T F_i(v_i) \frac{\text{tr} \left( J_i^D \right)}{E_{ri}/E_0} q_i(v) f_{-i}(v_{-i}) dv
\]

So we may write the fusion center’s objective function as,

\[
U_{FC}(p, q)
\]

\[= \int_T \left[ v_{FC} \text{tr} \left( \sum_{i=1}^{N} q_i(v) J_i^D + J^D \right) - \sum_{i=1}^{N} v_i \frac{\text{tr} \left( J_i^D \right)}{E_{ri}/E_0} q_i(v) \right] f(v) dv
\]

\[- \sum_{i=1}^{N} \int_T F_i(v_i) \frac{\text{tr} \left( J_i^D \right)}{E_{ri}/E_0} q_i(v) \frac{f(v)}{f_i(v_i)} dv - \sum_{i=1}^{N} U_i(p_i, q_i, b_i)
\]
In (4.14), $p_i$ appears only in the last term of the objective function. Thus, to maximize (4.14) subject to the constraints, we must have $U_i(p_i, q_i, b_i) = 0$ for $i \in \{1, \ldots, N\}$. Combining this condition with (4.5), (4.7) and (4.9), we get the formulation of the payment in (4.12) and the objective of (4.11) can be further derived as in the theorem.

**Implementation of the Proposed Mechanism**

We now consider the implementation of the proposed mechanism. Observe that, given $v$, the optimization problem (4.11) is a knapsack problem, where $Val(i) = v_{FC} \text{tr} \left(J_i^P\right) - \frac{\text{tr}(J_i^P)}{E_{r_i}/E_0} \left(v_i + \frac{E_i(v_i)}{f_i(v_i)}\right)$ maps to the value of each item, $M_i$ is equivalent to the weight of each item, and $R$ is mapped to the capacity of the knapsack.

The payment to each sensor can be calculated from (4.12), where the key point for the calculation is to find the threshold of the value estimate $w_i$ above which sensor $i$ will not be selected. For sensor $i$, we first set its payment $p_i = 0$. Assuming that its value estimate is $b_i$, we run our mechanism again, if it is still selected, we conclude that its value estimate threshold $w_i = b_i$, otherwise, $w_i$ must be between $v_i$ and $b_i$ and we apply bisection method until we find $w_i$. Then the payment can be calculated by $p_i = p_i + \frac{\text{tr}(J_i^P)}{E_{r_i}/E_0} w_i$.

**4.2.3 Simulation Results**

In this section, we study how our proposed incentive-based mechanism influences the utility of the FC as well as its impact on the energy efficiency of the network. In our simulation experiments, the size of the ROI is $5m \times 5m$ and the signal power at distance zero is $P_0 = 1000$. Sensor $i$ quantizes its measurement to $M_i$ bits where $M_i$ is randomly chosen from $\{1,2\}$ with equal probability. The quantization thresholds are designed as in [48]. We also assume that the prior pdf of the target location $x$ is $\mathcal{N}(\mu_0, \Sigma_0)$ with $\mu_0 = [1.25, 1.25]^T$ and $\Sigma_0 = \text{diag}[0.5^2, 0.5^2]$. The pdf of the value estimate of sensor $i$, $v_i$, is assumed to be uniformly distributed between $a_i$ and $b_i$ with $a_i = 0.1$ and $b_i = 0.6$. The performance of the location estimator is determined in terms of the mean square error (MSE) via 100 simulation runs.
Impact on the Utility of the FC

In this subsection, we show the utility of the FC and the corresponding estimation performance with the assumption that $E_{r_i} = E_0$ for each sensor $i$. It implies that FIM is the only factor that affects the auction mechanism.

In Fig. 4.1, we observe that the utility of the FC saturates. The value estimate of the FC is assumed to be $v_{FC} = 0.3$. We consider three different cases when the BL constraint of (4.6) is $R = 2, 6, 10$. We observe that when there are 9 sensors in the ROI, the utility of FC for $R = 6$ and $R = 10$ are relatively close to each other and higher than that for $R = 2$. It is because some sensors that make the utility of the FC negative are not selected, so that increase in $R$ does not help the FC to select more sensors or increase its utility. However, as the sensor density in the ROI increases, the chances of the FC selecting more informative sensors that require less payment increases. Payment decreases since the cutoff point of the integral in (4.12) decreases as sensor density increases. In other words, competition among sensors increases as sensor density increases, thereby making sensors participate with lesser payments. So the utility of the FC increases and the corresponding MSE decreases as the number of sensors in the ROI increases. Also, the FC’s utility and the MSE saturate when the number of sensors in the ROI is large. This is because, as has been observed in economics theory, a large number of competitors in a market correspond to a scenario of perfect competition resulting in the market prices to saturate. Further, more sensors are selected when we increase the BL constraint $R$, which makes the FIM increase and the payment decrease. However, the increase of the utility implies that the FIM dominates the payment. Thus, larger the $R$ is, the higher is the utility and lower is the MSE.

In Fig. 4.2, we show the utility of the FC and the corresponding FIM when the BL constraint of (4.6) is $R = 4$. As $v_{FC}$ increases, the FC’s valuation about the FIM increases, so the FC is more capable of buying information from more informative sensors, thus the utility of the FC and the FIM of the selected sensors increase and start approaching that of the FIM based scheme.
Fig. 4.1: The utility of the FC and the corresponding MSE as a function of the total number of sensors in the WSN

**Impact on Energy Efficiency**

In this subsection, we show the energy efficiency of our mechanism through the lifetime of the network, which is defined as the time step at which the network becomes non-functional. The network is non-functional when a specified percentage of the sensors die in the network [90]. A sensor is considered to be dead when its remaining energy is not enough to establish successful communication with the FC. In our work, we assume that the network is non-functional when half of the sensors die.

In Fig. 4.3, we show both the FIM based sensor selection where the sensors are selected based on the FIM with the bandwidth constraint and our auction based mechanism based sensor selection when 2 sensors are selected out of 9 sensors in a region of $20m \times 20m$. The FC is located at the center of the ROI. We assume that all the sensors quantize their measurements to $M = 3$ bits for simplicity. The value estimate of the FC is assumed to be $v_{FC} = 13$. For FIM based sensor selection, the FC repeatedly selects the 2 sensors with the maximum FIM until they die. Thus, the
Fig. 4.2: The utility of the FC and the corresponding FIM as a function of the total number of sensors in the WSN. Red-Circle: \( v_{FC} = 0.2 \), Green-Diamond: \( v_{FC} = 0.6 \), Blue-Triangle: \( v_{FC} = 0.9 \), Magenta-Plus: FIM based Sensor Selection.

more informative sensors die earlier than the less informative ones, resulting in abrupt decrease in the number of active sensors. For our auction mechanism based sensor selection, the sensors that have lower remaining energy are more expensive on an average when the FIM are more or less the same. So more informative sensors are not likely to be selected if their remaining energy is relatively low. In other words, our scheme achieves the tradeoff between selecting sensors with high FIM and making a large payment for selecting sensors with low energy, that is reflected in the energy efficiency of the network. Therefore, the average number of active sensors at each step under our auction mechanism method is larger than the FIM based sensor selection method.
4.3 Target Tracking via Crowdsourcing: A Mechanism Design Approach

In this section, we propose a crowdsourcing-based framework for myopic target tracking by designing an optimal incentive-compatible mechanism in a wireless sensor network (WSN) containing sensors that are selfish and profit-motivated. In typical WSNs which have limited bandwidth, the fusion center (FC) has to distribute the total number of bits that can be transmitted from the sensors to the FC among the sensors. In the formulation considered here, the FC conducts an auction by soliciting bids from the selfish sensors, which reflect how much they value their energy cost. The flowchart of the crowdsourcing-based bit allocation mechanism is given in Fig. 4.4. Furthermore, the rationality and truthfulness of the sensors are guaranteed in our model. The final problem is formulated as a multiple-choice knapsack problem (MCKP), which is solved by the dynamic programming method in pseudo-polynomial time. Simulation results show the effectiveness of our
proposed approach in terms of both the tracking performance and lifetime of the sensor network.

![Flowchart](image)

Fig. 4.4: Flowchart of the crowdsourcing-based target tracking mechanism. $T_s$ denotes the total number of tracking steps.

### 4.3.1 Target Tracking in Wireless Sensor Networks

**System Model**

The system model for target tracking in this section is as shown in Section 2.2, and we consider that the FC reimburses the sensors for energy spent for transmission. By assuming that there are no errors in data transmission, a simple model of energy consumption of sensor $i$ at time $t$ for transmitting $M_{i,t}$ bits over its distance to the FC $d_i^F$ (the sensors’ locations do not change with time) is given as [91]

$$E_{i,t}^c(R_{i,t}, h_i) = \epsilon_{amp} \times M_{i,t} \times d_i^{F^2},$$  \hspace{1cm} (4.15)
where $\epsilon_{\text{amp}}$ is assumed to be $10^{-8} \text{J/\text{bit/m}^2}$. The information criterion applied in this section, Fisher information (FI), and the particle filtering for target tracking are as introduced in Section 2.2.

### 4.3.2 Formulation of the Auction Design Problem

Our problem belongs to the general area of mechanism design [81]. Below we first describe the mechanism design problem in general before formulating our auction in the context of sensor management for tracking.

**Mechanism Design in Reverse Auction Context**

Consider $n$ bidders where each bidder $i \in \{1, \cdots, n\}$ competes to sell his object to the auctioneer. Bidder $i$ has some private information $v_i$, which is referred to as his value estimate, about his object. In a direct mechanism, each bidder directly reports his value estimate as $w_i$ (which may not be equal to his true value estimate $v_i$) to the auctioneer. Based on the vector of announced value estimates $w = (w_1, \cdots, w_n)$, the mechanism computes an output $q(w)$ (which would determine the winning bidder(s)), and a payment $p(w)$ (which would determine the payment to be made to each bidder). The utility of bidder $i$ is $p_i(w) - v_i q_i(w)$. The following two properties should be exhibited by the mechanism.

- **Incentive-Compatibility**: Each bidder’s utility is maximized by reporting his true value estimate $v_i$ to the auctioneer. In other words,

  $$p_i(v_i, w_{-i}) - v_i q_i(v_i, w_{-i}) \geq p_i(w_i, w_{-i}) - v_i q_i(w_i, w_{-i})$$

- **Individual Rationality**: The utility of a bidder should be non-negative, so that it is rational for him to participate in the game.
**The Auction Model**

The sensors, in our model, compete to sell the information contained in their measurements to the FC, and comprise the set of bidders (potential sellers) in the sensor network. For each sensor $i$, there is some quantity $v_i$ which is $i$’s value estimate for its unit energy cost, and that $v_i$ is sensor $i$’s true value estimate. Further, we assume that the FC will derive a benefit from performing location estimation and assume that the FC’s value estimate about the unit information of the selected sensors is $v_{FC}$. The FC is assumed to be unaware of the true value estimates of the sensors so that the sensors have to advertise their value estimates at the beginning of the target tracking process to the FC. This gives the sensors an opportunity to lie about their value estimates hoping for an extra benefit. For instance, a sensor may understate its value estimate in the hope of making the FC buy information with finer quantization (larger number of bits), which countervails its loss for announcing a value estimate that is lower than the truthful one. Or, it may exaggerate its value estimate that might increase the payment made to the sensor sufficiently to compensate for any resulting decrease in the resolution of the information bought.

We assume that the FC’s uncertainty about the value estimate of sensor $i$ can be described by a continuous probability distribution $f_i : [a_i, b_i] \rightarrow \mathbb{R}_+$, where $a_i$ is the lowest possible value which $i$ might assign to its data, and $b_i$ is the highest possible value which $i$ might assign to its data, and $-\infty \leq a_i \leq b_i \leq \infty$. $F_i : [a_i, b_i] \rightarrow [0, 1]$ denotes the cumulative distribution function, where $F_i(v_i) = \int_{a_i}^{v_i} f_i(u_i) du_i$. We let $T$ denote the set of all possible combinations of sensors’ value estimates:

$$T = [a_1, b_1] \times \ldots \times [a_n, b_n].$$

Also, for any sensor $i$, the set of all the combinations of the other sensors’ value estimates is

$$T_{-i} = [a_1, b_1] \times \ldots \times [a_{i-1}, b_{i-1}] \times [a_{i+1}, b_{i+1}] \times \ldots \times [a_n, b_n].$$

\(^4 v_{FC}, \) for instance, can reflect the value estimate of the entity trying to find the lost/stolen object as discussed in [33,35,37]
The value estimates of the \( N \) sensors are assumed to be statistically independent random variables. Thus, the joint pdf of the vector \( \mathbf{v} = [v_1, \ldots, v_N]^T \) is

\[
    f(\mathbf{v}) = \prod_{j \in \{1,2,\ldots,N\}} f_j(v_j).
\]  

(4.16)

We assume that sensor \( i \) treats other sensors’ value estimates in a similar way as the FC does. Thus, both the FC and the sensor \( i \) consider the joint pdf of the vector of values for all the sensors other than \( i \) \( \mathbf{v}_{-i} = [v_1, \ldots, v_{i-1}, v_{i+1}, \ldots, v_N]^T \) to be

\[
    f_{-i}(\mathbf{v}_{-i}) = \prod_{j \in \{1,\ldots,i-1,i+1,N\}} f_j(v_j).
\]  

(4.17)

**Problem Formulation**

Based on the above definitions and assumptions, we consider a direct revelation mechanism, where the sensors simultaneously and confidentially announce their value estimates to the FC. The FC then determines the number of bits it should buy from each sensor and how much it should pay them. We assume that the FC and the sensors are risk neutral, so that the expected value of the utility functions, which are defined as the gains minus the incurred costs, are considered in this paper. Our objective is to maximize the FC’s expected utility as a function of the bit allocations and the payment vector. We consider the Fisher information matrix (FIM) as the information metric for allocating bits among the sensors. From [92,93], we know that if a matrix optimization problem has an optimal solution, then the matrix optimization problem is equivalent to maximizing the trace of the matrix. Furthermore, by using the trace of the FIM as the metric of tracking performance, the sensors have additively separable utilities.
Utility Functions

At time step \( t \), we define the expected utility \( U^F_C(t) \) for the FC from the auction mechanism as the total value for its information gain minus the total payment it needs to make

\[
U^F_C(t, p, q) = \int_T v_{FC} \text{tr} \left( \sum_{i=1}^N \sum_{m=0}^M q_{i,m}(v) J_{i,t}^D(q_{i,m} = 1) + J_{i,t}^P \right) - \sum_{i=1}^N p_i(v) f(v) dv,
\]

(4.18)

where \( p = [p_1, \ldots, p_N]^T \) is the payment vector and \( p_i \) is the expected payment that the FC makes to sensor \( i \). \( q = [q_1^T, \ldots, q_N^T]^T \) and \( q_i = [q_{i,0}, \ldots, q_{i,m}, \ldots, q_{i,M}]^T \) are both Boolean vectors where \( q \) represents the bit allocation state of all the sensors and \( q_i \) represents the bit allocation state of sensor \( i \), i.e., \( q_{i,m} = 1 \) when sensor \( i \) transmits \( m \) bits, and \( q_{i,m} = 0 \) if sensor \( i \) does not transmit its data to the FC in \( m \) bits. Thus \( R_{i,t} = \sum_{m=0}^M mq_{i,m} \) is the number of bits allocated to sensor \( i \) at time step \( t \). Note that both \( p \) and \( q \) are functions of the vector of announced value estimates \( v = [v_1, \ldots, v_N] \), and we ignore the time index \( t \) for \( p, q \) for notational simplicity. Since sensor \( i \) knows that its value estimate is \( v_i \), its expected utility \( U_{i,t}(p_i, q_i, v_i) \) at time \( t \) is described as the payment it gets from the FC minus its total cost incurred by the energy consumption defined in (4.15),

\[
U_{i,t}(p_i, q_i, v_i) = \int_{T-i} \left[ p_i(v) - v_i E_{i,t}^c(q_i, v) \right] f_{-i}(v_{-i}) dv_{-i}, \tag{4.19}
\]

where \( dv_{-i} = dv_1 \ldots dv_{i-1} dv_{i+1} \ldots dv_n \). As shown in (4.15), \( E_{i,t}^c(q_i(v), h_i) = e_{amp} \times (\sum_{m=0}^M mq_{i,m}) \times h_i^2 \), where \( h_i \) is not a variable, so here we use a simplified notation for \( E_{i,t}^c(q_i(v), h_i) \) as \( E_{i,t}^c(q_i, v) \). On the other hand, if sensor \( i \) claimed that \( w_i \) was its value estimate when \( v_i \) was its true value estimate, its expected utility \( \tilde{U}_i \) would be

\[
\tilde{U}_i = \int_{T-i} \left[ p_i(w_i, v_{-i}) - v_i E_{i,t}^c(q_i, w_i, v_{-i}) \right] f_{-i}(v_{-i}) dv_{-i},
\]

where \((w_i, v_{-i}) = (v_1, \ldots, v_{i-1}, w_i, v_{i+1} \ldots v_n)\).
**The Optimization Problem**

Thus, the auction-based bit allocation problem at time step $t$ can be explicitly formulated as follows:

$$
\text{maximize} \quad U_t^{FC}(p, q)
$$

subject to

$$
U_{i,t}(p_i, q_i, v_i) \geq 0, \quad i \in \{1, \ldots N\} \quad (4.20a)
$$

$$
m \in \{0, \ldots R\}, \quad \forall v_i \in [a_i, b_i]
$$

$$
U_{i,t} \geq \tilde{U}_{i,t}, \quad i \in \{1, \ldots N\} \quad (4.20b)
$$

$$
\sum_{i=1}^{N} \sum_{m=0}^{R} mq_{i,m} \leq R \quad (4.20c)
$$

$$
\sum_{m=0}^{R} q_{i,m} = 1, \quad i \in \{1, \ldots N\} \quad (4.20d)
$$

$$
q_{i,m} \in \{0, 1\}, \quad i \in \{1, \ldots N\}, m \in \{1, \ldots R\} \quad (4.20e)
$$

Below we describe each constraint in detail.

- **Individual Rationality (IR) constraint** (4.20a): We assume that the FC cannot force a sensor to participate in an auction. If it did not participate in the auction, the sensor would not get paid, but also would not have any energy cost, so its utility would be zero. Thus, to guarantee that the sensors will participate in the auction, this condition must be satisfied.

- **Incentive-Compatibility (IC) constraint** (4.20b): We assume that the FC can not prevent any sensor from lying about its value estimate if the sensor is expected to gain from lying. Thus, to prevent sensors from lying, honest responses must form a Nash equilibrium in the auction game.

- **Bandwidth Limitation (BL) constraint** (4.20c): The FC can buy no more than $R$ bits from all the sensors.

- **Number of quantization Levels (NQL) constraint** (4.20d): Each sensor uses only one quanti-
zation level.

- $q_{i,m}$ (4.20e) is a Boolean variable.

### 4.3.3 Analysis of the Auction Design Problem

In this section, we analyze the optimization problem proposed in Section 4.3.2. We define

$$B_{i,t}(q_i, v_i) = \int_{T_{-i}} E^{c}_{i,t}(q_i, v) f_{-i}(v_{-i}) dv_{-i}$$  \hspace{1cm} (4.21)

at time step $t$ for any sensor $i$ with value estimate $v_i$. So $B_{i,t}(q_i, v_i)$ denotes the expected amount of energy that sensor $i$ would spend for communication with the FC conditioned on the value estimates of the other sensors $v_{-i}$.

Our first result is a simplified characterization of the IC constraint of the feasible auction mechanism.

**Lemma 4.3.1.** The IC constraint holds if and only if the following conditions hold:

1. if $v_i \leq w_i$ then $B_{i,t}(q_i, v_i) \geq B_{i,t}(q_i, w_i)$, \hspace{1cm} (4.22)

2. $U_{i,t}(p_i, q_i, v_i) = U_{i,t}(p_i, q_i, b_i) + \int_{v_i}^{b_i} B_{i,t}(q_i, r_i) dr_i$. \hspace{1cm} (4.23)

**Proof.** We first show the “only if” part. Without loss of generality, consider that $v_i \leq w_i$. We first consider the case that sensor $i$ claimed that $w_i$ is his value estimate, when $v_i$ is its true value
\[
U_{i,t}(p_i, q_i, v_i) = \int_{\nabla_i} \left[ p_i(v) - E_{i,t}^c(q_i, v) \right] f_{-i}(v) dv_{-i} 
\]
\[
\geq \int_{\nabla_i} \left[ p_i(v_{-i}, w_i) - E_{i,t}^c(q_i, v_{-i}, w_i) \right] f_{-i}(v_{-i}) dv_{-i} 
\]
\[
= \int_{\nabla_i} \left[ p_i(v_{-i}, w_i) - E_{i,t}^c(q_i, v_{-i}, w_i) \right] f_{-i}(v_{-i}) dv_{-i} + \int_{\nabla_i} \left[ w_i E_{i,t}^c(q_i, v_{-i}, w_i) \right] f_{-i}(v_{-i}) dv_{-i} - \int_{\nabla_i} \left[ v_i E_{i,t}^c(q_i, v_{-i}, w_i) \right] f_{-i}(v_{-i}) dv_{-i} = U_{i,t}(p_i, q_i, w_i) + (w_i - v_i) \int_{\nabla_i} E_{i,t}^c(q_i, v_{-i}, w_i) f_{-i}(v_{-i}) dv_{-i}. 
\]

So we can get,
\[
U_{i,t}(p_i, q_i, v_i) \geq U_{i,t}(p_i, q_i, w_i) + (w_i - v_i) B_{i,t}(q_i, w_i). \tag{4.24}
\]

Thus, the IC constraint is equivalent to (4.24). We now show that (4.24) implies (4.22) and (4.23). By switching the roles of \(v_i\) and \(w_i\), we have
\[
U_{i,t}(p_i, q_i, w_i) \geq U_{i,t}(p_i, q_i, v_i) + (v_i - w_i) B_{i,t}(q_i, v_i). \tag{4.25}
\]

Combining (4.24) and (4.25), we can see that
\[
(w_i - v_i) B_{i,t}(q_i, w_i) \leq U_{i,t}(p_i, q_i, v_i) - U_{i,t}(p_i, q_i, w_i) \leq (w_i - v_i) B_{i,t}(q_i, v_i),
\]
from which we can derive (4.22).

Define \(\delta = w_i - v_i\), these inequalities can be written for any \(\delta \to 0\)
\[
\delta B_{i,t}(q_i, v_i + \delta) \leq U_{i,t}(p_i, q_i, v_i) - U_{i,t}(p_i, q_i, v_i + \delta) \leq \delta B_{i,t}(q_i, v_i).
\]

Thus, \(B_{i,t}(x, v_i)\) is a decreasing function and it is, therefore, Riemann integrable. We then write
the utility function of sensor \( i \) for all \( v_i \in [a_i, b_i] \) as

\[
U_{i,t}(p_i, q_i, v_i) = U_{i,t}(p_i, q_i, b_i) + \int_{v_i}^{b_i} B_{i,t}(q_i, r_i) dr_i,
\]

which gives us (4.23).

Now we must show the “if” part of Lemma 4.3.1, i.e., the conditions in Lemma 4.3.1 also imply the IC constraint. Suppose \( v_i \leq w_i \), then (4.22) and (4.23) give us:

\[
U_{i,t}(p_i, q_i, v_i) = U_{i,t}(p_i, q_i, w_i) + \int_{v_i}^{w_i} B_{i,t}(q_i, r_i) dr_i
\geq U_{i,t}(p_i, q_i, w_i) + (w_i - v_i) B_{i,t}(q_i, w_i).
\]

Similarly, if \( v_i \geq w_i \),

\[
U_{i,t}(p_i, q_i, v_i) = U_{i,t}(p_i, q_i, w_i) - \int_{w_i}^{v_i} B_{i,t}(q_i, r_i) dr_i
\geq U_{i,t}(p_i, q_i, w_i) + (w_i - v_i) B_{i,t}(q_i, w_i).
\]

So (4.24) can be derived from (4.22) and (4.23). Thus, the conditions in Lemma 4.3.1 also imply the IC constraint. This proves Lemma 4.3.1.

**Optimal Auction-Based Bit Allocation Mechanism**

Based on Lemma 4.3.1, problem (4.20) can be simplified as follows.
Theorem 4.3.1. The optimal auction of (4.20) is equivalent to

\[
\text{maximize } \int_T \mathcal{V}(q, v) f(v) dv
\]

subject to

\[
\begin{align*}
\sum_{i=1}^{N} \sum_{m=0}^{R} mq_{i,m} & \leq R \\
\sum_{m=0}^{R} q_{i,m} & = 1, \quad i \in \{1, \ldots N\} \\
q_{i,m} & \in \{0, 1\}, \quad i \in \{1, \ldots N\}, m \in \{1, \ldots R\},
\end{align*}
\]

where

\[
\mathcal{V}(q, v) = v_{FC} \operatorname{tr} \left( \sum_{i=1}^{N} \sum_{m=0}^{R} q_{i,m}(v)J_i^D(q_{i,m} = 1) + J_i^P \right) - \sum_{i=1}^{N} E_{i,t}^c(q_i, v) \left( v_i + \frac{F_i(v_i)}{f_i(v_i)} \right),
\]

and the payment to sensor \(i\) is given by

\[
p_i(v) = v_i E_{i,t}^c(q_i, v) + \int_{v_i}^{b_i} E_{i,t}^c(q_i, v - r, r) dr.
\]

Proof. We may write the FC’s objective function (4.18) as

\[
\mathcal{U}_{t}^{FC}(p, q) = \int_T v_{FC} \left( \sum_{i=1}^{N} \sum_{m=0}^{R} q_{i,m}(v)J_i^D(q_{i,m} = 1) + J_i^P \right) f(v) dv
\]

\[
- \sum_{i=1}^{N} \int_T v_i E_{i,t}^c(q_i, v) f(v) dv - \left[ \sum_{i=1}^{N} \int_T (p_i(v) - v_i E_{i,t}^c(q_i, v)) f(v) dv \right].
\]
By (4.22) of Lemma 4.3.1, we know that:

\[
\int_{T} \left( p_i(v) - v_i E_{i,t}^c(q_i, v) \right) f(v) dv
\]

\[
= \int_{a_i}^{b_i} U_{i,t}(p_i, q_i, v_i) f_i(v_i) dv_i
\]

\[
= \int_{a_i}^{b_i} R_i B_{i,t}(q_i, r_i) dv_i dr_i
\]

\[
= \int_{a_i}^{b_i} F_i(r_i) B_{i,t}(q_i, r_i) dr_i
\]

\[
= \int_{a_i}^{b_i} F_i(v_i) E_{i,t}^c(q_i, v) f_i(v_i) dv_i
\]

(4.29)

Substituting (4.29) into (4.28) gives us:

\[
U_{FC}(p, q)
\]

\[
= \int_{T} \left[ \sum_{i=1}^{N} \sum_{m=0}^{R} q_{i,m}(v) J_{i,m}^P(q_i, m = 1) + J_p \right] - \sum_{i=1}^{N} v_i E_{i,t}^c(q_i, v) \right] f(v) dv
\]

\[
- \sum_{i=1}^{N} \int_{T} F_i(v_i) E_{i,t}^c(q_i, v) f_i(v_i) dv_i - \sum_{i=1}^{N} U_{i,t}(p_i, q_i, b_i)
\]

(4.30)

In (4.30), \( p \) appears only in the last term of the objective function. Also, by the IR constraint, we know that for each sensor \( i \), \( U_{i,t}(p_i, q_i, b_i) \geq 0 \). Thus, to maximize (4.30) subject to the constraints, we must have \( U_{i,t}(p_i, q_i, b_i) = 0 \). Combining this condition with (4.19), (4.21) and (4.23), we get

\[
U_{i,t}(p_i, q_i, v_i)
\]

\[
= \int_{v_i}^{b_i} B_{i,t}(q_i, v_i) dv_i
\]

(4.31)

Combine (4.31) with the definition of the utility function of sensor \( i \), we get the formulation of
the payment in (4.27). Thus, if the FC pays each sensor according to Equation (4.27), then the IR constraint is satisfied, as well as the best possible value of the last term in (4.30) is obtained, which is zero. So we can simplify the objective function of our optimization problem to (4.29) subject to the three remaining constraints. Thus, Theorem 4.3.1 follows.

4.3.4 Implementation of the Proposed Mechanism

In this section, we consider the algorithm to obtain the solution for the proposed mechanism. We first study the optimal algorithm to solve our optimization problem in (4.35), and then the case when sensors’ value estimates are dependent on their residual energy.

**Multiple-Choice Knapsack Problems**

The knapsack problem is one of the most important problems in discrete programming [94], and it has been intensively studied for both its theoretical importance and its applications in industry and financial management. The knapsack problem can be described as: given a set of \( n \) items with profit \( p_i \) and weight \( w_i \) and a knapsack with capacity \( c \), select a subset of the items so as to maximize the total profit of the knapsack while the total weights does not exceed \( c \)

\[
\begin{align*}
\text{maximize} \quad & \sum_{i=1}^{n} p_i x_i \\
\text{subject to} \quad & \sum_{i=1}^{n} w_i x_i \leq c \\
& x_i \in \{0, 1\}, \quad i \in \{1, \ldots, N\}.
\end{align*}
\]

(4.32)

There are several types of problems in the family of knapsack problems. The multiple-choice knapsack problem (MCKP) occurs when the set of items is partitioned into classes and the binary choice of taking an item is replaced by the selection of exactly one item out of each class of items [86]. Assume that \( m \) classes \( N_1, \ldots, N_m \) of items are to be packed in a knapsack with capacity \( c \). Each item \( j \in N_i \) has a profit \( p_{i,j} \) and weight \( w_{i,j} \). The problem is how to choose one
item from each class to maximize the total profit of the knapsack while the total weight does not exceed \( c \). The binary variables \( x_{i,j} \) are introduced to represent that item \( j \) is taken from class \( N_i \), the MCKP is formulated as [86] [87]:

\[
\begin{align*}
\text{maximize} & \quad \sum_{i=1}^{m} \sum_{j \in N_i} p_{i,j} x_{i,j} \\
\text{subject to} & \quad \sum_{i=1}^{m} \sum_{j \in N_i} w_{i,j} x_{i,j} \leq c \\
& \quad \sum_{j \in N_i} x_{i,j} = 1, \quad i \in \{1, \ldots, m\} \\
& \quad x_{i,j} \in \{0, 1\}, \quad i \in \{1, \ldots, N\}, m \in N_i,
\end{align*}
\]

(4.33)

where \( p_{i,j}, w_{i,j} \) and \( c \) are assumed to be nonnegative integers, with class \( N_i \) having size \( n_i \) so that the total number of items is \( n = \sum_{i=1}^{m} n_i \). By formulating a recursion form, the MCKP can be solved optimally by the dynamic programming method in pseudo-polynomial time with acceptable computation cost when the number of sensors and the bit constraint are not large.

**Optimal Solution by Dynamic Programming**

Substituting (4.15) into (4.26), the objective function \( \mathcal{Y}_i \) becomes:

\[
\begin{align*}
\mathcal{Y}_i(q, v) = \sum_{i=1}^{N} \sum_{m=0}^{R} q_{i,m}(v) & \left[ v_{FC} \text{ tr} (J_{i,t}^D(q_{i,m} = 1)) - m \epsilon_{amp} h_i^2 \left( v_i + \frac{F_i(v_i)}{f_i(v_i)} \right) \right] + v_{FC} \text{ tr} (J_i^P),
\end{align*}
\]

(4.34)

where the last term is not subject to the solutions of the optimization problem. Thus, by denoting \( V_{i,m} = v_{FC} \text{ tr} (J_{i,t}^D(q_{i,m} = 1)) - m \epsilon_{amp} h_i^2 \left( v_i + \frac{F_i(v_i)}{f_i(v_i)} \right) \), the optimization problem in (4.26) can be
written as:

\[
\begin{align*}
\max_{q} & \quad \int_{T} \left[ \sum_{i=1}^{N} \sum_{m=0}^{R} V_{i,m} q_{i,m} \right] f(v) \, dv \\
\text{subject to} & \quad \sum_{i=1}^{N} \sum_{m=0}^{R} mq_{i,m} \leq R \\
& \quad \sum_{m=0}^{R} q_{i,m} = 1, \quad i \in \{1, \ldots, N\} \\
& \quad q_{i,m} \in \{0, 1\}, \quad i \in \{1, \ldots, N\}, m \in \{1, \ldots, R\}.
\end{align*}
\] (4.35)

Observe that, given \(v\), (4.35) is a Multiple Choice Knapsack Problem (MCKP), which is an extension of the Knapsack Problem (KP) [86]. We interpret our optimal auction-based bit allocation problem as a MCKP as follows: In the WSN consisting of \(N\) sensors, information to be transmitted by each sensor \(i\) has \(R + 1\) variants (bits) where the \(m\)-th variant has weight \(w_{i,m} = m\) and utility value \(V_{i,m}\). As the network can carry only a limited capacity \(R\), the objective is to select one variant of each sensor such that the overall utility value is maximized without exceeding the capacity constraint.

The MCKP can be solved by the dynamic programming approach in pseudo polynomial time with \(O(NR)\) operations [94], [86]. Let \(b_{l}(y)\) denote the optimal solution to the MCKP defined on the first \(l\) sensors with restricted capacity \(y\):

\[
b_{l}(y) = \max \left\{ \sum_{i=1}^{l} \sum_{m=0}^{R} q_{i,m} V_{i,m} \mid \sum_{i=1}^{l} \sum_{m=0}^{R} mq_{i,m} \leq y, \right. \\
\left. \sum_{m=0}^{R} q_{i,m} = 1, \quad i \in \{1, \ldots, l\}, \quad q_{i,m} \in \{0, 1\}, \quad i \in \{1, \ldots, l\}, \quad m \in \{0, \ldots, R\} \right\},
\] (4.36)

and we assume that \(b_{l}(y) = -\infty\) if \(y \leq 0\), \(l > 0\) or \(y < 0\), \(l = 0\). Initially we set \(b_{0}(y) = 0\) for \(y = 0, \ldots, R\). We use the following recursion to compute \(b_{l}(y)\) for \(l = 1, \ldots, N\):

\[
b_{l}(y) = \max_{k=0, \ldots, y} \{b_{l-1}(y - k) + V_{i,k}\}
\] (4.37)
To explain the dynamic programming algorithm, we construct the trellis for \( N + 1 \) stages and \( R + 1 \) states associated with each stage [18]. Fig. 4.5 gives an example trellis for \( N = 5 \) and \( R = 3 \), which contains 6 stages and 4 states. For example, \( b_1(1) = \max \{ b_0(1) + V_{1,0}, b_0(0) + V_{1,1} \} \) and \( b_3(2) = \max \{ b_2(2) + V_{3,0}, b_2(1) + V_{3,1}, b_2(0) + V_{3,2} \} \). Thus, the optimal solution is found as \( b = b_N(R) \). Note that to get the optimal bit allocation, the solution \( q \) needs to be recorded at each step corresponding to the optimal \( b(y) \). On the other hand, the dynamic programming algorithm for our optimization problem is pseudo polynomial and has the complexity \( O(NR) \) [87]. Therefore, the optimality of the problem (4.35) is guaranteed and the rationality and the truthfulness properties of our incentive-based mechanism are maintained.

The payment to each sensor \( i \) can be calculated from (4.27). The total number of bits assigned to sensor \( i \) is determined through the optimal solution of (4.27), \( R_i^P = \sum_{m=0}^{R} m \hat{q}_{i,m}^{\text{opt}}(v) \). We first check the optimal solution of (4.26) with the value estimate vector \((b_i, v_{-i})\), and get the total
Algorithm 4.1 Payment Calculation

1: \( R^P_i = \sum_{m=0}^{R} mq_{i,m}^{opd}(\mathbf{v}) \)

2: \( \tilde{R}^P_i = \sum_{m=0}^{R} mq_{i,m}^{opt}(b_i, \mathbf{v}_{-i}) \)

3: if \( \tilde{R}^P_i = R^P_i \) then

4: \( p_i = b_i E^c_{i,t}(R^P_i) \).

5: else

6: \( n = 1, p_i = v_i E^c_{i,t}(R^P_i), r^0_i = v_i \)

7: while \( n \leq R^P_i \) do

8: Apply bisection method to find the maximum \( r^n_i \) in \((r^{n-1}_i, b_i)\), such that

9: \( \sum_{m=0}^{R} mq_{i,m}^{opd}(r^n_i, \mathbf{v}_{-i}) = R^P_i - (n - 1) \)

10: if \( r^n_i \) does not exist then

11: \( r^n_i = r^{n-1}_i \)

12: else

13: \( p_i = p_i + (r^n_i - r^{n-1}_i) E^c_{i,t}(R^P_i - (n - 1)) \)

14: \( n = n + 1 \)

15: end while

16: end if

number of bits \( \tilde{R}^P_i = \sum_{m=0}^{R} mq_{i,m}^{opt}(b_i, \mathbf{v}_{-i}) \). If \( \tilde{R}^P_i = R^P_i \), we claim that the payment of sensor \( i \) is \( p_i = b_i E^c_{i,t}(R^P_i) \). Otherwise, we apply bisection method to find the thresholds between \( v_i \) and \( b_i \), above which the sensors will be assigned different number of bits compared to the original optimal solution of (4.26). Note that the FC can pay the sensors cumulatively after the tracking process is finished. The pseudo-code of the detailed algorithm is presented in Algorithm 4.1.

Residual Energy-Dependent value estimates

So far, we have assumed that the value estimate of the sensors are invariant of the amount of residual energy of the sensors over time. We now relax this assumption and consider that the (true) value estimates of the sensors are dependent on their residual energy. Therefore, the remaining energy of the sensors are included in their utility functions,

\[
\hat{U}_{i,t}(p_i, q_i, v_i) = \int_{T_{-i}} \left[ p_i(v) - v_i g(e_{i,t-1}) E^c_{i,t}(q_i, v) \right] f_{-i}(v_{-i}) dv_{-i}, \tag{4.38}
\]
Fig. 4.6: Bit allocation with $M = 5$ and $M = 8$ (a) The number of selected sensors. (b) The utility of the FC. (c) MSE at each time step. (d) The utility of the FC with prior information excluded.

where $e_{i,t-1}$ is the remaining energy of sensor $i$ at the beginning of time $t - 1$, i.e., $e_{i,t-1} = e_{i,t-2} - E^c_{i,t-1}$, so that including $g(e_{i,t-1})$ makes the problem more general, and the new objective function $\hat{Y}_t$ of (4.26) becomes

$$\hat{Y}_t = v_{FC} \text{tr} \left( \sum_{i=1}^{N} \sum_{m=0}^{R} q_{i,m}(\nu)J^D_{i,t}(q_{i,m} = 1) + J^P_t \right) - \sum_{i=1}^{N} g(e_{i,t-1}) E^c_{i,t}(q_i, \nu) \left( v_i + \frac{F_i(v_i)}{f_i(v_i)} \right),$$

and the corresponding value of $V_{i,m}$ in (4.35) is

$$\hat{V}_{i,m} = v_{FC} \text{tr} \left( J^D_{i,t}(q_{i,m} = 1) \right) - h(e_{i,t-1}) m \epsilon_{\text{amp}} h_i^2 \left( v_i + \frac{F_i(v_i)}{f_i(v_i)} \right).$$
The target tracking problem with residual energy-based value estimate can also be mapped to a MCKP and solved by the dynamic programming method in pseudo-polynomial time. We assume that the FC knows the energy status of all the sensors at each time step, so the FC and the sensors decide how the value estimate of the sensors change with their remaining energy at the beginning of the tracking task.

4.3.5 Simulation Experiments

In this section, we study the dynamics of our proposed auction-based target tracking mechanism in a sensor network. In the experiments, \( N = 25 \) sensors are deployed uniformly in the ROI with the size \( 50m \times 50m \) and the FC is located at \( x_{FC} = -22, y_{FC} = 20 \). Note that our model can handle any sensor deployment pattern as long as the sensor locations are known to the FC in advance. The signal power at distance zero is \( P_0 = 1000 \). The target motion follows the white noise acceleration model with \( \tau = 2.5 \times 10^{-3} \). The variance of the measurement noise is selected as \( \sigma = 1 \). The prior distribution about the state of the target, \( p(x_0) \), is assumed to be Gaussian with the covariance matrix \( \Sigma_0 = diag[\sigma^2_x \sigma^2_x 0.01 0.01] \) where \( 3\sigma_x = 2 \) so that the initial location of the target stays in the ROI with high probability. The pdf of the value estimate of sensor \( i, v_i \), is assumed to be uniformly distributed between \( a_i \) and \( b_i \) with \( a_i = 0.1 \) and \( b_i = 1 \), and the value estimate of the FC is assumed to be \( v_{FC} = 1 \). The performance of the target location estimator is determined in terms of the mean square error (MSE) at each time step over 100 Monte Carlo trials and the number of particles of each Monte Carlo trial is \( N_s = 5000 \).

We first consider the implementation of our optimal auction-based target tracking procedure where the value estimates of the sensors do not vary with their residual energy. In the target motion model, measurements are assumed to be taken at regular intervals of \( D = 1.25 \) seconds and we observe the target for 20 time steps. The mean of the prior distribution about the target state is assumed to be \( \mu_0 = [-23 -23 2 2]^T \). Two different values of \( R \), namely 5 and 8, are considered to examine the impact of the total number of available bits. In Fig. 4.6(a), we show the number of sensors that are selected at each time step. And the corresponding tracking MSE is shown in
Fig. 4.6(c). We can see that around time steps 4, 9, 14 and 19, the target is relatively close to some sensor, and fewer sensors are activated. When the target is not relatively close to any sensor in the network, during time periods 5-8, 12-13 and 18-19, the uncertainty about the target is relatively high, which increases the estimation error, so that more sensors are activated. Fig. 4.6(b) shows the total utility of the FC at each time step. During the first ten time steps, the utility increases because the FC’s information about target track increases. As the FC has learned enough information about the target track, the Fisher information tends to saturate over the last ten time steps. Moreover, at time step 4, 9, 14, and 19, the target is relatively close to fewer sensors compared to the other time steps, so that fewer sensors are selected according to Fig. 4.6(a), and the total payment at these time steps is lower accordingly. Therefore, the utility of the FC has a sudden increase at these time steps. In Fig. 4.6(d), we also show the utility of the FC when the term due to prior FIM, $J^p_t$, is not included in the expression for the utility function given in (4.18). Due to the low noise environment and the accumulation of the information, $J^p_t$ contains more information and the contribution of the data to the utility function as a function of time diminishes. This is evident in Fig. 4.6(d) in that we observe a decreasing trend of the utility function as a function of time. Moreover, for all the results, we observe that when we have more number of bits (resources) to allocate, the performance in terms of tracking performance and the gains of the FC is better, i.e., results for $R = 8$ are better than those for $R = 5$.

In Fig. 4.7, we study the utility of the FC (Fig. 4.7(a)) and the corresponding MSE (Fig. 4.7(b)) when there are different number of sensors in the network. The figures show that as the number of sensors in the WSN increases, the utility of the FC increases, and the corresponding MSE decreases. It is because as the sensor density in the ROI increases, the chances of the FC selecting more informative sensors that require less payment increase at each tracking step. In other words, competition among sensors increases as sensor density increases, thereby making sensors participate with lesser payments. Also, the FC’s utility and the MSE saturate when the number of sensors in the ROI is large. This is because, as has been observed in economic theory, a large number of competitors in a market correspond to a scenario of perfect competition and result
in the market prices to saturate. Note that when \( N = 9 \) and 16, the utility of the FC decrease and the MSE diverges after a certain time. This is due to the fact that the number of sensors is not sufficient for accurate tracking over the large ROI.

Now we consider the case mentioned in Section 4.3.4 where the sensors value their remaining energy. In (2.2), we consider \( D = 1 \) second and the observation length is 40 seconds. The mean of the prior distribution is assumed to be \( \mu_0 = [-10 -11 2 2]^T \) and the other parameters are kept the same. The target moves back and forth between two different points. During the first and the third 10 second intervals, the target moves as described by model (2.1) in the forward direction. At other times during the second and fourth 10 second intervals, the target moves in the reverse direction with all other parameters fixed. For the function \( g(e_{i,t-1}) \) in (4.38), we take an example where the value estimates of the sensors increase as their remaining energy decreases according to \( g(e_{i,t-1}) = 1/(e_{i,t-1}/E_{i,0})^k \), where \( E_{i,0} \) is the initial energy of each sensor at the initial time step, and the power \( k \) controls the increasing speed. In Fig. 4.8, we show a) the remaining number of active sensors in the WSN of the FIM-based bit allocation algorithm in [18], b) our auction-based bit allocation without residual energy consideration, and, c) when residual energy is considered with different exponent \( k \). Note that in [18], the determinant of the FIM resulted in the suboptimality of the approximate dynamic programming method. Here, to compare with our work, we employ the trace of the FIM as the bit allocation metric to get the optimal solutions using dynamic programming. For FIM-based bit allocation algorithm and our algorithm without residual energy consideration, a specific bandwidth allocation maximizes the FC’s utility for a given target location, resulting in the same set of sensors to be repeatedly selected (as the target travels back to pre-visited locations) until the sensors die (sensors are defined to be dead when they run out of their energy). Thus, those sensors die earlier than the others and the number of active sensors decreases rapidly. However, the increase of the value estimate based on residual energy prevents the more informative sensors from being selected repeatedly because they become more expensive if they have already been selected earlier. In other words, on an average, sensors are allocated lesser number of bits as their residual energy decreases. Moreover, the larger the exponent \( k \) is, the
more the sensors value their remaining energy. We define the lifetime of the sensor network as the time step at which the network becomes non-functional (we say that the network is non-functional when a specified percentage $\alpha$ of the sensors die [90]). For example, we assume $\alpha = 0.6$, in the energy unaware case, the lifetime of the network is around 22. However, by our algorithm, the lifetime of the network gets extended to 30 when $k = 3$, and even gets extended to the last time step when the tracking task ends with $k = 15$ or $k = 30$, i.e., the network keeps functional until the last tracking step.

Corresponding to Fig. 4.8, in Fig. 4.9, we study the tradeoff of considering the function $g(e_{i,t-1} = 1/(e_{i,t-1}/E_{i,0})^k$ in terms of the utility of the FC (Fig. 4.9(a)) and MSE (Fig. 4.9(b)). As shown in Fig. 4.9(b), the FIM-based bit allocation algorithm gives the lowest tracking MSE because the sensors with highest Fisher information are always selected by the FC. For our algorithm without energy consideration and with residual energy considered as $k = 1$ and $k = 3$, the loss of the estimation error and the utility of FC are very small. However, the loss increases when $k$ increases to 15 and 30. This is because when the sensors increase their value estimates more aggressively, they become much more expensive after being selected for a few times. Then the FC, in order to maximize its utility, can only afford to select those cheaper (potentially non-informative) sensors and allocate bits to them. In other words, depending on the characteristics of the energy concerns of the participating sensors, the tradeoff between the estimation performance and the lifetime of the sensor network is automatically achieved.

4.4 Summary

In this chapter, we have proposed an incentive-compatible mechanism for 1) the sensor selection problem for estimating the location of a target, and 2) the dynamic bandwidth allocation problem in the myopic target tracking, by considering that the sensors are selfish and profit-motivated. To determine the proper management of the sensors and the pricing function for each sensor, the FC conducts an auction by soliciting bids from the sensors, which reflects how much they
value their energy cost. Furthermore, our model guaranteed the rationality and truthfulness of the sensors. The designed auction mechanism for the target tracking problem was formulated as multiple-choice knapsack problem (MCKP), which is solved by dynamic programming optimally. Also, we studied the trade-off between the utility of the FC and the lifetime of the sensor network when the value estimate of the sensors depend on their residual energy. In the simulation part, we implemented our mechanism with a computationally efficient algorithm, and the results show that our mechanism is promising and efficient.
Fig. 4.7: Impact of different number of sensors in the ROI (a) The utility of the FC. (b) The MSE at each time step.
Fig. 4.8: The remaining number of active sensors in the WSN
Fig. 4.9: Impact of residual energy consideration (a) The utility of the FC. (b) The MSE at each time step.
Chapter 5
Optimal Auction Design with Quantized Bids in Crowdsourcing based Sensor Networks

This section considers the design of an auction mechanism to sell the object of a seller when the buyers quantize their private value estimates regarding the object prior to communicating them to the seller. The designed auction mechanism maximizes the utility of the seller (i.e., the auction is optimal), prevents buyers from communicating falsified quantized bids (i.e., the auction is incentive-compatible), and ensures that buyers will participate in the auction (i.e., the auction is individually-rational).

5.1 Introduction

The field of mechanism design (also known as reverse game theory) aims to study how to implement desired objectives (social or individual) in systems comprised of multiple selfish and rational agents, with agents having private information that influence the solution [81]. Auction design [82], which falls in the category of mechanism design problems, seeks to investigate how
to allocate an object (such as, a resource) to a set of buyers, with buyers having private value estimates about the object, and to determine the price at which to trade the object via competition among the buyers. In general, auction design has been a well studied topic in the past. A good overview of the topic is provided in [82].

However, in traditional auction design (such as, in [82]), it has been assumed that the buyers send their private information, typically considered as analog values, to the seller in analog form. In contrast, in this chapter, we consider the design of an auction mechanism where the buyers quantize their private information, i.e., their private analog values, prior to communication. Quantization of analog private information prior to communication is practical, for example, 1) due to privacy concerns, the bidders may treat their value estimates as private information, so that they prefer sending their value estimates as categorical information (quantized values) instead of sending the exact values to the auctioneer; and 2) the bidders and the auctioneer may be operating in a resource (energy, bandwidth) constrained environment and, therefore, they communicate using quantized data. Some example scenarios of such environments include auction based resource allocation for sensor management in crowdsourcing based sensor networks [44, 95, 96], spectrum allocation in Cognitive Radio systems [97–100], and routing games in networks [101–103]. It should be noted that, design of an auction mechanism with quantized bids is not only complicated by the fact that the seller is unaware of the true value estimates of the bidders, but also by the fact that the seller only gets quantized data from the buyers that convey information about their private value estimates. The buyers (being selfish and rational entities) may intentionally falsify the quantized bids they transmit in order to acquire an additional advantage, which further complicates the problem. Moreover, as can be expected, choice of the quantization thresholds influences the outcome of the auction, so that the design of the optimal quantization thresholds becomes important.

In this chapter, we design an optimal auction mechanism where the buyers quantize their analog private value estimates regarding the traded object into quantized values prior to communication. To the best of our knowledge, this is the first work till date to investigate this problem. Our auction mechanism is comprised of three components- a) winner determination function, which
determines the bidder who wins the object, b) payment function, which determines the payment to be made by each bidder, and, c) quantization thresholds, which determine how the buyers will quantize their private analog value estimates. This chapter designs the aforementioned components of the auction so that the auction is optimal (i.e., maximizes the seller’s utility), individually-rational (i.e., rationalizes buyer participation), and incentive-compatible (i.e., prevents buyers from communicating falsified binary bids). We also study the influence of the quantization thresholds on the optimal mechanism via simulations. We implement the optimal auction mechanism with quantized value estimates through the target tracking problem in crowdsourcing based WSNs. We also study the sensor selection problem for the bidding process. The results show that having more sensors bidding with each sensor transmitting its value estimate in 1 bit has better performance than selecting fewer sensors with each sensor transmitting more bits.

5.2 Optimal Auction Design with Binary Bids

5.2.1 Auction Model

The seller, as an auctioneer in our model, has an object to sell to one of the $N$ potential buyers. The buyers, on the other hand, compete to buy the object from the seller, and comprise the set of bidders. Each buyer has a private (analog) value estimate regarding the object, which is unknown to the seller. The auction with quantized bids is conducted using the following steps: a) The seller designs an optimal auction mechanism and the corresponding optimal quantization thresholds; b) According to the rules set by the seller, the buyers transmit their quantized bids to the seller; c) The seller decides the winner of the auction and how much to charge for the object.

In this section, we consider the case where the buyers quantize their private value estimates and transmit binary bids to the seller to compete for the object. For each buyer $i$, there is some private (analog) value estimate $v_i$ for the object, and the corresponding quantized value estimate is denoted as $\omega_i$. We assume that the value estimate of buyer $i$ can be described by a probability density function $f_i : [a_i, b_i] \rightarrow \mathbb{R}_+$, where $a_i$ is buyer $i$’s lowest value estimate for the object, and
is his highest value estimate, and \(-\infty \leq a_i \leq b_i \leq \infty\). The binary value estimate of buyer \(i\) is defined as:

\[
\omega_i = \begin{cases} 
0 & a_i \leq v_i \leq \eta_i \\
1 & \eta_i < v_i \leq b_i 
\end{cases}
\] (5.1)

where, \(\eta_i\) is the quantization threshold of buyer \(i\). The seller’s uncertainty about the binary value estimate of buyer \(i\) can be described by the probability mass function (pmf) of \(\omega_i\)

\[
f_i(0) = \text{Pr}(\omega_i = 0) = F(a_i \leq v_i \leq \eta_i) = \int_{a_i}^{\eta_i} f_i(v_i)dv_i.
\] (5.2)

Let \(\Omega\) denote the set of all possible combinations of buyers’ binary value estimates \(\Omega = \{0, 1\}^N\), i.e., the vector \(\omega \in \Omega\). Similarly, we let \(\Omega_{-i}\) denote the set of all possible combinations of the value estimates of the buyers other than \(i\), so that the vector \(\omega_{-i} = [\omega_1, \ldots, \omega_{i-1}, \omega_{i+1}, \ldots, \omega_N]^T \in \Omega_{-i}\) where \(\Omega_{-i} = \{0, 1\}^{N-1}\).

The binary value estimates of the buyers are assumed to be statistically independent with each other. Thus, the joint pmf of the vector \(\omega\) is \(f(\omega) = \prod_{j=1,\ldots,N} f_j(\omega_j)\). We assume that buyer \(i\) treats the other buyers’ binary value estimates in a similar way as the seller does. Thus, both the seller and buyer \(i\) consider the joint pmf of the vector of value estimates for all the buyers other than \(i\), \(\omega_{-i}\), to be \(f_{-i}(\omega_{-i}) = \prod_{j=1,\ldots,i-1,i+1,\ldots,N} f_j(\omega_j)\). The seller’s personal value estimate for the object is denoted by \(v_0\).

### 5.2.2 Auction Design Problem Formulation

The auction design problem is to design the outcome functions \(q, p\), and the quantization thresholds \(\eta\)\(^1\) that maximize the seller’s expected utility subject to certain constraints, where \(q, p : \Omega \to \mathbb{R}^N\), \(q = [q_1, \ldots, q_N]^T\), \(p = [p_1, \ldots, p_N]^T\). Specifically, \(q_i(\omega)\) is the probability of buyer \(i\) being

\(^1\)In our work, the seller is the one who designs the optimal auction, where the design of the quantization thresholds is part of the design process. Thus, the seller can either set the thresholds before designing the optimal auction or determine optimal values for the thresholds during the auction design process.
selected by the seller, and \( p_i(\omega) \) is the amount that buyer \( i \) has to pay\(^2\). Further, in this section, we focus on the direct mechanism, where the buyers directly transmit their binary bids to the seller [104].

By assuming throughout this section that the seller and the buyers are risk neutral, we next define the utility functions of the seller and the buyers. The expected utility of the seller is the seller’s value estimate \( v_0 \) about the object if he were to keep it and not sell it to any bidders plus the total payment from the buyers.

\[
U_0(p, q, \eta) = \sum_{\omega \in \Omega} \left[ v_0 \left( 1 - \sum_{i=1}^N q_i(\omega) \right) + \sum_{i=1}^N p_i(\omega) \right] f(\omega) \tag{5.3}
\]

Since buyer \( i \) is aware of his actual value estimate \( v_i \in [a_i, b_i] \), his expected utility with the binary bid \( \omega_i \in \{0, 1\} \) is described as buyer \( i \)'s value estimate about the object if he gets the object minus the payment he needs to pay to the seller,

\[
U_i(p_i, q_i, v_i, \omega_i, \eta_{-i}) = \sum_{\omega_{-i} \in \Omega_{-i}} \left[ v_i q_i(\omega) - p_i(\omega) \right] f_{-i}(\omega_{-i}) \tag{5.4}
\]

where \( \eta_{-i} = [\eta_1, \ldots, \eta_{i-1}, \eta_{i+1}, \ldots, \eta_N]^T \). Consider now that buyer \( i \)'s actual value estimate \( v_i \) was supposed to be quantized to \( \omega_i \) according to (5.1), but he instead transmits a binary value estimate \( \tilde{\omega}_i \) \((\tilde{\omega}_i \in \{0, 1\}, \tilde{\omega}_i \text{ needs not to be equal to } \omega_i)\). Then, his expected utility would be

\[
\tilde{U}_i = \sum_{\omega_{-i} \in \Omega_{-i}} \left[ v_i q_i(\tilde{\omega}_i, \omega_{-i}) - p_i(\tilde{\omega}_i, \omega_{-i}) \right] f_{-i}(\omega_{-i}) \tag{5.5}
\]

The optimal auction mechanism is designed to maximize the seller’s expected utility while ensur-
maximize \( p, q, \eta \) \[ U_0(p, q, \eta) \]
subject to \[ U_i(p_i, q_i, v_i, \omega_i, \eta_i - \widetilde{\eta}_i) \geq 0 \] \qquad (5.6a)
\[ U_i \geq \bar{U}_i \] \qquad (5.6b)
\[ \sum_{i=1}^{N} q_i(\omega) \leq 1, \quad \forall \omega \in \Omega \] \qquad (5.6c)
\[ 0 \leq q_i(\omega) \leq 1, \quad i \in \{1, \ldots N\} \quad \forall \omega \in \Omega \] \qquad (5.6d)
\[ a_i \leq \eta_i \leq b_i, \quad i \in \{1, \ldots N\} \] \qquad (5.6e)

where \( v_i \in [a_i, b_i] \) and \( \bar{\omega}_i, \omega_i \in \{0, 1\} \). Below we describe each constraint in detail.

- **Individual Rationality (IR) constraint (5.6a):** We assume that the seller cannot force a buyer to participate in an auction. If the buyer does not participate in the auction, he would not get the object, but also would not pay the seller, so his utility would be zero. Thus, to make buyers participate in the auction, this condition must be satisfied.

- **Incentive-Compatibility (IC) constraint (5.6b):** We assume that the seller cannot prevent any buyer from lying about his binary value estimate if the buyer is expected to gain from lying. Thus, to make sure that no buyer has any incentive to lie about his value estimate, transmission of true binary value estimates must form a Bayesian Nash equilibrium of the game.

- **Probability constraints (5.6c) and (5.6d):** Since there is only one object, the seller can select at most one buyer to sell his object.

- **Threshold constraint (5.6e):** The quantization thresholds are between the lowest and highest value estimates of each buyer.
5.2.3 Optimal Auction Design for Given Quantization Thresholds

In this section, we analyze the optimal mechanism design problem when the quantization thresholds $\eta$ are given. We first state a lemma corresponding to the IC condition of (5.6b). We denote $Q_i \triangleq \sum_{\omega_{-i}} q_i(\omega_i = l, t_{-i}) f_{-i}(\omega_{-i})$ as the expected probability that buyer $i$ will be selected when he transmits his binary value estimate $l$ conditioned on all other buyers’ binary value estimates. Similarly, $P_i \triangleq \sum_{\omega_{-i}} p_i(\omega_i = l, t_{-i}) f_{-i}(\omega_{-i})$ is the expected payment buyer $i$ has to pay when he transmits his binary value estimate $l$ conditioned on all other buyers’ binary value estimates.

**Lemma 5.2.1.** The IC condition holds if and only if the following conditions hold:

1. $Q_i^1 - Q_i^0 \geq 0$ (5.7a)
2. $P_i^1 = P_i^0 + \eta_i(Q_i^1 - Q_i^0)$. (5.7b)

**Proof.** Recall (5.6b), we get the equivalent IC conditions as

$$v_iQ_i^0 - P_i^0 \geq v_iQ_i^1 - P_i^1, \quad \forall v_i \in [a_i, \eta_i]$$

$$w_iQ_i^1 - P_i^1 \geq w_iQ_i^0 - P_i^0, \quad \forall w_i \in (\eta_i, b_i)$$

Since (5.8a) and (5.8b) can be directly written as

$$v_i(Q_i^1 - Q_i^0) \leq P_i^1 - P_i^0, \quad \forall v_i \in [a_i, \eta_i]$$

$$w_i(Q_i^1 - Q_i^0) \geq P_i^1 - P_i^0, \quad \forall w_i \in (\eta_i, b_i)$$

If $Q_i^0 = Q_i^1$, (5.9a) and (5.9b) imply the condition that $P_i^0 = P_i^1$. If $Q_i^1 < Q_i^0$, (5.9a) and (5.9b) are equivalent to

$$v_i \geq \frac{P_i^1 - P_i^0}{Q_i^1 - Q_i^0}, \quad \forall v_i \in (a_i, \eta_i]$$

$$w_i \leq \frac{P_i^1 - P_i^0}{Q_i^1 - Q_i^0}, \quad \forall w_i \in (\eta_i, b_i)$$
Then the condition \( b_i \leq \frac{P_1^i - P_0^i}{Q_1^i - Q_0^i} \leq a_i \) must be satisfied, which is contradictory to our definition of buyers’ value estimates. With \( Q_1^i > Q_0^i \), we have

\[
    v_i \leq \frac{P_1^i - P_0^i}{Q_1^i - Q_0^i}, \quad \forall v_i \in [a_i, \eta_i]
\]

\[
    w_i \geq \frac{P_1^i - P_0^i}{Q_1^i - Q_0^i}, \quad \forall w_i \in (\eta_i, b_i].
\]

(5.10)

So that \( Q_1^i \geq Q_0^i \), and \( \eta_i = \frac{P_1^i - P_0^i}{Q_1^i - Q_0^i} \). Thus the lemma is proved.

Note that, the IC conditions in (5.7a) and (5.7b) imply the following: a) for buyer \( i \), the winning probability for transmitting 1 is no less than that for transmitting 0, b) if \( i \) wins, the expected amount he has to pay by transmitting 1 is larger than that of transmitting 0. Thus, the IC condition can be understood in the following way: if buyer \( i \)’s actual value estimate is supposed to be quantized to 0 according to his quantization threshold, then he does not have an incentive to transmit 1 and pay more for the object; if buyer \( i \) is supposed to quantize his actual value estimate to 1, he may have an incentive to transmit 0 and pay less (higher utility), however, transmitting 0 instead of 1 will decrease his probability to win the auction.

Based on Lemma 5.2.1, we can simplify the auction design problem in (5.6), when the quantization thresholds are given, as follows.

**Theorem 5.2.1.** The optimal mechanism design problem of (5.6), when the quantization thresholds are given, is equivalent to

\[
    \begin{align*}
    \text{maximize} & \quad \sum_{\omega \in \Omega} \left[ \sum_{i=1}^{N} u_i(\omega_i)q_i(\omega) \right] f(\omega) \\
    \text{subject to} & \quad \sum_{i=1}^{N} q_i(\omega) \leq 1, \quad \forall \omega \in \Omega \\
    & \quad 0 \leq q_i(\omega) \leq 1, \quad i \in \{1, \ldots N\} \quad \forall \omega \in \Omega
    \end{align*}
\]

(5.11a)

(5.6c)

(5.6d)
where
\[ u_i(\omega_i) = \begin{cases} \frac{a_i - (1 - \lambda_i)\eta_i}{\lambda_i} - v_0, & \omega_i = 0 \\ \eta_i - v_0, & \omega_i = 1 \end{cases} \] (5.12)

with \(\lambda_i = f_i(\omega_i = 0)\), and the payment to buyer \(i\) is given by
\[ p_i(\omega_i, \omega_{-i}) = \eta_i q_i(\omega_i, \omega_{-i}) - (\eta_i - a_i)q_i(\omega_i = 0, \omega_{-i}) \] (5.13)

**Proof.** The IR constraint of (5.6a) can be considered for the two cases as:
\[ U_i(p_i, q_i, v_i, \omega_i = 0) \geq 0, \quad \forall v_i \in [a_i, \eta_i] \] (5.14a)
\[ U_i(p_i, q_i, v_i, \omega_i = 1) \geq 0, \quad \forall v_i \in [\eta_i, b_i]. \] (5.14b)

We may write the seller’s objective function of (5.6) as
\[ U_0(p, q) \]
\[ = v_0 - \sum_{i=1}^{N} v_0 \left( \sum_{\omega \in \Omega} q_i(\omega) f(\omega) \right) + \sum_{i=1}^{N} \sum_{\omega \in \Omega} p_i(\omega) f(\omega) \]
\[ = v_0 - \sum_{i=1}^{N} v_0 \left[ \lambda_i Q_i^0 + (1 - \lambda_i)Q_i^1 \right] + \sum_{i=1}^{N} \left[ \lambda_i P_i^0 + (1 - \lambda_i)P_i^1 \right] \]

By (5.7b) in Lemma 5.2.1, we know that
\[ \lambda_i P_i^0 + (1 - \lambda_i)P_i^1 = \lambda_i P_i^0 + (1 - \lambda_i) \left[ P_i^0 + \eta_i(Q_i^1 - Q_i^0) \right] \]
\[ = P_i^0 + (1 - \lambda_i)\eta_i(Q_i^1 - Q_i^0) \]

The expected payment of buyer \(i\) for \(\forall v_i \in [a_i, \eta_i]\) is
\[ p_i^0 = -U_i(p_i, q_i, v_i, \omega_i = 0) + v_i Q_i^0, \]
with $v_i = a_i$,

$$P_i^0 = -U_i(p_i, q_i, v_i = a_i, \omega_i = 0) + a_iQ_i^0$$  \hfill (5.17)

Substituting (5.16) and (5.17) into (5.15) gives us:

$$U_0(p, q) = -\sum_{i=1}^{N} v_0 \left[ \lambda_i Q_i^0 + (1 - \lambda_i) Q_i^1 \right] + \sum_{i=1}^{N} \left[ (1 - \lambda_i) \eta_i (Q_i^1 - Q_i^0) + a_i Q_i^0 \right]$$

$$+ v_0 - \sum_{i=1}^{N} U_i(p_i, q_i, v_i = a_i, \omega_i = 0)$$

$$= \sum_{i=1}^{N} \left\{ \lambda_i \left[ \frac{a_i - (1 - \lambda_i) \eta_i}{\lambda_i} - v_0 \right] Q_i^0 + (1 - \lambda_i)(\eta_i - v_0) Q_i^1 \right\}$$

$$+ v_0 - \sum_{i=1}^{N} U_i(p_i, q_i, v_i = a_i, \omega_i = 0)$$  \hfill (5.18)

In (5.18), the payment vector only appears in the last term of the utility of the seller. Also, by the IR constraint (5.14a), we know that

$$U_i(p_i, q_i, v_i = a_i, \omega_i = 0) \geq 0, \quad i \in \{1, \ldots, N\}$$  \hfill (5.19)

Therefore, to maximize (5.18) subject to the constraints, the winning buyer must make payment to the seller according to:

$$U_i(p_i, q_i, v_i = a_i, \omega_i = 0) = 0$$  \hfill (5.20)

which, combined with (5.17) and (5.7b), gives the following payment functions

$$P_i^0 = a_iQ_i^0$$

$$P_i^1 = \eta_i Q_i^1 - (\eta_i - a_i)Q_i^0$$  \hfill (5.21)

From (5.21), we get the payment of buyer $i$ regarding his binary value estimate $\omega_i$ in (5.13). Further, substituting the payment functions (5.21) into the objective function (5.15), we get (5.11a) and (5.12).
To further check the condition in (5.7a), we observe that

\[ \eta_i \geq a_i - (1 - \lambda_i)\eta_i \lambda_i \]

(5.22)

So that \( u_i(\omega_i = 1) \geq u_i(\omega_i = 0) \), which means that whenever buyer \( i \) could win the auction by transmitting a binary value estimate 0, he could also win if he changed it to 1. That is, given other buyers’ binary value estimates, the expected probability that buyer \( i \) would win when he transmits his value estimate to be 1 is higher than that when he transmits 0, i.e., (5.7a) is satisfied. Moreover, (5.7b) is considered in (5.16), and the IR condition is satisfied as shown in (5.20). Therefore, the optimization problem considered in (5.6) is equivalent to maximizing the objective function (5.18) subject to the buyer selection probability constraints (5.6c) and (5.6d). This proves the theorem.

Based on Theorem 5.2.1, when the quantization thresholds are given, the optimal auction mechanism can be described as follows:

- For any set of realizations of the binary value estimates \( \omega \), the seller compares the corresponding \( u_i, i \in \{1, \cdots, N\} \) (defined based on (5.12)), and sells the object to the buyer with the highest \( u_i \). In other words, if \( u_i(\omega_i) \) is the highest among all the buyers, then the solution of the winning probability \( q \) is: \( q_i = 1, q_j = 0 \) for \( \forall j \in \{1, \cdots, i - 1, i + 1, \cdots, N\} \).

- Only the buyer that wins the auction needs to pay the seller for the object, and the payment is,

\[ p_i = a_i \] if buyer \( i \) wins by transmitting 0

\[ p_i = \eta_i - (\eta_i - a_i)q_i(\omega_i = 0, \omega_{-i}) \] if buyer \( i \) wins by transmitting 1

(5.23)

Note that, if buyer \( i \) wins the auction by transmitting 1, the seller needs to further determine \( q_i(\omega_i = 0, \omega_{-i}) \) to compute the payment, i.e., determine if buyer \( i \) would have still won the auction had his binary bid been 0 for the same set of binary bids of the other bidders.

- If there is a tie, i.e., multiple bidders have the highest \( u_i \), the seller can arbitrarily choose a
winner among them without affecting his own utility.

5.2.4 Optimal Quantization Thresholds

In Section 5.2.3, we have designed an optimal mechanism when the quantization thresholds \( \eta = [\eta_1, \ldots, \eta_N]^T \) are given. From Theorem 5.2.1, we observe that the thresholds influence the outcome of the auction mechanism. In this subsection, we investigate the design of the optimal quantization thresholds by assuming that the value estimates of the buyers are uniformly distributed.

We first study the case when there is only one buyer who is interested in the object. The value estimate of the buyer is assumed to be in \([a, b]\). With only one buyer, the objective function (5.11a) in the optimization problem (5.11) is (note that the indices have been omitted)

\[
\begin{align*}
&u(\omega = 0)q(0)\lambda + u(\omega = 1)q(1)(1 - \lambda) \\
&= \frac{(\eta - a)(\eta - b - v_0 + a)}{b - a} q(0) + \frac{(b - \eta)(\eta - v_0)}{b - a} q(1)
\end{align*}
\]

The seller maximizes (5.24) over \( \eta \) and \((q(0), q(1))\) subject to the constraints that \( a \leq \eta \leq b \), \( 0 \leq q(0) \leq 1 \), \( 0 \leq q(1) \leq 1 \). The following lemma gives the optimal quantization threshold \( \eta_{opt} \) when only one buyer is interested in the object with different parameter settings.

**Lemma 5.2.2.** The seller keeps the object instead of selling it to the buyer if \( v_0 > b \). When \( v_0 \leq b \), the seller designs the optimal quantization threshold \( \eta \): if \((b + v_0)/2 > a \), the optimal value for \( \eta \) is \((b + v_0)/2 \), otherwise, \( \eta \) can be any value in \([a, b]\). The details are shown in Table 5.2.

**Proof.** With only one buyer, the objective function in the optimization problem (5.11) is (note that
the index has been omitted)

\[
\begin{align*}
&u(\omega = 0)q(0)\lambda + u(\omega = 1)q(1)(1 - \lambda) \\
&= \lambda \left[ \frac{a - (1 - \lambda)\eta - v_0}{\lambda} \right] q(0) + (1 - \lambda)(\eta - v_0)q(1) \\
&= \frac{\eta^2 - (b + v_0)\eta + a\eta_0 + a(b - a)}{b - a}q(0) + \frac{-\eta^2 + (b + v_0)\eta - bv_0}{b - a}q(1) \\
&= \frac{(\eta - a)(\eta - b + a - v_0)}{b - a}q(0) + \frac{(b - \eta)(\eta - v_0)}{b - a}q(1) \\
&\triangleq Mq(0) + Nq(1)
\end{align*}
\]

(5.25)

where \( M \triangleq [(\eta - a)(\eta - b + a - v_0)]/(b - a) \) and \( N \triangleq [(b - \eta)(\eta - v_0)]/(b - a) \), and \( a \leq \eta \leq b \), \( 0 \leq q(0) \leq 1 \), \( 0 \leq q(1) \leq 1 \).

We observe that the optimal solutions for \( q_{opt}(0), q_{opt}(1) \), and \( \eta_{opt} \) depend on the relationship among the parameters of the system. Thus, we list all the conditions and the corresponding solutions in Table 5.1. From the table, it can be observed that when \( v_0 > b \) (row 1 of Table 5.1), the seller does not sell the object, so that the design of the quantization thresholds is irrelevant. Otherwise, if \( (b + v_0)/2 > a \) (rows 2, 3 and 4), then, since it can be shown that \( (b - v_0)^2/4(b - a) \geq a - v_0 \) for any real values of \( a, b, \) and \( v_0 \), the optimal quantization threshold can be set as \( (b + v_0)/2 \). However, if \( (b + v_0)/2 \leq a \) (row 5), any value in \([a, b]\) is equally good as a quantization threshold. This proves the lemma.

Next we consider the scenario when there are multiple bidders. The seller optimizes the expected value in (5.11a), where the problem of finding the optimal \( q \) when \( \eta \) is given in (5.11) is a linear programming problem, which can be solved with the MATLAB function “linprog”. To illustrate the influence of the quantization thresholds on the auction mechanism, we next provide some numerical examples. First, we assume that there are \( N = 2 \) buyers, \( v_0 = 10, a_1 = 2, b_1 = 8 \), and \( a_2 = 12, b_2 = 20 \). The expected utility as a function of \( \eta = [\eta_1, \eta_2]^T \) is shown in Fig. 5.1.

\[3\] Notation “\( \forall \)” for \( q_{opt} \) represents that \( q_{opt} \) can be either 1 or 0, and notation “\( \forall \)” for \( \eta_{opt} \) represents that \( \eta_{opt} \) can be any value between \([a, b]\).
Table 5.1: Optimal quantization thresholds with one buyer.

<table>
<thead>
<tr>
<th>$v_0, a, b$</th>
<th>$\eta$</th>
<th>$q_{\text{opt}}(0)$</th>
<th>$q_{\text{opt}}(1)$</th>
<th>$\eta_{\text{opt}}$</th>
<th>$\mathcal{U}^{\text{opt}}_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v_0 &gt; b ((b + v_0)/2 &gt; b)$</td>
<td>$a \leq \eta \leq b$</td>
<td>0</td>
<td>0</td>
<td>$\forall$</td>
<td>0</td>
</tr>
<tr>
<td>$a &lt; v_0 \leq \frac{b + v_0}{2} \leq b &lt; b - a + v_0$</td>
<td>$a &lt; \eta &lt; v_0$</td>
<td>0</td>
<td>0</td>
<td>$\forall$</td>
<td>$0$</td>
</tr>
<tr>
<td></td>
<td>$v_0 &lt; \eta &lt; b$</td>
<td>0</td>
<td>1</td>
<td>$\forall$</td>
<td>$(b + v_0)/2$</td>
</tr>
<tr>
<td></td>
<td>$\eta = b \text{ OR } \eta = v_0$</td>
<td>0</td>
<td>$\forall$</td>
<td>$\forall$</td>
<td>$(b - v_0)^2/(4(b - a))$</td>
</tr>
<tr>
<td></td>
<td>$\eta = a$</td>
<td>$\forall$</td>
<td>$\forall$</td>
<td>$\forall$</td>
<td>$0$</td>
</tr>
<tr>
<td>$v_0 = a$</td>
<td>$\eta = a \text{ OR } \eta = b$</td>
<td>$\forall$</td>
<td>$\forall$</td>
<td>$\forall$</td>
<td>$0$</td>
</tr>
<tr>
<td></td>
<td>$a &lt; \eta &lt; b$</td>
<td>0</td>
<td>1</td>
<td>$(b + v_0)/2$</td>
<td>$(b - v_0)^2/(4(b - a))$</td>
</tr>
<tr>
<td>$v_0 &lt; a &lt; \frac{b + v_0}{2} &lt; b - a + v_0$</td>
<td>$\eta = a \text{ OR } \eta = b - a + v_0$</td>
<td>$\forall$</td>
<td>1</td>
<td>$a \text{ OR } b - a + v_0$</td>
<td>$a - v_0$</td>
</tr>
<tr>
<td></td>
<td>$a &lt; \eta &lt; b - a + v_0$</td>
<td>0</td>
<td>1</td>
<td>$(b + v_0)/2$</td>
<td>$a - v_0$</td>
</tr>
<tr>
<td></td>
<td>$b - a + v_0 &lt; \eta &lt; b$</td>
<td>1</td>
<td>1</td>
<td>$\forall$</td>
<td>$(b + v_0)^2/(4(b - a))$</td>
</tr>
<tr>
<td></td>
<td>$\eta = b$</td>
<td>1</td>
<td>$\forall$</td>
<td>$b$</td>
<td>$a - v_0$</td>
</tr>
<tr>
<td>$v_0 &lt; b - a + v_0 \leq \frac{b + v_0}{2} \leq a &lt; b$</td>
<td>$\eta = a$</td>
<td>1</td>
<td>1</td>
<td>$\forall$</td>
<td>$a - v_0$</td>
</tr>
<tr>
<td></td>
<td>$\eta = b$</td>
<td>1</td>
<td>$\forall$</td>
<td>$b$</td>
<td>$a - v_0$</td>
</tr>
<tr>
<td></td>
<td>$a &lt; \eta &lt; b$</td>
<td>1</td>
<td>1</td>
<td>$\forall$</td>
<td>$a - v_0$</td>
</tr>
</tbody>
</table>
Table 5.2: Optimal thresholds with one buyer

<table>
<thead>
<tr>
<th>$v_0, a, b$</th>
<th>$\eta_{opt}$</th>
<th>$U_{opt}^0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(b + v_0)/2 &gt; a$</td>
<td>$(b + v_0)/2$</td>
<td>$(b - v_0)^2/(4(b - a))$</td>
</tr>
<tr>
<td>$(b + v_0)/2 \leq a$</td>
<td>$\forall$</td>
<td>$a - v_0$</td>
</tr>
</tbody>
</table>

Fig. 5.1: Utility of the seller as function of the nonidentical thresholds with $N = 2$, $v_0 = 10$, $a_1 = 2, b_1 = 8$, and $a_2 = 12, b_2 = 20$.

Since $v_0 > b_1$ and $b_1 < a_2$, the seller would always select buyer 2 as the winning bidder. In this case, it is irrelevant for the seller to consider buyer 1’s actual (binary) value estimate, and thus any $\eta_1 \in [a_1, b_1]$ is equally good for the seller. Therefore, as can be seen from Fig. 5.1, the expected utility of the seller changes only with buyer 2’s quantization threshold, and is invariant of buyer 1’s quantization threshold. From Fig. 5.1, it can also be seen that the optimal threshold for buyer 2 is $\eta_2 = 15$.

In Fig. 5.2, we study the scenario where $v_0 = 10, a_1 = 5, b_1 = 15$, and $a_2 = 8, b_2 = 20$. The expected utility of the seller is a function of both buyer’s quantization thresholds, since the interval of the two buyers’ value estimates are overlapped. As can be seen from the figure, the seller sets the quantization thresholds to be $\eta_1 = 13, \eta_2 = 15$ to obtain the optimal expected utility.
Fig. 5.2: Utility of the seller as function of the nonidentical thresholds with $N = 2$, $v_0 = 10$, $a_1 = 5$, $b_1 = 15$, and $a_2 = 8$, $b_2 = 20$.

5.3 Optimal Auction Design with Quantized Bids for Target Tracking

5.3.1 System Model

The model of the target tracking system is shown in Section 2.2.

Quantization

In a resource-limited WSN, the communication between the FC and the sensors must be constrained, and, therefore, the sensors send quantized bids and observations to the FC.

a) Quantization of Sensor Observations We assume that all the sensors quantize their measurements, $z_{i,t}$’s, into $m_i^D$ bits before transmitting to the FC. The quantized measurement of sensor $i$ at
time step \( t \), \( D_{i,t} \), is defined as:

\[
D_{i,t} = \begin{cases} 
0 & \eta_{i,0}^{D} < z_{i,t} < \eta_{i,1}^{D} \\
& \vdots \\
L_{i}^{D} - 1 & \eta_{i,L_{i}^{D}-1}^{D} \leq z_{i,t} < \eta_{i,L_{i}^{D}}^{D} ,
\end{cases}
\] (5.26)

where \( \eta_{i}^{D} = [\eta_{i,0}^{D}, \eta_{i,1}^{D}, \ldots, \eta_{i,L_{i}^{D}}^{D}]^{T} \) is the set of quantization thresholds for the measurements, and \( L_{i}^{D} = 2^{m_{i}} \) is the number of quantization levels of each sensor. Then, given the target state \( x_{t} \) at time step \( t \), the probability that \( D_{i,t} \) takes the value \( l \) is \( p(D_{i,t} = l | x_{t}) = F_{z} \left( \eta_{l+1}^{D} \right) - F_{z} \left( \eta_{l}^{D} \right) \), where \( F(.) \) is the cumulative distribution function (CDF) of \( z_{i,t} \). Given \( x_{t} \), the sensor measurements become conditionally independent, and the likelihood function of the vector \( D_{t} = [D_{1,t}, D_{2,t}, \ldots, D_{N,t}]^{T} \), where \( N \) is the number of sensors selected by the FC for the bidding process, can be written as the multiplication of each sensor \( i \)'s likelihood function of \( D_{i,t} \).

\[ b) \text{Quantization of Bids} \] The sensors’ bids represent their value estimate \( v_{i} \) per unit energy cost for participating in the target tracking task. We assume that the sensors have perfect information about their own value estimates, and they quantize their value estimates according to the quantization rules which are set by the FC prior to the tracking process. Regarding the problem that the FC does not know how much would the sensors ask for their unit energy cost while providing information, we assume that the FC’s uncertainty about each \( v_{i} \) is described by a probability density function (pdf) \( f^{v}(\cdot) \) over a finite interval \([a_{i}, b_{i}]\). The quantized value \( \omega_{i} \) of sensor \( i \)'s value estimate \( v_{i} \) is defined using a quantization rule that is similar to that for the sensor observations given in (5.26) with the quantization thresholds \( \eta_{i} = [\eta_{i,0}, \ldots, \eta_{i,L_{i}}]^{T} \), where each sensor quantizes its bid to \( m_{i} \) bits, and \( \eta_{i,0} = a_{i} \), \( \eta_{i,L_{i}} = b_{i} \). The FC’s uncertainty about the quantized value of the bid from sensor \( i \) can be described by the probability mass function (pmf) \( f^{\omega}(\cdot) \) of \( \omega_{i} \) with \( f^{\omega}(\omega_{i} = l) = F_{v} \left( \eta_{l+1} \right) - F_{v} \left( \eta_{l} \right) \).

Let \( \Omega \) denote the set of all possible combinations of bidders’ values, i.e., the vector \( \omega = (\omega_{1}, \omega_{2}, \ldots, \omega_{N})^{T} \in \Omega \). Similarly, we let \( \Omega_{-i} \) denote the set of all possible combinations of value estimates of bidders other than \( i \), so that the vector \( \omega_{-i} = (\omega_{1}, \ldots, \omega_{i-1}, \omega_{i+1}, \ldots, \omega_{N})^{T} \in \Omega_{-i} \).
The values of the $N$ sensors are assumed to be statistically independent of each other. Thus, the joint pdf of the vector $\omega$ is $f_\omega(\omega) = \prod_{j=1,\ldots,N} f_j^\omega(\omega_j)$. We assume that each sensor treats the other sensors’ quantized bid values in a similar way as the FC does. Thus, both the FC and the sensor $i$ consider the joint pdf of the vector of values for all the sensors other than $i$, $\omega_{-i}$, to be $f^\omega_{-i}(\omega_{-i}) = \prod_{j=1,i-1,i+1,\ldots,N} f_j^\omega(\omega_j)$. The FC’s personal value estimate per unit information gain is assumed to be $v_{FC}$. We assume that the FC has no private information about the object, so that $v_{FC}$ is known to all the bidders.

**Information Gain and Energy Cost**

In this work, we apply Fisher information (FI), as shown in Section 2.2, as the information criterion. FI is the inverse of the Cramer-Row lower bound of the estimation error. For a vector random variable, the Fisher information has a matrix form, and we consider the trace of the Fisher information matrix as the metric of the estimation performance gain. Denote $I_{i,t}^D = \text{tr} \left( \int x_t J_i^S(x_t) p(x_t) dx_t \right)$ as the information contribution of sensor $i$ at time step $t$, and $I_t^P = \text{tr}(J_t^P)$ as the information from the prior knowledge of the target. So that the total Fisher information $I_t = \text{tr}(J_t) = \sum_{i=1}^N I_{i,t}^D + I_t^P$.

By assuming that there are no errors in data transmission, a simple model of energy consumption of sensor $i$ at time $t$ for transmitting $m_i$ bits over its distance to the FC $d_{s_i}$ (the sensors and the FC do not move with time) is given as [91]

$$E_{i,t}(m_i, d_{s_i}) = \epsilon_{\text{amp}} \times m_i \times d_{s_i}^2,$$

where $\epsilon_{\text{amp}}$ is a constant.

**Model of the mechanism design system**

In this section, we consider an online auction mechanism, where the sensors bid for participating in the task of target tracking every time step. The model of our mechanism design system is as shown
in Fig. 5.3. Due to bandwidth limitation, the sensors send quantized bids to the FC in the auction, and the FC selects an optimal set of sensors and get their bids within the bandwidth constraint every $T_s$ time steps. Moreover, the sensors quantize their measurements before transmitting to the FC. The FC acts as auctioneer and conducts a reverse auction to further select a subset of sensors and get their observations about the target every time step. Based on the quantized measurements from the selected sensors, the FC estimates the state of the target at each time step.

In our model, the FC first selects sensors for bidding, and then selects sensors for measurement transmission. Since the FC gets benefit from estimating the state of the target based on the observations from the sensors, the utility functions of the FC and the sensors are defined for selecting sensors for measurement transmission. Thus, in our work, we design the auction mechanism to select sensors for measurement transmission and find payment to the winning sensors by maximizing the expected utility of the FC. And the criterion for selecting sensors for bidding will be studied after the auction mechanism is studied.

5.3.2 Sensor Selection for Measurement Transmission

In this subsection, we study the formulation of the optimal auction design problem for selecting a subset of sensors to transmit their measurements when the sensor set \( \{1, \cdots, N\} \) for bidding are given.

Utility Functions

Given the pmf of the sensors’ value estimates about their energy cost at each time step, the FC’s problem is to design an auction mechanism to maximize its own expected utility. By the revelation principle, we restrict our attention to the direct mechanism where the sensors report their value estimates truthfully. The outcome of the mechanism is a pair of functions \((p, q)\), where \(q, p : \Omega \to \mathbb{R}^N\), \(q = [q_1, \ldots, q_N]^T\), and \(p = [p_1, \ldots, p_N]^T\), such that \(q_i(\omega)\) is the probability of sensor \(i\) being selected by the FC and \(p_i(\omega)\) is the amount of money the FC must pay to sensor \(i\). Notice that we allow for the possibility that the FC might have to pay something to a sensor even if that
sensor is not selected.

By assuming throughout this section that the FC and the sensors are risk neutral, at each time step $t$, we define utility functions of the FC and the sensors as their expected revenue minus their expected costs. For ease of presentation, we ignore the time index $t$ for each $p$, $q$ and $\omega$, $\eta$.

\[
U^{FC}_{t}(p, q) = \sum_{\omega \in \Omega} \left[ v_{FC} \left( \sum_{i=1}^{N} q_i(\omega) I_{D,i,t} + I_{P,t} \right) - \sum_{i=1}^{N} p_i(\omega) \right] f^\omega(\omega) \tag{5.28}
\]

Since sensor $i$ does know that its actual value is $v_i$, its expected utility $U_{i,t}(p_i, q_i, v_i, \omega_i)$ with the quantized announced value $\omega_i$ is described as

\[
U_{i,t}(p_i, q_i, v_i, \omega_i) = \sum_{\omega_{-i} \in \Omega_{-i}} \left[ p_i(\omega) - v_i \left( E_{D,i,t} q_i(\omega) + E_{B,i,t} \right) \right] f^\omega_{-i}(\omega_{-i}) \tag{5.29}
\]

where $E_{D,i,t}$ is sensor $i$’s energy cost for sending the quantized measurements to the FC at time step $t$, and $E_{B,i,t}$ is the energy cost for sending a quantized bid. Note that among the sensors that send bids to the FC, only the selected ones need to send quantized measurements. On the other hand, if sensor $i$ announces $\bar{\omega}_i$ when its actual value $v_i$ was supposed to be quantized to $\omega_i$, its expected utility $\bar{U}_{i,t}$ would be

\[
\bar{U}_{i,t}(\bar{\omega}_i) = \sum_{\omega_{-i} \in \Omega_{-i}} \left[ p_i(\bar{\omega}_i, \omega_{-i}) - v_i \left( E_{D,i,t} q_i(\bar{\omega}_i, \omega_{-i}) + E_{B,i,t} \right) \right] f^\omega_{-i}(\omega_{-i}) \tag{5.30}
\]

**Optimization Problem**

We consider the mechanism design problem under the assumption that the quantization thresholds $\eta$ are given. Thus, the auction mechanism based sensor selection problem can be explicitly
formulated as follows:

\[
\begin{align*}
\text{maximize} & \quad \mathcal{U}_i^{FC}(p, q) \\
\text{subject to} & \quad \mathcal{U}_{i,t}(p_i, q_i, v_i, \omega_i) \geq 0 \\
& \quad \mathcal{U}_{i,t}(p_i, q_i, v_i, \omega_i) \geq \tilde{\mathcal{U}}_{i,t}(p_i, q_i, v_i, \tilde{\omega}_i) \\
& \quad \sum_{i=1}^{N} m_i^D q_i(\omega) \leq R, \quad \forall \omega \in \Omega \\
& \quad 0 \leq q_i(\omega) \leq 1, \quad i \in \{1, \ldots, N\} \quad \forall \omega \in \Omega
\end{align*}
\]

where \(v_i \in [a_i, b_i]\). Next, we describe each constraint in detail.

- **Individual-Rationality (IR) constraint (5.31a):** We assume that the FC cannot force a sensor to participate in an auction. If it did not participate in the auction, the sensor would not get paid, but also would not have any energy cost, so its utility would be zero. Thus, to guarantee that the sensors will participate in the auction, this condition must be satisfied.

- **Incentive-Compatibility (IC) constraint (5.31b):** We assume that the FC can not prevent any sensor from lying about its value if the sensor is expected to gain from lying. Thus, to prevent sensors from lying, honest responses must form a Nash equilibrium in the auction game.

- **Bandwidth limitation (BL) constraint (5.31c):** Sensor \(i\) quantizes its measurement to \(m_i^D\) bits, and the total bandwidth of the channel is \(R\) bits.

### 5.3.3 Analysis of the Optimal Auction Design Problem for Measurement Selection

In this section, we analyze the optimization problem. We define

\[
Q^l_i \triangleq \sum_{\omega_{-i}} q_i(\omega_l = l, \omega_{-i}) f_{-i}^\omega(\omega_{-i})
\]

\[(5.32)\]
to be the probability that bidder $i$ will be selected when it reports its quantized value estimate to be $l$ conditioning on all other sensors’ quantized values. Similarly, define

$$P_i^l \triangleq \sum_{\omega_{-i}} p_i(\omega_i = l, \omega_{-i}) f_{\omega_{-i}}(\omega_{-i})$$  \hspace{1cm} (5.33)$$

to be the expected payment of bidder $i$ when it transmits its quantized value $l$ conditioning on all other bidders’ quantized values. It is important to note that because the value estimates are independently distributed, both the probability and the expected payment of bidder $i$ conditioning on all other bidders’ quantized values depend only on the quantized value estimate it transmits to the buyer, no matter what its true value estimate is.

**Analysis of the IC Constraint**

We now present a lemma for the IC condition of (5.31b).

**Lemma 5.3.1.** The IC conditions hold if and only if the following conditions hold:

1. $Q_i^l - Q_i^{l+1} \geq 0$ \hspace{1cm} (5.34a)
2. $P_i^l = P_i^{l+1} + \eta_{i,l+1}(Q_i^l - Q_i^{l+1})E_{i,t}^D$ \hspace{1cm} (5.34b)

**Proof.** Without loss of generality, let us assume that sensor $i$’s true value estimate is $v_i \in [\eta_{i,l}, \eta_{i,l+1}]$, then sensor $i$ is supposed to send its quantized value estimate $l$ to the FC. Recall the IC condition in (5.31b), (5.32) and (5.33), we have

$$P_i^l - v_i E_{i,t}^D Q_i^l \geq P_i^m - v_i E_{i,t}^D Q_i^m, \hspace{0.5cm} 0 \leq \forall m \leq L - 1, m \neq l$$  \hspace{1cm} (5.35)$$

which is equivalent to

$$v_i E_{i,t}^D (Q_i^l - Q_i^m) \leq P_i^l - P_i^m, \hspace{0.5cm} 0 \leq \forall m \leq L - 1, m \neq l$$  \hspace{1cm} (5.36)$$
For $\forall m < l$, if $Q_i^m < Q_i^l$, then

$$\frac{P_i^l - P_i^m}{E_i^D(Q_i^l - Q_i^m)} \geq v_i$$  \hspace{1cm} (5.37)$$

Since $v_i \in [\eta_{i,l}, \eta_{i,l+1}]$,

$$\frac{P_i^l - P_i^n}{E_i^D(Q_i^l - Q_i^n)} \geq \eta_{l+1}$$  \hspace{1cm} (5.38)$$

Similarly, when $\forall n > l \Rightarrow Q_i^n > Q_i^l$, we get

$$\frac{P_i^l - P_i^n}{E_i^D(Q_i^l - Q_i^n)} \leq \eta_l$$  \hspace{1cm} (5.39)$$

Thus, the following condition can be derived when $Q_i^l$ is an increasing function with respect to $l$

$$\eta_{l+2} \leq \frac{P_i^{l+1} - P_i^l}{E_i^D(Q_i^{l+1} - Q_i^l)} \leq \eta_l$$  \hspace{1cm} (5.40)$$

which is a contradiction with the definition of the quantization method. Thus, $Q_i^l$ is a non-increasing function with respect to $l$, i.e., $Q_i^l - Q_i^{l+1} \geq 0, \forall l \in \{1, \cdots, L - 1\}$, and the first condition of the lemma is proved. Let us substitute this condition into (5.36), then

$$P_i^l - P_i^{l+1} = \eta_{l+1}(Q_i^l - Q_i^{l+1})E_i^D$$  \hspace{1cm} (5.41)$$

Thus the lemma is proved.

Lemma 5.3.1 implicates that, 1) the probability of each sensor to be selected conditioning on all other sensors’ quantized value estimates is a non-increasing function of its own quantized value estimate, 2) the payment of each sensor has specific relationship with its selection probability, quantization thresholds, and energy cost.

### 5.3.4 Optimization Problem Analysis

Based on Lemma 5.3.1, we further analyze the optimization problem in (5.31). For ease of presentation, we denote $f_i(\omega_i = l) = \lambda_{i,l}$. 

Theorem 5.3.1. The optimal auction of (5.31) is equivalent to finding the optimal solution of the following optimization problem

\[
\begin{align*}
\text{maximize} & \quad \sum_{\omega \in \Omega} \left[ \sum_{i=1}^{N} u_{i,t}(\omega_i) q_i(\omega) \right] f^\omega(\omega) \\
\text{subject to} & \quad Q^l_i \geq Q^{l+1}_i \quad (5.34a) \\
& \quad \sum_{i=1}^{N} m^D_i q_i(\omega) \leq R, \quad \forall \omega \in \Omega \quad (5.31c) \\
& \quad 0 \leq q_i(\omega) \leq 1, \quad i \in \{1, \ldots, N\} \quad \forall \omega \in \Omega \quad (5.31d)
\end{align*}
\]

where

\[
\begin{align*}
u_{i,t}(\omega_i = l) &= \begin{cases} 
  v_{FC} I_i^D - b_i E^B_i - E^D_i \eta_{i,1}, & l = 0 \\
  v_{FC} I_i^D - b_i E^B_i - E^D_i \left( \sum_{s=0}^{l} \lambda_{i,s} \eta_{i,l+1} - \left( \sum_{s=0}^{l-1} \lambda_{i,s} \right) \eta_{i,l} \right) / \lambda_{i,l}, & l \in \{1, \ldots, L - 1\}
\end{cases}
\end{align*}
\]

and the expected payment to bidder \( i \) is given by

\[
\begin{align*}
P^l_i &= P^{l+1}_i + \eta_{i,l+1}(Q^l_i - Q^{l+1}_i) E^D_i, \quad l \in \{0, \ldots, L - 2\} \\
P^L_i &= b_i E^D_i Q^{L-1}_i + b_i E^B_i, \quad l = L - 1
\end{align*}
\]

Proof. We may write the FC’s objective function of (5.31) as

\[
\begin{align*}
U^{FC}_t(p, q) &= \sum_{i=1}^{N} v_{FC} \left[ \left( \sum_{\omega \in \Omega} q_i(\omega) f^\omega(\omega) \right) I^D_{i,t} + I^P_t \right] - \sum_{i=1}^{N} \sum_{\omega \in \Omega} p_i(\omega) f^\omega(\omega) \\
&= \sum_{i=1}^{N} v_{FC} \left[ \left( \sum_{l=0}^{L-1} \lambda_{i,l} Q_i^l \right) I^D_{i,t} + I^P_t \right] - \sum_{i=1}^{N} \sum_{l=0}^{L-1} \lambda_{i,l} P_i^l 
\end{align*}
\]
By Lemma 5.3.1, we know that

\[
\sum_{l=0}^{L-1} \lambda_{i,l} P_i^l \\
= \sum_{l=0}^{L-2} \lambda_{i,l} \left( P_i^{l+1} + Q_i^{l+1} - Q_i^l \right) E_i^{D} + \lambda_{i,L-1} P_i^{L-1} \\
= \sum_{l=0}^{L-2} \left( \sum_{s=0}^{l} \lambda_{i,s} Q_i^{s+1} - Q_i^l \right) E_i^{D} + \left( \sum_{l=0}^{L-1} \lambda_{i,l} \right) P_i^{L-1} \\
= \sum_{l=0}^{L-2} \left( \sum_{s=0}^{l} \lambda_{i,s} Q_i^{s+1} - Q_i^l \right) E_i^{D} + P_i^{L-1}
\]

(5.46)

The expected payment of sensor \( i \) for \( \forall v_i \in (\eta_i, b_i] \) is

\[
P_i^{L-1} = U_i(p_i, q_i, v_i, \omega_i = L - 1) + v_i E_i^{D} Q_i^{L-1} + v_i E_i^{B}
\]

with \( v_i = b_i \),

\[
P_i^{L-1} = U_i(p_i, q_i, v_i = b_i, \omega_i = L - 1) + b_i E_i^{D} Q_i^{L-1} + b_i E_i^{B}
\]

(5.47)

Substituting (5.46) and (5.47) into (5.45) gives us:

\[
U_t^{FC}(p, q) = \sum_{i=1}^{N} v_{FC} \left( \sum_{l=0}^{L-1} \lambda_{i,l} Q_i^l \right) I_i^{D} + v_{FC} I_i^{P} \\
- \sum_{i=1}^{N} \sum_{l=0}^{L-2} \left( \sum_{s=0}^{l} \lambda_{i,s} Q_i^{s+1} - Q_i^l \right) E_i^{D} - \sum_{i=1}^{N} P_i^{L-1} \\
= \sum_{i=1}^{N} v_{FC} \left( \sum_{l=0}^{L-1} \lambda_{i,l} Q_i^l \right) I_i^{D} + v_{FC} I_i^{P} \\
- \sum_{i=1}^{N} \sum_{l=0}^{L-2} \left( \sum_{s=0}^{l} \lambda_{i,s} Q_i^{s+1} - Q_i^l \right) E_i^{D} \\
- \sum_{i=1}^{N} \left( b_i E_i^{D} Q_i^{L-1} + b_i E_i^{B} \right) - \sum_{i=1}^{N} U_i(p_i, q_i, v_i = b_i, \omega_i = L - 1)
\]

(5.48)
\[
= \sum_{i=1}^{N} \left\{ \lambda_{i,0} \left( v_{FC} I_{i,t}^{D} - E_{i,t}^{D} \right) Q_0^i \right. \\
+ \sum_{l=1}^{L-1} \lambda_{i,l} \left( v_{FC} I_{i,t}^{D} - E_{i,t}^{D} \right) \frac{\left( \sum_{s=0}^{l} \lambda_{i,s} \right) \eta_{i,l+1} - \left( \sum_{s=0}^{l-1} \lambda_{i,s} \right) \eta_{i,l}}{\lambda_{i,l}} \left. Q_l^i \right\} \\
+ v_{FC} I_t^P - \sum_{i=1}^{N} b_i E_{i,t}^{I} - \sum_{i=1}^{N} U_i(p_i, q_i, v_i = b_i, \omega_i = L - 1) \\
\triangleq \sum_{\omega \in \Omega} \left[ \sum_{i=1}^{N} u_{i,t,1(\omega)} q_i(\omega) \right] f^{\omega}(\omega) + v_{FC} I_t^P \\
- \sum_{i=1}^{N} U_i(p_i, q_i, v_i = b_i, \omega_i = L - 1) \\
\]

where \( \eta_{i,l} = b_i \) and \( u_{i,t,1(\omega_l = l)} \) is as described in (5.43). In (5.48), the payment only appears in the last term of the utility of the FC. Also, by the IR constraint (5.31a), we know that

\[
U_i(p_i, q_i, v_i = b_i, \omega_i = L - 1) \geq 0, \quad i \in \{1, \ldots, N\} \quad (5.49)
\]

Therefore, to maximize (5.48) subject to the constraints, the FC must make payment to the sensors according to:

\[
U_i(p_i, q_i, v_i = b_i, \omega_i = L - 1) = 0 \quad (5.50)
\]

which, combined with (5.47) and (5.34b), gives the payment functions in (5.44). Also, we get the objective function of (5.42a) by applying (5.50) into (5.48). This proves the theorem.

\[\square\]

Theorem 5.3.1 aims at finding the way to solve the optimization problem in (5.31), where the selection probabilities \( q \) are obtained from the optimization problem (5.42), and (5.44) provides the payments \( p \) as functions of the selection probabilities \( q \).

**Regularity Condition**

In the optimization problem in Theorem 5.3.1, (5.34a) requires the expected probability for each sensor \( i \) being selected by the FC to be a non-increasing function of its quantized bid \( \omega_i \).
**Definition 5.3.1.** Regularity condition: We say that the problem is regular if \( u_{i,t}(\omega_i) \) in (5.42a) is a non-increasing function of \( \omega_i \) for every sensor \( i \).

For any two quantized bids of sensor \( i \), \( a_i \leq \omega_i \leq \zeta_i \leq b_i \), and \( u_{i,t}(\omega_i) > u_{i,t}(\zeta_i) \), it means that if sensor \( i \) could win the auction by sending bid \( \zeta_i \), it can also win by submitting bid \( \omega_i \). That is, given the other sensors’s quantized bids, the expected probability that sensor \( i \) would win the auction when it reports its bid to be \( \omega_i \) is greater than that when it reports \( \zeta_i \), i.e., (5.34a) is satisfied. Thus, the regularity condition implies the condition of (5.34a), and we have the following corollary.

**Corollary 5.3.1.** The optimization problem (5.42) in Theorem 5.3.1 is equivalent to maximizing (5.42a) subject to the bandwidth limitation constraint (5.31c) and the probability constraint (5.31d) if the regularity condition is satisfied.

**Optimal Mechanism Design with Uniform Distributed Value Estimates**

In this subsection, we assume that the pdf of each sensor’s value estimate \( v_i \) is uniform, i.e., \( f^u(v_i) = \frac{1}{b_i - a_i} \), and \( \lambda_{i,t} = \frac{\eta_{i,t+1} - \eta_{i,t}}{b_i - a_i} \). In this case, the objective function \( u_i(\omega_i) \) in the optimization problem (5.42) can be further simplified. Since

\[
\begin{align*}
\left( \sum_{s=0}^{l} \lambda_{i,s} \right) \eta_{i,l+1} - \left( \sum_{s=0}^{l-1} \lambda_{i,s} \right) \eta_{i,l} & = \frac{\lambda_{i,t}}{\eta_{i,l+1} - \eta_{i,l}} \\
& = \frac{(\eta_{i,t+1} - \eta_{i,0})\eta_{i,l+1} - (\eta_{i,t} - \eta_{i,0})\eta_{i,l}}{\eta_{i,l+1} - \eta_{i,l}} \\
& = \eta_{i,l+1} + \eta_{i,l} + a_i
\end{align*}
\]

\( u_i(\omega_i) \) can be further written as

\[
u_{i,t}(\omega_i = l) = \begin{cases} 
  v_{FC} I_{i,t}^D - b_i E_{i,t}^B - E_{i,t}^P \eta_{i,l}, & l = 0 \\
  v_{FC} I_{i,t}^D - b_i E_{i,t}^B - E_{i,t}^P (\eta_{i,l+1} + \eta_{i,l} + a_i), & l \in \{1, \ldots, L-1\}
\end{cases}
\]
Thus, $u_{i,t}(\omega_i)$ decreases as $\omega_i$ increases, i.e., the regularity condition is satisfied when the value estimate of sensor $i$ is uniformly distributed. According to Corollary 5.3.1, the optimal mechanism selects the sensor with the highest $u_{i,t}$ as the winner when each sensor’s value estimate $v_i$ is uniformly distributed.

5.3.5 Sensor Selection for Bidding

In this section, we study the sensor selection criterion for the bidding process. As shown in Section 5.3.2 and Section 5.3.3, the sensors with higher $u_{i,t}(\omega_i)$ have higher probabilities to get selected by the FC for measurement transmission under the assumption that each sensor $i$ uniformly quantizes its value estimates in $(a_i, b_i)$ (so that the regularity condition is satisfied). In our work, we approximately consider the expected value of $u_{i,t}$, i.e., $\bar{u}_{i,t} = \sum_{\omega_i} u_{i,t}(\omega_i) f(\omega_i) = v_{FC} I_{i,t} - b_i (E_{B_{i,t}} + E_{D_{i,t}})$, as the criterion for selecting sensors for bidding, because the FC has no idea about the sensors’ value estimates before bidding.

Since the FC selects the sensors for bidding every $T_s$ time steps, the sensor with the highest $\sum_{t=k \times T_s}^{(k+1) \times T_s} \bar{u}_{i,t}$, where $k$ is an integer, are selected for every time window of $T_s$ time steps. Since $I_{i,t}^D$ and $E_{i,t}^D$ depend on the result of measurement selection only, the FC selects the sensors with the smallest $E_{i,t}^B$. If the number of bits for the sensors to quantize their value estimates are predetermined, then the FC directly chooses the sensors with the maximum $\bar{u}_{i,t}$. However, if the FC dynamically allocates the bandwidth to each sensor for bidding, the sensors which have the largest contribution should not be omitted from the bidding process in order to maximize the FC’s utility. Also, if any sensor is selected, its energy cost should be minimized. Thus, the FC would prefer selecting more sensors with each sensor quantizing its value estimate to 1 bit, than selecting fewer sensors with each sensor transmitting more bits.

5.3.6 Simulation Results

In this section, we conduct some simulation experiments to investigate the performance of our optimal mechanism in the target tracking task. In our simulations, we consider the WSN, as shown
in Fig. 5.4, containing 36 sensors deployed in the ROI of area $b^2 = 20 \times 20 \ m^2$. For the linear dynamical model of the target given in (2.1), the time interval is $D = 0.5$ seconds and the process noise parameter $q = 0.1$. The prior distribution about the state of the target, $p(x_0)$, is assumed to be Gaussian with mean $\mu_0 = [-9.2 \ -9.2 \ 2 \ 2]^T$ and covariance $\Sigma_0 = \text{diag}[\sigma_x^2 \ \sigma_x^2 \ 0.01 \ 0.01]$ where we select $\sigma_x = 2.4$. In the target tracking process, particle filtering is applied, where the initial $N_s = 5000$ particles are drawn from $p(x_0)$. The source power is $P_0 = 1000$ and the variance of the measurement noise is selected as $\sigma = 1$. The sensors quantize their value estimates and observations using uniform quantizers, where the quantization thresholds $[\eta_{i,1}, \cdots, \eta_{i,L-1}]$ are selected to be the values which evenly partition the interval $[a_i, b_i]$ and the quantization thresholds $[\eta_{D,i,1}, \cdots, \eta_{D,i,L-1}]$ are selected to be the values which evenly partition the interval $[-\sigma, \sigma + \sqrt{P_0}]$.

We assume that the total bandwidth is $R = 10$ bits, and all the sensors quantize their measurements to $m_i^D = 5$ bits. The value of the FC is assumed to be $v_{FC} = 100$, and $a_i = 5$, $b_i = 20$ for sensor $i \in \{1, \cdots, N\}$. The FC is located at $(-1.78, 1.08)$, and the parameter in the energy cost function is $\epsilon_{\text{amp}} = 10^{-3}$. The FC selects the sensors for bidding every $T_s$ time steps, and the total tracking time is $T = 20$. The mean square error (MSE) is used to measure errors between the ground truth and the estimates, and the MSE of the estimation at each time step of tracking is averaged over 100 Monte trials.

We first assume that the FC selects sensors for bidding every $T_s = 8$ time steps, and study the performance of the system when different number of sensors are available for bidding in each bidding window $T_s$ (since the total bandwidth of the channel is 10 bits, if the sensors are predetermined to quantize their value estimates to $m_i = 1, 2, 5$ bits, then there will be 10, 5, 2 sensors available in each bidding window). Fig. 5.5 shows the MSE and the utility of the FC under the above three scenarios. We observe that the scenario in which 10 sensors are available in each bidding window provides the best MSE performance and utility for the FC, and the case with 2 sensors available has the worst system performance\(^4\). The result can be explained by the fact that the objective of the sensor selection in the bidding process is to ensure that as many important sensors as possible

\(^4\)Note that, in both Fig. 5.5 and Fig. 5.6, the MSE first diverges and then drops at time step 16 because the candidate sensors are updated every 8 time steps
for the tracking process are included under the given resource constraint. Thus, as mentioned in Section 5.3.5, if the total bandwidth can be dynamically allocated among the sensors for bidding, the FC would choose to select as many sensors as possible with each sensor transmitting its value estimate with 1 bit.

In Fig. 5.6, the MSE and the utility of the FC is shown with the bidding time window being different $T_s = 1, 5, 8$. It is intuitive that our model performs better with smaller bidding time window $T_s$, which on the other hand motivates the use of online bidding. However, since the users who act as sensors may not be able to bid every time instant, the FC needs to consider the trade-off between the system performance and availability of the users to decide the bidding time window.

### 5.4 Summary

In this chapter, we first designed an optimal auction mechanism for an environment where bidders quantize their value estimates regarding the traded object into binary values prior to communicating them to the seller. The mechanism is designed to maximize the seller’s expected utility while ensuring the individual rationality (IR) and incentive-compatibility (IC) constraints. The chapter also investigated the design of the optimal quantization thresholds, using which buyers would quantize their private value estimates, such that the seller’s expected utility is maximized. Then we studied the optimal auction mechanism design problem for target tracking in crowdsourcing based wireless sensor networks (WSNs) where bidders quantize their value estimates based on their unit energy cost prior communicating them to the fusion center (FC). The mechanism is designed to maximize the FC’s expected utility while ensuring the IR and IC constraints. Numerical results show the efficiency of our model.

In this paper, we designed an optimal auction mechanism for the target tracking problem in crowdsourcing based wireless sensor networks (WSNs) where bidders quantize their value estimates based on their unit energy cost prior communicating them to the fusion center (FC). The mechanism is designed to maximize the FC’s expected utility while ensuring the individual ratio-
nality (IR) and incentive-compatibility (IC) constraints. Numerical results show the efficiency of our model. Future work will study the privacy models of the bidders in the optimal auction design problem.
Users install APP

$t = 1$

FC selects a sensor set for bidding

Selected sensors send quantized bids to the FC

FC selects a sensor subset for measurement transmission

Selected sensors send quantized measurements to the FC

FC estimates the target state

Is $t = T$?

Is $t = k \times T_s + 1$?

No

$t = t + 1$

Yes

FC pays the sensors cumulatively

Fig. 5.3: Flowchart of the crowdsourcing-based target tracking mechanism. The FC selects sensors for bidding every $T_s$ time steps, and $T$ denotes the total number of tracking steps.
Fig. 5.4: Illustration of WSN layout.

Fig. 5.5: MSE and utility of FC when different number of sensors are available in each bidding window.
Fig. 5.6: MSE and utility of the FC with different bidding window $T_s$. 

![Graph showing MSE and utility of the FC with different bidding window $T_s$.]
CHAPTER 6
CHARGING STATE AWARE OPTIMAL
AUCTION DESIGN FOR SENSOR
SELECTION IN CROWDSOURCING BASED
SENSOR NETWORKS

In this chapter, crowdsourcing based wireless sensor networks (WSNs) with rechargeable sensors are used for target localization. For rechargeable sensors, the state of charge (SOC) is one of the key factors that decides the sensors’ energy cost for the localization task. To conserve limited resources, the fusion center (FC) employs an optimal sensor selection scheme obtained through an auction design approach. The sensors compete to participate in the target localization task by sending bids based on their energy efficiency (analog data) and SOC (quantized data) to the FC. Aiming at maximizing the expected utility, the FC designs an optimal auction mechanism incorporating both analog information about sensors’ energy efficiency and quantized information of the SOC and decides on the winning sensor(s) as well as the payment to the winner(s). Simulation experiments show the effective performance of our framework by investigating the effect of the SOC of the sensors on the utility of the FC and the target localization mean square error (MSE).
6.1 Introduction

Auction models have been widely used in areas such as cognitive radios and sensor networks [105, 106], where selfish concerns of both the FC and the cognitive radios or sensors are addressed. In [99, 107], an optimal auction was designed for spectrum allocation among cognitive radios. The authors in [108] used a general auction-based negotiation model for the task allocation problem in mobile sensor networks. Distributed task allocation in resource limited WSNs has been investigated in [109] through an auction-based strategy. In Chapter 4 of this thesis, we designed optimal auction models for sensor management problems in wireless sensor networks, where the FC finds the optimal sensor management scheme by designing and conducting the optimal auction whose goal is to maximize the FC’s expected utility.

As shown in [110], the energy cost for the sensors to participate in any given task is different when their state of charge (SOC) is different. To the best of our knowledge, the SOC of the sensors has not been considered in auction based sensor management problems in the existing literature. In this chapter, we take the SOC of the sensors into consideration when designing the optimal auction model for the sensor selection problem in WSNs. Specifically, in our work, the FC’s task is to perform target localization by acquiring information about the target from a subset of sensors. The sensors compete to participate in the localization task for potential revenues through a bidding process. The problem of designing an optimal auction based sensor selection mechanism in such a scenario becomes challenging due to the following facts: 1) the resources of the WSN are limited, 2) the FC has incomplete information about the bids from the sensors, 3) the bids from the sensors contain both analog information about the sensors’ value estimates of unit energy cost and quantized information about the SOC, 4) sensors, being selfish in nature, may dishonestly provide their bids hoping for an extra profit. Thus, the FC decides on the winning bidder(s) and the payment it has to make by designing an optimal auction with probabilistic information being available about the sensors’ bids. In the optimal auction mechanism, the expected utility, which incorporates both the sensors’ value estimates of unit energy cost and their SOC, is maximized, and the constraints a) resource limitation, b) individual rationality (IR, to rationalize sensor participation), and c)
incentive-compatibility (IC, to ensure strategy-proofness) are satisfied. Simulation experiments show the effectiveness of our framework where we study the impact of the SOC of the sensors on the utility of the FC and mean square error (MSE) of target localization.

The rest of the chapter is organized as follows. In Section 6.2, we present the system model and our problem formulation. In Section 6.3, we analyze the incentive compatibility condition of the mechanism and the optimization problem. We perform simulation experiments in Section 6.4, and a summary is presented in Section 6.5.

6.2 Formulation of the Auction Design Problem

6.2.1 System Model

In this section, the system model, the corresponding Fisher information (FI) criterion, and Monte Carlo method for target localization are as presented in Section 2.3.

We employ an energy-efficient on-off keying scheme, where only the sensors that are selected by the FC need to sense the target power and transmit their quantized measurements to the FC. For mobile devices, it is natural to assume that the energy consumption when the SOC of the device is high is less than that when the SOC of the device is low [110].

6.2.2 Probabilistic Bid Information

The sensors compete to sell their measurements to the FC, and act as bidders or potential sellers in the sensor network. One problem that the FC has is due to the fact that it has no idea how much the sensors are willing to sell their information for. Each sensor determines the minimum amount it sells its information for based on its value estimate $v_i$ per unit of energy cost and its SOC $c_i$.

We assume that sensor $i$ quantizes its SOC to $\{0, 1, \cdots, C_i\}$, $i = 1, \cdots, N$, before sending it to the FC. However, we consider that the FC is unaware of the true value estimate $v_i$ and the SOC $c_i$, which gives an opportunity to the sensors to lie about these values, and assume that the FC’s
uncertainty about the value estimate and the SOC of sensor \(i\) can be described by some probability density functions. Specifically, let \(f^v_i(v_i) : [a_i, b_i] \rightarrow \mathbb{R}_+\) (positive real number field) represent the pdf for sensor \(i\)'s value estimate over a finite interval \([a_i, b_i]\), where \(-\infty \leq a_i \leq b_i \leq \infty\); and let \(f^c_i(c_i) : \{0, 1, \cdots, C_i\} \rightarrow \mathbb{R}_+\) represent the probability mass function (pmf) for sensor \(i\)’s SOC. We assume that the value estimates and the SOC are statistically independent across the sensors, and that each sensor \(i\) treats the other sensors’ value estimates in a similar way as the FC. Thus, both the FC and sensor \(i\) consider the joint pdf of the vector of the value estimates and SOC for all the sensors other than \(i\), i.e., \(v_{-i} = (v_1, \ldots, v_{i-1}, v_{i+1}, \ldots, v_N)\) and \(c_{-i} = (c_1, \ldots, c_{i-1}, c_{i+1}, \ldots, c_N)\), to be \(f^v_{v_{-i}}(v_{-i}) = \prod_{j \in \{1, \ldots, i-1, i+1, \ldots, N\}} f^v_j(v_j)\) and \(f^c_{c_{-i}}(c_{-i}) = \prod_{j \in \{1, \ldots, i-1, i+1, \ldots, N\}} f^c_j(c_j)\), respectively.

Further, we assume the FC to derive a benefit from performing the location estimation and consider that the value estimate of the FC per unit of information about the target is \(v_{FC}\).

### 6.2.3 Problem Formulation

Given the above definitions and assumptions, the FC’s problem is to design an auction mechanism to maximize its own expected utility. We consider a direct revelation mechanism [83], where the sensors simultaneously and confidentially announce their value estimates and SOC to the FC. The FC then determines from whom it should buy the data and how much it should pay to each sensor. Thus, our objective is to maximize the FC’s utility as a function of the user selection scheme variables and the payment vector. We assume that the FC and the users are risk neutral and have additively separable utility functions [89]. Then the expected utility of the FC from the auction mechanism is defined as

\[
U_{FC}(p, q) = \sum_{C} \int_{T} \left[ v_{FC} \operatorname{tr} \left( \sum_{i=1}^{N} q_i(v, c) J_i^P + J^P \right) - \sum_{i=1}^{N} p_i(v, c) \right] f^v(v) dv f^c(c) \tag{6.1}
\]

where, \(p = [p_1, \ldots, p_N]\) is the payment vector and \(p_i\) is the payment that the FC makes to sensor \(i\) which is a function of the vector of announced value estimates \(v = [v_1, \ldots, v_N]\) and SOC \(c = [c_1, \ldots, c_N]\); \(q = [q_1, \ldots, q_N]\) is a Boolean vector which represents the selection state of the
bidders, i.e., \( q_i = 1 \) when sensor \( i \) is selected and \( q_i = 0 \) when it is not; \( \text{tr} \left( \sum_{i=1}^{N} q_i(v, c)J_i^P + J^P \right) \) denotes the trace of the FI matrix obtained from the selected sensors [77, 78]; \( C \) and \( T \) denote the sets of all possible combinations of the sensors’ SOC and value estimates; \( f^v(v) \) is the pdf of the value estimate vector and \( dv = dv_1 \ldots dv_n \); and \( f^c(c) \) is the pmf of the SOC vector. Since sensor \( i \) knows that its value estimate is \( v_i \), and its SOC is \( c_i \), its expected utility \( U_i(p_i, q_i, v_i, c_i) \) is described as

\[
U_i(p_i, q_i, v_i, c_i) = \sum_{C_{-i}} \int_{T_{-i}} \left[ p_i(v, c) - v_i q_i(v, c) (E_i^T + E_i^S(c_i)) \right] f^v_i(v_{-i}) dv_{-i} f^c_i(c_{-i})
\]

(6.2)

where, \( C_{-i} \) and \( T_{-i} \) denote the sets of all possible combinations of the sensors’ SOC and value estimates other than sensor \( i \); \( dv_{-i} = dv_1 \ldots dv_{i-1} dv_{i+1} \ldots dv_n \); \( E_i^T \) is the energy cost for data transmission, and \( E_i^S \) is the sensing energy cost of sensor \( i \), which is a function of its SOC.

The FC is assumed to be unaware of the true value estimates and SOC of the sensors, so that the users have to advertise their value estimates and SOC to the FC, which give the sensors opportunities to lie for potentially an extra benefit. Sensor \( i \) claims that \( \tilde{v}_i \) is its value estimate while \( v_i \) is its true value estimate given its SOC, or claims its SOC is \( \tilde{c}_i \) when its true SOC is \( c_i \) given its value estimate \( v_i \), then its expected utility \( \tilde{U}_i \) would be

\[
\tilde{U}_i(p_i, q_i, \tilde{v}_i, \tilde{c}_i) = \sum_{C_{-i}} \int_{T_{-i}} \left[ p_i(\tilde{v}_i, v_{-i}, \tilde{c}_i, c_{-i}) \right. \\
- v_i q_i(\tilde{v}_i, v_{-i}, \tilde{c}_i, c_{-i}) (E_i^T + E_i^S(c_i)) \left. \right] f^v_i(v_{-i}) dv_{-i} f^c_i(c_{-i})
\]

(6.3)

where \( (\tilde{v}_i, v_{-i}) = (v_1, v_{i+1} \ldots v_N) \), and \( (\tilde{c}_i, c_{-i}) = (c_1, c_{i+1} \ldots c_N) \). The
auction mechanism based sensor selection problem can be explicitly formulated as follows:

$$\max_{\mathbf{q}} \quad \mathcal{U}_{FC}(\mathbf{p}, \mathbf{q})$$  \quad (6.3a)

s.t.  
$$\sum_{i=1}^{N} M q_i(v, c) \leq R \quad i \in \{1, \ldots, N\}, \quad \forall v \in T, c \in C$$  \quad (6.3b)

$$\mathcal{U}_i(p_i, q_i, v_i, c_i) \geq 0$$  \quad (6.3c)

$$\mathcal{U}_i(p_i, q_i, v_i, c_i) \geq \tilde{\mathcal{U}}_i(p_i, q_i, \tilde{v}_i, c_i)$$  \quad (6.3d)

$$\mathcal{U}_i(p_i, q_i, v_i, c_i) \geq \tilde{\mathcal{U}}_i(p_i, q_i, v_i, \tilde{c}_i)$$  \quad (6.3e)

$$i \in \{1, \ldots, N\}, \forall v_i, \tilde{v}_i \in [a_i, b_i], c_i, \tilde{c}_i \in \{0, 1, \cdots, C_i\}$$

Since each sensor transmit its measurement through a $M$-bit channel, and the total bandwidth for data transmission is $R$ bits, the first constraint (6.3b) guarantees that the FC can select no more than $R/M$ sensors; we call it the bandwidth limitation (BL) constraint. We assume that the FC cannot force a sensor to participate in an auction. If it did not participate in the auction, the sensor will not get paid, but also would not have any energy cost, so its utility would be zero. Thus, to guarantee that the sensors will participate in the auction, the \textit{individual-rationality} (IR) condition, which is shown in the second constraint (6.3c), must be satisfied. Finally, we assume that the FC can not prevent any sensor from lying about its value estimate and SOC if the sensor is expected to gain from lying. Thus, to prevent sensors from lying, honest responses must form a Nash equilibrium in the auction game. This constraint is addressed in in (6.3d) and (6.3e), which is called the \textit{incentive-compatibility} (IC) constraint. If the above constraints are all satisfied, we say that our auction mechanism is feasible.
6.3 Problem Analysis

In this section, we analyze the optimization problem proposed in Section 6.2.3. We define

$$Q_i(q_i, v_i, c_i) = \sum_{c_{-i}} \int_{T_{-i}} q_i(v, c) f_{-i}^v(v_{-i}) dv_{-i} f_{-i}^c(c_{-i})$$ (6.4)

$$P_i(p_i, v_i, c_i) = \sum_{c_{-i}} \int_{T_{-i}} p_i(v, c) f_{-i}^v(v_{-i}) dv_{-i} f_{-i}^c(c_{-i})$$ (6.5)

for sensor $i$ based on value estimate $v_i$ and SOC $c_i$. So $Q_i(q_i, v_i, c_i)$ denotes the expected probability that sensor $i$ is going to be selected by the FC conditioned on the value estimates and SOC of all the other sensors. Similarly, $P_i(p_i, v_i, c_i)$ denotes the expected payment from the FC to sensor $i$ conditioned on the value estimates and SOC of all the other sensors.

In our analysis, we assume the SOC of each sensor to be binary and characterized as “high” and “low” for simplicity, i.e., the SOC of each sensor is binary, $c_i \in \{0, 1\}$. Our first result is a simplified characterization of the IC constraint for the mechanism design problem.

**Lemma 6.3.1.** The IC constraint shown in (6.3d) with $c_i$ being binary holds if and only if the following conditions hold for $v_i \in [a_i, b_i]$:

1) $Q_i(q_i, v_i, c_i = 1) \geq Q_i(q_i, v_i, c_i = 0)$, \hspace{1cm} (6.6a)

2) $U_i(p_i, q_i, v_i, c_i = 1) = U_i(p_i, q_i, v_i, c_i = 0)$ \hspace{1cm} (6.6b)

$$+ v_i (E_{i}^{SH} - E_{i}^{SL}) \left[ \alpha_i Q_i(p_i, v_i, c_i = 0) 
+ (1 - \alpha_i) Q_i(q_i, v_i, c_i = 1) \right], \alpha_i \in [0, 1]$$

where $E_{i}^{SH}$ and $E_{i}^{SL}$ are the sensing energy costs of sensor $i$ when its charge is high and low, and $E_{i}^{SL} \geq E_{i}^{SH}$ [110]. In (6.6b), $\alpha_i$ is any number between 0 and 1, so that $(E_{i}^{SL} - E_{i}^{SH}) \left[ \alpha_i Q_i(q_i, v_i, c_i = 0) + (1 - \alpha_i) Q_i(q_i, v_i, c_i = 1) \right]$ represents any value between $(E_{i}^{SL} - E_{i}^{SH}) Q_i(q_i, v_i, c_i = 0)$ and $(E_{i}^{SL} - E_{i}^{SH}) Q_i(q_i, v_i, c_i = 1)$. Also, the condition in (6.3e) with $c_i$ being binary is equivalent to
the following conditions for \( \forall c_i \in \{0, 1\} \):

1) \( Q_i(p_i, q_i, v_i, c_i) \) is a non-increasing function of \( v_i \) \hfill (6.7a)

2) \( U_i(p_i, q_i, v_i, c_i) = U_i(p_i, q_i, b_i, c_i) \) \hfill (6.7b)

\[ + \int_{v_i}^{b_i} Q_i(q_i, s_i, c_i)[E_i^T + c_i E_i^{SH} + (1 - c_i) E_i^{SL}]ds_i \]

Proof. For any given \( v_i \), when \( c_i = 1 \) and \( \bar{c}_i = 0 \), using (6.3e), we have,

\[ U_i(p_i, q_i, v_i, c_i = 1) = P_i(p_i, v_i, c_i = 1) - v_i Q_i(q_i, v_i, c_i = 1)(E_i^T + E_i^{SH}) \]
\[ \geq P_i(p_i, v_i, c_i = 0) - v_i Q_i(q_i, v_i, c_i = 0)(E_i^T + E_i^{SH}) \]
\[ = U_i(p_i, q_i, v_i, c_i = 0) + v_i Q_i(q_i, v_i, c_i = 0)(E_i^{SL} - E_i^{SH}) \] \hfill (6.8)

Since \( E_i^{SL} \geq E_i^{SH} \), \( U_i(p_i, q_i, v_i, c_i = 1) \geq U_i(p_i, q_i, v_i, c_i = 0) \). On the other hand, when \( c_i = 0 \) and \( \bar{c}_i = 1 \), again using (6.3e), we have,

\[ U_i(p_i, q_i, v_i, c_i = 0) = P_i(p_i, v_i, c_i = 0) - v_i Q_i(q_i, v_i, c_i = 0)(E_i^T + E_i^{SL}) \]
\[ \geq P_i(p_i, v_i, c_i = 1) - v_i Q_i(q_i, v_i, c_i = 1)(E_i^T + E_i^{SL}) \]
\[ = U_i(p_i, q_i, v_i, c_i = 1) - v_i Q_i(q_i, v_i, c_i = 1)(E_i^{SL} - E_i^{SH}) \] \hfill (6.9)
Thus, we have

\begin{align}
Q_i(q_i, v_i, c_i = 1) &\geq Q_i(q_i, v_i, c_i = 0) \\
v_iQ_i(q_i, v_i, c_i = 0)(E_i^{SL} - E_i^{SH}) &
\leq U_i(p_i, q_i, v_i, c_i = 1) - U_i(p_i, q_i, v_i, c_i = 0) \\
v_i(E_i^T + E_i^{SH})(Q_i(q_i, v_i, c_i = 1) - Q_i(q_i, v_i, c_i = 0)) &
\leq P_i(p_i, v_i, c_i = 1) - P_i(p_i, v_i, c_i = 0) \\
v_i(E_i^T + E_i^{SL})(Q_i(q_i, v_i, c_i = 1) - Q_i(q_i, v_i, c_i = 0)) &
\leq P_i(p_i, v_i, c_i = 1) - P_i(p_i, v_i, c_i = 0)
\end{align}

(6.10b) implies that for any given \(v_i \in [a_i, b_i]\),

\begin{align}
U_i(p_i, q_i, v_i, c_i = 1) &\geq U_i(p_i, q_i, v_i, c_i = 0) + v_i(E_i^{SL} - E_i^{SH}) \\
&\geq \left[ \alpha_i Q_i(q_i, v_i, c_i = 0) + (1 - \alpha_i)Q_i(q_i, v_i, c_i = 1) \right]
\end{align}

(6.11)

where \(\alpha_i\) can be any number between 0 and 1, so that \((E_i^{SL} - E_i^{SH})\left[ \alpha_i Q_i(q_i, v_i, c_i = 0) + (1 - \alpha_i)Q_i(q_i, v_i, c_i = 1) \right]\) represents any value between \((E_i^{SL} - E_i^{SH})Q_i(q_i, v_i, c_i = 0)\) and \((E_i^{SL} - E_i^{SH})Q_i(q_i, v_i, c_i = 1)\). Similarly, (6.10c) implies the following payment function:

\begin{align}
P_i(p_i, v_i, c_i = 1) &\geq P_i(p_i, v_i, c_i = 0) + \left[ Q_i(q_i, v_i, c_i = 1) \\
&- Q_i(q_i, v_i, c_i = 0) \right] \left[ v_iE_i^T + v_i[\alpha_iE_i^{SH} + (1 - \alpha_i)E_i^{SL}] \right]
\end{align}

(6.12)

Also, for any given \(c_i\), we can show in a similar manner [111] that \(Q_i(q_i, v_i, c_i)\) is a decreasing function of \(v_i\), and

\begin{align}
U_i(p_i, q_i, v_i, c_i) &\geq U_i(p_i, q_i, b_i, c_i) + \int_{v_i}^{b_i} Q_i(q_i, s_i, c_i)[E_i^T + c_iE_i^{SH} + (1 - c_i)E_i^{SL}]ds_i
\end{align}

(6.13)
In Lemma 6.3.1, we obtained equivalent conditions for the IC constraint, which can be used for further analysis of our optimization problem in (6.3).

**Theorem 6.3.1.** The optimal auction of (6.3) with \( c_i \) being binary is equivalent to maximizing

\[
\sum_C \int_T \left\{ \sum_{i=1}^N u_i(v_i, c_i) q_i(v, c) \right\} f^w(v) d
\]

where,

\[
\begin{align*}
\left\{ \begin{array}{l}
u_i(v_i, c_i = 0) = v_{FC} \text{tr}(J_i^D) + v_i(E_i^T + E_i^{SL}) - \frac{(E_i^T + E_i^{SL}) f_i^w(v_i)}{f_i^w(c_i = 0)} \\
\quad - \alpha_i v_i (E_i^{SL} - E_i^{SH}) \frac{f_i^c(c_i = 1)}{f_i^c(c_i = 0)} \\
\quad \quad - \alpha_i v_i (E_i^{SL} - E_i^{SH}) \frac{f_i^c(c_i = 1)}{f_i^c(c_i = 0)} \\
\nu_i(v_i, c_i = 1) = v_{FC} \text{tr}(J_i^D) + v_i(E_i^T + E_i^{SH}) - (1 - \alpha_i) v_i (E_i^{SL} - E_i^{SH})
\end{array} \right.
\] (6.15)

subject to the constraints (6.3b), (6.6a) and (6.7a). In (6.14) and (6.15), \( u_i(v_i, c_i) \) is defined as "virtual valuation" of each sensor. The payment to sensor \( i \) is given by

\[
p_i(v, c_{-i}, c_i) = (E_i^T + E_i^{SL}) v_i q_i(v, c_{-i}, c_i = 0) - \int_{s_i}^{b_i} q_i(v_{-i}, s_i, c_{-i}, c_i = 0) ds_i
\]

\[
p_i(v, c_{-i}, c_i = 1) = (E_i^T + E_i^{SL}) v_i q_i(v, c_{-i}, c_i = 1) + v_i \alpha_i (E_i^{SL} - E_i^{SH}) q_i(v, c_{-i}, c_i = 0) - (E_i^T + E_i^{SL}) \int_{s_i}^{b_i} q_i(v_{-i}, s_i, c_{-i}, c_i = 0) ds_i
\] (6.16)

\[\text{1}^\text{We call it "virtual valuation" because the FC performs optimal auction based on } u_i(v_i, c_i) \text{ which is function of the "value estimate" } v_i \text{ and SOC } c_i.\]
Proof. We first relate the utilities of the FC and the sensors,

\[ U_{FC}(p, q) = \sum_C \int_T \left[ v_{FC} \text{tr} \left( \sum_{i=1}^{N} q_i(v, c)J_i^D + J^P \right) \right. \]

\[ + \sum_{i=1}^{N} v_i q_i(v, c) \left( E_i^T + c_i E_i^{SH} + (1 - c_i) E^{NC} \right) \int f^v(v)dv f^c(c) \]

\[ - \left[ \sum_{i=1}^{N} \sum_C \int_T \left[ p_i(v, c) - v_i q_i(v, c) \left( E_i^T + c_i E_i^{SH} + (1 - c_i) E^{NC} \right) \right] f^v(v)dv f^c(c) \right] \]  

(6.17)

where

\[ \sum_C \int_T \left[ p_i(v, c) - v_i q_i(v, c) \left( E_i^T + c_i E_i^{SH} + (1 - c_i) E^{NC} \right) \right] f^v(v)dv f^c(c) \]

\[ = \sum_{c_i} \int_{a_i}^{b_i} U_i(p_i, q_i, v_i, c_i) f^v_i(v_i)dv_i f^c_i(c_i) \]

\[ = \int_{a_i}^{b_i} \left[ U_i(p_i, q_i, v_i, c_i = 1) f^c_i(c_i = 1) + U_i(p_i, q_i, v_i, c_i = 0) f^c_i(c_i = 0) \right] f^v_i(v_i)dv_i \]  

(6.18)

We substitute (6.11) about the relationship between \( U_i(p_i, q_i, v_i, c_i = 1) \) and \( U_i(p_i, q_i, v_i, c_i = 0) \) into equation (6.18), so that (6.18) is further equivalent to

\[ \int_{a_i}^{b_i} \left[ U_i(p_i, q_i, v_i, c_i = 0) + v_i (E_i^{SL} - E_i^{SH}) \left[ \alpha_i Q_i(q_i, v_i, c_i = 0) \right. \right. \]

\[ + (1 - \alpha_i) Q_i(q_i, v_i, c_i = 1) \left. \right] f^c_i(c_i = 1) \int f^v_i(v_i)dv_i \]

\[ = U_i(p_i, q_i, b_i, c_i = 0) + \int_{a_i}^{b_i} \int_{a_i}^{b_i} Q_i(q_i, s_i, c_i = 0) (E_i^T + E_i^{SL}) ds_i f^v_i(v_i)dv_i \]

\[ + v_i (E_i^{SL} - E_i^{SH}) \int_{a_i}^{b_i} \left[ \alpha_i Q_i(q_i, v_i, c_i = 0) + (1 - \alpha_i) Q_i(q_i, v_i, c_i = 1) \right] f^c_i(c_i = 1) f^v_i(v_i)dv_i \]
\[ \begin{align*}
= \mathcal{U}_i(p_i, q_i, b_i, c_i = 0) + \int_{a_i}^{b_i} (E_i^T + E_i^{SL}) F_i(s_i) Q_i(q_i, s_i, c_i = 0) ds_i \\
+ v_i(E_i^{SL} - E_i^{SH}) \int_{a_i}^{b_i} \left[ \alpha_i Q_i(q_i, v_i, c_i = 0) + (1 - \alpha_i) Q_i(q_i, v_i, c_i = 1) \right] f^c_i(c_i = 1) f_i^p(v_i) dv_i \\
= \mathcal{U}_i(p_i, q_i, b_i, c_i = 0) + \int_{a_i}^{b_i} \left[ (E_i^T + E_i^{SL}) f_i^p(v_i) \\
+ \alpha_i v_i(E_i^{SL} - E_i^{SH}) f_i^c(c_i = 1) f_i^p(v_i) \right] Q_i(q_i, v_i, c_i = 0) \\
+ (1 - \alpha_i) v_i(E_i^{SL} - E_i^{SH}) f_i^c(c_i = 1) f_i^p(v_i) Q_i(q_i, v_i, c_i = 1) \right] dv_i \\
\end{align*} \]

(6.19)

Thus,

\[ \mathcal{U}_{FC}(p, q) \]

\[ \begin{align*}
= \sum_C \int_T \left[ v_{FC} \text{ tr} \left( \sum_{i=1}^{N} q_i(v, c) \mathbf{J}_i^D + \mathbf{J}_i^p \right) \\
+ v_i(E_i^T + c_i E_i^{SH} + (1 - c_i) E_i^{NC}) \right] f^e(v) dv f^c(c) \\
- \sum_{i=1}^{N} \sum_C \int_T \left[ \left( (E_i^T + E_i^{SL}) F_i^p(v_i) \right) + \alpha_i v_i(E_i^{SL} - E_i^{SH}) \frac{f_i^c(c_i = 1)}{f_i^c(c_i = 0)} \right] \mathbb{1}\{c_i = 0\} \\
+ (1 - \alpha_i) v_i(E_i^{SL} - E_i^{SH}) \mathbb{1}\{c_i = 1\} q_i(v, c) f^e(v) dv f^c(c) - \sum_{i=1}^{N} \mathcal{U}_i(p_i, q_i, b_i, c_i = 0) \\
= \sum_{i=1}^{N} \sum_C \int_T \left[ v_{FC} \text{ tr}(\mathbf{J}_i^D) + v_i(E_i^T + c_i E_i^{SH} + (1 - c_i) E_i^{NC}) \right. \\
\left. - \left[ (E_i^T + E_i^{SL}) F_i^p(v_i) \right] \frac{f_i^c(c_i = 1)}{f_i^c(c_i = 0)} \mathbb{1}\{c_i = 0\} \right] \\
- (1 - \alpha_i) v_i(E_i^{SL} - E_i^{SH}) \mathbb{1}\{c_i = 1\} q_i(v, c) f^e(v) dv f^c(c) \\
-v_{FC} \text{ tr}(\mathbf{J}_i^p) - \sum_{i=1}^{N} \mathcal{U}_i(p_i, q_i, b_i, c_i = 0) \\
\end{align*} \]

(6.20)

Since the IR condition requires \( \mathcal{U}_i(p_i, q_i, b_i, c_i = 0) \geq 0 \), we must have \( \mathcal{U}_i(p_i, q_i, b_i, c_i = 0) = 0 \), in
order to maximize the utility of the FC in (6.20). Recall (6.11),

$$\mathcal{U}_i(p_i, q_i, v_i, c_i = 0) = \mathcal{U}_i(p_i, q_i, v_i, c_i = 0) - \int_{v_i}^{b_i} Q_i(q_i, s_i, c_i = 0)(E^T_i + E^{SL}_i)ds_i$$

$$= P_i(p_i, v_i, c_i = 0) - v_iQ_i(q_i, v_i, c_i = 0)(E^T_i + E^{SL}_i) - \int_{v_i}^{b_i} Q_i(q_i, s_i, c_i = 0)(E^T_i + E^{SL}_i)ds_i$$

(6.21)

So that

$$P_i(p_i, v_i, c_i = 0) = (E^T_i + E^{SL}_i)\left[v_iQ_i(q_i, v_i, c_i = 0) - \int_{v_i}^{b_i} Q_i(q_i, s_i, c_i = 0)ds_i\right]$$

(6.22)

With the relationship between $P_i(p_i, v_i, c_i = 0)$ and $P_i(p_i, v_i, c_i = 1)$ in (6.12), we have

$$P_i(p_i, v_i, c_i = 1) = [v_iE^T_i + v_i(\alpha_iE^{SH}_i + (1 - \alpha_i)E^{SL}_i)]Q_i(q_i, v_i, c_i = 1)$$

$$+ v_i\alpha_i(E^{SL}_i - E^{SH}_i)Q_i(q_i, v_i, c_i = 0) - (E^T_i + E^{SL}_i)\int_{v_i}^{b_i} Q_i(q_i, s_i, c_i = 0)ds_i$$

(6.23)

Thus, we have proved the theorem.

Lemma 6.3.1 and Theorem 6.3.1 present the optimal auction mechanism for the FC to select the optimal sensor(s) for the task of target localization. The sensors send their bids, which include their value estimates per unit energy $v_i$ and their SOC $c_i$, $i \in \{1, \cdots, N\}$, to the FC and compete for participation opportunities for localization. Then the FC solves the optimization problem given in Theorem 6.3.1 based on the bids from the sensors and decides the winning sensor(s) and the corresponding payment.

### 6.4 Simulation Experiments

In this section, we study the performance of our proposed framework through some simulation experiments. In order to implement our auction mechanism, we first find the following sufficient
conditions for (6.6a) and (6.7a) by investigating the virtual valuation of each sensor in (6.14) and (6.15):

\[ u_i(v_i, c_i = 1) \geq u_i(v_i, c_i = 0), \quad \forall v_i \in [a_i, b_i] \quad \text{(6.24a)} \]

\[ u_i(v_i, c_i) \text{ is a non-increasing function of } v_i, \quad \forall c_i \in \{0, 1\} \quad \text{(6.24b)} \]

With the assumption that \( v_i \) is uniformly distributed in \([a_i, b_i]\), the derivative of \( u_i(v_i, c_i = 0) \) is

\[
(E_i^T + E_i^{SL})(1 - \frac{1}{f_i^c(c_i = 0)}[f_i^v(v_i)]^2) - \alpha_i \frac{f_i^c(c_i = 1)}{f_i^c(c_i = 0)} (E_i^{SL} - E_i^{SH}) \\
= (E_i^T + E_i^{SL})(1 - \frac{1}{f_i^c(c_i = 0)}) - \alpha_i \frac{f_i^c(c_i = 1)}{f_i^c(c_i = 0)} (E_i^{SL} - E_i^{SH}) \quad \text{(6.25)}
\]

Since \( 1 - \frac{1}{f_i^c(c_i = 0)} \leq 0 \) and \( E_i^{SL} - E_i^{SH} \geq 0 \), (6.25) is non-positive. So that \( u_i(v_i, c_i = 0) \) is a a non-increasing function of \( v_i \). Also, it is straightforward to show that the condition \( \alpha_i \frac{f_i^c(c_i = 1)}{f_i^c(c_i = 0)} \geq 1 - \alpha_i \)
implies (6.24a). Thus, if
\[
\alpha_i f^c_i(c_i = 1) \geq 1 - \alpha_i \quad (6.26a)
\]

\[
E^T_i + E^{SH}_i - (1 - \alpha_i)(E^{SL}_i - E^{SH}_i) \leq 0 \quad (6.26b)
\]

then (6.6a) and (6.7a) are satisfied. As defined in Lemma 6.3.1, \(\alpha_i\) can be any value between 0 and 1, so that we assume that \(\alpha_i = 0.99\) for \(i \in \{1, \cdots, N\}\) in our experiments. We consider that the energy consumption at sensor \(i\) for transmitting \(M_i\) bits over distance \(d^F_i\) (\(d^F_i\) is the distance between sensor \(i\) and the FC) is modeled as [88]

\[
E^T_i = E_{\text{elec}} \times M_i + \epsilon_{\text{amp}} \times M_i \times d^F_i^2 \quad (6.27)
\]

where \(E_{\text{elec}} = 0.05\), \(\epsilon_{\text{amp}} = 1 \times 10^{-4}\). For the sensing energy cost, we simply assume that \(E^{SL}_i = 40\) and \(E^{SH}_i = 0.05\). Under our assumptions, (6.26b) is satisfied, and in the following experiments, we always choose \(f^c_i(c_i = 1)\) (it is straightforward to see that \(f^c_i(c_i = 0) = 1 - f^c_i(c_i = 1)\)) to satisfy (6.26a). Thus, the FC decides the winning bidder by maximizing (6.14) subject to constraint (6.3b). In other words, the FC simply chooses the sensor(s) with maximal virtual valuation \(u_i\) in (6.15) as the winning bidder(s) and pays the winner(s) according to (6.16).

In our simulation experiments, the size of the ROI is \(20m \times 20m\), the FC is located at the center of the ROI, and the signal power at distance zero is \(P_0 = 1000\). Sensors in the ROI quantize their measurements to \(M_i = 3\) bits. The quantization thresholds are designed as in [48]. We also assume that the prior pdf of the target location \(x\) is \(N(\mu_0, \Sigma_0)\) with \(\mu_0 = [1.25, 1.25]^T\) and \(\Sigma_0 = \text{diag}[2^2, 2^2]\). The pdf of the value estimate of sensor \(i\), \(v_i\), is assumed to be uniformly distributed between \(a_i\) and \(b_i\) with \(a_i = 0.1\) and \(b_i = 0.6\). The performance of the location estimator is determined in terms of the MSE via 500 simulation runs.

We first consider a WSN with \(N = 25\) sensors in the ROI as shown in Fig. 6.1, where the sensor indices are shown above each sensor. Since research on the effect of sensors’ value estimates per
unit energy has been investigated in Chapter 4, we focus only on the study of sensors’ SOC in this chapter. We consider three different scenarios: 1) all the 25 sensors in the ROI have large probabilities that their SOC are “high”, i.e., \( f_{c_i}(c_i = 1) = 0.95 \) for \( i \in \{1, \cdots, N\} \). 2) all the 25 sensors have large probabilities that their SOC are “low”, i.e., \( f_{c_i}(c_i = 0) = 0.95 \) (\( f_{c_i}(c_i = 1) = 0.05 \)) for \( i \in \{1, \cdots, N\} \), and 3) a fraction of the sensors have large probabilities that their SOC are “high”. Note that, in case 3), we assume that sensors 12-15 and sensors 17-20 have \( f_{c_i}(c_i = 0) = 0.95 \), and the other sensors have \( f_{c_i}(c_i = 1) = 0.95 \), i.e., sensor 12-15 and sensor 17-20 have large probabilities that their SOC are “low”, and all the other sensors have large probabilities that their SOC are “high”. Sensors 12-15 and sensors 17-20 are relatively close to the expected location of the target, and have larger contribution to the FI gain compared with the other sensors. However, the sensing energy costs are much higher when the sensors’ SOC are “low” than that when their SOC are “high” since we have assumed that \( E_{SL_i}^S = 40 \) and \( E_{SH_i}^S = 0.05 \). Thus, we expect more interesting results on the trade-off between information gain and energy cost by considering the above three scenarios.

In Fig. 6.2, we show the utility of the FC and the total payment made by the FC to all the selected sensors when the number of selected sensors increases, and the value estimate per unit information of the FC is \( v_{FC} = 100 \). When all the sensors have \( f_{c_i}(c_i = 1) = 0.95 \) or all the sensors have \( f_{c_i}(c_i = 0) = 0.95 \), the sensors with relatively high contribution on the FI gain are selected by the FC. Since the energy costs are higher when the sensors’ SOC are “low”, the FC has to pay more to the winning sensors (as shown in Fig. 6.2(b)), and the utility of the FC is lower accordingly when all the sensors have \( f_{c_i}(c_i = 1) = 0.95 \) compared to the case when all the sensors have \( f_{c_i}(c_i = 0) = 0.95 \) (as shown in Fig. 6.2(a)). As mentioned, sensors 12-15 and sensors 17-20 contribute more on the FI gain. However, their energy cost is much higher if their SOC are “low”. When few sensors can be selected because of the bandwidth constraint, the FC prefers more informative sensors. However, when the FC is relatively rich in bandwidth, those sensors which are relatively far away and have “high” SOC (thus have low energy cost) are also included in the selected sensor set. Thus, the utility of the FC for the case that a fraction of sensors
have $f_i^c(c_i = 1) = 0.95$ is similar to that when all the sensors have $f_i^c(c_i = 0) = 0.95$ if the FC has low bandwidth limitation, and is between the case when all the sensors have $f_i^c(c_i = 1) = 0.95$ and the case when all the sensors have $f_i^c(c_i = 0) = 0.95$ if the FC is relatively rich in bandwidth. Similar performance can be observed for the total payment to the selected sensors.

In Fig. 6.3, we present the MSE of the target localization task. Fig. 6.3(a) corresponds to the scenario that the FC has a large gain in utility for every unit FI, i.e., the value estimate per unit information of the FC is $v_{FC} = 100$, and Fig. 6.3(b) shows the MSE for $v_{FC} = 1$. The FC prefers more informative sensors when it has relatively large $v_{FC}$. So that, with $v_{FC} = 100$, the selected sensor set when all the sensors have $f_i^c(c_i = 1) = 0.95$ is similar to that when all the sensors have $f_i^c(c_i = 0) = 0.95$. For the scenario where a fraction of the sensors have $f_i^c(c_i = 1) = 0.95$, those few sensors which are most informative are still selected by the FC. In other words, the MSE for the three cases considered are all similar when $v_{FC}$ is large. This is because $v_{FC}$ is large so that the FC can always afford to select the sensors that are more informative without regarding to their SOC. However, when the FC has a relatively small $v_{FC} = 1$, as shown in Fig. 6.3(b), the FC gains less for every unit FI compared with Fig. 6.3(a), and prefers the sensors with relatively low energy cost. So that the FC always prefers those sensors that have $f_i^c(c_i = 1) = 0.95$ or the sensors which are relatively far away from the FC (they have lower sensing energy cost under our assumption). Therefore, the case when all the sensors have $f_i^c(c_i = 1) = 0.95$ provides the best MSE performance, and the case when all the sensors have $f_i^c(c_i = 0) = 0.95$ provides the worst MSE performance.

We then study the performance of our framework while varying the total number of sensors in the ROI in Fig. 6.4, and 4 sensors are selected by the FC in each case. The scenario where a fraction of the sensors have $f_i^c(c_i = 1) = 0.95$ are not considered in here since different densities of the WSNs are investigated. We observe that the utility of the FC increases as the total number of sensors in the ROI increases. The reason is that as the WSN becomes denser, the chance for the FC to get more FI gain with the similar amount of energy cost increases. Also, in Fig. 6.4, we observe similar insights with Fig. 6.2(a) that the utility of the FC when all the sensors have
\[ f^c_i(c_i = 1) = 0.95 \text{ is greater than that when all the sensors have } f^c_i(c_i = 0) = 0.95. \]

### 6.5 Summary

In this chapter, we have investigated the target localization problem in crowdsourcing based wireless sensor networks (WSNs). Due to limited bandwidth, the fusion center (FC) selects a set of sensors to acquire information by designing an optimal auction mechanism. The sensors compete to sell their measurements about the target to the FC by sending out bids which are based on the energy efficiency and state of charge (SOC) of the sensors. The FC selects the winning sensor(s) and make payment to the winner(s) based on the strategy given by the optimal auction mechanism. The results in the simulation experiments show the dynamics of the proposed mechanism and its efficiency. Future work will consider bandwidth allocation problem for target tracking when rechargeable sensors serve in crowdsourcing based WSNs.
Fig. 6.2: Performance with relatively large $v_{FC}$: (a) utility of the FC, (b) total payment made by the FC to the selected sensors.
Fig. 6.3: MSE of localization: (a) $v_{FC} = 100$, (b) $v_{FC} = 1$. 
Fig. 6.4: Utility of the FC as the total number of sensors in the ROI increases.
In this chapter, we introduce “cloud sensing” as a paradigm for enabling sensing-as-a-service in the context of target localization in wireless sensor networks (WSNs). We present a bilateral trading mechanism consisting of a sensing service provider (fusion center) that “sells” information regarding the target through sensor management, and a user who seeks to “buy” information regarding the target. Our mechanism, aware of resource costs involved in service provisioning, maximizes the expected total gain from the trade while assuring individual rationality and incentive compatibility. The impossibility of achieving ex post efficiency is also shown in the chapter. Design of the mechanism enables the study of the tradeoff between information gain and the costs of the WSN for sensor management. Simulation results provide insights into the dynamics of the proposed model.

7.1 Introduction

Wireless sensor networks (WSNs) are composed of a large number of densely deployed sensors. When programmed and networked properly, WSNs are very useful in many application areas in-
cluding battlefield surveillance, environment monitoring, industrial process or health monitoring and control. In this chapter, we assume that the task of the WSN is to locate a target in a given region of interest (ROI). There exists a central node called the fusion center (FC), which is responsible for the final inference, acquires measurements from the sensors in the WSN. As the novel aspect of this chapter, we consider that the FC “sells” the inferred information from the WSN as a service. Users, on the other hand, “buy” the service from the FC, for example, to find their lost item using the provided sensing service.

We refer to our model as Cloud Sensing, which is illustrated in Fig. 8.1. In the cloud sensing framework, sensing is delivered as a service through cloud computing. The model of sensing as a service was introduced as a new concept in [112], where various sensing services are provided using mobile phones for a large number of cloud users. Some general requirements for the model are described in the chapter. In [113], sensing as a service was applied for smart cities supported by the “internet of things”. However, [112] and [113] only talk about the general properties of the model. In this chapter, we focus on the specific formulation of the cloud sensing framework.

We model the cloud sensing process as a bilateral trading problem, where we face the following fundamental challenges:

- how to define the utilities of the user and the FC,
- under what condition shall the FC sell the inferred information to the user,
- at what price shall the FC sell the information to the user,
- due the limited resources, the tradeoff between the information gain and the costs of the WSN has to be considered.

Thus, we design an optimal mechanism that includes consideration of the above issues.

For the mechanism design problem, it is important to focus on the optimality of the mechanism while satisfying the constraints such as individual-rationality (IR) that rationalizes user participation, incentive-compatibility (IC) that ensures honest reporting, and efficiency [81–83, 114, 115].
Vickrey [116], D’Aspremont and Gerard-Varet [117] and Chatterjee [118] studied different conditions while designing mechanisms for bilateral trading. By using some techniques similar to those in [83], Myerson and Satterthwaite characterized a more general set of allocation mechanisms that are incentive compatible and individually rational, and studied the possibility of ex post efficiency. Ex post efficiency is defined after the agents give their bids, the mechanism is ex post efficient if the buyer gets the service whenever its total valuation is higher. Also, in [104], the expected total gain from the trade is maximized while computing the mechanism.

In this chapter, we design the mechanism for our cloud sensing paradigm where the sensing service is traded between the user and the FC in order to estimate the location of the target in the WSN. Formally, the key contributions of the chapter are as follows.

- By properly defining the utilities of the FC and the user regarding the information gain and the resource costs of the WSN, we study the IR and IC properties of the mechanism.

- Aware of the tradeoff between the information gain and the resource costs of the WSN, we prove that a mechanism that satisfies IR and IC constraints cannot be ex post efficient, in the sense that the FC does not provide the service to the user whenever its valuation for the resource costs is higher than the user’s valuation for the information gain.

- We design the optimal mechanism to maximize the expected total gains from the trade, subject to the IR and IC constraints.
• Through the optimal mechanism, we derive the trading condition between the FC and the user and the payment the user has to pay for the service.

• The optimal mechanism provides the metric for the FC to select the optimal subset of sensors such that the best tradeoff between the information gain and the costs of the WSN is obtained.

The rest of the chapter is organized as follows. In Section 7.2, we introduce the basic assumptions and formulate the problem. In Section 7.3, we analyze the IR, IC, and the ex post efficiency properties of the mechanism, showing the impossibility condition for the mechanism to satisfy these three conditions simultaneously. We design the optimal mechanism that maximizes the expected total gains from trade in Section 7.4. A specific WSN is considered and the corresponding simulation results are presented in Section 7.5. We provide some concluding remarks in Section 7.6.

7.2 Problem Formulation

7.2.1 Basic Assumptions

We consider a WSN consisting of $N$ sensors in the ROI. The FC acquires the measurements from a subset of sensors for information gain, based on which the location of a target (e.g., the lost item of a user) will be determined. We denote the selection state of each sensor by a Boolean vector $\alpha = [\alpha_1, \alpha_2, \ldots, \alpha_N]$. The information gain from the subset of sensors and the corresponding cost are dependent on the selection state of the sensors, which are denoted as $G(\alpha)$ and $C(\alpha)$ respectively. We consider that the FC and the user have private knowledge about their valuations $v_f$ and $v_c$, where $v_f$ (FC’s valuation per unit cost) represents how much the FC values the resource costs for providing the service, and $v_c$ (user’s valuation per unit gain) represents how much the user values the information gain. We assume that $v_f$ and $v_c$ are independent random variables, and each of them can be described by a probability density function (pdf) over a finite interval, $v_f \sim f_f : [a_f, b_f] \to \mathbb{R}_+$ and $v_c \sim f_c : [a_c, b_c] \to \mathbb{R}_+$, where $a_f$ and $a_c$ are the lowest valuations
of the FC and the user respectively, and $b_f$ and $b_c$ are the respective highest valuations. Note that the pdfs of $v_f$ and $v_c$ are assumed to be common knowledge for the participating individuals. Let $F_f$ and $F_c$ be the cumulative distribution functions (cdfs) of the valuations $v_f$ and $v_c$.

### 7.2.2 Motivation for Incentive Compatibility Constraint

The bilateral trading between the FC and the user defines a Bayesian game [119]. To ensure that the bidders would not revise their strategies\(^1\), it is important to find the Nash equilibrium of the game. We focus on the direct mechanism [83], where the bidders directly report their valuations about the item under trading instead of reporting their strategy plans. Thus, we use the terms “valuation” and “bid” interchangeably in the rest of the chapter.

A direct mechanism is incentive compatible if honest reporting forms a Bayesian Nash equilibrium [104]. We restrict our attention to the incentive compatible mechanism because of a key result in economics, the “revelation principle” [120].

**Proposition (Revelation Principle) 7.2.1.** Given a mechanism and an equilibrium of that mechanism, there exists a direct mechanism, in which

- the outcomes are the same as in the equilibrium of the original mechanism,
- it is a Bayesian Nash equilibrium for each bidder to report its valuation truthfully.

**Remark:** By construction, each individual reports its truthful valuation and makes sure the outcome is the same as if they had submitted their strategies in the original game. If the individuals deviate from the truthful valuation, then the corresponding strategy would not be in equilibrium of the original game.

Thus, in this chapter, we restrict our attention to the direct mechanism where the individuals report their private valuations truthfully.

---

\(^1\)In game theory, a strategy refers to the rules that a player uses to choose between the available actionable options.
7.2.3 Expected Utility Functions

The FC and the user have private information about their own valuations ($v_f$ and $v_c$), and they treat the other’s valuations to be random. The outcome of the direct mechanism is represented by two functions: $p_v(v_f, v_c)$ and $x_v(v_f, v_c)$, where $p_v$ is the probability of the FC acquiring information from the WSN and providing the service to the user, and $x_v$ is the payment that the user has to make to the FC for the service. We assume that the individuals in our work are risk neutral\(^2\) so that the expected values of the gains are considered. Given the pdfs of the valuations and the selection state of each sensor ($\alpha$ is given), we define the expected utilities of both the FC and the user as follows. For the FC,

$$U_f(v_f) = \bar{x}_f(v_f) - \bar{p}_f(v_f) [v_f C(\alpha)]$$ \hspace{1cm} (7.1)

where, $\bar{x}_f$ is the expected payment that the FC gets from the user, and $\bar{p}_f$ is the expected probability that the FC sells the service to the user,

$$\bar{x}_f(v_f) = \int_{v_c} x_v(v_f, v_c) f_c(v_c) dv_c$$ \hspace{1cm} (7.2)

$$\bar{p}_f(v_f) = \int_{v_c} p_v(v_f, v_c) f_c(v_c) dv_c.$$ \hspace{1cm} (7.3)

Similarly, the expected utility of the user given its valuation $v_c$ and the vector $\alpha$ is,

$$U_c(v_c) = \bar{p}_c(v_c) [v_c G(\alpha)] - \bar{x}_c(v_c)$$ \hspace{1cm} (7.4)

where, $\bar{x}_c$ is the expected payment that the user has to pay to the FC, and $\bar{p}_c$ is the expected probability that the user gets the service from the FC,

$$\bar{x}_c(v_c) = \int_{v_f} x_v(v_f, v_c) f_f(v_f) dv_f$$ \hspace{1cm} (7.5)

\(^2\)The risk neutral agents only care about the expected value of their profit, even if it is risky.
\[
\bar{p}_c(v_c) = \int_{v_f} p_v(v_f, v_c) f_f(v_f) dv_f. \tag{7.6}
\]

Thus, the expected total gain of the FC and the user from the trade is:

\[
E[U_{Total}] = \int_{a_f}^{b_f} U_f(v_f) f_f(v_f) dv_f + \int_{a_c}^{b_c} U_c(v_c) f_c(v_c) dv_c \tag{7.7}
\]

\[
= \int_{a_c}^{b_c} \int_{a_f}^{b_f} [v_c G(\alpha) - v_f C(\alpha)] p_v(v_f, v_c) f_f(v_f) f_c(v_c) dv_f dv_c.
\]

7.3 Constraints of Individual Rationality, Incentive Compatibility, and Ex Post Efficiency

Having the utility functions of the FC and the user, we define the properties of the mechanism in this section.

7.3.1 Individual Rationality and Incentive Compatibility

To guarantee that each individual is willing to participate in the mechanism, the IR constraint needs to ensure that each individual obtains a non-negative expected gain from the trade in the mechanism, regardless of its valuation [104]. Thus, based on the expected utility functions, the IR constraint is

\[
U_f(v_f) \geq 0 \quad \text{and} \quad U_c(v_c) \geq 0. \tag{7.8}
\]

We say that the mechanism \((p, x)\) is incentive-compatible if and only if given that \(v_f\) and \(v_c\) are the honest valuations, for any \(w_f \in [a_f, b_f]\) and any \(w_c \in [a_c, b_c]\),

\[
U_f(v_f) \geq \bar{x}_f(w_f) - \bar{p}_f(w_f) [v_f C(\alpha)] \tag{7.9}
\]

\[
U_c(v_c) \geq \bar{p}_c(w_c) [v_c G(\alpha)] - \bar{x}_c(w_c). \tag{7.10}
\]
The IC constraint ensures that neither the FC nor the user can expect to gain from lying about their valuations.

**Theorem 7.3.1.** A mechanism is incentive-compatible and individually rational if and only if \( \bar{p}_f \) is decreasing, \( \bar{p}_c \) is increasing, and

\[
U_f(b_f) + U_c(a_c) \geq 0, \tag{7.11}
\]

where \( U_f(b_f) \) is the minimum value of \( U_f(v_f) \) for \( v_f \in [a_f, b_f] \), and \( U_c(a_c) \) is the minimum value of \( U_c(v_c) \) for \( v_c \in [a_c, b_c] \), and

\[
U_f(b_f) + U_c(a_c) = \int_{a_c}^{b_c} \int_{a_f}^{b_f} \left[ G(\alpha) \left( v_c - \frac{1 - F_c(v_c)}{f_c(v_c)} \right) - C(\alpha) \left( v_f + \frac{F_f(v_f)}{f_f(v_f)} \right) \right] \times p(v_f, v_c) f_f(v_f) f_c(v_c) d v_f d v_c. \tag{7.12}
\]

**Proof.** We first show the “only if” part of the theorem. For the FC, the IC condition is equivalent to

\[
U_f(v_f) = \max_{w_f \in [a_f, b_f]} \left\{ \bar{x}_f(w_f) - \bar{p}_f(w_f) [v_f C(\alpha)] \right\}, \tag{7.13}
\]

so that \( U_f(\cdot) \) is the maximum of a family of affine functions of the true valuation \( v_f \), therefore, \( U_f \) is a convex function [82]. The definition of its utility (7.1) and its IC condition (7.9) give that

\[
U_f(v_f) \geq U(w_f) + (w_f - v_f) \bar{p}_f(w_f) C(\alpha), \tag{7.14}
\]

where \( C(\alpha) \) is only a function of the vector \( \alpha \), so that the convexity of \( U_f(\cdot) \) and (7.14) imply that \( U'(v_f) = -\bar{p}_f(v_f) C(\alpha) \) and \( \bar{p}_f(\cdot) \) is decreasing. Thus,

\[
U_f(v_f) = U_f(b_f) + C(\alpha) \int_{v_f}^{b_f} \bar{p}_f(t_f) dt_f. \tag{7.15}
\]
In a similar way, we can conclude for the user that \( U'_e(v_e) = \tilde{p}_e(v_e)G(\alpha) \), \( \tilde{p}_e(\cdot) \) is increasing, and

\[
U_e(v_e) = U_e(a_c) + G(\alpha) \int_{a_c}^{v_e} \tilde{p}_e(t_c)dt_c. \tag{7.16}
\]

(7.15) and (7.16) imply that \( U_f(v_f) \) is decreasing and \( U_e(v_e) \) is increasing, so that

\[
\min_{v_f \in [a_f, b_f]} (U_f(v_f)) = U_f(b_f), \quad \min_{v_e \in [a_e, b_e]} (U_e(v_e)) = U_e(a_e).
\]

Thus, to satisfy the IR condition (7.8), the minimum of \( U_f \) and \( U_e \) has to be nonnegative, which gives (7.11). Furthermore, in (7.7),

\[
\begin{align*}
\int_{a_c}^{b_c} \int_{a_f}^{b_f} & [v_eG(\alpha) - v_fC(\alpha)] p_e(v_f, v_e) f_f(v_f) f_e(v_e)dv_fdv_e \\
& = \int_{a_f}^{b_f} U_f(v_f) f_f(v_f)dv_f + \int_{a_c}^{b_c} U_e(v_e) f_e(v_e)dv_e \\
& = U_f(b_f) + C(\alpha) \int_{a_f}^{b_f} \int_{v_f}^{b_f} p_f(t_f)dt_f f_f(v_f)dv_f + U_e(a_c) + G(\alpha) \int_{a_c}^{b_c} \int_{a_c}^{v_e} \tilde{p}_e(t_c)dt_c f_e(v_e)dv_e \\
& = U_f(b_f) + C(\alpha) \int_{a_f}^{b_f} F_f(v_f) \tilde{p}_f(v_f)dv_f + U_e(a_c) + G(\alpha) \int_{a_c}^{b_c} (1 - F_e(v_e)) \tilde{p}_e(v_e)dv_e \\
& = U_f(b_f) + U_e(a_c) + \int_{a_c}^{b_c} \int_{a_f}^{b_f} \left[ C(\alpha) \frac{F_f(v_f)}{f_f(v_f)} + G(\alpha) \frac{1 - F_e(v_e)}{f_e(v_e)} \right] p(v_f, v_e) f_f(v_f) f_e(v_e)dv_fdv_e,
\end{align*}
\]

which gives (7.12), and the “only if” part of the theorem is proved. We now prove the “if” part, i.e., given that \( \tilde{p}_f \) is decreasing, \( \tilde{p}_e \) is increasing, and (7.11) is satisfied, we need to prove the IR and IC conditions. To prove this, we first construct the payment function \( x_e(v_f, v_e) \) such that the mechanism is individually rational and incentive-compatible. There exist many such functions, we consider the following one

\[
x_e(v_f, v_e) = \int_{t_e = a_c}^{v_e} \tau_e G(\alpha) d[\tilde{p}_e(t_e)] - \int_{t_f = a_f}^{v_f} \tau_f C(\alpha) d[\tilde{p}_f(t_f)] + a_c G(\alpha) \tilde{p}_e(a_c) + \int_{t_f = a_f}^{b_f} \tau_f [1 - F_f(t_f)] C(\alpha) d[\tilde{p}_f(t_f)], \tag{7.17}
\]
where the integrals are non-negative because $\bar{p}_f(\cdot)$ is decreasing and $\bar{p}_c(\cdot)$ is increasing.

With the payment function (7.17), we have

$$U_c(a_c) = \bar{p}_c(a_c)[a_cG(\alpha)] - \bar{x}_c(a_c)$$

$$= \bar{p}_c(a_c)[a_cG(\alpha)] - \int_{v_f=a_f}^{b_f} x(v_f, a_c)f_f(v_f)dv_f.$$

With $v_c = a_c$, the first term in (7.17) becomes 0. For the second term,

$$\int_{v_f=a_f}^{b_f} \int_{t_f=a_f}^{v_f} t_fC(\alpha)d[-\bar{p}_f(t_f)]f_f(v_f)dv_f$$

$$= \int_{t_f=a_f}^{b_f} \left\{ \int_{v_f=t_f}^{b_f} f_f(v_f)dv_f \right\} t_fC(\alpha)d[-\bar{p}_f(t_f)]$$

$$= \int_{t_f=a_f}^{b_f} t_f[1 - F_f(t_f)]C(\alpha)d[-\bar{p}_f(t_f)],$$

which is equal to the last term of (7.17). Thus, $U_c(a_c) = 0$. Since we already assumed (7.11), we get $U_f(b_f) \geq 0$. Combined with the properties of $\bar{p}_f(\cdot)$ and $\bar{p}_c(\cdot)$, the IR condition is satisfied.

Further, we compare the utility of the user with the truthful valuation $v_c$ and the untruthful valuation $w_c$, when $v_c \geq w_c$,

$$U_c(v_c) - U_c(w_c)$$

$$= \left[ \bar{p}_c(v_c)[v_cG(\alpha)] - \bar{p}_c(w_c)[v_cG(\alpha)] \right] - \left[ \bar{x}_c(v_c) - \bar{x}_c(w_c) \right]$$

$$= G(\alpha) \left[ v_c \int_{t_c=w_c}^{v_c} d[\bar{p}_c(t_c)] - \int_{t_c=w_c}^{v_c} t_c d[\bar{p}_c(t_c)] \right]$$

$$= G(\alpha) \int_{t_c=w_c}^{v_c} (v_c - t_c)d[\bar{p}_c(t_c)] \geq 0. \quad (7.18)$$

It is straightforward to see that $U_c(v_c) - U_c(w_c)$ also holds for $v_c < w_c$. The proof of the IC condition for the FC is analogous. Therefore, the proof for Theorem 7.3.1 is complete.
7.3.2 Ex Post Efficiency

The mechanism is ex post efficient if and only if [104]

\[ p_v(v_f, v_c) = \begin{cases} 1 & \text{if } v_f C(\alpha) < v_c G(\alpha) \\ 0 & \text{if } v_f C(\alpha) > v_c G(\alpha) \end{cases} \]  

(7.19)

That is, in an ex post efficient mechanism for a WSN, the user gets the service whenever its valuation for the information gain is higher, and the FC prefers not to provide the service whenever its total cost for providing the service is higher.

**Theorem 7.3.2.** If the parameters of the model have the following relationship

\[ a_c G(\alpha) < b_f C(\alpha) < b_c G(\alpha), \]  

(7.20)

then, an incentive-compatible mechanism which is ex post efficient can not be individually rational unless the following amount

\[ \int_{a_c}^{b_f C(\alpha)} G(\alpha) \left[ 1 - F_c(v_c) \right] F_f \left( v_c \frac{G(\alpha)}{C(\alpha)} \right) dv_c \]  

(7.21)

is reimbursed by a third party.

**Proof.** For an incentive-compatible mechanism which is ex post efficient, we check the IR condi-
tion in (7.11),

\[ U_f(b_f) + U_c(a_c) \]

\[ = \int_{a_c}^{b_c} \int_{a_f}^{b_f} \left[ G(\alpha) \left( v_c - \frac{1 - F_e(v_c)}{f_e(v_c)} \right) - C(\alpha) \left( v_f + \frac{F_f(v_f)}{f_f(v_f)} \right) \right] \times p(v_f, v_c) f_f(v_f) f_e(v_c) dv_f dv_c \]

\[ = \int_{a_c}^{b_c} \int_{a_f}^{b_f} \left[ G(\alpha) \left( v_c - \frac{1 - F_e(v_c)}{f_e(v_c)} \right) - C(\alpha) \left( v_f + \frac{F_f(v_f)}{f_f(v_f)} \right) \right] f_f(v_f) f_e(v_c) dv_f dv_c \]

\[ = \int_{a_c}^{b_c} G(\alpha) \left[ v_c f_e(v_c) + F_e(v_c) - 1 \right] F_f \left( \frac{G(\alpha)}{C(\alpha)} \right) dv_c \]

\[ - \int_{a_c}^{b_c} \int_{a_f}^{b_f} C(\alpha) \left[ v_f f_f(v_f) + F_f(v_f) \right] f_f(v_f) f_e(v_c) dv_f dv_c \]

\[ = \int_{a_c}^{b_c} G(\alpha) \left[ v_c G(\alpha) - b_f C(\alpha) \right] f_e(v_c) dv_c + \int_{a_c}^{b_c} G(\alpha) \left[ F_e(v_c) - 1 \right] F_f \left( \frac{G(\alpha)}{C(\alpha)} \right) dv_c \]

\[ = - \int_{a_c}^{b_c} G(\alpha) \left[ 1 - F_e(v_c) \right] F_f \left( \frac{G(\alpha)}{C(\alpha)} \right) dv_c \leq 0. \]

(7.22)

Thus, with condition (7.20), \( U_f(b_f) + U_c(a_c) \) is negative, and (7.21) is the smallest amount that is needed from a third party to create a Bayesian mechanism which is individually rational and ex post efficient. This proves the theorem. \( \square \)

### 7.4 Sensor Selection in Optimal Mechanism

In this section, we design the optimal mechanism for a WSN whose cost is no higher than the gain for each sensor state vector \( \alpha \), i.e., \( C(\alpha) \leq G(\alpha) \). Also, we assume that \( v_f \) and \( v_c \) are both uniformly distributed on \([0, 1]\) (\( a_c = a_f = 0 \) and \( b_c = b_f = 1 \)). In this scenario, the condition (7.20) is satisfied, and the ex post efficiency is contradictory with the IR condition for an incentive-
compatible mechanism.

### 7.4.1 Optimal Trading Rule and Corresponding Payment

We design our optimal mechanism by maximizing the expected total utility of the FC and the user (7.7) subjected to the IR and IC constraints as shown in Theorem 7.3.1. The optimization problem is formulated as

$$\text{maximize} \quad \int_0^1 U_f(v_f) f_f(v_f) dv_f + \int_0^1 U_c(v_c) f_c(v_c) dv_c$$

subject to \[ U_f(1) + U_c(0) \geq 0 \] (7.23)

as well as ensuring that $\bar{p}_f$ is decreasing and $\bar{p}_c$ is increasing. The following theorem provides the optimal solution.

**Theorem 7.4.1.** The optimal mechanism that maximizes the expected total utility of the FC and the user subject to the IR and IC constraints gives the following trading rule between the FC and the user

$$p_e(v_f, v_c) = \begin{cases} 
1 & \text{if } 4v_fC'(\alpha) < (4v_c - 1)G(\alpha) \\
0 & \text{if } 4v_fC'(\alpha) > (4v_c - 1)G(\alpha).
\end{cases}$$

(7.24)

Furthermore, the corresponding payment in (7.17) is

$$x_e(v_f, v_c) = \frac{G^2(\alpha)}{C'(\alpha)} \left( \frac{1}{2} v_c^2 \right) - \frac{C^2(\alpha)}{G(\alpha)} \left( \frac{1}{2} v_f^2 - \frac{1}{6} \right).$$

(7.25)

**Proof.** The optimization problem (7.23) can be solved through Karush-Kuhn-Tucker (KKT) opti-
mality conditions [70].

\[
\begin{align*}
\text{maximize} & \quad \int_0^1 U_f(v_f) f_f(v_f) dv_f + \int_0^1 U_c(v_c) f_c(v_c) dv_c \\
& \quad + \lambda \left[U_f(1) + U_c(0)\right]
\end{align*}
\]

subject to \[ U_f(1) + U_c(0) \geq 0 \] \hspace{1cm} (7.26)

\[ \lambda \geq 0 \]

\[ \lambda \left[U_f(1) + U_c(0)\right] = 0, \]

where the Lagrangian (objective function in (7.26)) can be further written as

\[
L(\lambda) = (1 + \lambda) \int_0^1 \int_0^1 \left[G(\alpha) \left(v_c - \frac{\lambda}{1 + \lambda}(1 - v_c)\right) - C(\alpha) \left(v_f + \frac{\lambda}{1 + \lambda}v_f\right)\right] p(v_f, v_c) dv_f dv_c
\]

(7.27)

Thus, the solution \((p^*, \lambda^*)\) that maximizes the Lagrangian (7.27) as well as satisfying

\[
\lambda^* > 0 \quad \text{} \quad (7.28a)
\]

\[
\int_0^1 \int_0^1 \left[(2v_c - 1) G(\alpha) - 2v_f C(\alpha)\right] p^*(v_f, v_c) dv_f dv_c = 0. \quad (7.28b)
\]

must be the optimal solution of (7.23). Since the trading probability between the FC and the user can only be 0 or 1, the following function for \(p(v_f, v_c)\) maximizes the Lagrangian (7.27) as well as satisfying that \(\bar{p}_f\) is decreasing and \(\bar{p}_c\) is increasing,

\[
p_e(v_f, v_c) = \begin{cases} 
1 & \text{if } C(\alpha) \left(v_f + \frac{\lambda}{1 + \lambda}v_f\right) < G(\alpha) \left(v_c - \frac{\lambda}{1 + \lambda}(1 - v_c)\right) \\
0 & \text{if } C(\alpha) \left(v_f + \frac{\lambda}{1 + \lambda}v_f\right) > G(\alpha) \left(v_c - \frac{\lambda}{1 + \lambda}(1 - v_c)\right).
\end{cases} \quad (7.29)
\]
Thus, (7.28b) is equivalent to

\[
0 = \int_{\frac{1}{1+2\lambda}}^{1} \int_{0}^{\frac{G(\alpha)}{C(\alpha)}} (2v_c - 1) \ G(\alpha) - 2v_f C(\alpha) \ dv_f dv_c
\]

\[
= \frac{G^2(\alpha)}{C(\alpha)} \frac{(2\lambda - 1)(\lambda + 1)}{6(1 + 2\lambda)^2},
\]

combining with (7.28a), we get \( \lambda = \frac{1}{2} \).

Therefore, \( p(v_f, v_c) \) in (7.29) with \( \lambda = \frac{1}{2} \) gives (7.24), which is the optimal solution for (7.23).

So that

\[
\bar{p}_f(v_f) = \int_{\frac{G(\alpha)}{C(\alpha)}v_f + \frac{1}{4}}^{1} dv_c = \frac{3}{4} - \frac{C(z)}{G(\alpha)} v_f
\]

and

\[
\bar{p}_c(v_c) = \int_{0}^{\frac{G(\alpha)}{C(\alpha)}(v_c - \frac{1}{4})} dv_f = \frac{G(z)}{C(\alpha)} (v_c - \frac{1}{4}).
\]

Then the payment function in (7.17) can be written as,

\[
x_v(v_f, v_c) = \frac{G^2(\alpha)}{C(\alpha)} \int_{t_c=0}^{v_c} t_c dt_c - \frac{C^2(\alpha)}{G(\alpha)} \int_{t_f=0}^{v_f} t_f dt_f + \frac{C^2(\alpha)}{G(\alpha)} \int_{t_f=0}^{1} \left[ t_f - t_f \right] dt_f,
\]

which gives (7.25). So that the theorem is proved. \( \square \)
7.4.2 Sensor Selection Problem

With the trading rule in (7.24), the expected total gain of the FC and the user is

\[
E[U_{Total}]^{\text{opt}} = \int_0^1 U_f(v_f) f_f(v_f) dv_f + \int_0^1 U_c(v_c) f_c(v_c) dv_c
\]

\[
= \int_0^1 \int_0^1 [v_c G(\alpha) - v_f C(\alpha)] p_v(v_f, v_c) f_f(v_f) f_c(v_c) dv_f dv_c
\]

\[
= \int_{\frac{1}{2}}^1 \int_{0}^{\frac{G_c(\alpha)}{C(\alpha)}(v_c - \frac{1}{4})} [v_c G(\alpha) - v_f C(\alpha)] dv_f dv_c
\]

\[
= \frac{G^2(\alpha)}{C(\alpha)} \left( \frac{1}{6} v_c^3 - \frac{1}{32} v_c \right) \bigg|_{\frac{1}{2}}^{1},
\]

where the vector \( \alpha \) represents the selection state of the sensors in the WSN. So that \( \alpha \) can be further designed to maximize the expected total gain of the FC and the user. Therefore, for the designed optimal mechanism, we select an optimal subset of sensors in the WSN by solving the following optimization problem

\[
\alpha^* = \arg \max_{\alpha} \frac{G^2(\alpha)}{C(\alpha)}.
\]

With this optimization problem, the FC can determine beforehand which sensor(s) to select from the WSN, according to the sensor locations and the prior knowledge of the target, if she decides to provide service to the user.

7.5 Simulation Experiments

In this section, we consider a specific WSN where \( N \) sensors are randomly distributed in the ROI whose size is 20m x 20m. We select a subset of sensors for target localization through the designed optimal mechanism. The target is assumed to emit an isotropic signal from its location, so that the
sensor measurement is given by
\[ z_i = \sqrt{\frac{P_0}{1 + d_i^2}} + n_i, \quad (7.36) \]
where \( P_0 \) is the signal power at distance zero, which is assumed to be 1000 in the experiment, and \( d_i \) is the distance between the target \( \theta = (\theta_x \theta_y)^T \) and the \( i \)th sensor \( \theta_i = (\theta_x^i \theta_y^i)^T \). The noises \( n_i \) are independent across time steps and across sensors and modeled as standard Gaussian \( N(0, 1) \).

The sensor measurements \( r \) are quantized locally to \( M = 3 \) bits as in [121]. We assume that the prior pdf of the target location \( x \) is \( N(\mu_0, \Sigma_0) \) with \( \mu_0 = [1.25, 1.25]^T \) and \( \Sigma_0 = diag[0.5^2 \ 0.5^2] \).

Based on the quantized sensor measurements \( D = (D_1, \ldots, D_N)^T \), the FC estimates the location of the target through the importance sampling based Monte Carlo method as is also done in [13] and [95]. The posterior pdf of the target location \( x \) is \( N(\mu_0, \Sigma_0) \) with \( \mu_0 = [1.25, 1.25]^T \) and \( \Sigma_0 = diag[0.5^2 \ 0.5^2] \).

The performance of the location estimator is determined in terms of the mean square error (MSE) via 100 simulation runs.

In this chapter, the Fisher information is applied as the metric of the information gain of the WSN. From [13], the overall Fisher information matrix (FIM) can be written as the sum of the standard FIMs of the individual sensors and the FIM due to the prior information, \( J = \sum_{i=1}^{N} J^D_i + J^P \). So that we consider the normalized version of the determinant of the FIM as the total gain of the selected sensors \( G(\alpha) \),

\[
G(\alpha) = \frac{\det \left\{ \sum_{i=1}^{N} \alpha_i J^D_i + J^P \right\}}{\det \left\{ \sum_{i=1}^{N} J^D_i + J^P \right\}}. \quad (7.37)
\]
We define the resource costs of the WSN as the normalized number of selected sensors, i.e.,

$$
C(\alpha) = \frac{\sum_{i=1}^{N} \alpha_i}{N}.
$$

(7.38)

Having the gain and cost functions, a subset of sensors are selected such that the following tradeoff function between the determinant of the Fisher information and the number of selected sensors is maximized

$$
\frac{N \left[ \det \left\{ \sum_{i=1}^{N} \alpha_i J_i^D + J_P \right\} \right]^2}{\left[ \sum_{i=1}^{N} \alpha_i \right] \left[ \det \left\{ \sum_{i=1}^{N} J_i^P \right\} \right]^2}.
$$

(7.39)

We use the exhaustive search method to solve the optimization problem (7.35) to investigate the performance of our model.

In Fig. 7.2 and Fig. 7.3, we show the sensor selection results for two different WSNs with $N = 9$ sensors randomly distributed in the network. In Fig. 7.2, the sensors are distributed in a layout such that one of the sensors is relatively close to the target compared to all the other sensors as shown in Fig. 7.2(a). By solving the optimization problem (7.35), sensor 6 is selected (as shown by the box around sensor 6 in Fig. 7.2). Given the number of sensors to be selected, the exhaustive method selects the optimal set of sensors of the WSN, the values of the gain $G(\alpha)$, the cost $C(\alpha)$, and the tradeoff $g(\alpha)$ for the optimal set of sensors are shown in Fig. 7.2(b). Since sensor 6 is the only sensor that is relatively close to the target, selecting more than one sensor would improve the gain $G(\alpha)$ a little while increasing the cost $C(\alpha)$ more. Thus, selecting only sensor 6 gives the best tradeoff between the gain and cost of the WSN as shown in Fig. 7.2(b). Similar results can be found in Fig. 7.3(a) and Fig. 7.3(b), where sensors 3, 4, and 7 are relatively close to the target compared to the other sensors. Thus, selecting these three sensors gives the best tradeoff between the gain and cost of the WSN.

We take Fig. 7.2 as an example to illustrate the trading between the FC and the user. The designed optimal mechanism indicates that in order to maximize the expected total gain from the trade, sensor 6 has to be selected by the FC if she is going to sell its information to the user, and
the cost is $1/9$. The user, on the other hand, can get 0.38 units of information gain if she buys the service from the FC. Recall (7.24) and (7.17), for the scenario that $v_f$ and $v_c$ are both uniformly distributed on $[0, 1]$, the trading rule between the FC and user with a WSN in Fig. 7.2(a) is

$$p_v(v_f, v_c) = \begin{cases} 1 & \text{if } \frac{4}{9}v_f < 0.38(4v_c - 1) \\ 0 & \text{if } \frac{4}{9}v_f > 0.38(4v_c - 1), \end{cases}$$

(7.40)

and the corresponding payment is

$$x_v(v_f, v_c) = 0.65v_c^2 - 0.02v_f^2 + 0.005.$$  

(7.41)

According to the design process of the mechanism,

- the rationality of the FC and the user is considered,

- no one has an incentive to lie about their valuations,

- though the ex post efficiency cannot be satisfied, the optimal trading rule given the optimal mechanism and the corresponding payment ensures that the social welfare is maximized.

So it is the best choice for both the FC and the user to follow the rules provided in the optimal mechanism. Therefore, when the trade starts, the FC and the user give out their truthful valuations about the cost and the information gain, $v_f$ and $v_c$, no matter what the other individual’s valuation is. Then the condition (7.40) is checked, if $p(\cdot)$ is 1, then the FC sells the information provided by sensor 6 in the WSN to the user, and the user has to make a payment to the FC according to (7.41).

### 7.6 Summary

In this chapter, we designed an optimal bilateral mechanism for our cloud sensing paradigm where the sensing-as-a-service framework is enabled to estimate the target location in wireless sensor
networks (WSNs). The fusion center (FC), sells information regarding the target through sensor management to the user, who seeks to buy information regarding the target. Aware of the tradeoff between the information gain and costs of the WSN for sensor management, the general impossibility of achieving ex post efficiency as well as individually rationality (IR) and incentive-compatibility (IC) are shown. The optimal bilateral mechanism maximizes the expected total gains from the trade while assuring IR and IC constraints. In simulation experiments, we investigated the efficiency of our mechanism. In the future, we will consider designing an optimal mechanism for the scenario when multiple service providers sell service to multiple users.
Fig. 7.2: WSN with $N = 9$ randomly distributed sensors: Case 1 (a) Sensors and the target, the sensor with a square is selected. (b) The optimal gain, cost, and tradeoff of the WSN as a function of the number of selected sensors.
Fig. 7.3: WSN with $N = 9$ randomly distributed sensors: Case 2 (a) Sensors and the target, the sensors with squares are selected. (b) The optimal gain, cost, and tradeoff of the WSN as a function of the number of selected sensors.
CHAPTER 8
MARKET BASED SENSOR MOBILITY MANAGEMENT FOR TARGET LOCALIZATION

In this chapter, we propose a framework for the mobile sensor scheduling problem in target location estimation by designing an equilibrium based two-sided market model where the fusion center (FC) is modeled as the consumer and the mobile sensors are modeled as the producers. To accomplish the task, the FC provides incentives to the sensors to motivate them to optimally relocate themselves in a manner that maximizes the information gain for estimating the location of the target. On the other hand, the sensors calculate their own best moving distances that maximize their profits. Price adjustment rules are designed to compute the equilibrium prices and moving distances, so that a stable solution is reached. Simulation experiments show the effectiveness of our model.
8.1 Introduction

Many of today’s sensing applications allow a number of users carrying devices with built-in sensors, such as sensors built on automobiles to contribute towards an inference task with their sensing measurements. Moreover, the users that carry the devices are oftentimes mobile, so that it is easy to change the deployment of the sensors compared with the traditional fixed wireless sensor networks (WSN). As a specific example shown in Fig. 8.1, the platform has a sensing task in a large region, where the street view cars with installed sensors, such as google street view cars, are quite active. Thus, instead of placing a large number of sensors with high cost, the platform would prefer to recruit the cars from the companies to move to the expected region of the task and perform sensing. An advantage of such mobile architectures is that they do not need a dedicated sensing infrastructure for different inference tasks, thereby providing cost effectiveness. Such architectures are poised to revolutionize many sectors of our life, including social networks, environmental monitoring, and healthcare.

Existing sensing applications and systems, however, assume voluntary participation of users. While participating in a sensing task, users consume their own resources such as energy (due to movement, communication etc.), processing power, and may even have concerns regarding their privacy. Consider the example in Fig. 8.1, with their own work in hand, it is not realistic for the street view cars to move to the place that the platform is interested in for free. Therefore, the sensors may not participate in sensing and inference tasks unless suitable incentives are provided to them. Thus, there is a need to design flexible sensing architectures that can provide appropriate incentives to the users to motivate their participation in sensing tasks by suitably compensating them for their consumed resources.

Some past work has considered the issue of mobility. In [122], the mobile sensor network is controlled by maximizing the mutual information between the sensor measurements as a function of the control inputs and the target state. The mobile sensor network is steered to an optimal deployment by the proposed motion coordination algorithms in [123]. In [124], the sensor relocation problem is studied in mobile sensor networks, where the sensors are relocated to deal with sensor
Fig. 8.1: Example of market based mobile sensor network

failure or respond to new events and the redundant sensors are identified and relocated. However, to the best of our knowledge, the selfish concerns of mobile users have not been considered in a mobile sensing context.

Market based mechanisms for sensor management problems (where sensors are, however, considered fixed) have started to gain attention only recently [43, 46, 125–127]. In [43], the authors explored the possibility of using economics concepts for sensor management. The authors in [125] use the concept of the Walrasian equilibrium [46] to model market based sensor management. They also proposed algorithms to compute an approximate equilibrium when finding the equilibrium prices and allocations is computationally prohibitive. The authors in [126] studied the market based bit allocation problem for target tracking in the energy constrained WSN.

In this chapter, we focus on the design of a market model for the mobility management of the sensor network where we consider both the fusion center (FC) and the users (sensors) in the network are selfish. As opposed to conventional sensor networks, the problem at hand portrays two unique characteristics— a) How to decide the optimal locations of the sensors given the fact that the sensors’ moving distances not only decide how much the FC needs to spend but also the sensors’ participatory costs?, and, b) How to price the sensors’ moving distance such that both the FC and the sensors’ interests are maximized? To answer both questions, we model both the FC and the mobile sensors as agents in a two-sided market, where the mobile sensors are modeled as producers who produce the moving distance, which is then the supply of the market, and the FC is modeled
as a consumer who wants to buy the sensors’ moving distance, which is the demand of the market. Then the basic idea of determining the price of the moving distance is to balance the global supply and demand of the market. Since the FC’s payment to the sensors is restricted by its endowment, and the estimation performance about the target is dependent on the location of sensors, thus it is straightforward for the FC to determine its demand by maximizing estimation performance subject to its endowment constraint. On the other hand, the amount of moving distance of each sensor and its final location determine its participatory cost, it is then straightforward for the sensor to calculate its best supply through its utility as a function of its revenue from the FC and its cost. In this way, our approach gives a natural way for the FC and the sensors to interact with each other about their supply and demand. Thus, the market will converge to the equilibrium point, which is comprised of the optimal moving distance and the corresponding price dynamically.

The rest of the chapter is organized as follows. In Section 8.2, we formulate and analyze the market based mobile sensor scheduling problem. Simulation experiments are presented in Section 8.3, and we conclude our work in Section 8.4.

### 8.2 Market Based Mobile Sensor Scheduling

In this section, we first introduce our system model. And then describe how the FC and the sensors formulate their own optimization problem to determine their demand and supply, based on which we find the market equilibrium.

#### 8.2.1 System Model

We consider a WSN consisting of \( N \) selfish mobile sensors in a square region of interest (ROI) of size \( b^2 \). Based on the sensor measurements, the FC estimates the target location. We assume that the target and all the sensors are based on flat ground and have the same height, so that we can formulate the problem with a 2-D model. The target is assumed to emit an isotropic signal from its location, so that the sensor measurements are given by
\[ z_i = h_i(x, \theta_i) + n_i, \quad h_i(x, \theta_i) = \sqrt{\frac{P_0}{1 + d_i^2}} \]  

(8.1)

where \( P_0 \) is the signal power at distance zero and \( d_i \) is the distance between the target \( x = [x \ y]^T \) and the \( i \)th sensor \( \theta_i = [\theta_{ix} \ \theta_{iy}]^T \), i.e., \( d_i = \sqrt{(\theta_{ix} - x)^2 + (\theta_{iy} - y)^2} \). The noises \( n_i \) are independent across sensors and modeled as standard Gaussian \( \mathcal{N}(0, \sigma^2) \). The sensor measurements \( z_i \) are quantized locally to \( D_i \),

\[
D_i = \begin{cases} 
0 & -\eta_0 < z_i < \eta_1 \\
1 & \eta_1 < z_i < \eta_2 \\
\vdots \\
L - 1 & \eta_{(L-1)} < z_i < \eta_L 
\end{cases} \tag{8.2}
\]

where \( \eta = [\eta_0, \eta_1, \ldots, \eta_L]^T \) is the set of quantization thresholds with \( \eta_0 = -\infty \) and \( \eta_L = \infty \) and \( L = 2^M \) is the number of quantization levels. For simplicity, the quantization thresholds are assumed to be identical at each sensor and are designed according to the Fisher Information based heuristic quantization as in [48]. Then, given the target state, the probability that \( D_i \) takes value \( l \) is,

\[
p(D_i = l|x) = Q\left(\frac{\eta_l - h_i(\theta_i)}{\sigma}\right) - Q\left(\frac{\eta_{l+1} - h_i(\theta_i)}{\sigma}\right) \tag{8.3}
\]

where \( Q(.) \) is the complementary distribution function of the standard normal distribution. Given \( x \), the sensor measurements become conditionally independent, so the likelihood function of \( D = [D_1, D_2, \ldots, D_N]^T \) can be written as \( p(D|x) = \prod_{i=1}^{N} p(D_i|x) \).

Having the system model introduced, the FC determines its estimation performance gain and the sensors determine their energy cost. And the optimization problems for the FC and the mobile sensors are formulated accordingly.

### 8.2.2 Optimization Problem for the Fusion Center

In our market based model, the FC has to make payment to the sensors to purchase their moving distance, which is subject to its initial endowment \( W_{FC} \) (the maximum amount of money the FC
can spend). Since the sensors’ measurements are dependent on their locations, which, therefore, determines the estimation performance. Let \( \mathbf{p} = [p_1, \ldots, p_i, \ldots, p_N]^T \) denote the price vector where \( p_i \) is the price paid to sensor \( i \) for it to move a distance of \( d_i^M(\theta_i) \) from its initial location \( [\theta_{ix}^0, \theta_{iy}^0]^T \), i.e., \( d_i^M(\theta_i) = \sqrt{(\theta_{ix} - \theta_{ix}^0)^2 + (\theta_{iy} - \theta_{iy}^0)^2} \). Given \( \mathbf{p} \), the FC maximizes its estimation performance as a function of the location vector of all the sensors \( \Theta \) subject to its initial endowment,

\[
\begin{align*}
\text{maximize} & \quad J(\Theta) \\
\text{subject to} & \quad \sum_{i=1}^{N} p_i d_i^M(\theta_i) \leq W_{FC} 
\end{align*}
\]

where, \( J(\Theta) \) represents the estimation performance of the FC, for which we apply Fisher information as a metric in our work.

**Fisher Information Calculation**

Based on the received quantized data \( \mathbf{D} \) and the prior probability density function of the target state \( \mathbf{x} \), \( p(\mathbf{x}) \), the posterior Cramer-Rao lower bound on the mean square error (MSE) is,

\[
E\{[\hat{\mathbf{x}} - \mathbf{x}][\hat{\mathbf{x}} - \mathbf{x}]^T|\Theta\} \geq J^{-1} \tag{8.5}
\]

where \( J \) is the Fisher information matrix (FIM) which can be written as

\[
\begin{align*}
J(\Theta) & = E[-\Delta^x \log p(\mathbf{D}, \mathbf{x})] \\
& = E[-\Delta^x \log p(\mathbf{D}|\mathbf{x})] + E[-\Delta^x \log p(\mathbf{x})] \\
& = \sum_{i=1}^{N} J^D_i(\theta_i) + J^P \\
& = \sum_{i=1}^{N} \int_{\mathbf{x}} J^S_i(\theta_i) f(\mathbf{x}) d\mathbf{x} + J^P \tag{8.6}
\end{align*}
\]

where, the expectation is taken with respect to the joint distribution \( p(\mathbf{D}, \mathbf{x}) \), and \( \Delta^x \) is the second order derivative operator. The FIM can be decomposed into two parts where \( J^D_i \) is obtained from
the measurements of sensor $i$ and $J^P$ represents the prior information. With Monte Carlo method, given the state vector $x$, $J_i^S$ is the standard FIM of sensor $i$ and can be written as,

$$ J_i^S(\theta_i) = \frac{1}{N_s} \sum_{s=1}^{N_s} \frac{n^2 \alpha^2 h_i^2(\theta_i) d_i^{2n-4}(\theta_i)}{8\pi \sigma^2} \kappa(\theta_i, x) \begin{bmatrix} (\theta_{ix} - x_s)^2 & (\theta_{ix} - x_s)(\theta_{iy} - y_s) \\ (\theta_{ix} - x_s)(\theta_{iy} - y_s) & (\theta_{iy} - y_s)^2 \end{bmatrix} \tag{8.7} $$

where

$$ \kappa(\theta_i, x) = \sum_{l=0}^{L} \left[ e^{-\frac{(\eta_l - h_i(\theta_i))^2}{2\sigma^2}} - e^{-\frac{(\eta_{l+1} - h_i(\theta_i))^2}{2\sigma^2}} \right]^2 $$

The detailed derivation about the FIM can be found in [48] and [13].

We know that if (8.4) has an optimal solution, then it is equivalent to maximizing the trace of the FIM with the same constraints [128]. As shown in (8.6), since $J^P$ does not depend on the decision variables, (8.4) can be transformed to maximizing $\sum_{i=1}^{N} \text{tr}(J_i^D(\theta_i))$ subject to the constraints, and

$$ \sum_{i=1}^{N} \text{tr}(J_i^D) = \frac{1}{N_s} \sum_{s=1}^{N_s} \sum_{i=1}^{N} \frac{n^2 \alpha^2 h_i^2(\theta_i) d_i^{2n-4}(\theta_i)}{8\pi \sigma^2} \kappa(\theta_{ix}, \theta_{iy}, x_s, y_s) \left[ (\theta_{ix} - x_s)^2 + (\theta_{iy} - y_s)^2 \right] \tag{8.8} $$

We can see that (8.8) is nonlinear but a continuous and differentiable function of the location vector $[\theta_{ix}, \theta_{iy}]^T$. Thus, the interior-point method is a well-suited optimization tool to solve the problem. Also, since only a locally optimal solution can be found by the interior-point method, a feasible starting point is necessary for finding the optimal solution of (8.4).

### 8.2.3 Optimization Problem for the Sensors

As mentioned earlier, the sensors are treated as producers in the market who produce the supply – distance. Since moving to another location induces energy costs, part of which is induced by movement and the other part is induced by communicating with the FC. We consider that the
energy cost induced by movement is a linear function of the distance moved,

\[ E_i^M(\theta_i) = M_d d_i^M(\theta_i) \] (8.9)

where, \( M_d \) is the energy cost per unit distance of moving. And we consider a simple model of energy consumption of sensor \( i \) for transmitting \( M_i \) bits over the distance between sensor \( i \) and the FC [88]

\[ E_i^T(M_i, \theta_i) = \epsilon_{amp} \times M_i \times \left( (\theta_{ix} - x_{FC})^2 + (\theta_{iy} - y_{FC})^2 \right) \] (8.10)

where, \((x_{FC}, y_{FC})\) is the location of the FC, and \( \epsilon_{amp} \) is the transmitter amplifier. Thus, the total energy consumption of sensor \( i \) for transmission and moving is

\[ E_i(M_i, \theta_i) = E_i^T(M_i, \theta_i) + E_i^M(\theta_i) \] (8.11)

Thus, it is straightforward for the sensor to calculate its own optimal location by maximizing its utility function while ensuring that its total energy cost does not exceed its initial energy. Here we define the sensors’ utility function simply as the amount of money it gets from the FC minus its total energy cost. Therefore, given the price \( p_i \), sensor \( i \)'s optimization problem can be formulated as,

\[
\begin{align*}
\text{maximize} & \quad p_i d_i^M(\theta_i) - v_i E_i(M_i, \theta_i) \\
\text{subject to} & \quad p_i d_i^M(\theta_i) \geq v_i E_i(M_i, \theta_i) \quad \text{and} \quad E_i(M_i, \theta_i) \leq E_0
\end{align*}
\] (8.12)

where, \( v_i \) is sensor \( i \)'s cost per unit energy. Here we assume that the sensors do not have private information about \( v_i \), so that the vector \( \mathbf{v} = [v_i, \ldots, v_N]^T \) is known to both the FC and all the sensors.
Algorithm 8.1

1. Set \( v = 0 \), and price vector \( p = p^0 \)
2. Given \( p^v \), solve (8.4) and (8.12) to find the solution vector \( \Theta_F \) and \( \Theta_S \)
3. IF \( \text{mean}(d_F) \approx \text{mean}(d_S) \), ENDIF
4. Given \( \Theta_F \) and \( \Theta_S \), update prices.
   For all \( i \in \{1, 2, \ldots, N\} \), do
   IF \( d_{iF} > d_{iS}, p_{i}^{v+1} = p_{i}^{v}(1 + \delta) \) // FC needs the sensors to move more than the sensors would like to, increase price
   ELSE IF \( d_{iF} < d_{iS}, p_{i}^{v+1} = p_{i}^{v}/(1 + \delta) \) // The sensors would like to move more than the FC needs, decrease price
   ELSE \( p_{i}^{v+1} = p_{i}^{v} \) ENDIF.
5. Set \( v = v + 1 \) and go to Step (2)

8.2.4 The Market Equilibrium

In economics, demand, which is the total quantity bought by all consumers, is a decreasing function of price; and supply, which is the total quantity sold by producers, is an increasing function of price. At the equilibrium price, supply equals demand. Thus, the market is clear since there is neither excess nor shortage [129, 130]. If in some cases, the supply of the market exceeds the demand, i.e., there is a glut, the excess of the product would be sold at lower price. Then less efficient producers would be unable to trade, which drives down the price until the market reaches equilibrium. Alternatively, if the demand of the market exceeds the supply, i.e., there is a shortage, then the buyers would pay more than market price for the scarce products, which would drive up the price. This property of our model is shown in Fig. 8.2 in the simulation part.

In our problem, given the price vector \( p \), we denote the solution of (8.4) as \( \Theta_F \) and the solutions of the \( N \) separate optimization problems given in (8.12) as \( \Theta_S \). \( d_F \) and \( d_S \) are the corresponding distance vectors, which are treated as the product quantities in the market. The market equilibrium is reached when \( d_F = d_S \), in which case we say that the market is clear. Since “Zero Arbitrage" does not appear in the real world [131], we use the average moving distance of the sensors to define the market equilibrium, i.e., the market considered to be clear when \( \overline{d}_F \approx \overline{d}_S \). Updating the prices can be explained as follows. If the FC requires the sensors to move more than what the sensors want to move for a given price, i.e., demand exceeds supply, then the price per unit distance would
be driven up to motivate the sensors to move additional distance, i.e., supply increases. Otherwise, the price would be driven down until the market equilibrium is reached. In this chapter, the price is modified in an iterative manner as shown in Algorithm 8.1 to find the market equilibrium. Notice that, one can modify the value of $\delta$ to control the rate of change of the price vector. Further, it should also be noted that a large $\delta$ may help the FC to reach the equilibrium point faster, but at the cost of sacrificing the accuracy of the solution.

8.3 Simulation Results

In this section, we study the characteristics of the proposed marked model using simulations. In our simulation experiments, $N = 3$ sensors are deployed in a ROI with a size of $20m \times 20m$. Initially, the sensors are located around the FC, which is located at $[-8, -8]^T$. Also, all the sensors quantize their measurements to $M = 3$ bits. We assume that the prior pdf of the target location $x$ is $N(\mu_0, \Sigma_0)$ with $\mu_0 = [8, 8]^T$ and $\Sigma_0 = diag[1, 1]$ and the signal power at distance zero is $P_0 = 1000$. The $M_d$ in (8.9) is assumed to be $5J/m$ and the $\epsilon_{amp}$ in (8.10) is assumed to be $5 J/\text{bit/m}^2$. Also, the initial energy of each sensor is assumed to be $E_0 = 4000J$ and we assume that the sensors’ cost per unit energy is $v_i = 0.01$.

We first study the equilibrium point and the supply-demand curve of the FC and the sensors where the FC is assumed to have an endowment $W_0 = 30$. In Fig. 8.2, the line with the circles is the average moving distance of the sensors demanded by the FC and the one with the triangles is the corresponding average distance that the sensors want to move (i.e., the supply). Notice that, the demand curve is a decreasing function of price, and the supply curve is an increasing function of price, as can be noted in (8.4) and (8.12). The supply curve saturates because of the energy constraint of the sensors. The intersection point gives the equilibrium distance, at which the supply equals the demand, i.e., the market clears. Thus, our goal is to find the equilibrium point in terms of the final prices and locations of the sensors to localize the target.

In Fig. 8.3, we study the performance of the FC as a function of its budget $W_{FC}$. In Fig. 8.3(a),
we can see that when the budget of the FC increases, the average distance of moving demanded by the FC increases and finally saturates. The reason can be found from (8.4), which implies that the FC can afford to move the sensors more when the budget of the FC increases. However, when the budget is large enough, the information gain may not have scope for improvement by moving the sensors more (as they may, for instance, already be close to the target). In this case, the average distance saturates as it is not necessary to move the sensors closer to the target any further. Fig. 8.3(b) shows the FIM corresponding to the moving distance demanded by the FC.

Finally, in Fig. 8.4, we study the equilibrium solutions and the estimation MSE as functions of the initial average distance of the sensors to the FC. For this figure, we choose the initial locations of the sensors, so that, as the initial locations of the sensors move further from the FC, they get closer to the prior mean coordinates of the target. From Fig. 8.4(a), we can see that when the sensors are initially closer to the expected region of the target, they need to move less to satisfy the requirement of the FC. However, at the same time, as the sensors get further from the FC, the energy costs of communicating with the FC (quadratic function of the distance to the FC) get much larger and dominate the moving energy (linear to the distance of moving costs). Thus, as can be noted from Fig. 8.4(a), the payment that the FC has to make to the sensors increases overall as the sensors’ initial locations become further away from the FC (and closer to the target), so the FC has to pay more to motivate the sensors. Also, since the sensors are more likely to give better observations, the corresponding MSE gets better on average as shown by Fig. 8.4(b).

8.4 Summary

In this chapter, we proposed a market based sensor placement mechanism by considering the FC and the sensors to be self interested agents in the two-sided market. With the endowment constraint, the FC calculates the optimal moving distance for all the sensors, and the sensors calculate their own optimal location that maximize their profits. Using an iterative procedure, we computed the optimal distance the sensors should move and the prices they should be paid, in equilibrium.
The simulation experiments investigated the properties of the market model as well as studied the characteristics of the equilibrium solutions.
Fig. 8.2: Supply and Demand curve of the FC and the sensors

Fig. 8.3: The demanded distance and the corresponding FIM of the FC as a function of the budget
<table>
<thead>
<tr>
<th>Initial Mean Distance to the FC</th>
<th>Final Price</th>
<th>Final Distance</th>
<th>MSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.4</td>
<td>2.8</td>
<td>4</td>
</tr>
<tr>
<td>1.4</td>
<td>2</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>2.8</td>
<td>3</td>
<td>6</td>
<td>8</td>
</tr>
<tr>
<td>4.2</td>
<td>4</td>
<td>8</td>
<td>10</td>
</tr>
<tr>
<td>5.6</td>
<td>5</td>
<td>10</td>
<td>12</td>
</tr>
<tr>
<td>7.1</td>
<td>6</td>
<td>12</td>
<td>14</td>
</tr>
<tr>
<td>8.5</td>
<td>7</td>
<td>14</td>
<td>16</td>
</tr>
<tr>
<td>9.9</td>
<td>8</td>
<td>16</td>
<td>18</td>
</tr>
<tr>
<td>11.3</td>
<td>9</td>
<td>18</td>
<td>20</td>
</tr>
<tr>
<td>12.7</td>
<td>10</td>
<td>20</td>
<td>22</td>
</tr>
</tbody>
</table>

Fig. 8.4: The equilibrium price and distance and the estimating MSE as a function of the initial location of the sensors
CHAPTER 9
CONCLUSION AND FUTURE RESEARCH DIRECTIONS

9.1 Concluding Remarks

Target localization and tracking problems often require coverage of broad areas and a large number of sensors that can be densely deployed over the region of interest (ROI). This results in new challenges when the resources (bandwidth and energy) are limited. In such situations, it is inefficient to utilize all the sensors in the ROI including the uninformative ones, which hardly contribute to the inference task at hand but still consume resources. This issue has been investigated and addressed via the development of sensor management schemes, whose goal is to properly manage the sensors for the localization/tracking task while satisfying some performance and/or resource constraints [6].

Multiobjective optimization based dynamic sensor selection in classical WSNs: In the literature, sensor selection problems assume that the number of sensors to be selected is \textit{a priori}. In Chapter 3, we have considered in a more practical view point that the number of sensors is to be decided by the algorithm. We have proposed a sensor management strategy for target localization and tracking problems in Wireless Sensor Networks by formulating it as a multiobjective
optimization problem (MOP). We obtained tradeoff solutions between two conflicting objectives: minimization of the number of selected sensors and minimization of the information gap between the information gain when all the sensors transmit measurements and the information gain when only the selected sensors transmit their measurements based on the sensor selection strategy.

**Financial portfolio selection theory based sensor selection:** We proposed a portfolio theory based sensor selection framework in Section 3.4 of Chapter 3 in WSNs when there is uncertainty associated with sensor observations in the sense that sensor observations may be unreliable (they may probabilistically contain only noise). Such a consideration complicates the sensor selection problem since it introduces the risk of selecting sensors that may provide only noise. In other words, while selecting sensors, it becomes necessary to consider not only the maximization of information gain from the sensors, but to also consider the minimization of the risk (the reliability of the selected sensors) involved. Moreover, the dependence among the sensors also affect the sensor selection result. Therefore, our main objective was to find a sensor selection scheme that considers: 1) the expected information gain of each sensor, 2) the reliability of the sensor observations, and 3) the dependence among the sensors.

**Optimal mechanism design for sensor management problems in crowdsourcing based WSNs:** We design an optimal incentive compatible mechanism for sensor management problems in Chapter 4 to Chapter 7 in WSNs containing sensors that are selfish and profit-motivated. In typical WSNs which have limited bandwidth, the fusion center (FC) has to perform proper sensor management. In the formulation considered here, the FC conducted an auction by soliciting bids from the selfish sensors, which reflects how much they value their energy cost. Furthermore, the rationality and truthfulness of the sensors are guaranteed in our model. The mechanism designed in Chapter 4 is based on the assumption that the sensors send analog bids to the FC, where we first limited our focus on the design of an incentive-based mechanism for sensor selection problem in target localization, and then we studied the more general problem of designing an incentive-based mechanism for dynamic bit allocation in target tracking process. In Chapter 5, practical consideration was included where the sensors were only allowed to send quantized bids to the
FC because of communication bandwidth constraint or some privacy issues. Further, in Chapter 6, the state of charge (SOC) of the sensors, which affects the sensors’ energy cost in the task, were considered in the mechanism. Chapter 7 studied a two-sided auction mechanism for sensor management problems in the target localization problem.

**Market supply-demand model based sensor mobility management:** We proposed a framework for the mobile sensor scheduling problem in target location estimation in Chapter 8 by designing an equilibrium based two-sided market model where the FC was modeled as the consumer and the mobile sensors were modeled as the producers. To accomplish the task, the FC was assumed to provide incentives to the sensors to motivate them to optimally relocate themselves in a manner that maximizes the information gain for estimating the location of the target. On the other hand, the sensors calculated their own best moving distances that maximize their profits. Price adjustment rules were designed to compute the equilibrium prices and moving distances, so that a stable solution was reached.

### 9.2 Directions for Future Research

**Fully autonomous sensor networks:** The popularity of WSNs implies an increase in battery utilization in both traditional and crowdsourcing based WSNs. Moreover, the battery’s self-discharge becomes an issue when it has to be stored in the device for a long time [132]. The energy harvesting technology makes it possible for the sensors to be always ready to operate. Autonomous wireless sensors can be powered by batteries as well as the energy coming from the environment. Instead of concerning the generation of energy, a future direction is how to manage the autonomous sensors so that the overall consumption of resources in the network is minimized while the performance of the system is guaranteed.

**Machine learning in WSNs:** Research presented in this thesis focused on target tracking and localization problems based on parametric models. In some scenarios, it is not practical to know the exact prior knowledge of the system, which makes it difficult to get appropriate statistical models.
Machine learning algorithms provide models for nonparametric methods. Moreover, learning of the large set of data from large WSNs helps the network designer understand the characteristics of the network, for example, sensor correlation and uncertainty of the environment, etc. In both centralized and distributed WSNs, incorporation of learning methods is potentially an attractive approach for tracking, localization, and fault detection tasks.
REFERENCES


VITA

NAME OF AUTHOR:  Nianxia Cao

PLACE OF BIRTH: Taizhou, Jiangsu, China

DATE OF BIRTH: Jan. 05, 1987

GRADUATE AND UNDERGRADUATE SCHOOLS ATTENDED:

   Northwestern Polytechnical University, Xi’an, China

DEGREES AWARDED:

   M. S., 2012, Northwestern Polytechnical University, Xi’an, China
   B. S., 2009, Northwestern Polytechnical University, Xi’an, China