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Three–Dimensional Quantum Gravity Coupled to Gauge Fields

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ABSTRACT: We show how to simulate $U(1)$ gauge fields coupled to three–dimensional quantum gravity and then examine the phase diagram of this system. Quenched mean field theory suggests that a transition separates confined and deconfined phases (for the gauge matter) in both the negative curvature phase and the positive curvature phase of the quantum gravity, but numerical simulations find no evidence for such transitions.

Introduction

In two dimensions, dynamically triangulated random surfaces have proven to be a useful discretization of quantum gravity. An analogous approach in three dimensions has been formulated [1,2]. However, this system can only be studied in the grand canonical ensemble because no set of ergodic moves have been found for fixed volume. The action can be written as

$$S = \alpha N_0 - \beta N_3 \tag{1}$$

where N_0 is the number of nodes and N_3 is the the number of tetrahedra (i.e. the volume). β is therefore the cosmological constant while α acts like Newton's constant, since at fixed volume varying the number of nodes varies the curvature. In two dimensions, the topological classification of manifolds allows one to prove that at fixed Euler character there is an exponential bound on the number of triangulations as a function of the number of triangles. This implies that a chemical potential can control the volume. In three dimensions there is no corresponding classification and therefore no corresponding proof of an exponential bound. The naive result is factorial growth which would mean no partition function could be defined. It was a significant discovery that a chemical potential (β) does control the volume in three dimensions [3,4]. The volume diverges at a particular critical value of β. Given that the thermodynamic limit can be taken, the next issue is the nature of the phase diagram. In particular, is there a second order phase transition where a non–trivial continuum limit could be taken? It turns out that there are two phases separated by a transition at α^*. For smaller values of α the system is in a negative curvature phase where the coordination number of the nodes diverges. For larger values of alpha, the system is in a positive curvature phase where the number of sites is proportional to the volume. Unfortunately, the transition between these two phases, where one might have hoped for a continuum limit with zero curvature, appears to be first order so that there is

no interesting continuum limit.

This doesn't necessarily spell doom, since an expanded phase diagram (with more coupling constants) might still contain an appropriate transition. A simple and interesting way to expand the phase diagram is to couple matter to the system. Recently, studies of Ising matter coupled to three–dimensional quantum gravity have been completed [5-7]. If the matter is placed on the nodes, the phase diagram consists of the gravitational transition and an Ising transition. The two cross at zero Ising coupling so that the Ising matter does not change the nature of the gravitational transition. If a dual formulation is employed in which the matter fields are placed on the elementary volumes, the gravitational transition is also unchanged.

In this paper, we couple gauge fields to three–dimensional quantum gravity. (Our simplicial lattices correspond to triangulations of S^3). Aside from the possibility of finding a non–trivial continuum limit, this system is interesting because it couples gauge fields and quantum gravity. In two–dimensions, the analogous system is topological — quantum gravity has no interesting effect [8]. Three dimensions is thus the lowest dimension in which coupling these two theories gives a non–trivial result. In what follows, we restrict ourselves to the U(1) gauge theory.

Mean Field Theory

A quenched mean field theory proved useful in the case of Ising matter coupled to three–dimensional quantum gravity and it proves useful for gauge matter as well. The action for U(1) gauge fields coupled to three–dimensional quantum gravity is

$$S = \alpha N_0 - \beta N_3 + \lambda \sum_{\Delta} \cos \phi_{\Delta} \tag{2}$$

where the sum is over all triangles and ϕ is the sum of gauge angles about the given triangle. A simple approximation to the phase diagram can be obtained by doing mean

field theory for the gauge variables while fixing the volume and the number of nodes. This allows us to consider the influence of the background metric on the matter, but neglects the influence of the matter on the metric. The simplest mean field theory [9] for gauge theories begins with a trial action

$$S_H = H \sum_{\text{links}} \cos\theta \tag{3}$$

for which the free energy per link is simple

$$u(H) = \ln I_0(H) \tag{4}$$

where I_0 is a modified Bessel function. Adding and subtracting the trial action to the gauge part of the full action gives

$$\begin{aligned} Z = \mathrm{Tr} e^S = Z(H) < \exp(\lambda \sum_{\Delta} \cos\phi_\Delta - H \sum_{\text{links}} \cos\theta) >_H \\ \geq Z(H) \exp < \lambda \sum_{\Delta} \cos\phi_\Delta - H \sum_{\text{links}} \cos\theta >_H \end{aligned} \tag{5}$$

where Z is the partition function for the gauge part of the full action, $Z(H)$ is the partition function for the trial action and the subscript H denotes that the trial action is used as the weight in the expectation values. The resulting free energy per unit volume is

$$F(H, \lambda) = (uN_1 - Hu'N_1 + \lambda(u')^3 N_2)/N_3 \tag{6}$$

(the prime denotes the derivative with respect to H, N_1 is the number of links, and N_2 is the number of triangles). This must be maximized with respect to the field H. Maximizing gives a first order transition at $\lambda^* = 0.794$ in the negative curvature phase and $\lambda^* = (1 + 1/c) * 0.794$ in the positive curvature phase, where $c = N_3/N_0$ as the volume is taken large. (A more sophisticated gauge invariant approach leads to the same results). This result suggests the thick line in the phase diagram in figure 1. Note that mean field theory cannot be taken too seriously and is given here just to guide the later

numerical calculations. In three dimensions on a fixed lattice, for instance, the theory can be rewritten as a Coulomb gas that is in an ionized phase for all temperatures. Mean field theory mistakes a crossover for a phase transition.

Placement of the gravitational transition in the phase diagram is guided by contact with the pure gravity theory that can be made in the limits of large and small λ. Note that (using $N_1 = N_0 + N_3$)

$$\prod_{\text{links}} \left(\int_0^{2\pi} d\theta \right) = (2\pi)^{N_1} = e^{N_1 \ln 2\pi} = e^{(N_0+N_3)\ln 2\pi} \tag{7}$$

so that

$$Z(\alpha, \beta, \lambda = 0) = Z_{\text{pure grav}}(\alpha + \ln 2\pi, \beta - \ln 2\pi) \tag{8}$$

implying that the gravitational transition is shifted to $\alpha^* \sim 1.91 - 2.16$ [4]. For very large λ the theory also approaches a pure gravity theory. In this limit, (since $N_2 = 2N_3$)

$$\sum_{\Delta} \lambda \cos(\phi_\Delta) \sim \lambda N_2 = 2\lambda N_3 \tag{9}$$

implying

$$Z(\alpha, \beta, \lambda) \sim Z_{\text{pure grav}}(\alpha, \beta - 2\lambda) \quad (\lambda \text{ such that} \cos\phi_\Delta \sim 1) \tag{10}$$

This final equation suggests that the gravitational transition is only weakly dependent on λ as indicated by the dotted line in figure 1.

Detailed Balance

The task now is to numerically check figure 1 and to determine the order of the phase transitions suggested by mean field theory. In order to simulate this theory, we must derive the detailed balance relations in the grand–canonical ensemble. In three dimensions, any triangulation can be reached from any other triangulation through a series of local moves chosen from a set of four possibilities. These moves are labelled by the number of tetrahedra

before and after the move. The (1,4) move (figure 2) corresponds to the insertion of a node into a tetrahedron and the (4,1) move is its inverse. The (2,3) move (figure 3) replaces a triangle separating two tetrahedra with a new link at the boundary of three tetrahedra. The (3,2) move is its inverse. In contrast to the situation with Ising matter, where only the (1,4) detailed balance relations were modified by the addition of matter, here all the relations are modified. First, consider the (2,3) move. Here, it is necessary to assign a value for the gauge field on a potential new link. Label the state with two tetrahedra A and the state with the new link θ, where θ is an angle between 0 and 2π. Then consider attempting a $2 \to 3$ move with 50% probability and a $3 \to 2$ move with 50% probability. In the $2 \to 3$ case choose the triangle by picking a tetrahedron at random and then choosing one of its four sides. Note that there are two ways of picking the same triangle with this procedure. In the $3 \to 2$ case, the link is chosen from a list of all possible links with coordination number 3. If l_3 is the number of such links, the resulting detailed balance equation is

$$\frac{1}{2}\frac{1}{N_3}\frac{1}{4}2e^{S_A}P(A \to \theta) = \frac{1}{2}\frac{1}{l_3}e^{S_\theta}P(\theta \to A) \tag{11}$$

The acceptance rate can be increased by integrating over the possible values of θ so that it is first decided whether to update the triangulation or not and then later the gauge field values are determined. Integrating over all possible values of θ and assuming that $P(\theta \to A)$ can be chosen independent of θ gives

$$\frac{l_3}{2N_3}P(A \to B) = P(B \to A)\int_0^{2\pi} e^{S_\theta - S_A}d\theta \tag{12}$$

where B denotes the state with a new link regardless of its value. Define

$$I_{23} = \int_0^{2\pi} e^{S_\theta - S_A}d\theta \tag{13}$$

(which will turn out to be a gauge invariant function of the loops on the surface of the cluster of the two or three tetrahedra under consideration) and denote the ratio

$$R_{23} = l_3/2N_3 \tag{14}$$

then detailed balance is satisfied by

$$P(A \to B) = \frac{I_{23}}{R_{23} + I_{23}} \tag{15}$$

$$P(B \to A) = \frac{R_{23}}{R_{23} + I_{23}} \tag{16}$$

Now consider detailed balance for the (1,4) move. In this case, attempt a $1 \to 4$ move or a $4 \to 1$ move each with a 50% probability. In the former case, just pick a tetrahedron at random. In the latter case, pick from a list of sites known to have coordination number four. Detailed balance requires

$$\frac{1}{2}\frac{1}{N_3} e^{S_A} P(A \to \Theta) = \frac{1}{2}\frac{1}{l_4} e^{S_\Theta} P(\Theta \to A) \tag{17}$$

where A represents the state with no node, Θ represents the four gauge degrees of freedom living on the four new links, and l_4 represents the number of nodes with coordination number four. Again, define

$$I_{14} = \int e^{S_\Theta - S_A} \tag{18}$$

where now integration is over four angles, and

$$R_{14} = l_4 / N_3 \tag{19}$$

then detailed balance is satisfied by

$$P(A \to B) = \frac{I_{14}}{R_{14} + I_{14}} \tag{20}$$

$$P(B \to A) = \frac{R_{14}}{R_{14} + I_{14}} \tag{21}$$

where now B represents the presence of a node without specifying the four new gauge degrees of freedom associated with it. These update probabilities tell us whether to put in or take out a link and whether to put in or take out a node. They do not indicate what the gauge degrees of freedom should be. This is taken care of by a subsequent Metropolis

or heat bath step. One caveat should be added. The ratio in R_{14} is interpreted differently depending on whether a $1 \to 4$ move is being contemplated or a $4 \to 1$ move is being contemplated. In the former case, N_3 is the number of tetrahedra to begin with while l_4 is the number of sites with coordination number 4 if the move is accepted. In the latter case, l_4 is the number of sites with coordination number 4 to begin with while N_3 is the number of tetrahedra there will be if the move is accepted (i.e. the current number of tetrahedra minus 3). Similar considerations are required in the case of the (2,3) move.

At first sight, these equations suggest that a simulation of gauge fields coupled to quantum gravity in this grand–canonical ensemble would be impractical. An integration needs to be done every time there is a local update. We will see that the integral I_{23} is not really very troublesome. The integral I_{14} is much more demanding computationally, but it turns out to be within the realm of practicality.

Consider first the action difference for the former integral (the (2,3) move).

$$S_\theta - S_A = -\beta + \lambda(t_1 + t_2 + t_3 - t_0) \tag{22}$$

where 1, 2, and 3 label the new triangles associated with the new link and 0 labels the old triangle that has been eliminated. The quantities t_i are the cosines of the sum of the gauge angles about the specified loop. The needed gauge integral is therefore

$$I_{23} = e^{-\beta - \lambda t_0} \int_0^{2\pi} e^{\lambda(t_1+t_2+t_3)} d\theta \tag{23}$$

If the links, α_i, involved in the integral are labeled as in figure 3 and x_i is defined such that

$$x_1 \equiv \alpha_1 + \alpha_2, \quad x_2 \equiv \alpha_3 + \alpha_4, \quad x_3 \equiv \alpha_5 + \alpha_6 \tag{24}$$

then

$$t_1 + t_2 + t_3 = \cos(\theta + x_1) + \cos(\theta + x_2) + \cos(\theta + x_3) \tag{25}$$

$$= r\cos(\theta + \omega) \tag{26}$$

where ω is independent of θ and

$$r^2 = \left(\sum_{i=1}^{3} \cos x_i\right)^2 + \left(\sum_{i=1}^{3} \sin x_i\right)^2 \tag{27}$$

Finally,

$$I_{23} = 2\pi \; I_0(\lambda r) \; e^{-\beta - \lambda t_0} \tag{28}$$

Is this result gauge invariant? The values of the gauge fields enter only through r. It is easy to see that r is gauge invariant. Change variables to

$$z_1 = x_1 - x_2, \quad z_2 = x_1 - x_3, \quad z_3 = x_2 - x_3 \tag{29}$$

then

$$r^2 = 3 + 2(\cos z_1 + \cos z_2 + \cos z_3) \tag{30}$$

Since each of the z_i is a gauge loop, r is manifestly gauge invariant.

I_{14} is more complicated. Here there are four gauge variables to be integrated over, labeled as in figure 2. The action difference is

$$S_\Theta - S_A = \alpha - 3\beta + \lambda \sum_{i=1}^{i=6} \cos p_i \tag{31}$$

where the p_i are the new plaquettes. In terms of the gauge variables

$$p_1 = \theta_1 + \theta_2 + x_1 \tag{32}$$

$$p_2 = \theta_1 + \theta_3 - x_5 \tag{33}$$

$$p_3 = \theta_1 - \theta_4 - x_3 \tag{34}$$

$$p_4 = \theta_2 - \theta_3 - x_4 \tag{35}$$

$$p_5 = \theta_2 + \theta_4 - x_2 \tag{36}$$

$$p_6 = \theta_3 + \theta_4 - x_6 \tag{37}$$

The action ratio can now be rewritten

$$e^{S_\Theta - S_A} = e^{\alpha - 3\beta} \prod_{i=1}^{i=6} \left(\sum_{k_i=-\infty}^{\infty} I_{k_i}(\lambda) e^{ik_i p_i} \right) \tag{38}$$

The exponential can be rewritten as i times

$$\theta_1(k_1 + k_2 + k_3) + \theta_2(k_1 + k_4 + k_5) + \theta_3(k_2 - k_4 + k_6) + \theta_4(-k_3 + k_5 + k_6)$$

$$+x_1 k_1 - x_5 k_2 - x_3 k_3 - x_4 k_4 - x_2 k_5 - x_6 k_6 \tag{39}$$

Integration over θ_1, θ_2, and θ_3 gives a factor

$$(2\pi)^3 \; \delta(k_1 + k_2 + k_3) \; \delta(k_1 + k_4 + k_5) \; \delta(k_2 - k_4 + k_6) \tag{40}$$

Notice that these three δ–functions automatically make the coefficient of θ_4 zero. This is a consequence of gauge invariance under transformations at the center node. Integration over θ_4 just gives another 2π. The δ–functions give

$$k_3 = -k_1 - k_2, \qquad k_5 = -k_1 - k_4, \qquad k_6 = k_4 - k_2 \tag{41}$$

so that the x_i terms become

$$k_1(x_1 + x_2 + x_3) + k_2(-x_5 + x_3 + x_6) + k_4(-x_4 + x_2 - x_6) \tag{42}$$

Each coefficient of the k_i is a gauge invariant loop on the surface of the tetrahedron. If the coefficients of the k_i are relabeled as z_1, z_2, and z_3 respectively then the final result is

$$I_{14} = (2\pi)^4 e^{\alpha - 3\beta} \sum_{m_1=-\infty}^{\infty} \sum_{m_2=-\infty}^{\infty} \sum_{m_3=-\infty}^{\infty} I_{m_1}(\lambda)$$

$$I_{m_2}(\lambda) I_{m_3}(\lambda) I_{m_1+m_2}(\lambda) I_{m_1+m_3}(\lambda) I_{m_3-m_2}(\lambda) e^{i(m_1 z_1 + m_2 z_2 + m_3 z_3)} \tag{43}$$

Invariance under sign changes in m_1, m_2, and m_3 make it clear that the right hand side is real — the exponent can be replaced with a cosine. (Computations are faster if the expression is rewritten to take advantage of this fact).

Results

One parameter that has to be tuned is the ratio, r, of update sweeps of the gauge field to the update sweeps of the metric. In two dimensions, when bosons are coupled to quantum gravity, the expectation value of the bosonic action can be calculated exactly due to scale invariance. This can be used to check whether the bosons are in equilibrium and then the equilibrium of the gravity sector can be checked by varying the above ratio and checking to see that the results are stable. Here, there is no analytical check of either the matter or the gravity sector. We can only check various expectation values as a function of the ratio and look for stability. For a fixed lattice (that has not been warmed up) one update pass on the gauge fields is insufficient. We suspect that on a dynamical lattice, where the connectivity is constantly changing, that more than one gauge update is required per update sweep through the metric. Consider the expectation values of the plaquette, the specific heat, and the number of nodes, as well as the value of β required to maintain a given volume. As r is varied from 1 to 16, the values of all of these quantities change, but reach a plateau near $r = 8$. This is illustrated in fig. 4 with a plot of the expectation value of the number of nodes versus r. The expectation value of the plaquette has a similar behavior, varying from 0.62 to 0.67 as r varies from 1 to 16, but varying by less than a part per thousand as r is increased from 8 to 16. $r = 8$ is the value we used. A number of runs at $r = 1$ were also done. The results differed numerically from the $r = 8$ results, but they implied the same conclusions described below.

The phase diagram suggested by the quenched mean field theory and the partition function relations can be tested numerically. The easiest thing to check is the implication of eqn. (10) that the gravitational transition is independent of λ. As in the case of pure gravity [4], the expectation value of the number of nodes scales with the volume as

$$< N_0 > \sim N_3^{\delta} \tag{44}$$

where δ is one in the phase with finite coordination number and less than one in the phase where the coordination number diverges. For $\lambda = 2$, we computed δ for $\alpha = 1.6$, $\alpha = 1.8$, and $\alpha = 2.1$ on lattices of volume 2000, 4000, 8000, and 16000. The results for δ are 0.55(1), 0.84(2), and 1.007(1) respectively. The error bars are statistical only. The confidence level of the (log) fits to a straight line were not high, suggesting that larger lattices are necessary to reach scaling, but the trend is consistent with a gravitational transition still in the region $\alpha = 1.91 - 2.16$ of the $\lambda = 0$ transition.

The gauge transition is more difficult. To search for the gauge transitions we computed the gauge part of the specific heat as a function of λ for various fixed values of α on both sides of the gravitational transition and looked for a peak growing with volume. Figures 5 and 6 show the results. There is a maximum in the specific heat in the region predicted by mean field theory, but there is no sharp peak, it does not grow with volume, and there is no evidence for critical slowing down. The same results were obtained for large and smaller values of α. This suggests that the gauge sector is in a strong coupling phase for all λ. Other operators show consistent behavior. For instance, the expectation value of the plaquette is almost a straight line as a function of λ in the region of λ we considered and had no significant volume dependence on lattices up to a volume of 16000. One could attempt to verify that the entire phase is in strong coupling by examining Wilson loops for large λ and testing for an area law, but this is much more difficult on a dynamical triangulation than on a fixed hypercubic lattice and we have not tried to do it.

A simpler way to try to get information about what is going on in the gauge sector is to look at monopoles and their clusters. Consider a three–dimensional elementary volume (a tetrahedron here) with a surface of oriented plaquettes, P_i (i.e. $\sum P_i = 0$). For each P_i define n_i such that

$$P_i = 2\pi n_i + \phi_i \tag{45}$$

where $-\pi < \phi < \pi$. Then the monopole number in this volume is

$$m = \sum_i n_i \tag{46}$$

Generally, a certain value of n_i requires $p_i > (2n_i - 1)\pi$. Summing this inequality over the s sides of the elementary volume gives a limit on the monopole number:

$$m < s/2 \tag{47}$$

For a cube, this gives the familiar result that $m \in \{-2, -1, 0, 1, 2\}$. For a tetrahedron there are fewer possibilities: $m \in \{-1, 0, 1\}$. A monopole density is defined as

$$\rho = (N_+ + N_-)/V \tag{48}$$

where N_m is the number of tetrahedra with monopole number m and V is the volume (the number of tetrahedra). Clusters are defined as groups of neighboring monopoles (signs are ignored and monopoles are neighbors if the surfaces of their tetrahedra share a face). An interesting order parameter for clusters is the ratio

$$M = \left\langle \frac{N_{\rm max}}{N_{\rm tot}} \right\rangle \tag{49}$$

where $N_{\rm max}$ is the number of monopoles in the largest cluster and $N_{\rm tot}$ is the total number of monopoles. M is a good order parameter for a percolation transition, a transition from a phase with many clusters to a phase with one cluster. In the three-dimensional quantum gravity plus $U(1)$ gauge fields system, the monopole density is 1/3 when $\lambda = 0$ and it slowly and smoothly decreases as λ is increased. (On a cubic lattice the monopole density is 0.43 at $\lambda = 0$ and decreases for larger λ.) The density has very little sensitivity to the value of α. The cluster order parameter, M, is small for $\lambda = 0$ and α in the region of the gravitational transition and it decreases smoothly as λ is increased. Eventually it increases again, but this is only because the density becomes so small that there are of order unity monopoles (i.e. $N_{\rm max} \sim N_{\rm tot}$). M also has no sensitivity to the value of α

and this is surprising. In general, one expects a low percolation threshold for lattices with a high coordination number (such as those generated in the negative curvature phase). Apparently, it is the coordination number of the tetrahedron that counts (since that is where the monopole lives) rather than that of the sites. A tetrahedron has a coordination number of four, the same as a site in a two–dimensional square lattice, so one might guess that the threshold for the tetrahedral lattice is roughly that of the square lattice, namely 0.59 [10]. This guess can be confirmed by artificially placing monopoles of a given density on the dynamical lattices and measuring the threshold directly. Consequently, the physical monopole density is always below the percolation threshold and they provide no useful signal.

In conclusion, we have shown how to couple gauge fields to three-dimensional quantum gravity and made an effort to find an interesting continuum limit. No appropriate phase transition was found in the space of actions we considered.

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FIGURE CAPTIONS

[1] The quenched mean field diagram for three–dimensional quantum gravity coupled to U(1) gauge fields.

[2] The tetrahedra involved in a (1,4) move.

[3] The tetrahedra involved in a (2,3) move.

[4] The expectation value of the number of nodes, $N0$, versus the ratio of the number of gauge update sweeps to the number of gravity update sweeps. $\alpha = 2.1$, $\lambda = 1.0$, and $N3 = 4000$.

[5] The gauge component of the specific heat as a function of λ at $\alpha = 1.8$ for two lattice sizes: $N_3 = 2000$ (crosses) and $N_3 = 4000$ (boxes).

[6] The gauge component of the specific heat as a function of λ at $\alpha = 2.2$ for two lattice sizes: $N_3 = 2000$ (crosses) and $N_3 = 4000$ (boxes).