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Supersymmetric lattices[1]

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Abstract. Discretization of supersymmetric theories is an old problem in lattice field theory. It has resisted solution until quite recently when new ideas drawn from orbifold constructions and topological field theory have been brought to bear on the question. The result has been the creation of a new class of lattice gauge theory in which the lattice action is invariant under one or more supersymmetries. The resultant theories are local and free of doublers and in the case of Yang-Mills theories also possess exact gauge invariance. In principle they form the basis for a truly non-perturbative definition of the continuum supersymmetric field theory. In this talk these ideas are reviewed with particular emphasis being placed on $\mathcal{N} = 4$ super Yang-Mills theory.

1. Introduction

The problem of formulating supersymmetric theories on lattices has a long history going back to the earliest days of lattice gauge theory. However, after initial efforts failed to produce useful supersymmetric lattice actions the topic languished for many years. Indeed a folklore developed that supersymmetry and the lattice were mutually incompatible. However, recently, the problem has been re-examined using new tools and ideas such as topological twisting, orbifold projection and deconstruction and a class of lattice models have been constructed which maintain one or more supersymmetries exactly at non-zero lattice spacing.

While in low dimensions there are many continuum supersymmetric theories that can be discretized this way, in four dimensions there appears to a unique solution to the constraints – $\mathcal{N} = 4$ super Yang-Mills. The availability of a supersymmetric lattice construction for this theory is clearly very exciting from the point of view of exploring the connection between gauge theories and string/gravitational theories. But even without this connection to string theory it is clearly of great importance to be able to give a non-perturbative formulation of a supersymmetric theory via a lattice path integral in the same way that one can formally define QCD as a limit of lattice QCD as the lattice spacing goes to zero and the box size to infinity. From a practical point of view one can also hope that some of the technology of lattice field theory such as strong coupling expansions and Monte Carlo simulation can be brought to bear on such supersymmetric theories.

In this talk I will outline some of the key ingredients that go into these constructions, the kinds of applications that have been considered so far and highlight the remaining difficulties.

First, let me explain why discretization of supersymmetric theories resisted solution for so long. The central problem is that naive discretizations of continuum supersymmetric

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theories break supersymmetry completely and radiative effects lead to a profusion of relevant supersymmetry breaking counterterms in the renormalized lattice action. The coefficients to these counterterms must then be carefully fine tuned as the lattice spacing is sent to zero in order to arrive at a supersymmetric theory in the continuum limit. In most cases this is both unnatural and practically impossible – particularly if the theory contains scalar fields. Of course, one might have expected problems – the supersymmetry algebra is an extension of the Poincaré algebra which is explicitly broken on the lattice. Specifically, there are no infinitesimal translation generators on a discrete spacetime so that the algebra $\{Q, \overline{Q}\} = \gamma_\mu p_\mu$ is already broken at the classical level. Equivalently it is a straightforward exercise to show that a naive supersymmetry variation of a naively discretized supersymmetric theory fails to yield zero as a consequence of the failure of the Leibniz rule when applied to lattice difference operators[2]. In the last five years or so this problem has been revisited using new theoretical tools and ideas and a set of lattice models have been constructed which retain exactly some of the continuum supersymmetry at non-zero lattice spacing. The basic idea is to maintain a particular subalgebra of the full supersymmetry algebra in the lattice theory. The hope is that this exact symmetry will constrain the effective lattice action and protect the theory from dangerous susy violating counterterms.

Two approaches have been pursued to produce such supersymmetric actions; one based on ideas drawn from the field of topological field theory [7, 8, 9, 10] and another pioneered by David B. Kaplan and collaborators using ideas of orbifolding and deconstruction [4, 5, 6]. Remarkably these two seemingly independent approaches lead to the same lattice theories – see [11, 12, 13, 14] and the recent reviews [15, 16]. This convergence of two seemingly completely different approaches leads one to suspect that the final lattice theories may represent essentially unique solutions to the simultaneous requirements of locality, gauge invariance and at least one exact supersymmetry. We will only have time to discuss the approach via topological twisting in this talk.

2. Topological twisting

Perhaps the simplest way to understand how this subalgebra emerges is to reformulate the target theory in terms of "twisted fields". The basic idea of twisting goes back to Witten in his seminal paper on topological field theory [17] but actually had been anticipated in earlier work on staggered fermions [18]. In our context the idea is decompose the fields of the theory in terms of representations not of the original (Euclidean) rotational symmetry $SO_{\rm rot}(D)$ but a twisted rotational symmetry which is the diagonal subgroup of this symmetry and an $SO_{\rm R}(D)$ subgroup of the R-symmetry of the theory.

$$SO(D)' = \mathrm{diag}(SO_{\rm Lorentz}(D) \times SO_{\rm R}(D)) \tag{1}$$

To be explicit consider the case where the total number of supersymmetries is $Q = 2^D$. In this case I can treat the supercharges of the twisted theory as a $2^{D/2} \times 2^{D/2}$ matrix q. This matrix can be expanded on products of gamma matrices

$$q = \mathcal{Q} I + \mathcal{Q}_\mu \gamma_\mu + \mathcal{Q}_{\mu\nu} \gamma_\mu \gamma_n u + \ldots \tag{2}$$

The 2^D antisymmetric tensor components that arise in this basis are the twisted supercharges and satisfy a corresponding supersymmetry algebra following from the original algebra

$$\mathcal{Q}^2 \quad = \quad 0 \tag{3}$$

[2] Significant work has gone into generalizing the Leibniz rule to finite difference operators in the context of non-commutative models using the techniques of Hopf algebras see [1, 2, 3]. This approach will not be discussed in this talk

$$\{\mathcal{Q}, \mathcal{Q}_\mu\} = p_\mu \tag{4}$$
$$\cdots \tag{5}$$

The presence of the nilpotent scalar supercharge $\mathcal{Q}$ is most important; it is the algebra of this charge that we can hope to translate to the lattice. The second piece of the algebra expresses the fact that the momentum is the $\mathcal{Q}$-variation of something which makes plausible the statement that the energy-momentum tensor and hence the entire action can be written in $\mathcal{Q}$-exact form[3]. Notice that an action written in such a $\mathcal{Q}$-exact form is trivially invariant under the scalar supersymmetry provided the latter remains nilpotent under discretization.

The rewriting of the supercharges in terms of twisted variables can be repeated for the fermions of the theory and yields a set of antisymmetric tensors $(\eta, \psi_\mu, \chi_{\mu\nu}, \ldots)$ which for the case of $Q = 2^D$ matches the number of components of a real Kähler-Dirac field. This repackaging of the fermions of the theory into a Kähler-Dirac field is at the heart of how the discrete theory avoids fermion doubling as was shown by Becher, Joos and Rabin in the early days of lattice gauge theory [19, 20].

It is important to recognize that the transformation to twisted variables corresponds to a simple change of variables in flat space – one more suitable to discretization. A true topological field theory only results when the scalar charge is treated as a true BRST charge and attention is restricted to states annihilated by this charge. In the language of the supersymmetric parent theory such a restriction corresponds to a projection to the vacua of the theory. It is *not* employed in these lattice constructions.

3. An example: 2D super Yang-Mills

This theory satisfies our requirements for supersymmetric latticization; its R-symmetry possesses an $SO(2)$ subgroup corresponding to rotations of the its two degenerate Majorana fermions into each other. Explicitly the theory can be written in twisted form as

$$S = \frac{1}{g^2}\mathcal{Q}\int \mathrm{Tr}\left(\chi_{\mu\nu}\mathcal{F}_{\mu\nu} + \eta[\overline{\mathcal{D}}_\mu, \mathcal{D}_\mu] - \frac{1}{2}\eta d\right) \tag{6}$$

The degrees of freedom are just the twisted fermions $(\eta, \psi_\mu, \chi_{\mu\nu})$ previously described and a complex gauge field $\mathcal{A}_\mu$. The latter is built from the usual gauge field and the two scalars present in the untwisted theory $\mathcal{A}_\mu = A_\mu + iB_\mu$ with corresponding complexified field strength $\mathcal{F}_{\mu\nu}$.

Notice that the original scalar fields transform as vectors under the original R-symmetry and hence become vectors under the twisted rotation group while the gauge fields are singlets under the R-symmetry and so remain vectors under twisted rotations. This structure makes possible the appearance of a complex gauge field in the twisted theory. Notice though,that the theory is only invariant under the usual $U(N)$ gauge symmetry and not its complexified cousin.

The nilpotent transformations associated with $\mathcal{Q}$ are given explicitly by

$$\begin{aligned}
\mathcal{Q}\,\mathcal{A}_\mu &= \psi_\mu \\
\mathcal{Q}\,\psi_\mu &= 0 \\
\mathcal{Q}\,\overline{\mathcal{A}}_\mu &= 0 \\
\mathcal{Q}\,\chi_{\mu\nu} &= -\overline{\mathcal{F}}_{\mu\nu} \\
\mathcal{Q}\,\eta &= d \\
\mathcal{Q}\,d &= 0
\end{aligned}$$

[3] Actually in the case of $\mathcal{N} = 4$ there is an additional $\mathcal{Q}$-closed term needed

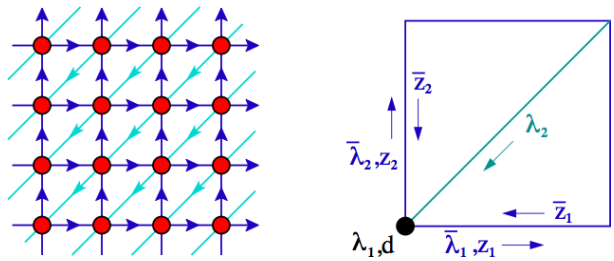

Figure 1. 2d four supercharge lattice

Performing the $\mathcal{Q}$-variation and integrating out the auxiliary field d yields

$$S = \frac{1}{g^2}\int \mathrm{Tr}\left(-\overline{\mathcal{F}}_{\mu\nu}\mathcal{F}_{\mu\nu} + \frac{1}{2}[\overline{\mathcal{D}}_\mu, \mathcal{D}_\mu]^2 - \chi_{\mu\nu}\mathcal{D}_{[\mu}\psi_{\nu]} - \eta\overline{\mathcal{D}}_\mu\psi_\mu\right) \tag{7}$$

To untwist the theory and verify that indeed in flat space it just corresponds to the usual theory one can do a further integration by parts to produce

$$S = \frac{1}{g^2}\int \mathrm{Tr}\left(-F_{\mu\nu}^2 + 2B_\mu D_\nu D_\nu B_\mu - [B_\mu, B_\nu]^2 + L_F\right) \tag{8}$$

where $F_{\mu\nu}$ is the usual Yang-Mills term. It is now clear that the imaginary parts of the gauge fields B_μ can now be given an interpretation as the scalar fields of the usual formulation. Similarly one can build spinors out of the twisted fermions and write the action in the manifestly Dirac form

$$L_F = \begin{pmatrix} \chi_{12} & \frac{\eta}{2} \end{pmatrix} \begin{pmatrix} -D_2 - iB_2 & D_1 + iB_1 \\ D_1 - iB_1 & D_2 - iB_2 \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \tag{9}$$

4. Discretization

The prescription for discretization is somewhat natural. (Complex) gauge fields are represented as complexified Wilson gauge links $\mathcal{U}_\mu(x) = e_\mu^{\mathcal{A}}(x)$ living on links of a lattice which for the moment we can think of as hypercubic. These transform in the usual way under $U(N)$ lattice gauge transformations

$$\mathcal{U}_\mu(x) \to G(x)\mathcal{U}_\mu(x)G^\dagger(x) \tag{10}$$

Supersymmetric invariance then implies that $\psi_\mu(x)$ live on the same links and transform identically. The scalar fermion $\eta(x)$ is clearly most naturally associated with a site and transforms accordingly

$$\eta(x) \to G(x)\eta(x)G^\dagger(x) \tag{11}$$

The field $\chi_{\mu\nu}$ is slightly more difficult. Naturally as a 2-form it should be associated with a plaquette. In practice we introduce diagonal links running through the center of the plaquette and choose $\chi_{\mu\nu}$ to lie *with opposite orientation* along those diagonal links. This choice of orientation will be necessary to ensure gauge invariance. Figure 1. shows the resultant lattice theory (with $\psi_\mu \to \overline{\lambda}_\mu$ and $\eta \to \lambda_1$ and $\chi_{12} \to \lambda_2$)

To complete the discretization we need to describe how continuum derivatives are to be replaced by difference operators. A natural technology for accomplishing this in the case of adjoint fields was developed many years ago and yields expressions for the derivative operator applied to arbitrary lattice p-forms [21]. In the case discussed here we need just two derivatives given by the expressions

$$\mathcal{D}_\mu^{(+)} f_\nu = \mathcal{U}_\mu(x) f_\nu(x+\mu) - f_\nu(x)\mathcal{U}_\mu(x+\nu) \tag{12}$$

$$\overline{\mathcal{D}}_\mu^{(-)} f_\mu = f_\mu(x)\overline{\mathcal{U}}_\mu(x) - \overline{\mathcal{U}}_\mu(x-\mu) f_\mu(x-\mu) \tag{13}$$

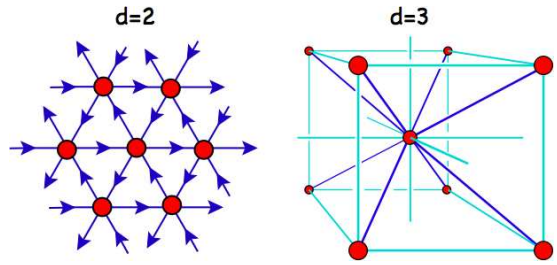

Figure 2. Other $Q = 8$ supersymmetric lattices

The lattice field strength is then given by the gauged forward difference $\mathcal{F}_{\mu\nu} = D_\mu^{(+)}\mathcal{U}_\nu$ and is automatically antisymmetric in its indices. Furthermore it transforms like a lattice 2-form and yields a gauge invariant loop on the lattice when contracted with $\chi_{\mu\nu}$. Similarly the covariant backward difference appearing in $\overline{\mathcal{D}}_\mu\mathcal{U}_\mu$ transforms as a 0-form or site field and hence can be contracted with the site field η to yield a gauge invariant expression.

This use of forward and backward difference operators guarantees that the solutions of the theory map one-to-one with the solutions of the continuum theory and hence fermion doubling problems are evaded [19]. Indeed, by introducing a lattice with half the lattice spacing one can map this Kähler-Dirac fermion action into the action for staggered fermions [22]. Notice that, unlike the case of QCD, there is no rooting problem in this supersymmetric construction since the additional fermion degeneracy is already required by the continuum theory.

Many other examples of supersymmetric lattices exist. Figure 2. shows two such lattices arising in the case of eight supercharges – a two dimensional triangular lattice and a generalized hypercubic lattice (including body and face links) in three dimensions. Notice that in all cases almost all fields live on links with the exception of a small number of fermion site fields – the number of those corrresponding to the number of exact supersymmetries preserved in the lattice theory. Furthermore, in all cases the number of fermions exactly fills out multiples of a basic Kähler-Dirac field in the corresponding number of dimensions.

5. Twisted $\mathcal{N} = 4$ super Yang-Mills

In four dimensions the constraint that the target theory possess 16 supercharges singles out a single theory for which this construction can be undertaken – $\mathcal{N} = 4$ SYM.

The continuum twist of $\mathcal{N} = 4$ that is the starting point of the twisted lattice construction was first written down by Marcus in 1995 [23] although it now plays a important role in the Geometric-Langlands program and is hence sometimes called the GL-twist [24]. This four dimensional twisted theory is most compactly expressed as the dimensional reduction of a five dimensional theory in which the ten (one gauge field and six scalars) bosonic fields are realized as the components of a complexified five dimensional gauge field while the 16 twisted fermions naturally span one of the two Kähler-Dirac fields needed in five dimensions. Remarkably, the action of this theory contains a $\mathcal{Q}$-exact piece of precisely the same form as the two dimensional theory given in eqn. 6 provided one extends the field labels to run now from one to five. In addition the Marcus twist requires a new $\mathcal{Q}$-closed term which was not possible in the two dimensional theory.

$$S_{\text{closed}} = -\frac{1}{8}\int \text{Tr}\, \epsilon_{mnpqr}\chi_{qr}\overline{\mathcal{D}}_p\chi_{mn} \tag{14}$$

The supersymmetric invariance of this term then relies on the Bianchi identity $\epsilon_{mnpqr}\mathcal{D}_p\mathcal{F}_{qr} = 0$.

The four dimensional lattice that emerges from examining the moduli space of the resulting discrete theory is called A_4^* and is constructed from the set of five basis vectors v_a pointing

out from the center of a four dimensional equilateral simplex out to its vertices together with their inverses $-v_a$. It is the four dimensional analog of the 2D triangular lattice. Complexified Wilson gauge link variables $\mathcal{U}_a$ are placed on these links together with their $\mathcal{Q}$-superpartners ψ_a. Another 10 fermions are associated with the diagonal links $v_a + v_b$ with $a > b$. Finally, the exact scalar supersymmetry implies the existence of a single fermion for every lattice site. The lattice action corresponds to a discretization of the Marcus twist on this A_4^* lattice and can be represented as a set of traced closed bosonic and fermionic loops. It is invariant under the exact $\mathcal{Q}$ scalar susy, lattice gauge transformations and a global permutation symmetry S^5 and can be proven free of fermion doubling problems as discussed before. The $\mathcal{Q}$-exact part of the lattice action is again given by eqn. 7 where the indices μ, ν now correspond to the indices labeling the five basis vectors of A_4^*.

While the supersymmetric invariance of this $\mathcal{Q}$-exact term is manifest in the lattice theory it is not clear how to discretize the continuum $\mathcal{Q}$-closed term. Remarkably, it is possible to discretize eqn. 14 in such a way that it is indeed exactly invariant under the twisted supersymmetry!

$$S_{\text{closed}} = -\frac{1}{8}\sum_{\mathbf{x}} \text{Tr}\, \epsilon_{mnpqr}\chi_{qr}(\mathbf{x}+\mu_m+\mu_n+\mu_p)\overline{\mathcal{D}}_p^{(-)}\chi_{mn}(\mathbf{x}+\mu_p) \tag{15}$$

and can be seen to be supersymmetric since the lattice field strength satisfies an exact Bianchi identity [21].

$$\epsilon_{mnpqr}\mathcal{D}_p^{(+)}\mathcal{F}_{qr} = 0 \tag{16}$$

6. Numerical results

The lattice theory may be simulated using standard algorithms and techniques drawn from lattice QCD. The first step is to integrate out the twisted fermions which results in a Pfaffian of the twisted fermion kinetic operator $\text{Pf}\left(M(\mathcal{U},\overline{\mathcal{U}})\right)$. Assuming that the phase of this operator can be neglected[4] this Pfaffian can be represented by the integral

$$\text{Pf}M(\mathcal{U},\overline{\mathcal{U}}) = \int \mathcal{D}F\mathcal{D}F^\dagger e^{-F^\dagger\left(M^\dagger M\right)^{\frac{1}{4}}F} \tag{17}$$

where the *bosonic* pseudofermion field F carries the same labels as its fermion counterpart. The fractional inverse power is then approximated to whatever accuracy is desired by a partial fraction expansion whose coefficients are determined using the remez algorithm to minimize the error over some interval in the spectrum. A standard RHMC algorithm can then be used to simulate the full partition function of the resultant theory.

6.1. Exact supersymmetry

One of the first things to check is the of course the exact supersymmetry. Table 1. compares the Monte Carlo measurements of the bosonic action against the exact value computed assuming the exact supersymmetry (the bosonic action can be derived using an exact $\mathcal{Q}$ Ward identity). Results are shown for $SU(2)$ and several lattice couplings κ for both the two dimensional model with 4 supercharges and $\mathcal{N} = 4$ super Yang-Mills in four dimensions with 16 supercharges [25].

These results confirm rather convincingly that the lattice theory does indeed possess exact supersymmetry.

[4] Numerical and analytical arguments suggest that this is the case

κ	κS_B	exact
1.0	4.40(2)	4.5
10.0	4.47(2)	4.5
100.0	4.49(1)	4.5

$\mathcal{Q} = 4$

κ	κS_B	exact
1.0	13.67(4)	13.5
10.0	13.52(2)	13.5
100.0	13.48(2)	13.5

$\mathcal{Q} = 16$

Table 1. Tests of exact SUSY - bosonic action for $SU(2)$ theory at various lattice couplings

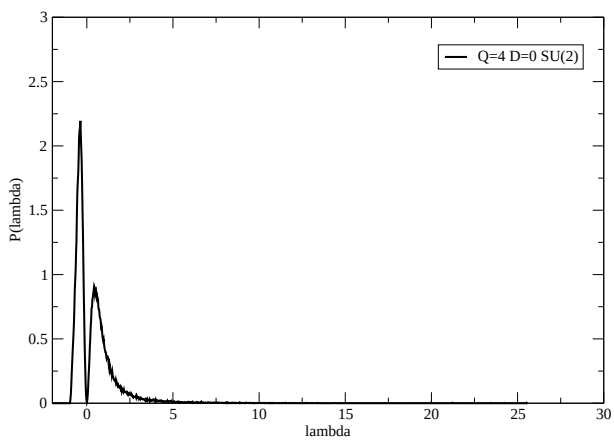

Figure 3. Distribution of eigenvalues of $\overline{\mathcal{U}}_\mu \mathcal{U}_\mu - I$ for $Q = 4$ SU(2) theory

6.2. Moduli space

Next we turn to the moduli space. In the continuum and on the lattice both the 4 and 16 supercharge theories possesses an infinite set of classical vacua corresponding to constant commuting $\mathcal{U}$ matrices. Furthermore, since the complex fields are in principle unbounded from above one might worry that the partition function would be unbounded. In the usual formulation the usual statement is that the scalar fields can run off to infinity along these flat directions. In practice we find that the partition function and low moments of the scalar field distribution are well defined in these theories. Figures 3. and 4. show the distribution of eigenvalues of the scalar fields (given by $\overline{\mathcal{U}}_\mu \mathcal{U}_\mu - 1$) for the 4 and 16 supercharge $SU(2)$ models. While one observes long power law tails particularly in the 4 supercharge case the fields remain localized around the origin in the moduli space, the partition function is finite and no instability is seen in the simulations.

6.3. Fermion eigenvalues

It is interesting to examine the eigenvalues of the fermion operator accumulated over the same set of Monte Carlo configurations used in the moduli space analysis. These are shown in figures 5. and 6. for 4 and 16 supercharges respectively.

Notice these picture show only the region close to the origin – the full spectrum extends to larger values and shows a clustering along the imaginary (vertical) axis corresponding to eigenvalues of the derivative operator appearing in the fermion kinetic terms. Nevertheless, these pictures reveal one strong difference between the 4 and 16 supercharge cases; the relative scarcity of eigenvalues close to the origin in the 16 supercharge case. The absence of a zero

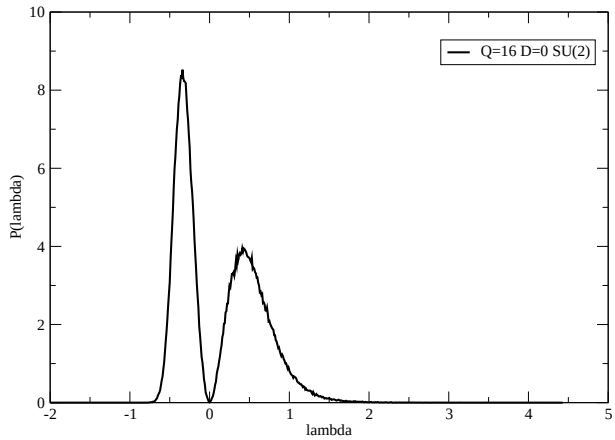

Figure 4. Distribution of eigenvalues of $\overline{\mathcal{U}}_\mu \mathcal{U}_\mu - I$ for $Q = 16$ SU(2) theory

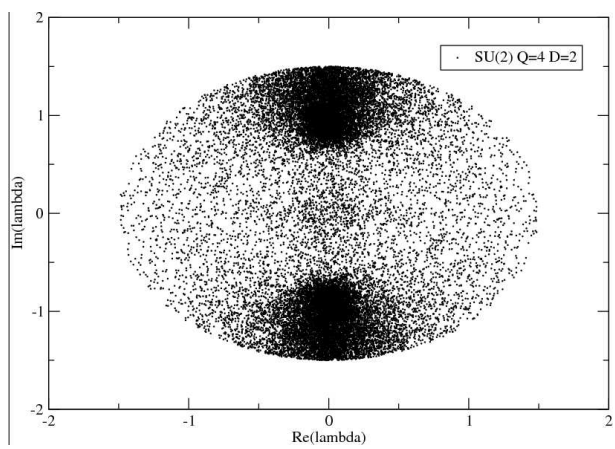

Figure 5. Scatter plot of real and imaginary parts of fermion eigenvalues for $Q = 4$ $SU(2)$ model on 2×2 lattice

eigenvalue in the fermion spectrum indicates by supersymmetry a corresponding absence of boson zero modes – those associated with the classical flat directions. In the context of the Monte Carlo this is consistent with the observation that the scalars do not penetrate far down the classical flat directions but rather correspond to a wavefunction which localizes close to the origin of moduli space. In contrast the scalars in the 4 supercharge theory can be found at relatively large distance from the origin (the tail of the eigenvalue distribution falls only as a single inverse power of the eigenvalue and thus the variance of the eigenvalues actually diverges logarithmically). At such large distances there is a near bosonic zero corresponding to translations along the flat directions and corresponding a near zero fermion eigenvalue as is seen in the spectrum. Indeed, the presence of a nearly massless fermion mode in the spectrum is a necessary condition for supersymmetry breaking where it would play the role of a Goldstino. An order parameter for supersymmetry breaking is the Witten index. The measured value of W is close to zero in this model (it can be measured via the phase of the Pfaffian in the

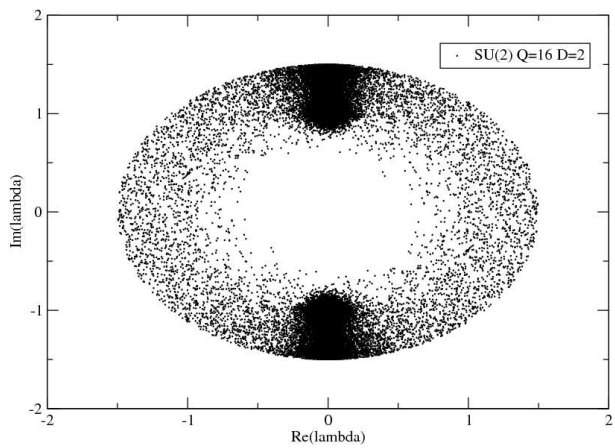

Figure 6. Scatter plot of real and imaginary parts of fermion eigenvalues for $Q = 16$ $SU(2)$ model on 2×2 lattice

phase quenched theory we simulate). Thus we conclude that our simulations yield tantalizing evidence for dynamical supersymmetry breaking in the two dimensional model. No such breaking apparently occurs for the four dimensional theory in line with expectations.

7. Prospects

One of the key issues that still remains to be explored is the question of how much residual fine tuning will be required to achieve a continuum limit in which full supersymmetry is restored. This is controlled by the flows in all relevant operators which could be induced in the effective action as a result of quantum corrections. We have used the exact lattice symmetries together with power counting to enumerate the possible set of such lattice operators.

Only four terms appear in this counter term analysis and three of these correspond to wavefunction renormalizations of kinetic terms already present in the bare lattice action. There is one additional term of the form

$$\mathcal{Q}(\eta \mathcal{U}_\mu \overline{\mathcal{U}}_\mu) \tag{18}$$

which leads to supersymmetric mass terms for the fermions and scalars. However, such a term would lead to a lifting of the classical moduli space which does not happen in the continuum theory. We have computed the effective action to one loop in the lattice theory and find that it vanishes as a consequence of exact $\mathcal{Q}$ supersymmetry just as in the continuum. Indeed, this result can be generalized to any finite order of perturbation theory using the $\mathcal{Q}$-exact property of the lattice action and indicates that such a dangerous radiative correction is *not* induced to all orders of perturbation theory [26]. This is a strong and useful result as it indicates at worst a logarithmic tuning of the other terms in the action will be needed.

Having eliminated all such mass terms the remaining question concerning the restoration of full supersymmetry then rests on whether the ratios of the coefficients to the renormalized kinetic operators flow away from their classical values as the lattice spacing is decreased. A one loop calculation is in progress which should shed light on this issue.

Beyond this issue it would be very interesting to use Monte Carlo simulation to test aspects of the AdS/CFT conjecture. Parallel code has now been developed to study $\mathcal{N} = 4$ super Yang-Mills and a program of numerical investigations of this theory is ongoing. At finite temperature

this theory and its dimensional reductions and deformations should be dual to a variety of black hole solution in supergravity – see the recent work reported in [27, 28] and complementary numerical work using non-lattice methods reported in [29, 30, 31, 32, 33, 34]. One would hope that the results of such simulations could be useful in the quest to understand how aspects of the quantum geometry can be understood in terms of the dual gauge theory.

8. Acknowledgments

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