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Isotropization in Brane Gas Cosmology

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Brane Gas Cosmology (BGC) is an approach to unifying string theory and cosmology in which matter is described by a gas of strings and branes in a dilaton gravity background. The Universe is assumed to start out with all spatial dimensions compact and small. It has previously been shown that in this context, in the approximation of neglecting inhomogeneities and anisotropies, there is a dynamical mechanism which allows only three spatial dimensions to become large. However, previous studies do not lead to any conclusions concerning the isotropy or anisotropy of these three large spatial dimensions. Here, we generalize the equations of BGC to the anisotropic case, and find that isotropization is a natural consequence of the dynamics.

I. INTRODUCTION

The Standard Big Bang (SBB) cosmology has become an extremely successful model that has been well tested by experiment. However, the model is incomplete. The underlying theory of classical general relativity and the description of matter as an ideal gas breaks down at the high temperatures of the early Universe, and the solutions of the theory in fact have an initial singularity. Moreover, SBB does not address many important cosmological questions such as the observed homogeneity, spatial flatness, and the origin of structure in the Universe. Cosmological inflation (see e.g. [1,2] for textbook treatises and [3,4] for shorter reviews) builds on SBB cosmology providing a solution to some of these issues, but it (at least in the context of scalar field-driven inflation) suffers from the same initial singularity problem [5] and other conceptual problems [4], which indicate that inflation cannot be the complete story of early Universe cosmology.

In recent years, many models motivated by string theory and M-theory have emerged as possible solutions to the outstanding problems of early Universe and inflationary cosmology (see e.g. [6,8] for recent but incomplete reviews). Beginning with the work on Pre-Big-Bang cosmology [9,10] it was realized that a dynamical dilaton should play an important role in the very early Universe. More recently, models have become prominent in which our Universe consists of a 3-brane embedded in a higher dimensional bulk space, with the standard model constrained to live on the brane [11–17]. Although these models can resolve a number of issues, such as the hierarchy problem, they introduce several other difficulties in the process. For example, large extra dimensions should be explained by classical general relativity, and it has been shown this results in problems stabilizing the brane [18]. More importantly, in most of these models the six "extra" spatial dimensions are taken to be compactified, *a priori*, with no explanation provided for how this

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could come about dynamically. Although it appears to be an important concern for the naturalness of these models, this issue is rarely discussed in the literature.

An alternative approach to string/M-theory cosmology is the string gas or BV scenario. This model began with works [19,20] in which the effects of string gases on the cosmological evolution of the low energy effective string theory background geometry including the dilaton were explored. The most important result to emerge from these works is a dynamical mechanism, tied to the existence of string winding modes, which yields a nonsingular cosmology and may explain why at most three spatial dimensions can become large if the initial state is chosen to correspond to a Universe which is small in all spatial directions.

This work has been generalized to include the cosmological effects of p-brane gases and leads to the current model of Brane Gas Cosmology (BGC) [21]. In BGC, the Universe starts analogous to the SBB picture, i.e. hot, dense, and with all fundamental degrees of freedom in thermal equilibrium. The Universe is assumed to be toroidal in all nine spatial dimensions and filled with a p-brane gas. The assumption of toroidal geometry of the background space leads to the existence of string winding modes, since the background space admits cycles on which branes of the relevant dimensionalities (in particular one branes) can wrap. This wrapping is associated with a winding energy which - in the context of dilaton gravity - acts as a confining potential for the scale factor preventing further expansion of the spatial dimensions. Also associated with the brane are oscillatory modes described by scalar fields and momentum modes which correspond to the center of mass motion of the brane. The momentum modes are related by T-duality to the winding modes and this duality results in the non-singular behavior of the model. In order for dimensions to decompactify, p-brane winding modes must annihilate with anti-winding modes and it is argued that this only occurs in a maximum of $2p+1$ dimensions [21]. Since strings ($p=1$) are the lowest dimensional objects that admit winding modes, since they are the lightest of all winding modes and hence fall out of equilibrium later than other winding modes, it follows that the number of large space-time dimensions can be at most (3+1).

In the context of the background equations of dilaton gravity, the winding modes yield a confining potential for the scale factor which also gives rise to a period of cosmological loitering (expansion rate near zero) for the three large spatial dimensions. This is due to the time needed for winding modes to annihilate and produce closed strings or loops [22]. This is of great interest since loitering can explain the horizon and relic problems of standard cosmology without resorting to inflation [1]. It was also shown in [22] that by considering loop production at late times BGC naturally evolves into the SBB, with a $3+1$ dimensional, radiation dominated Universe.

There are important issues that remain to be addressed within BGC. The fact that toroidal geometry was assumed for the background space was used for the existence and topological stability of winding modes. However, K3 or Calabi-Yau manifolds are more realistic choices for backgrounds within string theory and they do not admit 1-cycles (necessary to have topologically stable winding modes). Promising results have recently appeared which indicate that the conclusions of BGC extend to a much wider class of spatial background, including backgrounds which are K3 or Calabi-Yau manifolds [7,24].

Perhaps the main issue to be addressed in BGC is that of spatial inhomogeneities. That is, we would

[1]However, to obtain a solution of the flatness and entropy problems, a phase of inflation following the decoupling of the three large spatial dimensions may be required

naturally expect fluxes and p-brane sources to lead to the possibility of catastrophic instabilities of spatial fluctuation modes. Although we do not address this issue here, we plan to study the role of inhomogeneities in followup work.

Other important issues for BGC are the questions of stabilization of the six small extra dimensions and isotropization of the three dimensions that grow large. Although these topics may seem to be unrelated, they both can be addressed by generalizing the BGC scenario to the anisotropic case. This paper will concentrate on the isotropization of the three large dimensions, but the generalization of BGC to the anisotropic case achieved in this paper will be valuable to address the issue of stabilization in later work.

II. BRANE GAS COSMOLOGY

We begin with a brief review of BGC, for more details the reader is referred to [21]. Consider compactification of 11-dimensional M-theory on S^1, which yields type II-A string theory in 10-dimensions. The fundamental degrees of freedom in the theory are 0-branes, strings, 2-branes, 5-branes, 6-branes, and 8-branes. The low energy effective action of the theory is that of supersymmetrized dilaton gravity,

$$S_{bulk} = \frac{1}{2\kappa^2}\int d^{10}x\sqrt{-G}e^{-2\phi}\left[R + 4G^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi - \frac{1}{12}H_{\mu\nu\alpha}H^{\mu\nu\alpha}\right], \tag{2.1}$$

where G is the determinant of the background metric $G_{\mu\nu}$, ϕ is the dilaton given by the radius of the S^1 compactification, H denotes the field strength corresponding to the bulk antisymmetric tensor field $B_{\mu\nu}$, and κ is determined by the ten-dimensional Newton constant.

Fluctuations of each of the p-branes are described by the Dirac-Born-Infeld (DBI) action [25] and are coupled to the ten-dimensional action via delta function sources. The DBI action is

$$S_p \,=\, T_p\int d^{p+1}\zeta e^{-\phi}\sqrt{-det(g_{mn}+b_{mn}+2\pi\alpha' F_{mn})}\,, \tag{2.2}$$

where T_p is the tension of the brane, g_{mn} is the induced metric on the brane, b_{mn} is the induced antisymmetric tensor field, and F_{mn} is the field strength tensor of gauge fields A_m living on the brane. The constant $\alpha' \sim l_{st}^2$ is given by the string length scale l_{st}.

In this paper we will concentrate on the role of fundamental strings ($p = 1$), since strings play the critical role in the dynamics [21]. We will also ignore the effects of fluxes and string oscillatory modes since we are working in the low energy regime. Our basic approach is to consider the string gas as a matter source for the dilaton-gravity background. The string winding modes play a key role. The main point is that we want to find the first departures from the standard FRW picture resulting from considering *stringy* effects. This should result from the winding modes of the strings and the presence of the dilaton in the equations of motion.

In this paper, we generalize the equations of BGC to the anisotropic case. Thus, we take the metric to be of the form

$$ds^2 \,=\, dt^2 - \sum_{i=1}^{D} e^{2\lambda_i}(dx_i)^2\,, \tag{2.3}$$

where the label i runs over the D spatial indices, x_i are the comoving spatial coordinates, t is physical time, and the scale factor in the i'th direction is $log(\lambda_i)$. We again stress that inhomogenities are of vital importance, however we will leave their investigation to future work and take the string gas to be homogeneous.

By varying the total action we obtain the following equations,

$$-\sum_{i=1}^{D}\dot{\lambda}_i + \dot{\varphi}^2 = e^{\varphi}E \tag{2.4}$$

$$-\sum_{i=1}^{D}\dot{\lambda}_i + \ddot{\varphi} = \frac{1}{2}e^{\varphi}E \tag{2.5}$$

$$\ddot{\lambda}_i - \dot{\varphi}\dot{\lambda}_i = \frac{1}{2}e^{\varphi}P_i, \tag{2.6}$$

where E is the total energy of the strings and P_i is the pressure in the ith direction, and we have introduced a "shifted" dilaton field

$$\varphi \,=\, 2\phi - \sum_{i=1}^{D}\lambda_i\,. \tag{2.7}$$

We can see immediately from the first equation that φ can not change sign. To insure that the low energy approximation remains valid we choose $\varphi < 0$ and $\dot{\varphi} > 0$. This causes the dilaton to have a damping effect in the equations of motion [2]. Another important observation is that these equations reduce to the standard FRW equations for fixed dilaton as was discussed in [20]. This is comforting because at late times it is expected that the dilaton will gain a mass, perhaps associated with supersymmetry breaking and as a result we naturally regain the usual SBB cosmology. The string winding states will appear at late times as solitons [21].

A. Winding Modes

The energy and pressure terms follow from varying the brane action (2.2). The energy associated with the winding modes in the ith direction is given by (taking the winding strings to be straight for simplicity)

$$E_i^w = \mu N_i(t)a_i(t) = \mu N_i(t)e^{\lambda_i}, \tag{2.8}$$

where N_i is the number of winding modes in the ith direction and μ is the mass per unit length of the string multiplied by the initial spatial dimension. It follows that the total energy is given by,

$$E_T^w = \sum_{i=1}^{D}\mu N_i(t)e^{\lambda_i}. \tag{2.9}$$

[2] Alternatively we could consider $\varphi > 0$ and one might anticipate that this could lead to inflation. However this is a false conclusion, since the presence of winding modes act as a confining potential which can be seen from the negative pressure term in equation (2.6).

In the same approximation that the winding strings are straight, the corresponding pressure terms are given by [21],

$$P_i^w = -\mu N_i(t) e^{\lambda_i} \tag{2.10}$$

$$P_j^w = 0 \ \ (j \neq i)\,. \tag{2.11}$$

It follows then by inserting (2.10) into (2.6) that winding strings give rise to a confining potential for the scale factor, and hence prevent expansion. In order for a spatial dimension to be able to become large, the winding modes in that direction must be able to annihilate.

B. Annihilation and Loop Production

As argued earlier, the fact that strings are likely to annihilate in a maximum of three space dimensions leads to three of the initial nine spatial dimensions growing large. As the background continues to expand the three dimensional space will be filled with loops resulting from the annihilation of the winding states. The corresponding energy loss in winding states and the energy and pressure of the creation of loops must be taken into account when considering the dynamics. The strings in the expanding space will become of macroscopic size and the required pressure and energy terms will be analogous to that of a cosmic string network [26,27].

We know that the energy in winding modes is given by (2.9). Considering the time rate of change of this energy we find,

$$\dot{E}_T^w = \sum_{i=1}^{D} \mu \dot{N}_i(t) e^{\lambda_i} + \sum_{i=1}^{D} \mu N_i(t) e^{\lambda_i} \dot{\lambda}_i. \tag{2.12}$$

The first term on the right hand side of this equation corresponds to energy loss into string loops or radiation, the second term corresponds to the gain in total energy of the strings due to the stretching by the expansion of space.

The energy in loops can be written as [22]

$$E^{loops} = g(t) V_0, \tag{2.13}$$

where $g(t)$ is the energy density in string loops (plus radiation) per initial comoving volume (and is a constant if no loop production or energy loss from winding strings into radiation occurs), and $V_0 = \exp(\sum \lambda_{0i})$ is the initial volume, λ_{0i} being the initial values of the logarithms of the scale factors. Since sting loops are likely to be produced relativistically, we will use the equation of state of radiation to describe them. This approximation also allows us to treat the string loops and other radiation together. This approximation could easily be relaxed without changing our basic conclusions. Thus, we use the equation of state of relativistic radiation $p = (1/3)\rho$ (ρ and p denoting energy density and pressure, respectively) which implies,

$$P^{loops} = \frac{1}{3} g(t) V_0. \tag{2.14}$$

To find how $g(t)$ evolves we equate the loss in winding energy due to energy transfer (the first term on the right hand side of 2.12) with the change in loop energy,

$$-\sum_{i=1}^{D}\mu\dot{N}_i(t)e^{\lambda_i} = \dot{g}(t)V_0. \tag{2.15}$$

Thus, the loop production is determined by the change in the number of winding modes, as expected:

$$\dot{g}(t) = -V_0^{-1}\sum_{i=1}^{D}\mu\dot{N}_i(t)e^{\lambda_i}. \tag{2.16}$$

It remains to determine the rate of winding mode annihilation, but before doing so let us consider the specific case of a 2+1 anisotropic Universe.

III. ANISOTROPIC GENERALIZATION

As three of the nine spatial dimensions grow large there is no *a priori* reason to expect that this should happen in an isotropic manner. Moreover, since the dimensions expanding correspond to the annihilation of winding modes due to string intersections, we might expect this process to occur at differing rates leading to anisotropic dimensions. However, we might also expect that as the winding modes annihilate in one dimension and that dimension begins to expand faster there are then fewer winding modes left to annihilate. In this way the remaining dimensions are given an opportunity to isotropize. To explore if this is indeed the case, let us consider the case of a 2+1 anisotropic background, whose metric we write in the form

$$ds^2 \ = \ dt^2 - e^{2\lambda}(dx^2 + dy^2) - e^{2\nu}dz^2\,. \tag{3.1}$$

The scale factor corresponding to λ is denoted $a(t)$, the one corresponding to ν is denoted $b(t)$.

The equations of motion (2.4) then become,

$$\begin{aligned} -2\dot{\lambda}^2 - \dot{\nu}^2 + \dot{\varphi}^2 &= e^{\varphi}E & (3.2)\\ -2\dot{\lambda}^2 - \dot{\nu}^2 + \ddot{\varphi} &= \frac{1}{2}e^{\varphi}E & (3.3)\\ \ddot{\lambda} - \dot{\varphi}\dot{\lambda} &= \frac{1}{2}e^{\varphi}P_\lambda & (3.4)\\ \ddot{\nu} - \dot{\varphi}\dot{\nu} &= \frac{1}{2}e^{\varphi}P_\nu\,. & (3.5) \end{aligned}$$

The energy and pressure terms are given by

$$\begin{aligned} E = E_T^w + E^{loops} &= 2\mu N(t)e^{\lambda} + \mu M(t)e^{\nu} + g(t)e^{2\lambda_0+\nu_0}, & (3.6)\\ P_\lambda &= -\mu N(t)e^{\lambda} + \frac{1}{3}g(t)e^{2\lambda_0+\nu_0}, & (3.7)\\ P_\nu &= -\mu M(t)e^{\nu} + \frac{1}{3}g(t)e^{2\lambda_0+\nu_0}, & (3.8) \end{aligned}$$

where $N(t)$ and $M(t)$ are the number of winding modes in the λ and ν directions, respectively.

We now consider the effect of loop production as a result of winding mode annihilation. To simplify the analysis, we will assume that winding modes in λ and ν directions form two separate, noninteracting gases. We expect that this approximation will *reduce* the isotropization of winding modes, and hence that including the interactions we omit will lead to accelerated isotropization. This issue is being studied currently [28].

Winding mode annihilation results from string intersections, which depends [26,27] on the square of the number of modes, inversely on the cross-sectional area available for the interaction and is proportional to the Hubble length of the string. Since causality plays a role in the interaction, we must scale our dimensions by the Hubble time t. Hence,

$$\dot{N}(t) = -\tilde{c}_N N^2(t)\Big(\frac{t^2}{Area}\Big)t^{-1} = -\tilde{c}_N N^2(t)\Big(\frac{t^2}{a(t)b(t)}\Big)t^{-1} = -c_N t N^2(t) e^{-\lambda-\nu}\,, \tag{3.9}$$

where $\tilde{c}_N$ is a dimensionless constant, and c_N is this same constant rescaled by the basic length dimensions related to a and b. Similarly, for $M(t)$ we have

$$\dot{M}(t) = -c_M t M^2(t) e^{-2\lambda}. \tag{3.10}$$

Using these expressions in (2.16) gives us the evolution of loop production,

$$\dot{g}(t) = \mu t e^{-2\lambda_0-\nu}\Big(2c_N N^2(t)e^{-\nu} + c_M M^2(t) e^{-2\lambda+\nu}\Big). \tag{3.11}$$

For computational simplicity we define $l(t) \equiv \dot{\lambda}$, $q(t) \equiv \dot{\nu}$, and $f(t) \equiv \dot{\varphi}$, which leaves us with the following set of first order ordinary differential equations,

$$-2l^2 - q^2 + f^2 = e^{\varphi}\Big(2\mu N(t)e^{\lambda} + \mu M(t)e^{\nu} + g(t)e^{2\lambda_0+\nu_0}\Big) \tag{3.12}$$

$$\dot{f} = 2l^2 + q^2 + \frac{1}{2}e^{\varphi}\Big(2\mu N(t)e^{\lambda} + \mu M(t)e^{\nu} + g(t)e^{2\lambda_0+\nu_0}\Big) \tag{3.13}$$

$$\dot{l} = l\varphi + \frac{1}{2}e^{\varphi}\Big(-\mu N(t)e^{\lambda} + \frac{1}{3}g(t)e^{2\lambda_0+\nu_0}\Big) \tag{3.14}$$

$$\dot{q} = q\varphi + \frac{1}{2}e^{\varphi}\Big(-\mu M(t)e^{\nu} + \frac{1}{3}g(t)e^{2\lambda_0+\nu_0}\Big) \tag{3.15}$$

$$\dot{N}(t) = -c_N t N^2(t)e^{-\lambda-\nu} \tag{3.16}$$

$$\dot{M}(t) = -c_M t M^2(t)e^{-2\lambda} \tag{3.17}$$

$$\dot{g}(t) = \mu t e^{-2\lambda_0-\nu}\Big(2c_N N^2(t)e^{-\nu} + c_M M^2(t)e^{-2\lambda+\nu}\Big)V_0^{-1} \tag{3.18}$$

$$l(t) = \dot{\lambda} \tag{3.19}$$

$$q(t) = \dot{\nu} \tag{3.20}$$

$$f(t) = \dot{\varphi}. \tag{3.21}$$

The system (3.12-3.21) is over-determined (there are more equations than unknowns). Therefore we take (3.12) as a constraint on the initial conditions. We find from numerical analysis that the winding modes $N(t)$ and $M(t)$ approach zero as the system evolves in time, as expected from winding mode annihilation (see Fig. 1). As a result loop production continues until all the winding modes are annihilated at which time $g(t)$ approaches a constant (see Fig. 2). Note from Figure 1 that before converging to zero, the two winding numbers converge to eachother. This is a necessary condition for isotropization. This result seems reasonable, since the rate of annihilation depends on the number of winding states present. Thus, as the number of winding modes decreases in one direction, the other direction should have an larger rate of annihilation. This suggests a sort of equilibration process due to the presence of winding modes.

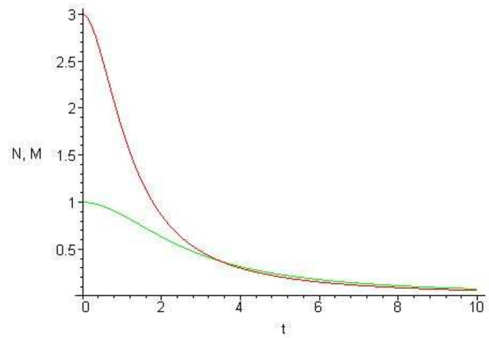

FIG. 1: A plot of the numbers N (upper curve for small values of t) and M (lower curve) as a function of time t. We see that the number of winding modes approaches zero as winding annihilation continues.

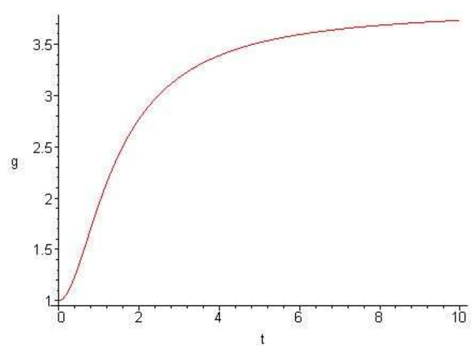

FIG. 2: A plot of the comoving energy density in loops g as a function of time t. As the winding modes annihilate, loop production continues until all the winding modes have vanished and the loop energy becomes constant.

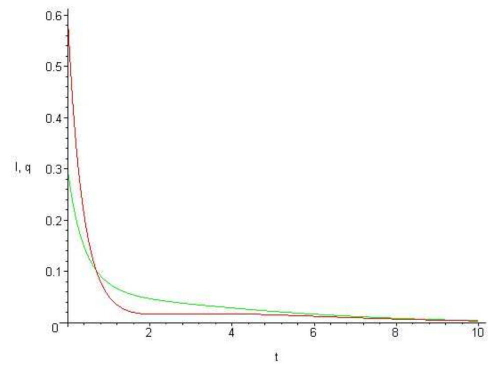

FIG. 3: A plot of the two expansion rates l (upper curve for small values of t) and q as a function of time t. It is seen that as the winding modes annihilate, the expansion rates converge to each other before converging to zero.

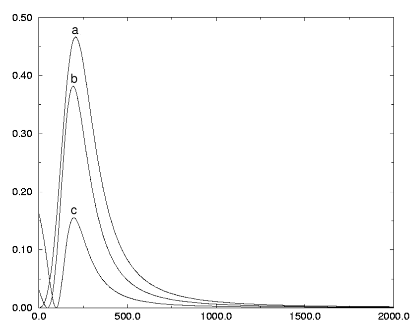

FIG. 4: A plot of the anisotropy parameter $\mathcal{A}$ as a function of time t for a range of initial anisotropies. Curve a represents no initial anisotropy, but varying winding numbers. Curve b has an initial anisotropy of $A = 0.025$ and curve c has $A = 0.160$. We see that the anisotropy parameter reaches a maximum early on in the evolution and in all cases tends to zero at late times.

However, the above alone is not enough evidence to prove that isotropization of the dimensions occurs. To prove that isotropization occurs we need to study the geometry of the background. We can see from Figure

3 that the expansion rates, $l(t)$ and $q(t)$, converge at late times. Note, however, that at the same time the ratio of scale factors continues to increase. A more quantitative definition of isotropization can be obtained by defining the average *Hubble parameter* $\bar{\lambda}$ and the *anisotropy parameter* $\mathcal{A}$ [29],

$$\bar{\lambda} \equiv \frac{1}{D}\sum_{i=1}^{D}\lambda_i \tag{3.22}$$

$$\mathcal{A} \equiv \frac{1}{D}\sum_{i=1}^{D}\frac{(\lambda_i - \bar{\lambda})^2}{\bar{\lambda}^2}. \tag{3.23}$$

In our case we have $D = 3$ and the anisotropy parameter becomes,

$$\mathcal{A} = 2\frac{(l(t) - q(t))^2}{(2l(t) + q(t))^2}. \tag{3.24}$$

We find that for any amount of initial anisotropy, $\mathcal{A}$ approaches a maximum value and then goes to zero, as can be seen from Figure 4. Furthermore, for the case of isotropic initial expansion but inequivalent winding mode numbers we find that $\mathcal{A}$ reaches a larger maximum but the conclusion of isotropization at later times is unchanged.

IV. CONCLUSION

We have generalized the equations of BGC to the anisotropic case including the effects of winding state annihilation and loop production. We address the issue of isotropization of the three large dimensions quantitatively by introducing the anisotropy parameter $\mathcal{A}$. Our analysis indicates that for an arbitrary amount of initial anisotropy, the anisotropy will reach a maximum early in the evolution and then approach zero at later times. Thus, we have explained how isotropy arises as a natural consequence of BGC.

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