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September 10, 1994

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Abstract

In the predicate calculus, variables provide a flexible indexing service that selects the actual arguments to a predicate letter from among possible arguments that precede the predicate letter (in the parse of the formula). In the process of selection, the possible arguments can be permuted, repeated (used more than once), and skipped. If this service is withheld, so that arguments must be the immediately preceding ones, taken in the order in which they occur, the formula is said to be *fluted*. Quine showed that if a fluted formula contains only homogeneous conjunction (conjoins only subformulas of equal arity), then the satisfiability of the formula is decidable. It remained an open question whether the satisfiability of a fluted formula without this restriction is decidable. This paper answers that question.

1 Introduction

In 1960, in "Variables explained away" [6], Quine presented his Predicate Functor Logic (PFL), a system equivalent to predicate logic, but without variables. Quine sought to explicate the notion of *variable* by carefully delineating the roles that variables play in predicate logic. He did this by introducing predicate functors that provided the various services normally provided by variables.

Quine returned to PFL in a number of his papers and books in the following years (e.g., [8, 9, 10]). The set of predicate functors varied in different versions of PFL. One could try to make do with as few as possible, or try to make the functors individually as simple as possible. A set that achieves the latter goal is the following.

$\exists$ (crop), $\neg$, $\wedge$, perm, Perm, pad, ref

This set falls naturally into two subsets:

(i) the alethic functors, $\exists$, $\neg$, $\wedge$; and

(ii) the combinatory functors, perm, Perm, pad, ref.

The formulas (or schemas) that can be formed using only predicate letters and the alethic functors were named *fluted* formulas by Quine. In 1969 in "On the limits of decision" [7], Quine showed that if the fluted formulas are restricted to conjoin only subformulas of the same arity (called *homogeneous* conjunction), then their satisfiability is decidable. However the method used (an extension of the method used by Herbrand to show monadic logic decidable) breaks down when the restriction on conjunction is relaxed (see Noah [3]). It remained an open question whether satisfiability of unrestricted fluted formulas is decidable.

This paper answers the latter question in the affirmative.

2 Preliminaries

This paper assumes the usual definition of the pure predicate calculus. The set of predicate symbols typically will be defined by some given finite set of formulas or *premises*. The finite set of predicate symbols will be referred to as the *lexicon*. Let L be a lexicon and $R \in L$. Then $ar(R)$ denotes the arity of R. Define $ar(L) := max\{ar(R) : R \in L\}$. Without loss of generality, we will assume $ar(L) > 0$.

A standard result from predicate calculus is given here without proof.

THEOREM 1 (The Principle of Monotonicity) *Let θ be a subformula, not in the scope of $\neg$, that occurs as a conjunct in formula ϕ. Then ϕ' can be inferred from ϕ, where ϕ' is obtained from ϕ by deleting θ.*

The empty conjunction is defined to be equivalent to $\top$ (verum).

An *interpretation* $\mathcal{I}$ of a lexicon L consists of a set $\mathcal{D}$, the *domain* of $\mathcal{I}$, and a mapping that assigns to each $R \in L$ a subset of $\mathcal{D}^{ar(R)}$. If ϕ is a formula over L with free variables among $\{x_1, \ldots, x_k\}$, and ϕ is satisfied in $\mathcal{I}$ by the assignment to variables $\{x_i \mapsto a_i\}_{1 \leq i \leq k}$, we write $a_1 \cdots a_k \models \phi$.

3 Fluted formulas

Let L be a finite set of predicate symbols. Let $X_m := \{x_1, \ldots, x_m\}$ be a set of m variables where $m \geq 0$. An *atomic fluted formula* of L over X_m is $Rx_{m-n+1} \cdots x_m$, where $R \in L$ and $ar(R) = n \leq m$. The set of all atomic fluted formulas of L over X_m will be denoted $Af_L(X_m)$. Define $Af_L(X_0) := \{\top\}$.

A *fluted formula* of L over X_m is defined inductively.

(i) An atomic fluted formula of L over X_m is a fluted formula of L over X_m.

(ii) If ϕ is a fluted formula of L over X_m, then $\exists x_m \phi$ and $\forall x_m \phi$ are fluted formulas of L over X_{m-1}.

(iii) If ϕ and ψ are fluted formulas of L over X_m, then $\phi \wedge \psi$, $\phi \vee \psi$, $\phi \rightarrow \psi$, and $\neg\phi$ are fluted formulas of L over X_m.

This definition can be generalized as follows. Call the fluted formulas just defined *standard* fluted formulas. Now any formula that is alpha-equivalent to a standard fluted formula is defined to be a fluted formula.

The fluted formulas of L are a proper subset of the formulas of the pure predicate calculus with predicate symbols L. The semantics of the fluted formulas of L is defined to coincide with the usual semantics of the pure predicate calculus. In connection with standard fluted formulas, $abc \cdots \models \phi$ will always mean that ϕ is satisfied (in the interpretation given by the context) by the assignment to variables $\{x_1 \mapsto a, x_2 \mapsto b, x_3 \mapsto c, \ldots\}$.

It might be noted in passing that in the predicate calculus restricted to fluted formulas, it would be possible to dispense with variables entirely, since the arity and position of a predicate symbol completely determines the sequence of variables that follow the predicate symbol. However, variables will be retained to make the presentation more familiar and more explicit.

4 Fluted constituents

The set of conjunctions in which for each $\rho \in Af_L(X_m)$ either ρ or $\neg\rho$ (but not both) occurs as a conjunct will be denoted $\Delta Af_L(X_m)$ (cf. Rantala [11]). Note that if $\Delta Af_L(X_m) = \{\theta_1, \ldots, \theta_l\}$, and ϕ is any quantifier-free formula over $Af_L(X_m)$, then

(i) $\neg(\theta_i \wedge \theta_j)$ for $i \neq j$,

(ii) $\theta_1 \vee \cdots \vee \theta_l$, and

(iii) either $\theta_i \rightarrow \phi$, or $\theta_i \rightarrow \neg\phi$, for $1 \leq i \leq l$

are tautologies.

Let $\mathbf{N}$ be the natural numbers, and $\mathbf{N}^*$ the set of finite strings over $\mathbf{N}$. String concatenation is denoted by juxtaposition. The empty string is ε. If $\alpha = i_1 \cdots i_n \in \mathbf{N}^*$, then for $k \leq n$, $(k : \alpha) := i_1 \cdots i_k$ is the k-prefix of α. We define a (balanced) tree domain $\mathcal{T} \subseteq \mathbf{N}^*$ with a height function h as follows. $w(\alpha)$ is the number of immediate descendants of α.

(i) $\varepsilon \in \mathcal{T}$ and $h(\varepsilon) = 0$.

(ii) If $\alpha \in \mathcal{T}$, then $\alpha 1, \ldots, \alpha w(\alpha) \in \mathcal{T}$ and $h(\alpha 1) = \cdots = h(\alpha w(\alpha)) = h(\alpha) + 1$.

(iii) If $\alpha, \beta \in \mathcal{T}$ and $w(\alpha) = w(\beta) = 0$, then $h(\alpha) = h(\beta) =: h(\mathcal{T})$.

If $w(\alpha) = 0$, then α is *terminal* in $\mathcal{T}$. If $0 < h(\alpha) < h(\mathcal{T})$, then α is *internal* in $\mathcal{T}$. The subtree of $\mathcal{T}$ rooted on α will be denoted $(\alpha]$. The path in $\mathcal{T}$ from ε to α will be denoted $[\alpha)$.

Let $\mathcal{T}$ be a tree domain. The labelled tree domain $\mathcal{T}_L$ is defined to be $\mathcal{T}$ with a formula $\theta_\alpha \in \Delta Af_L(X_{h(\alpha)})$ associated with each $\alpha \in \mathcal{T}$. The subtree of $\mathcal{T}_L$ rooted on α will be denoted $(\theta_\alpha]$. The path in $\mathcal{T}_L$ from ε to α will be denoted $[\theta_\alpha)$. The subtree $(\theta_\alpha]$ is given the following interpretation.

(i) If α is terminal, then $(\theta_\alpha]$ denotes θ_α.

(ii) If α is nonterminal with height k, then $(\theta_\alpha]$ denotes $\theta_\alpha \wedge \exists x_{k+1}(\theta_{\alpha 1}] \wedge \cdots \wedge \exists x_{k+1}(\theta_{\alpha w(\alpha)}] \wedge \forall x_{k+1}((\theta_{\alpha 1}] \vee \cdots \vee (\theta_{\alpha w(\alpha)}])$.

The formula denoted by $(\theta_\alpha]$ is a *fluted constituent of L of height* $h(\mathcal{T}) - h(\alpha)$ *over the variables* $X_{h(\alpha)}$. If $h(\alpha) = 0$, the formula denoted by $(\theta_\alpha]$ is a *constituent sentence.*

The path $[\theta_\alpha)$ denotes $\theta_\varepsilon \wedge \theta_{1:\alpha} \wedge \theta_{2:\alpha} \wedge \cdots \wedge \theta_\alpha$. In the remainder of this paper, $(\theta_\alpha]$ and $[\theta_\alpha)$ will not be distinguished from the formulas they denote. If $\theta_\varepsilon = \neg\top$, then $\mathcal{T}_L$ is *trivial.* In the remainder of this paper, $\mathcal{T}_L$ will always be assumed to be nontrivial. Under this assumption, θ_ε can usually be elided.

Let $\alpha \in \mathcal{T}$ and $\theta_\alpha \in \Delta Af_L(X_{h(\alpha)})$. Define $g(\alpha) := max(1, 1 + h(\alpha) - ar(L))$. Then the variables occurring in θ_α are precisely $x_{g(\alpha)}, \ldots, x_{h(\alpha)}$.

If ϕ is a constituent or path, then define:

(i) $\phi^{[-k]}$ is ϕ with the last k variables eliminated;

(ii) $\phi_{[-k]}$ is ϕ with the first k variables eliminated.

Here elimination of a variable is accomplished by removing all atomic formulas in which that variable occurs, as well as the quantifier, if any, associated with that variable.

If ϕ is a fluted formula (including tree and path), containing occurrences of variables $x_l, \ldots, x_k$, then $\phi^\dagger := \phi\{x_l \mapsto x_1, \ldots, x_k \mapsto x_{k-l+1}\}$ is the *standardization* of ϕ.

Fluted constituents are related to Hintikka constituents of the second kind (see [11]). Indeed, the main results for Hintikka constituents hold for fluted constituents.

THEOREM 2 (The Fundamental Property of Constituents) *If ϕ and ψ are fluted constituents of L of height k over the variables X_l, and $\phi \neq \psi$, then $\phi \wedge \psi$ is inconsistent.*

proof: See [11], Theorem 3.10 (i). □

THEOREM 3 *Let ϕ be a standard fluted formula of L containing variables X_m, where variables $X_k \subseteq X_m$ are free. Then ϕ is logically equivalent to a disjunction of fluted constituents of height $m-k$ over X_k.*

proof: See [11], Theorem 4.1. □

5 Trivial inconsistency

Note that if ϕ is a constituent sentence, $\phi \to \phi^{[-k]}$ and $\phi \to \phi_{[-k]}$ by the Principle of Monotonicity. Hence $\phi \to (\phi^{[-k]} \wedge \phi_{[-k]})$. Moreover, $\phi^{[-k]}$ and $\phi_{[-k]}$ are constituent sentences of the same height. It follows from the Fundamental Property of Constituents, that either $\phi^{[-k]}$ and $\phi_{[-k]}$ are identical (up to possible repetition of constituents, order of conjunction and disjunction, and alpha-equivalence), or ϕ is inconsistent. In the latter case, ϕ is said to be *trivially inconsistent* (cf. Hintikka [1, 2]).

Let $\mathcal{T}_L$ be a fluted constituent of height h, and suppose that $\mathcal{T}_L$ is not trivially inconsistent. Assume further that $ar(L) > 1$. ($ar(L) = 1$ yields monadic logic, the decidability of which is well-known.) These assumptions impose a significant constraint on the syntax of $\mathcal{T}_L$. Two properties arising from this constraint, which will be used in Section 6, are described next.

The first property is that the constituent $\mathcal{T}_L^{[-h+1]}$ is 'embedded' in every elementary subtree of $\mathcal{T}_L$. Precisely stated, for any nonterminal $\alpha \in \mathcal{T}$,

$$\{([\theta_{\alpha j})_{[-h(\alpha)]})^{\dagger} : 1 \leq j \leq w(\alpha)\} = \{\theta_j : 1 \leq j \leq w(\varepsilon)\}.$$

If this property fails, then for some $\alpha \in \mathcal{T} : \mathcal{T}_{L[-h(\alpha)]} \not\leftrightarrow \mathcal{T}_L^{[-h(\alpha)]}$, in which case $\mathcal{T}_L$ is trivially inconsistent.

The second property is that elementary subtrees are 'repeated' throughout $\mathcal{T}_L$ according to a certain pattern. This property is precisely stated as follows. For any internal $\alpha \in \mathcal{T}$, $\exists! \gamma \in \mathcal{T}$, such that $h(\gamma) < h(\alpha)$ and

(i) $[\theta_\gamma) = ([\theta_\alpha)_{[-g(\alpha)]})^{\dagger}$, and

(ii) $\{[\theta_{\gamma j}) : 1 \leq j \leq w(\gamma)\} = \{([\theta_{\alpha j})_{[-g(\alpha)]})^{\dagger} : 1 \leq j \leq w(\alpha)\}$.

If this property fails, then for some $\alpha \in \mathcal{T} : \mathcal{T}_{L[-g(\alpha)]} \not\leftrightarrow \mathcal{T}_L^{[-g(\alpha)]}$, in which case $\mathcal{T}_L$ is trivially inconsistent.

6 Satisfiability of fluted constituents

Every fluted formula can be expressed as a disjunction of fluted constituents of sufficient depth of the lexicon of that formula. Therefore, the question of satisfiability of a fluted formula reduces to the question of satisfiability of a fluted constituent. The following theorem, which provides a decision procedure for the latter question, is the main result of the paper.

THEOREM 4 *A fluted constituent is unsatisfiable iff it is trivially inconsistent.*

proof: The 'if' direction is obvious. The 'only-if' direction is proved in its contrapositive form. Let $\mathcal{T}_L$ be a fluted constituent of height h, and assume that $\mathcal{T}_L$ is not trivially inconsistent. We first define an interpretation $\mathcal{I}$ of L in the domain $\mathcal{D} := \{a_\alpha : (\alpha \in \mathcal{T}) \wedge (\alpha \neq \varepsilon)\}$. Then we show that $\mathcal{I}$ satisfies $\mathcal{T}_L$.

It suffices to interpret the $\theta_\alpha \in \mathcal{T}_L$, since this fixes a unique interpretation of the elements of L. $\mathcal{I}$ is defined in two parts. First, a basis for the definition is given as follows.

For each $\alpha \in \mathcal{T}$, define $a_{1:\alpha} \cdots a_\alpha \models \theta_\alpha$.

It follows that for each $\alpha \in \mathcal{T}$, $a_{1:\alpha} \cdots a_\alpha \models [\theta_\alpha)$.

Second, the basis is extended inductively, ordered by height. The following property is to be maintained by this induction.

If $h(\alpha) = k$ and $a_{\beta_1} \cdots a_{\beta_k} \models [\theta_\alpha)$, then

(i) $\forall a_\beta \in \mathcal{D} : a_{\beta_1} \cdots a_{\beta_k} a_\beta \models \theta_{\alpha 1} \vee \cdots \vee \theta_{\alpha w(\alpha)}$, and

(ii) for $1 \leq j \leq w(\alpha) : \exists a_\beta \in \mathcal{D} : a_{\beta_1} \cdots a_{\beta_k} a_\beta \models \theta_{\alpha j}$.

For the first step, $k = 0$, we extend the interpretation of θ_j, where $1 \leq j \leq w(\varepsilon)$, as follows. For each $\beta \in \mathcal{T}$, define $a_\beta \models \theta_j$ iff $([\theta_\beta)_{[-h(\beta)+1]})^\dagger = \theta_j$. Since $\mathcal{T}_L$ is not trivially inconsistent, $\forall \beta \in \mathcal{T} : (([\theta_\beta)_{[-h(\beta)+1]})^\dagger = \theta_1) \vee \cdots \vee (([\theta_\beta)_{[-h(\beta)+1]})^\dagger = \theta_{w(\varepsilon)})$.

This follows from the first property given in Section 5. Hence $\forall a_\beta \in \mathcal{D} : a_\beta \models (\theta_1 \vee \cdots \vee \theta_{w(\varepsilon)})$. From the basis, $\exists a_\beta : a_\beta \models \theta_j$. Thus the inductive property holds for $k = 0$.

Proceeding inductively, let $h(\alpha) = k > 0$ and consider $\theta_{\alpha j}$, where $1 \leq j \leq w(\alpha)$. From the basis, $a_{1:\alpha} \cdots a_\alpha \models [\theta_\alpha)$. We extend the interpretation of $\theta_{\alpha j}$ as follows. For each $\beta \in \mathcal{T}$, if

(i) $a_{1:\alpha} \cdots a_\alpha a_\beta \models [\theta_{\alpha j})_{[-g(\alpha)]}$, and

(ii) $1 \leq l \leq w(\alpha)$ implies $a_{1:\alpha} \cdots a_\alpha a_\beta \not\models \theta_{\alpha l}$,

then define $a_{1:\alpha} \cdots a_\alpha a_\beta \models \theta_{\alpha j}$. Note that if (ii) fails for $l = j$, then the extension under consideration has already been made; if (ii) fails for $l \neq j$, then the extension under consideration cannot be made without introducing inconsistency. Now since $a_{1:\alpha} \cdots a_\alpha \models [\theta_\alpha)$, it follows that $a_{1:\alpha} \cdots a_\alpha \models [\theta_\alpha)_{[-g(\alpha)]}$, since $[\theta_\alpha) \rightarrow [\theta_\alpha)_{[-g(\alpha)]}$. Hence $a_{1:\delta} \cdots a_\delta \models ([\theta_\alpha)_{[-g(\alpha)]})^\dagger$, where δ is the suffix of α defined $\alpha = (g(\alpha) : \alpha)\delta$. Since $\mathcal{T}_L$ is not trivially inconsistent, by the second property given in Section 5, $\exists! \gamma \in \mathcal{T}$, such that $h(\gamma) < k$ and

(i) $[\theta_\gamma) = ([\theta_\alpha)_{[-g(\alpha)]})^\dagger$, and

(ii) $\{[\theta_{\gamma j}) : 1 \leq j \leq w(\gamma)\} = \{([\theta_{\alpha j})_{[-g(\alpha)]})^\dagger : 1 \leq j \leq w(\alpha)\}$.

Therefore, $a_{1:\delta} \cdots a_\delta \models [\theta_\gamma)$. By the inductive property,

(i) $\forall a_\beta \in \mathcal{D} : a_{1:\delta} \cdots a_\delta a_\beta \models \theta_{\gamma 1} \vee \cdots \vee \theta_{\gamma w(\gamma)}$, and

(ii) for $1 \leq j \leq w(\gamma) : \exists a_\beta \in \mathcal{D} : a_{1:\delta} \cdots a_\delta a_\beta \models \theta_{\gamma j}$.

But then

(i) $\forall a_\beta \in \mathcal{D} : a_{1:\delta} \cdots a_\delta a_\beta \models ([\theta_{\alpha 1})_{[-g(\alpha)]})^\dagger \vee \cdots \vee ([\theta_{\alpha w(\alpha)})_{[-g(\alpha)]})^\dagger$, and

(ii) for $1 \leq j \leq w(\alpha) : \exists a_\beta \in \mathcal{D} : a_{1:\delta} \cdots a_\delta a_\beta \models ([\theta_{\alpha j})_{[-g(\alpha)]})^\dagger$.

That is,

(i) $\forall a_\beta \in \mathcal{D} : a_{1:\alpha} \cdots a_\alpha a_\beta \models [\theta_{\alpha 1})_{[-g(\alpha)]} \vee \cdots \vee [\theta_{\alpha w(\alpha)})_{[-g(\alpha)]}$, and

(ii) for $1 \leq j \leq w(\alpha) : \exists a_\beta \in \mathcal{D} : a_{1:\alpha} \cdots a_\alpha a_\beta \models [\theta_{\alpha j})_{[-g(\alpha)]}$.

From the definition of the extension given above,

(i) $\forall a_\beta \in \mathcal{D} : a_{1:\alpha} \cdots a_\alpha a_\beta \models \theta_{\alpha 1} \vee \cdots \vee \theta_{\alpha w(\alpha)}$, and

(ii) for $1 \leq j \leq w(\alpha) : \exists a_\beta \in \mathcal{D} : a_{1:\alpha} \cdots a_\alpha a_\beta \models \theta_{\alpha j}$.

Notice that the inductive property now holds for the 'point' $a_{1:\alpha} \cdots a_\alpha$.

Finally, this extension is copied to other points that satisfy $[\theta_\alpha)$ as follows. Suppose $a_{\beta_1} \cdots a_{\beta_k} \models [\theta_\alpha)$. For each $\beta \in \mathcal{T}$, if $a_{1:\alpha} \cdots a_\alpha a_\beta \models \theta_{\alpha j}$, then define $a_{\beta_1} \cdots a_{\beta_k} a_\beta \models \theta_{\alpha j}$. Thus the inductive property holds at height k.

This concludes the definition of the interpretation $\mathcal{I}$. It remains to prove that $\mathcal{I}$ satisfies $\mathcal{T}_L$. The proof is by induction on the depth $d = h - k$, where k is the height of $\alpha \in \mathcal{T}$. The induction hypothesis is:

if depth of α is $< d$, and $a_{\beta_1} \cdots a_{\beta_k} \models [\theta_\alpha)$, then $a_{\beta_1} \cdots a_{\beta_k} \models (\theta_\alpha]$.

For the basis step, $d = 0$, θ_α is at height h. Here $(\theta_\alpha] = \theta_\alpha$, and so the induction hypothesis is trivially true.

For the induction step, $d > 0$, θ_α is at height $k = h - d$. Suppose $a_{\beta_1} \cdots a_{\beta_k} \models [\theta_\alpha)$. By the inductive property,

(i) $\forall a_\beta \in \mathcal{D} : a_{\beta_1} \cdots a_{\beta_k} a_\beta \models \theta_{\alpha 1} \vee \cdots \vee \theta_{\alpha w(\alpha)}$, and

(ii) for $1 \leq j \leq w(\alpha) : \exists a_\beta \in \mathcal{D} : a_{\beta_1} \cdots a_{\beta_k} a_\beta \models \theta_{\alpha j}$.

By the induction hypothesis, if $a_{\beta_1} \cdots a_{\beta_k} a_\beta \models [\theta_{\alpha j})$, then $a_{\beta_1} \cdots a_{\beta_k} a_\beta \models (\theta_{\alpha j}]$. Therefore,

(i) $\forall a_\beta \in \mathcal{D} : a_{\beta_1} \cdots a_{\beta_k} a_\beta \models (\theta_{\alpha 1}] \vee \cdots \vee (\theta_{\alpha w(\alpha)}]$, and

(ii) for $1 \leq j \leq w(\alpha) : \exists a_\beta \in \mathcal{D} : a_{\beta_1} \cdots a_{\beta_k} a_\beta \models (\theta_{\alpha j}]$.

Thus $a_{\beta_1} \cdots a_{\beta_k} \models (\theta_\alpha]$. $\square$

COROLLARY 5 *If a fluted constituent of L of height h is satisfiable, it is satisfiable in a finite domain, whose cardinality is bounded above by $2^{h\ card(L)}$.*

If ϕ is a fluted formula, Theorem 3 states that ϕ is equivalent to the disjunction of its constituents. Moreover, the proof of Theorem 3 provides an effective method of transforming ϕ into the disjunction of its constituents. Obviously ϕ is satisfiable iff one of its constituents is satisfiable. Theorem 4 states that a constituent is satisfiable iff it is not trivially inconsistent. Trivial inconsistency can be decided by a finite number of tests on the syntax of the constituent. Theorems 3 and 4 therefore yield the following conclusion.

THEOREM 6 *The satisfiability of a fluted formula is decidable.*

7 Discussion

Theorem 6 locates the boundary between decidable and undecidable logic more precisely than heretofore, putting fluted logic on the same side as monadic logic and homogeneous fluted logic. Quine's conjecture that PFL (and general quantification theory) gets its 'escape velocity' from the combinatory functors is given further support.

But fluted logic may have an importance beyond its relation to the limits of decidability. It may be related to natural language in a way that sheds light on natural language reasoning.

Natural language does not contain variables. When inter-sentence linking is required, anaphoric pronouns are used, but these cannot be considered simply as variables (see Purdy [5] and references cited there). This observation has inspired a number of variable-free formal languages, whose syntax is designed to closely parallel that of natural language (e.g., Suppes [14], Sommers [12], Purdy [4]). However, to match the expressive power of predicate calculus, they incorporate devices equivalent to the combinatory functors of PFL, and thereby deviate from natural language.

It was noted (in Section 3) that variables play no essential role in fluted formulas, even though fluted formulas are deprived of the services of the combinatory functors. Moreover, it appears that much of natural language reasoning is conducted within the constraints of fluted logic. Many examples can be found in [12]. Even the infamous Schubert's Steamroller (Stickel [13]) can be stated in fluted formulas. The most complex premise is:

> Every animal either likes to eat all plants or all animals much smaller than itself that like to eat some plants.

This can be rendered by the fluted sentence:

$$\forall x_1(Ax_1 \rightarrow (\forall x_2(Px_2 \rightarrow Ex_1x_2) \vee \forall x_2((Ax_2 \wedge Mx_1x_2 \wedge \forall x_3(Px_3 \wedge Ex_2x_3)) \rightarrow Ex_1x_2)))$$

Perhaps it is no coincidence that fluted logic falls close to or at the boundary of decidability. If this intuition is correct, one can expect to find that there exists a reasonably efficient decision procedure for satisfiability of fluted formulas.

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