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A hydrodynamic approach to the Bose-Glass transition

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Abstract

Nonlinear hydrodynamics is used to evaluate disorder-induced corrections to the vortex liquid tilt modulus for *finite* screening length and arbitrary disorder geometry. Explicit results for aligned columnar defects yield a criterion for locating the Bose glass transition line at all fields.

Key words: Vortex matter, hydrodynamics, Bose glass, tilt modulus.

The vortex phase diagram of high temperature superconductors shows a rich diversity of phases, including vortex lattices, liquids and glasses. Novel types of glasses are also possible because of pinning in disordered samples (for a review, see Refs. [1–3]). Much progress in the theoretical understanding of vortex behavior at low fields has been made by employing the formal analogy of the statistical mechanics of $(2 + 1)$ -dimensional directed lines with the quantum mechanics of $2d$ bosons.[4] In this mapping, the vortex lines traversing the sample along the direction z of the external field, $\mathbf{H}_0 = \hat{\mathbf{z}}H_0$, correspond to the imaginary-time world lines of the quantum particles.[4] The thickness, L , of the superconducting sample is the inverse temperature, $\beta_B\hbar$, of the bosons, the vortex line tension, $\tilde{\epsilon}_1$, represents the boson mass, m , and thermal fluctuations in the vortex state $\propto k_B T$ map onto quantum fluctuations $\propto \hbar$ in the boson system. This “boson mapping” has been particularly useful for understanding the properties of vortices in superconducting samples with aligned damage tracks from heavy-ion irradiation. This type of disorder yields a low-temperature Bose-glass phase where every vortex is trapped on a columnar defect [5,6] and vortex pinning is strongly enhanced.[7] The transition at T_{BG} from the entangled vortex liquid to the Bose-glass is continuous and is signalled by the vanishing of the linear resistivity and of the inverse tilt modulus, c_{44}^{-1} .

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The drawback of the boson mapping in the form used by Nelson and coworkers [4,6,8] is that intervortex interactions are assumed to be strictly *local* in z , the magnetic field direction. This restricts the application of the results to low fields. It also renders problematic the evaluation of the tilt modulus c_{44} , which measures the linear response to a transverse tilting field \mathbf{H}_\perp normal to \mathbf{H}_0 . The long-wavelength tilt modulus of the vortex liquid can be written as $c_{44} = c_{44}^v + B^2/(4\pi)$, where $c_{44}^v = n_0\tilde{\epsilon}_1$, with $n_0 = B/\phi_0$ the vortex density, is the single vortex part and the second term represents a compressive contribution. By assuming local interactions along z , the compressive part of c_{44} , which dominates at high fields, is neglected entirely. With this approximation, the mathematical analogy between vortices and $2d$ bosons can be exploited further to show that the inverse tilt modulus maps onto the superfluid density, n_s , of the bosons, with

$$\frac{n_0^2}{c_{44}^v} = \frac{n_s}{m}. \quad (1)$$

The transition as $T \rightarrow T_{BG}^+$ from the entangled vortex liquid to the Bose glass corresponds then to the transition from a boson superfluid to a localized normal phase of bosons. Täuber and Nelson have evaluated perturbatively the reduction of n_s due to various types of disorder.[8] They also showed that for aligned columnar disorder such a perturbative calculation yields a useful criterion for locating the transition line $T_{BG}(B)$ at low fields.

Larkin and Vinokur [9] argued that a generalization of Eq. (1) that incorporates the compressive part of c_{44} can be obtained by using the *nonlocal* mapping on vortex lines onto $2d$ bosons introduced some time ago by Feigel'man and collaborators.[10] These authors showed that the fully nonlocal London model of interacting vortex lines can be mapped to a system of $2d$ *charged* bosons coupled to a massive photon field. The duality between vortices and bosons translates into

$$c_{44} = \frac{B^2}{4\pi} + \frac{n_0^2\tilde{\epsilon}_1}{n_s}, \quad (2)$$

where n_s is defined here in terms of the polarization function of the fictitious gauge field. Near a Bose glass transition where $n_s \rightarrow 0$ the second term of Eq. (2) dominates and results in the divergence of the tilt modulus. Feigel'man et al. carried out a perturbative calculation of the reduction of n_s from intervortex interactions in a clean material (in the limit $\lambda \rightarrow \infty$) [10], but the calculation of the reduction of n_s from disorder in the context of the charged boson model is cumbersome and does not provide much physical insight.

An alternative approach for modeling interacting vortex arrays is hydrodynamics, which has proved very useful to describe the long wavelength proper-

ties of flux-line liquids at high fields.[11] Hydrodynamics provides a physically transparent formulation that naturally incorporates the nonlocality of the intervortex interaction. The goal of this note is to show how hydrodynamics can be used in a transparent way to evaluate disorder-induced corrections to the wave-vector dependent tilt modulus for finite values of the screening length λ and arbitrary disorder geometry. The connection of the hydrodynamic formulation to the charged boson formalism will also be discussed. Explicit results are presented for aligned columnar defects.

The hydrodynamic free energy of the flux-line liquid is a functional of two coarse-grained fields, the local areal density of vortices, $n(\mathbf{r})$, and the tilt field, $\mathbf{t}(\mathbf{r})$, which measures the local deviation of a volume of flux liquid from the direction of the external field, \mathbf{H}_0 . [12] It is given by $F = F_0 + F_D$, where F_0 is the free energy in the absence of disorder and F_D describes the coupling to quenched defects. The disorder-free contribution is

$$F_0 = \int_{\mathbf{r}} \frac{\tilde{\epsilon}_1 |\mathbf{t}(\mathbf{r})|^2}{2n(\mathbf{r})} + \frac{1}{2n_0^2} \int_{\mathbf{q}} \left\{ c_{44}^{c0}(\mathbf{q}) |\mathbf{t}(\mathbf{q})|^2 + c_{11}^0(\mathbf{q}) |\delta n(\mathbf{q})|^2 \right\}, \quad (3)$$

where $\delta n(\mathbf{r}) = n(\mathbf{r}) - n_0$. The density and tilt field are related by the familiar constraint that flux lines cannot start nor stop inside the sample,

$$\partial_z n(\mathbf{r}) + \nabla_{\perp} \cdot \mathbf{t}(\mathbf{r}) = 0. \quad (4)$$

The bare elastic constants, c_{44}^{c0} (the compressive part of the tilt modulus) and c_{11}^0 (the bare compressional modulus), are determined by the intervortex interaction. For isotropic materials they are simply $c_{44}^{c0}(\mathbf{q}) = c_{11}^0(\mathbf{q}) = (B^2/4\pi)/(1 + q^2\lambda^2)$, with λ the screening length.[13] Quenched disorder from material defects couples to the flux-line density and gives a contribution

$$F_D = \int_{\mathbf{r}} V_D(\mathbf{r}) \delta n(\mathbf{r}). \quad (5)$$

The random potential $V_D(\mathbf{r})$ is taken to be Gaussian, statistically homogeneous, and isotropic in the xy plane so that

$$\overline{\delta V_D(\mathbf{q}) \delta V_D(\mathbf{q}')} = \Delta(q_{\perp}, q_z) \Omega \delta_{\mathbf{q}_{\perp} + \mathbf{q}'_{\perp}, 0}, \quad (6)$$

where $\delta V_D(\mathbf{r}) = V_D(\mathbf{r}) - \overline{V_D(\mathbf{r})}$, and the overbar represents the disorder average. The correlator $\Delta(q_{\perp}, q_z)$ depends on the geometry of disorder and will be specified below for the case of interest.

The hydrodynamic free energy F_0 goes beyond the Gaussian hydrodynamic model commonly used in the vortex literature [11] as the first term on the right

hand side of Eq. (3), describing the “kinetic energy” part of the vortex interaction, incorporates non-Gaussian terms. In a recent publication we showed that this non-Gaussian hydrodynamics is precisely equivalent to the charged boson model of Feigel’man and coworkers.[10] Such a “nonlinear hydrodynamics” was used in Ref. [12] to evaluate perturbatively the enhancement of c_{44} from interactions in a clean material. It was shown there that nonlocality is crucial to yield a correction to c_{44} that remains finite for $L \rightarrow \infty$. The same model is used here to evaluate corrections to c_{44} from disorder. In particular, for aligned columnar defects our calculation yields a criterion for locating the transition line $T_{BG}(B)$ *at all fields*.

The tilt modulus, c_{44} , can be expressed in terms of the tilt field autocorrelation function, $T_{ij}(\mathbf{q}) = \langle t_i(\mathbf{q})t_j(-\mathbf{q}) \rangle$, where the brackets denote a thermal average with weight $\sim \exp[-(F_0 + F_D)/k_B T]$, to be carried out subject to the constraint (4). It is given by

$$\frac{n_0^2 k_B T}{c_{44}} = \lim_{q_z \rightarrow 0} \lim_{q_\perp \rightarrow 0} P_{ij}^T(\hat{\mathbf{q}}_\perp) T_{ij}(\mathbf{q}), \quad (7)$$

with $P_{ij}^T(\hat{\mathbf{q}}_\perp) = \delta_{ij} - \hat{q}_{\perp i} \hat{q}_{\perp j}$ the familiar transverse projection operator, and $\hat{\mathbf{q}}_\perp = \mathbf{q}_\perp / q_\perp$. When only quadratic terms in the fluctuations δn and \mathbf{t} are retained, the resulting Gaussian free energy, F_G , is $F_G = F_{0G} + F_D$, where F_{0G} is obtained from Eq. (3) by the replacement $|\mathbf{t}|^2/n(\mathbf{r}) \rightarrow |\mathbf{t}|^2/n_0$. The first two terms on the RHS of Eq. (3) can then be combined to define a bare tilt modulus $c_{44}^0 = n\tilde{\epsilon}_1 + c_{44}^{0c}$ and Eq. (7) is simply an identity. To Gaussian order there is no coupling between the density field and the transverse part of the tilt field that determines c_{44} . As a result, disorder, which couples to the density, does not change the tilt modulus. Non-Gaussian terms in the hydrodynamic free energy of Eq. (3) do introduce a coupling between transverse tilt and density, yielding a renormalization of the tilt modulus. Specifically, the free energy is written as $F = F_G + \delta F$, with

$$\delta F = - \int_{\mathbf{r}} \frac{\tilde{\epsilon}_1 |\mathbf{t}(\mathbf{r})|^2}{2n_0} \frac{\delta n(\mathbf{r})}{n(\mathbf{r})}. \quad (8)$$

By treating the non-Gaussian part δF perturbatively, we obtain

$$\frac{n_0^2}{c_{44}} = \frac{n_0^2}{c_{44}^0} \left[1 - \frac{\tilde{\epsilon}_1/n_0}{c_{44}^0/n_0^2} \frac{n_n}{n_0} \right], \quad (9)$$

with

$$\frac{n_n}{n_0} = \frac{1}{n_0^2} \int_{\mathbf{q}} \left\{ \left[1 - \frac{\overline{|\mathbf{t}(\mathbf{q})|^2}_G}{2n_0 \tilde{\epsilon}_1 k_B T} \right] \frac{\overline{|\delta n(\mathbf{q})|^2}_G}{\langle |\delta n(\mathbf{q})|^2 \rangle_G} - \frac{|\langle \hat{\mathbf{q}}_\perp \cdot \mathbf{t}(\mathbf{q}) \delta n(-\mathbf{q}) \rangle_G|^2}{2n_0 \tilde{\epsilon}_1 k_B T} \right\}. \quad (10)$$

The brackets $\overline{\langle \dots \rangle_G}$ in Eq. (10) denote a thermal average with weight $\sim \exp[-F_G/k_B T]$, subject to the constraint (4), followed by the average over quenched disorder. The correction has been denoted by n_n because it has the suggestive interpretation of a normal-fluid density. Notice that its expression is formally identical to that obtained in [12] in the absence of disorder. Disorder only enters here in the expressions for the Gaussian correlators, which can be found for instance in Ref. [12]. A more general expression for the renormalized tilt modulus at finite wave vector can also be obtained by the same methods.[14] In the dilute limit, $\lambda \ll a$, with $a = 1/\sqrt{n_0}$ the intervortex spacing, the z -nonlocality becomes insignificant and Eq. (10) reduces to the result obtained for instance by Täuber and Nelson using the local boson mapping.[8]

We now focus on the case of correlated disorder created by heavy ion irradiation yielding rectilinear damage tracks aligned with the field \mathbf{H}_0 . This corresponds to $\Delta(\mathbf{q}) = \Delta L \delta_{q_z, 0}$, where $\Delta \approx U_0^2 b^4 / d^2$, with U_0 the depth of the pinning potential well, b its radius, and d the average distance between columnar defects.[6] In this case the disorder contribution to n_n from disorder takes a particularly simple form, given by

$$\frac{n_n^D}{n_0} = -\frac{\Delta}{2\tilde{\epsilon}_1^2} \int_{\mathbf{q}_\perp} \frac{q_\perp^4}{[\epsilon_B(q_\perp)/k_B T]^4} G(q_\perp), \quad (11)$$

where $G(q_\perp) = 1 + c_{44}^0(q_\perp, q_z = 0)/c_{44}^0(q_\perp, q_z = 0)$, and $\epsilon_B(q_\perp)$ corresponds to the spectrum of $2d$ bosons with screened interactions,

$$\frac{\epsilon_B(q_\perp)}{k_B T} = \sqrt{\frac{n_0 q_\perp^2 V(q_\perp)}{\tilde{\epsilon}_1} + \left(\frac{k_B T q_\perp^2}{2\tilde{\epsilon}_1}\right)^2}, \quad (12)$$

with $V(q_\perp) = V_0/(1 + \tilde{\lambda}_\perp^2 q_\perp^2)$ the screened boson interaction and $V_0 = \phi_0^2/4\pi$.

The function $G(q_\perp)$ depends only weakly on q_\perp , varying between 1 and 2, and is the only manifestation of the z -nonlocality of the intervortex interaction. If we let $G(q_\perp) = 1$ and neglect the screening, i.e., $V(q_\perp) = V_0$, our result becomes identical to the disorder-induced renormalization of $1/c_{44}$ obtained by others. [15,8] As expected, for the case of aligned columnar defects the z -nonlocality is not very important. The screening of the boson interaction incorporated in $V(q_\perp)$ is, however, important at high vortex densities. This becomes clear by plotting the boson spectrum as a function of wave vector, as shown in Fig. 1. At high density the screening of the interaction yields a flat region in the spectrum. The wave vector integral in Eq. (11) is dominated by $q_\perp \sim k_{BZ}$. At low density the main contribution to the integral comes therefore from a region where the spectrum is phonon-like, i.e., $\epsilon_B(q_\perp)/(k_B T) \approx q_\perp \sqrt{n_0 \tilde{V}(q_\perp, T)}/\tilde{\epsilon}_1$, as obtained from the theory of uncharged boson superfluids.[16] At high density, however, the main contribution to the

integral comes from a region of wave vectors where the spectrum is plasmon-like, i.e., $\epsilon_B(q_\perp)/(k_B T) \approx \sqrt{4\pi n_0}$, as appropriate for a charged superfluid. This was actually recognized earlier by Larkin and Vinokur, who in order to use the results of the local boson theory at high fields proposed an ad hoc formula for the Bogoliubov spectrum that captures the dense liquid physics by interpolating between these two limits.[9] Hydrodynamics naturally provides a simple and unified description of the behavior of flux-line liquids in the presence of disorder that applies over a wide range of densities.

As discussed in Ref. [12], the perturbation theory breaks down at high fields and temperatures ($k_B T/\tilde{\epsilon}_1 a > 1$). An approximate expression for the tilt moduls of the form proposed by Larkin and Vinokur [9] can be obtained by treating the nonlinearities in a mean field approximation, as done by Feigel'man and collaborators for the charged boson superfluid.[10] The tilt-tilt autocorrelation function is evaluated by retaining only transverse fields and using a Hartree-type approximation for the non-Gaussian terms.[14] This yields

$$\frac{n_0^2}{c_{44}} \approx \frac{1}{c_{44}^{c_0}/n_0^2 + \tilde{\epsilon}_1/(n_0 - n_n)}, \quad (13)$$

with n_n given approximately by Eq. (10).

The Bose glass transition line $B_{BG}(T)$ can now be evaluated as the locus of points in the (B, T) plane where $n_n^D/n_0 = 1$, corresponding to $c_{44} \rightarrow \infty$. In general the integral in Eq. (11) has to be evaluated numerically. The resulting $B_{BG}(T)$ phase line is shown in Fig. 2. Analytical results can be obtained in the limit of low and high field. For a dilute liquid ($\lambda \gg a$), the screening of the interaction can be neglected and the Bogoliubov spectrum can be approximated as linear in q_\perp . The integral can then be evaluated and gives

$$B_{BG}(T) \approx B_\phi \frac{\ln(1/n\lambda^2)}{8\pi^2} \left(\frac{T^*}{T}\right)^4 \sim \left(\frac{T_c - T}{T}\right)^4, \quad (14)$$

where $B_\phi = \phi_0/d^2$ is the matching field and $k_B T^* = b(\tilde{\epsilon}_1 U_0)^{1/2}$ is a characteristic pinning energy scale, in agreement with earlier work by other authors.[15,9,8] In dense liquids ($\lambda \gg a$), the Bogoliubov spectrum is approximately q_\perp -independent. Again, the transition line can be obtained analytically, with

$$B_{BG}(T) \approx B_\phi \frac{U_0 b^2}{64\pi\tilde{\epsilon}_1 d^2} \left(\frac{T^*}{T}\right)^6 \sim \left(\frac{T_c - T}{T}\right)^6, \quad (15)$$

which agrees with the earlier result of Larkin and Vinokur[9].

Hydrodynamics provides a physically transparent framework for evaluating disorder-induced corrections to the elastic constants of the vortex liquid. It naturally incorporates the full nonlocality of the intervortex interaction and it allows us to compute finite-wavevector elastic constants for arbitrary disorder geometry. In particular, the nonlocality in the field (z) direction is expected to be important for splayed columnar defects, where terms coupling disorder directly to the tilt field may need to be incorporated in the coarse-grained theory. This work will be presented elsewhere.[14]

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Figure Captions

Fig. 1. The boson spectrum $\epsilon^* = [\epsilon_B(q_\perp)/(k_B T k_{BZ})]^2$ versus wavevector $K = q_\perp/k_{BZ}$ for $n_0\lambda^2 = 0.1$ (solid line) and $n_0\lambda^2 = 10$. (dashed line).

Fig. 2. The Bose glass transition line as obtained from the condition $n_n^D/n_0 = 1$. The vertical axis is $B^* = B_{BG}(T)/B_\phi$ and the horizontal axis is $t = T/T_c$. The parameter values used correspond to $B_\phi = 2.3T$, $b = 20\text{\AA}$, and $T^* = 0.7T_c$ at $T = 0$. The transition line is well approximated by $B_{BG} \sim 1/T^6$ as in Eq. (15) for all but extremely low fields. The dashed line represents the the mean field H_{c2} .



