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Decision Fusion in a Wireless Sensor Network with a Large Number of Sensors

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Abstract – For a wireless sensor network (WSN) with a large number of sensors, a decision fusion rule using the total number of detections reported by local sensors for hypothesis testing, is proposed and studied. Based on a signal attenuation model where the received signal power decays as the distance from the target increases, the system level detection performance, namely probabilities of detection and false alarms, are derived and calculated. Without the knowledge of local sensors’ performances and at low signal to noise ratio (SNR), this fusion rule can still achieve very good system level detection performance if the number of sensors is sufficiently large. The problem of designing an optimum local sensor level threshold is investigated. For various system parameters, the optimal thresholds are found numerically. Guidelines on selecting the optimal local threshold have been presented.

Keywords: Wireless sensor networks, distributed detection, decision fusion, signal attenuation model.

1 Introduction

Wireless sensor networks (WSN) have gained much attention recently. Usually a WSN consists of a large number of low-cost and low-energy sensors, which are deployed in the environment to collect observations and pre-process the observations. Each sensor node has its own communication capability to communicate with other sensor nodes or the central node (fusion center) via a wireless channel. Normally, there is a fusion center that fuses data from sensors and forms a global situational assessment. Due to their high flexibility, surveillance coverage, robustness and cost effectiveness, WSNs are very suitable for battlefield surveillance and environment monitoring.

One of the most important tasks a WSN needs to perform is target detection, typically in a distributed manner. There are already numerous papers in the literature on the conventional distributed detection (decision fusion) problem. In [1, 2], optimum fusion rules have been obtained under the conditional independence assumption. Decision fusion with correlated observations has been studied in [3, 4, 5, 6]. There also exist many papers dedicated to the problem of distributed detection with constrained system resources [7, 8, 9, 10, 11, 12, 13]. Specifically, these papers have proposed solutions to optimal bit allocation (or sensor selection) given a constraint on the total amount of communications.

However, most of these results are based on the assumption that the local sensors’ performances are known. For a dynamic target and passive sensors, it is very hard to estimate local sensors’ performances via experiments because these performances are time-varying as the target moves. Even if the local sensors can somehow estimate their detection performances in real time, it will be very expensive to transmit them to the fusion center. For a WSN with a very large number of inexpensive sensors, it is important to limit the communication within the WSN to save precious system resources (both energy and bandwidth). Hence, the knowledge of the local sensors’ performances can not be taken for granted in a WSN and a fusion rule that does not require local sensors’ performances is highly preferable.

In this paper, we propose a fusion rule that uses the total number of detections (“1”s) transmitted from local sensors as the statistic. Based on the assumption that the signal power decays as a function of the distance from the target, we will analyze this detector’s performance at the system (fusion center) level. In addition, we will give guidelines on how to choose an optimum threshold at the local sensor to maximize system level probability of detection.

In Section 2, basic assumptions of the WSN are made and the signal attenuation model is provided. In Section 3, the fusion rule based on the total number of local detections is proposed. Analytical methods to derive and calculate the system level detection performance are presented in Section 4. In addition, the asymptotic detection performance is studied. Simulation results are also provided to confirm our analyses. The problem of designing an optimum local sensor level threshold is investigated in Section 5. There, the optimum thresholds for varying system parameters are found numerically. Conclusions and some suggestions for future work are discussed in Section 6.

2 Problem Formulation

As shown in Fig. 1, a total of N sensors are randomly deployed in the region of interest (ROI), which is a square with area $a^2$. The locations of sensors are unknown to the WSN, but it is assumed that they are i.i.d. and follow a uniform distribution in the ROI:

$$f(x_i, y_i) = \frac{1}{a^2} (-\frac{a}{2} \leq x_i, y_i \leq \frac{a}{2}) \quad (1)$$

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$$f(x_i, y_i) = \frac{1}{a^2} (-\frac{a}{2} \leq x_i, y_i \leq \frac{a}{2}) \quad (1)$$
for \( i = 1, \cdots, N \), where \((x_i, y_i)\) are the coordinates of sensor \( i \).

Because the noise has unit variance, it is evident that the SNR at local sensor \( i \) is

\[
\text{SNR}_i = \frac{\alpha_i^2}{1 + \alpha d_i^n} = \frac{\frac{P_0}{1 + \alpha d_i^n}}{1 + \alpha d_i^n}
\]

We define the SNR at distance zero as

\[
\text{SNR}_0 = 10 \log_{10} (P_0)
\]

Assume that all the local sensors use the same threshold \( \tau \) to make a decision, or equivalently make a two-level quantization. The threshold \( \tau \) and the false alarm rate \( p_{fa} \) satisfy the following relationship:

\[
p_{fa} = \int_{\tau}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt = Q(\tau)
\]

or

\[
\tau = Q^{-1} (p_{fa})
\]

where \( Q(\cdot) \) is the complementary distribution function of the standard Gaussian, i.e.,

\[
Q(x) = \int_{x}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt
\]

The probability of detection at local sensor \( i \) is

\[
p_{d_i} = \int_{\tau}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt = Q(\tau - \sqrt{\frac{P_0}{1 + \alpha d_i^n}})
\]

### 3 Decision Fusion

We denote the binary data from local sensor \( i \) as \( I_i = \{0, 1\} \). \( I_i \) takes the value 1 when there is a detection; otherwise, it takes 0. The data from all the sensors are denoted as \( \mathbf{I} = \{I_i, i = 1, \cdots, N\} \). After collecting data \( \mathbf{I} \), the fusion center makes a final decision about a target’s presence or absence.

We know the optimal decision fusion rule is the Chair-Varshney fusion rule [1], and its statistic is:

\[
\Lambda_1 = \sum_{i=1}^{N} \left[ I_i \ln \frac{p_{d_i}}{p_{fa_i}} + (1 - I_i) \ln \frac{1 - p_{d_i}}{1 - p_{fa_i}} \right]
\]

The probability of false alarm at each sensor is known \((p_{fa_i} = p_{fa})\) from (9), as long as we know the threshold \( \tau \). However, at each sensor, it is very difficult to calculate \( p_{d_i} \) and \( p_{fa} \) since according to (11), \( p_{d_i} \) is decided by each sensor’s distance to the target and the amplitude of the target’s signal. An alternative scheme is that each sensor transmits raw data \( s_i \) to the fusion center, and the fusion center will make a decision based on these raw measurements. However, the transmission of raw data will be very expensive especially for a typical WSN with very limited energy and bandwidth. Therefore, it is desirable to transmit only binary data to the fusion center. Without the knowledge of \( p_{d_i} \), the fusion center is forced to treat every sensor equally. An intuitive
choice is to use the total number of “1”s as a statistic since the information about which sensor reports a “1” is of little use to the fusion center.

If we set \( p_d = p_f \) and \( p_f = p_f \) in (12) and assume that \( p_d > p_f \), the statistic \( \Lambda_1 \) in (12) can be simplified as

\[
\Lambda_2 = \sum_{i=1}^{N} I_i
\]  

(13)

This result coincides with our intuition. The fusion rule at the fusion center is therefore:

\[
\Lambda_2 = \sum_{i=1}^{N} \frac{H_1}{H_0} I_i \geq T
\]  

(14)

So the hypothesis testing problem is to first count the number of detections made by local sensors and then compare it with a threshold \( T \).

4 Performance Analysis

In this section, the system performances, namely the probability of false alarm \( P_{fa} \) and probability of detection \( P_d \) at the fusion center will be derived.

4.1 Calculation of \( P_{fa} \)

At the fusion center level, the probability of false alarm \( P_{fa} \) is

\[
P_{fa} = P_r\{ \Lambda_2 = \sum_{i=1}^{N} I_i \geq T | H_0 \}\]

(15)

Obviously, under hypothesis \( H_0 \), the total number of detections \( \Lambda_2 = \sum_{i=1}^{N} I_i \) follows a Binomial \( (N, p_{fa}) \) distribution. Therefore, for a given threshold \( T \), the false alarm rate can be calculated as the following:

\[
P_{fa} = \sum_{i=1}^{N} \binom{N}{i} p_{fa}^i (1 - p_{fa})^{N-i}
\]  

(16)

When \( N \) is large enough, \( P_{fa} \) or Eq. (16) can be calculated by using Laplace-DeMoivre approximation [15]:

\[
P_{fa} \approx Q \left( \frac{T - N p_{fa}}{\sqrt{N p_{fa}(1 - p_{fa})}} \right)
\]  

(17)

4.2 Calculation of \( P_d \)

To obtain the ROC curve, for a given \( T \), we also need the corresponding \( P_d \) value. Because of the nature of this problem, different local sensors will have different \( p_d \), which is a function of \( d_i \) as shown in (11). Therefore, under hypothesis \( H_1 \), the total number of detections \( \Lambda_2 \) no longer follows a Binomial distribution. It is very difficult to derive an analytical expression for the distribution of \( \Lambda_2 \). Instead, we will obtain the \( P_d \) either through approximation by using Central Limit Theorem (CLT) or through simulation.

We know that \( I_i \) follows a Bernoulli distribution with \( p_d \) as its probability of success. Because \( \{ I_1, \ldots, I_N \} \) are mutually independent, when \( N \) is large enough, according to CLT [15], the distribution function of \( \Lambda_2 \) approaches a Gaussian distribution with mean \( \eta \) and variance \( \sigma^2 \), where

\[
\eta = \frac{1}{N} \sum_{i=1}^{N} E\{ I_i \} = \sum_{i=1}^{N} p_d
\]  

(18)

and

\[
\sigma^2 = \frac{1}{N} \sum_{i=1}^{N} \text{var}\{ I_i \} = \sum_{i=1}^{N} p_d (1 - p_d)
\]  

(19)

Remember that \( p_d \) is a function of \( x_i, y_i, x_t, y_t \). When \( N \) is large, the summation in (18) can be approximated by:

\[
\eta(x_t, y_t) = \sum_{i=1}^{N} p_d(x_i, y_i, x_t, y_t)
\]  

\[
\approx \sum_{i=1}^{N} E\{ p_d(x_i, y_i, x_t, y_t) \}
\]  

(20)

where

\[
p_d(x, y, x_t, y_t) = \frac{1}{\alpha^2} \int_{-\frac{\alpha}{2}}^{\frac{\alpha}{2}} \int_{-\frac{\alpha}{2}}^{\frac{\alpha}{2}} p_d(x, y, x_t, y_t) dx dy
\]  

(21)

and

\[
Q\left( \tau - \sqrt{\frac{1}{1 + \alpha ((x - x_t)^2 + (y - y_t)^2)^{\frac{1}{2}}} \right) = 0
\]  

(22)

Similarly,

\[
s^2(x_t, y_t) = N \sigma^2(x_t, y_t)
\]

\[
= \frac{1}{\alpha^2} \int_{-\frac{\alpha}{2}}^{\frac{\alpha}{2}} \int_{-\frac{\alpha}{2}}^{\frac{\alpha}{2}} (1 - p_d(x, y, x_t, y_t)) \times p_d(x, y, x_t, y_t) dx dy
\]  

(23)

Therefore,

\[
P_d(x_t, y_t) = P_r\{ \Lambda_2 \geq T \}
\]

\[
\approx Q\left( \frac{T - \eta(x_t, y_t)}{\sigma(x_t, y_t)} \right)
\]  

(24)

Assuming that the target’s location follows a uniform distribution within the ROI, the average \( P_d \) is

\[
P_d = \frac{1}{\alpha^2} \int_{-\frac{\alpha}{2}}^{\frac{\alpha}{2}} \int_{-\frac{\alpha}{2}}^{\frac{\alpha}{2}} P_d(x_t, y_t) dx_t dy_t
\]  

(25)
We can make some reasonable assumptions to further simplify the calculation. We assume that the ROI is very large and the signal power of the target decays very fast as the distance increases (\(\alpha\) is large). Based on these assumptions, only within a very small fraction of the ROI, which is the area surrounding the target, the received signal power is significantly larger than the area surrounding the target, the received signal power is much smaller. Hence, the system level threshold is in the form of

\[
P_d = \frac{T - N\hat{P}_a}{\sqrt{N\sigma^2}}
\]

where

\[
\hat{P}_a = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \rho_a(x, y, 0, 0) \, dx \, dy
\]

and

\[
\rho_a(x, y) \approx \rho_a(0, 0)
\]

(26)

(27)

(28)

(29)

(30)

(31)

4.3 Evaluation of System Performance via Simulations

The system level \(P_d\) and \(P_{fa}\) can also be estimated by simulations. In Figs. 2 and 3, the receiver operative characteristic (ROC) curve obtained by using approximations in Section 4.1 and 4.2 and that obtained by simulations are plotted. The simulation results are based on \(10^4\) runs. From Figs. 2 and 3, it is clear that the results by using approximations are very close to those obtained by simulations. Note that as discussed in Section 4.2, the border effect of the ROI is ignored in both the calculation and the simulation.

Fig. 2: ROC curves obtained by calculation and simulations. \(N = 1000, n = 2, \alpha = 100, \alpha = 200, \text{ and } \tau = 0.1\).

Fig. 3: ROC curves obtained by calculation and simulations. \(n = 4, \alpha = 100, \alpha = 4, SNR_0 = 27dB\) (or \(P_0 = 500\)), and \(\tau = 0.1\).

4.4 Asymptotic Analysis

Assume that the system level threshold is in the form of \(T = \beta N\). From (17), we have

\[
P_{fa} = Q\left(\frac{\beta - \hat{P}_{fa}}{\sqrt{N\sigma^2}}\right)
\]

(32)

Similarly, from (31), we have

\[
P_d = Q\left(\frac{\beta - \hat{P}_a}{\sqrt{N\sigma^2}}\right)
\]

(33)
Therefore, when $N \rightarrow \infty$, if $\beta < p_{fa}$, $P_{fa} = P_d = Q(-\infty) = 1$; if $p_{fa} < \beta < \tilde{\beta}_d$, $P_{fa} = Q(\infty) = 0$ and $P_d = Q(-\infty) = 1$; if $\beta > \tilde{\beta}_d$, $P_{fa} = P_d = Q(\infty) = 0$. As a result, as long as $\beta$ takes a value between $p_{fa}$ and $\tilde{\beta}_d$, as the sensor number $N$ goes to $\infty$, the WSN’s detection performance will be perfect with $P_d = 1$ and $P_{fa} = 0$.

In Figs. 4 and 5, $P_d$ and $P_{fa}$ as functions of the total number of sensors $N$ are plotted. It is clear that the $P_d$ converges to 1 as $N$ increases and $P_{fa}$ converges to 0. In this example, we set $\beta = \frac{p_{fa} + \tilde{\beta}_d}{2}$. From Figs. 4 and 5, the asymptotic results also show that when there are enough sensors in the ROI ($N$ is large), even though the $SNR_0$ is small, the system can still achieve a very good detection performance.

![Figure 4](image4.png)

**Fig. 4:** System level $P_d$ as a function of $N$. $n = 2, \alpha = 100, \alpha = 200$, and $\tau = 0.12$.

![Figure 5](image5.png)

**Fig. 5:** System level $P_{fa}$ as a function of $N$. $n = 2, \alpha = 100, \alpha = 200$, and $\tau = 0.12$.

5 Determination of the Threshold for Local Sensors

In the above sections, we have assumed that the threshold $\tau$ (or equivalently $p_{fa}$) is given. However, this is a parameter that can be designed to achieve a better system level performance.

Assume that we are using a Neyman-Pearson detector at the fusion center. From (17), for a given system level $P_{fa}$, we have

$$T = \frac{Q^{-1}(P_{fa})\sqrt{Np_{fa}(1-p_{fa})} + Np_{fa}}{\sqrt{\sigma^2}}$$

Substituting (34) into (31), it follows that

$$P_d = Q\left(\frac{Q^{-1}(P_{fa})\sqrt{p_{fa}(1-p_{fa})} + \sqrt{N(p_{fa} - \tilde{\beta}_d)}}{\sqrt{\sigma^2}}\right)$$

From (9), (28) and (30), it is clear that $p_{fa}$, $\tilde{\beta}_d$ and $\sigma^2$ are functions of the local sensor threshold $\tau$. Therefore, $P_d$ is a function of $\tau$:

$$P_d(\tau) = Q(C(\tau))$$

where

$$C(\tau) = \frac{Q^{-1}(P_{fa})\sqrt{p_{fa}(1-p_{fa})} + \sqrt{N(p_{fa} - \tilde{\beta}_d)}}{\sqrt{\sigma^2}}$$

The optimum $\tau$ can be found by maximizing $P_d(\tau)$ with respect to $\tau$ or by minimizing $C(\tau)$ with respect to $\tau$, since $Q(x)$ is a monotone decreasing function of $x$. The optimization problem is therefore:

$$\min_{\tau} C(\tau)$$

In Fig. 6, for $P_{fa} = 0.01$, $C(\tau)$ and $P_d(\tau)$ are plotted. As we can see, there exists an optimal $\tau$ (0.24) that maximizes the system level $P_d$. By employing this optimum $\tau_{opt}$, a significant improvement in $P_d$ can be achieved.

![Figure 6](image6.png)

**Fig. 6:** $C(\tau)$ and $P_d(\tau)$. $N = 1000, n = 2, \alpha = 100, \alpha = 200$, $SNR_0 = 30dB$, and $P_{fa} = 0.01$. $\tau_{opt}$ as a function of system level $P_{fa}$ is shown in Fig. 7. As $P_{fa}$ increases, the optimal $\tau_{opt}$ decreases, meaning that at the local sensors the corresponding $p_{fa}$ increases. Therefore, a system level detector with higher $P_{fa}$ can tolerate a higher $p_{fa}$ at the local sensor level.
Nominal function of the distance from the target, we have received signal power is inversely proportional to a polynomial function of $N$. $SNR_0 = 30dB$.

In Fig. 8, $\tau_{opt}$ as a function of $SNR_0$ is shown. As $SNR_0$ increases, the optimal $\tau_{opt}$ increases. This is because with a stronger target signal, the local sensors can still detect the signal even with a higher threshold. From Fig. 9, it is clear that when the total number of sensors $N$ is large, a lower $\tau$ gives better system level performance.

According to these results, it is clear that if the pre-specified system level $P_{fa}$ is high, $SNR_0$ is low, and $N$ is large, a lower $\tau$ should be chosen; otherwise, we should adopt a higher threshold $\tau$ at the local sensor.

As a function of $SNR_0$, $N = 1000$, $n = 2$, $\alpha = 100$, $\alpha = 200$, and $P_{fa} = 0.001$.

6 Conclusions and Discussion

In this paper, we have proposed a decision fusion rule based on the total number of detections made by local sensors, for a WSN with a large number of sensors. Assuming that the received signal power is inversely proportional to a polynomial function of the distance from the target, we have derived and calculated the system-level probabilities of detection and false alarms. Our analysis shows that even at very low SNR, this fusion rule can achieve very good system level detection performance given that there are a sufficiently large number of sensors deployed in the ROI. The number of sensors needed for a pre-specified system level performance can be easily calculated based on our formulas.

We have shown that an optimum threshold at the local sensor can be found to maximize the system-level detection performance. The optimal thresholds are calculated numerically for various system parameters. If the pre-specified system level $P_{fa}$ is high, $SNR_0$ is low, and $N$ is large, a lower local threshold $\tau$ should be chosen; otherwise, a higher $\tau$ should be employed to achieve a better performance.

In this paper, the total number of sensors $N$ is assumed known. In the future, this assumption will be relaxed and the case where $N$ is a random variable, e.g. a Poisson random variable, will be investigated.

References


