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Baltagi, Badi H.; Song, Seuck Heun; and Kwon, Jae Hyeok, "Testing for Heteroskedasticity and Spatial Correlation in a Random Effects Panel Data Model" (2008). *Center for Policy Research.* Paper 59.
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ISSN: 1525-3066

Center for Policy Research
Working Paper No. 108

Testing for Heteroskedasticity and Spatial Correlation in a Random Effects Panel Data Model

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July 2008

$5.00

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Abstract

A panel data regression model with heteroskedastic as well as spatially correlated disturbancesis considered, and a joint LM test for homoskedasticity and no spatial correlation is derived. In addition, a conditional LM test for no spatial correlation given heteroskedasticity, as well as a conditional LM test for homoskedasticity given spatial correlation, are also derived. These LM tests are compared with marginal LM tests that ignore heteroskedasticity in testing for spatial correlation, or spatial correlation in testing for homoskedasticity. Monte Carlo results show that these LM tests as well as their LR counterparts perform well even for small N and T. However, misleading inference can occur when using marginal rather than joint or conditional LM tests when spatial correlation or heteroskedasticity is present.

JEL Code: C12 and C23

Keywords: Panel data; Heteroskedasticity; Spatial Correlation; Lagrange Multiplier tests; Random Effects.

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1 Introduction

The standard error component panel data model assumes that the disturbances have homoskedastic variances and no spatial correlation, see Hsiao (2003) and Baltagi (2005). These may be restrictive assumptions for a lot of panel data applications. For example, the cross-sectional units may be varying in size and as a result may exhibit heteroskedasticity. Also, for trade flows across a panel of countries, there may be spatial effects affecting this trade depending on the distance between these countries. The standard error components model has been extended to take into account spatial correlation by Anselin (1988), Baltagi, Song and Koh (2003), and Kapoor, Kelejian and Prucha (2007), to mention a few. This model has also been generalized to take into account heteroskedasticity by Mazodier and Trognon (1978), Baltagi and Griffin (1988), Li and Stengos (1994), Lejeune (1996), Holly and Gardiol (2000), Roy (2002) and Baltagi, Bresson and Pirotte (2006) to mention a few. For a review of these papers, see Baltagi (2005). However, these strands of literature are almost separate in the panel data error components literature. When one deals with heteroskedasticity, spatial correlation is ignored, and when one deals with spatial correlation, heteroskedasticity is ignored.

LM tests for spatial models are surveyed in Anselin (1988, 2001) and Anselin and Bera (1998), to mention a few. For a joint test of the absence of spatial correlation and random effects in a panel data model, see Baltagi, Song and Koh (2003). However, these tests ignore the heteroskedasticity in the disturbances. On the other hand, Holly and Gardiol (2000) derived an LM statistic which tests for homoskedasticity of the disturbances in the context of a one-way random effects panel data model. However, this LM test ignores the spatial correlation in the disturbances. This paper extends the Holly and Gardiol (2000) model to allow for spatial correlation in the remainder disturbances. It derives a joint LM test for homoskedasticity and no spatial correlation. The

restricted model is the standard random effects error component model. It also derives a conditional LM test for no spatial correlation given heteroskedasticity, as well as, conditional LM test for homoskedasticity given spatial correlation. The paper then contrasts these LM tests with marginal LM tests that ignore heteroskedasticity in testing for spatial correlation, or spatial correlation in testing for homoskedasticity, again in the context of a random effects panel data model. These LM tests are computationally simple. Monte Carlo results show that misleading inference can occur when using marginal rather than joint or conditional LM tests when spatial correlation or heteroskedasticity is present. It is important to note that this paper does not consider alternative forms of spatial lag dependence. It also does not allow for endogeneity of the regressors and requires the normality asssumption to derive the LM tests. The rest of the paper is organized as follows: Section 2 reviews the general heteroskedastic one-way error component model with spatially autocorrelated residual disturbances. Section 3 derives the joint and conditional LM tests described above. Section 4 performs Monte Carlo simulations comparing the size and power of these LM tests along with their LR counterparts. Section 5 concludes.

2 The Model

Consider the following panel data regression model

$$y_{ti} = X'_{ti}\beta + u_{ti}, \;\; i = 1, \cdots, N; t = 1, \cdots T, \tag{2.1}$$

where y_{ti} is the observation on the ith region for the tth time period. X_{ti} denotes the $k \times 1$ vector of observations on the non-stochastic regressors and u_{ti} is the regression disturbance. In vector form, the disturbance vector of (2.1) is assumed to have random region effects as well as spatially autocorrelated residual disturbances, see Anselin (1988):

$$u_t = \mu + \varepsilon_t, \tag{2.2}$$

with

$$\varepsilon_t = \lambda W \varepsilon_t + v_t, \tag{2.3}$$

where $u' = (u_{t1}, \ldots, u_{tN})$, $\varepsilon_t' = (\varepsilon_{t1}, \ldots, \varepsilon_{tN})$ and $\mu' = (\mu_1, \ldots, \mu_N)$ are assumed independent and normally distributed according to:

$$\mu_i \sim N(0, \sigma_{\mu i}^2), \quad \sigma_{\mu i}^2 = \sigma_\mu^2 h(f_i' \theta), \quad i = 1, \ldots, N. \tag{2.4}$$

$h(.)$ is an arbitrary non-indexed (strictly) positive twice continuously differentiable function satisfying $h(.) > 0$, $h(0) = 1$, $h^{(1)}(0) \neq 0$, where $h^{(1)}(0)$ denote the first derivative of $h(.)$ evaluated at zero. f_i is a $(p \times 1)$ vector of strictly exogenous regressors which determine the heteroskedasticity of the individual specific effects. θ is$(p \times 1)$ vector of parameters, and λ is the scalar spatial autoregressive coefficient with $|\lambda| < 1$. W is a known $N \times N$ spatial weight matrix whose diagonal elements are zero. W also satisfies the condition that $(I_N - \lambda W)$ is nonsingular for all $|\lambda| < 1$. $v_t' = (v_{t1}, \ldots, v_{tN})$, where v_{ti} is *i.i.d.* over i and t and is assumed to be $N(0, \sigma_v^2)$. The v_{ti} process is also independent of the μ_i process. One can rewrite (2.3) as

$$\varepsilon_t = (I_N - \lambda W)^{-1} v_t = B^{-1} v_t, \tag{2.5}$$

where $B = I_N - \lambda W$ and I_N is an identity matrix of dimension N. The model (2.1) can be rewritten in matrix notation as

$$y = X\beta + u, \tag{2.6}$$

where y is now of dimension $NT \times 1$, X is $NT \times k$, β is $k \times 1$ and u is $NT \times 1$. X is assumed to be of full column rank and its elements are assumed to be asymptotically bounded in absolute value. Equation(2.2) can be written in vector form as:

$$u = (\iota_T \otimes I_N)\mu + (I_T \otimes B^{-1})v, \tag{2.7}$$

where $v' = (\nu_1', \ldots, \nu_T')$, ι_T is a vector of ones of dimension T, I_N is an identity matrix of dimension T and $\otimes$ denotes the kronecker product. Under these assumptions, the variance covariance matrix of u can be written as

$$\begin{aligned} \Omega &= E(uu') = (\iota_T \otimes I_N)(diag[\sigma_\mu^2 h(f_i'\theta)])(\iota_T' \otimes I_N) + \sigma_v^2 (I_T \otimes (B'B)^{-1}) \\ &= \sigma_\mu^2 \left(J_T \otimes diag\left(h\left(F\theta\right)\right)\right) + \sigma_v^2 (I_T \otimes (B'B)^{-1}), \end{aligned} \tag{2.8}$$

where J_T is a matrix of ones of dimension T and F is $(N \times p)$ matrix of regressors that determine the heteroskedasticity. $diag(h(F\theta))$ denotes a diagonal $(N \times N)$ matrix with its ith diagonal element

being the ith element of the $(N \times 1)$ vector $h(F\theta)$. Note that the computational difficulty in dealing with this Ω is only hampered by the inversion of $(B'B)$. Smirnov (2005) has designed an algorithm for computing the information matrix without storing $(B'B)^{-1}$. He developed a sparse version of the conjugate gradient method for which he reports good numerical stability and modest computational requirements.

3 LM Tests

3.1 Joint LM Test

In this subsection, we derive the joint LM test for testing for no heteroskedasticity and no spatial correlation in a random effects panel data model. The null hypothesis is given by $H_o^a:\ \theta_1 = \cdots = \theta_p = 0$ and $\lambda = 0|\ \sigma_\mu^2 > 0,\ \sigma_v^2 > 0$. The log-likelihood function under normality of the disturbances is given by

$$L(\beta, \varphi) = constant - \frac{1}{2}\log|\Omega| - \frac{1}{2}u'\Omega^{-1}u, \tag{3.1}$$

where $\varphi' = \left(\sigma_v^2,\, \sigma_\mu^2,\, \lambda, \theta_1, \ldots, \theta_p\right)$. The information matrix is block-diagonal between β and φ. Since H_0^a involves only φ, the part of the information due to β is ignored, see Breusch and Pagan (1980). In order to obtain the joint LM statistic, we need $D(\varphi) = (\partial L/\partial\varphi)$ and the information matrix $J(\varphi) = E[-\partial^2 L/\partial\varphi\partial\varphi']$ evaluated at the restricted ML estimator $\widetilde{\varphi}$. Under the null hypothesis H_0^a, the variance-covariance matrix reduces to

$$\Omega_0 = \sigma_\mu^2\,(J_T \otimes I_N) + \sigma_v^2(I_T \otimes I_N) = \sigma_1^2\,(\bar{J}_T \otimes I_N) + \sigma_v^2(E_T \otimes I_N), \tag{3.2}$$

where $E_T = I_T - \bar{J}_T$ with $\bar{J}_T = J_T/T$ and $\sigma_1^2 = \left(T\sigma_\mu^2 + \sigma_v^2\right)$. It is the familiar form of the random effects error component model with no spatial correlation or heteroskedasticity, see Baltagi (2005). Its inverse is given by

$$\Omega_0^{-1} = \frac{1}{\sigma_1^2}\,(\bar{J}_T \otimes I_N) + \frac{1}{\sigma_v^2}(E_T \otimes I_N). \tag{3.3}$$

The score $\widetilde{D}_{\varphi}$, and information matrix $\widetilde{J}_a(\varphi)$,under the null hypothesis H_0^a are derived in Appendix 1. The resulting LM statistic for H_0^a is given by:

$$LM_{\lambda\theta} = \widetilde{D}'_{\varphi}\widetilde{J}_a^{-1}(\varphi)\,\widetilde{D}_{\varphi} = \frac{D(\widetilde{\lambda})^2\widetilde{\sigma}_1^4}{\left(\widetilde{\sigma}_v^4 + \widetilde{\sigma}_1^4(T-1)\right)b} + \frac{1}{2\widetilde{\sigma}_1^4}S'\underline{F}\left(\underline{F}'\underline{F}\right)^{-1}\underline{F}'S, \tag{3.4}$$

where $b = tr(W^2 + W'W)$,

$$\widetilde{D}(\lambda) = \frac{\partial L}{\partial \lambda}\Big|_{H_0^a} = \frac{1}{2\widetilde{\sigma}_v^2}\left[\widetilde{u}'(E_T \otimes (W+W'))\widetilde{u}\right] + \frac{\widetilde{\sigma}_v^2}{2\widetilde{\sigma}_1^4}\left[\widetilde{u}'(\bar{J}_T \otimes (W+W'))\widetilde{u}\right], \tag{3.5}$$

with $\tilde{u} = y - X\tilde{\beta}_{MLE}$ denoting the restricted maximum likelihood residuals under H_0^a, i.e., under a random effects panel data model with no spatial correlation or heteroskedasticity. S is an $(N \times 1)$ vector with typical element $S_i = \tilde{u}_i'\bar{J}_T\tilde{u}_i$ and $\underline{F} = (I_N - \bar{J}_N)F$. Here, $\tilde{u}_i' = (\widetilde{u}_{i1}, .., \widetilde{u}_{iT})$, i.e., the residuals are re-ordered according to time for each individual. $\underline{F} = (I_N - \bar{J}_N)F$, and $\widetilde{\sigma}_1^2$ and $\widetilde{\sigma}_v^2$ are the restricted maximum likelihood estimates of σ_1^2 and σ_v^2 given by $\widetilde{\sigma}_1^2 = \frac{\widetilde{u}'(\bar{J}_T \otimes I_N)\widetilde{u}}{N}$ and $\widetilde{\sigma}_v^2 = \frac{\tilde{u}'(E_T \otimes I_N)\tilde{u}}{N(T-1)}$, respectively.

Under the null hypothesis H_0^a, $LM_{\lambda\theta}$ should be asymptotically distributed as χ^2_{p+1}. Although we do not explicitly derive the asymptotic distribution of our test statistics, they are likely to hold under a similar set of primitive assumptions developed by Kelejian and Prucha (2001). Also, note that for large N, tricks for computing the trace(W^2) can be approximated as in Barry and Pace (1999) to any desired accuracy using an O(N) algorithm.

3.2 Conditional LM Tests

The joint LM test derived in the previous section is useful especially when one does not reject the null hypothesis H_0^a. However, if the null hypotheses is rejected, one can not infer whether the presence of heteroskedasticity, or the presence of spatial correlation, or both factors caused this rejection. Alternatively, one can derive two conditional LM tests. The first one tests for the absence of spatial correlation assuming that heteroskedasticity of the individual effects might be present. The second one tests for homoskedasticity assuming that spatial correlation might be present. All in the context of a random effects panel data model.

For the first conditional LM test, the null hypothesis is given by $H_0^b : \lambda = 0$ (assuming at least one element of θ is not zero). Under the null hypothesis H_0^b , the variance-covariance matrix in (2.8)

reduces to

$$\Omega_0 = \sigma_\mu^2 (J_T \otimes diag\,(h\,(F\theta)) + \sigma_v^2\,(I_T \otimes I_N)\,. \tag{3.6}$$

Replacing J_T by $T\bar{J}_T$ and I_T by $(E_T + \bar{J}_T)$, where $E_T = I_T - \bar{J}_T$, and $\bar{J}_T = J_T/T$, see Wansbeek and Kapteyn (1982), one gets:

$$\Omega_0 = \left[(\bar{J}_T \otimes diag\,(g_i)) + \sigma_v^2\,(E_T \otimes I_N)\right], \tag{3.7}$$

where $g_i = T\sigma_\mu^2 h(f_i'\theta) + \sigma_v^2$. This also implies that

$$\Omega_0^{-1} = \left(\bar{J}_T \otimes diag\left(\frac{1}{g_i}\right)\right) + \frac{1}{\sigma_v^2}(E_T \otimes I_N)\,. \tag{3.8}$$

Appendix 2 derives the score $\widetilde{D}_\varphi$ and information matrix $\widetilde{J}_b\,(\varphi)$ under H_0^b. The resulting conditional LM statistic for H_0^b is given by

$$LM_{\lambda|\theta} = \widetilde{D}_\varphi' \widetilde{J_b}^{-1} \widetilde{D}_\varphi = \frac{2D(\widetilde{\lambda})^2}{\widetilde{\sigma}_v^4 d + 2\,(T-1)b}, \tag{3.9}$$

where $b = tr(W^2 + W'W)$,

$$d = tr\left[diag\left(\frac{1}{g_i}\right)(W' + W)\,diag\left(\frac{1}{g_i}\right)(W' + W)\right], \tag{3.10}$$

$$D\left(\widetilde{\lambda}\right) = \frac{1}{2\widetilde{\sigma}_v^2}\left[\widetilde{u}'(E_T \otimes (W' + W))\widetilde{u}\right] + \frac{\widetilde{\sigma}_v^2}{2}\left[\widetilde{u}'(\bar{J}_T \otimes diag\left(\frac{1}{\widetilde{g}_i}\right)(W' + W) diag\left(\frac{1}{\widetilde{g}_i}\right))\widetilde{u}\right], \tag{3.11}$$

with $\tilde{u} = y - X\tilde{\beta}_{MLE}$ denoting the restricted maximum likelihood residuals under H_0^b, i.e., under a random effects panel data model with no spatial correlation but possible heteroskedasticity, $\widetilde{g}_i = (T\widetilde{\sigma}_\mu^2 h(f_i'\widetilde{\theta}) + \widetilde{\sigma}_v^2)$, where $\widetilde{\theta}$, $\widetilde{\sigma}_\mu^2$ and $\widetilde{\sigma}_v^2$ are the restricted MLE of θ, σ_μ^2 and σ_v^2, under H_0^b. Under the null hypothesis H_0^b, $LM_{\lambda|\theta}$ should be asymptotically distributed as χ_1^2.

For the second conditional LM test, the null hypothesis is given by: $H_0^c : \theta = 0$ (assuming λ may not be zero). Under the null hypothesis H_0^c, the variance-covariance matrix in (2.8) reduces to

$$\Omega_0 = \sigma_\mu^2\,(J_T \otimes I_N) + \sigma_v^2 (I_T \otimes (B'B)^{-1}). \tag{3.12}$$

Replacing J_T by $T\bar{J}_T$ and I_T by $(E_T + \bar{J}_T)$, where $E_T = I_T - \bar{J}_T$, and $\bar{J}_T = J_T/T$, see Wansbeek and Kapteyn (1982), one gets:

$$\Omega_0 = \sigma_v^2\left[\bar{J}_T \otimes \left(T\phi I_N + (B'B)^{-1}\right) + E_T \otimes (B'B)^{-1}\right], \tag{3.13}$$

where $\phi = \sigma_\mu^2/\sigma_v^2$. This also implies that

$$\Omega_0^{-1} = \frac{1}{\sigma_v^2}\left[\bar{J}_T \otimes Z + E_T \otimes (B'B)\right], \tag{3.14}$$

where $Z = \left(T\phi I_N + (B'B)^{-1}\right)^{-1}$. Appendix 3 derives the score $\widetilde{D}_\varphi$ and information matrix $\widetilde{J}_c(\varphi)$ under H_0^c, i.e., under a random effects panel data model with no heteroskedasticity but possible spatial correlation. The resulting conditional LM statistic for H_0^c is given by

$$LM_{\theta|\lambda} = \widetilde{D}_\varphi' \widetilde{J}_c^{-1}(\varphi)\, \widetilde{D}_\varphi, \tag{3.15}$$

and did not have a simple expression as $LM_{\lambda|\theta}$. Under the null hypothesis H_0^c, the LM statistic should be asymptotically distributed as χ_p^2.

4 Monte Carlo Results

The experimental design for the Monte Carlo simulations is based on the format extensively used in earlier studies in the spatial regression model by Anselin and Rey (1991) and Anselin and Florax (1995), and in the heteroskedastic panel data model by Roy (2002).

The model is set as follows:

$$y_{it} = \alpha + x_{it}'\beta + u_{it} \quad i = 1, \cdots, N,\ t = 1, \cdots, T,$$

where $\alpha = 5$ and $\beta = 0.5$. x_{it} is generated by a similar method to that of Nerlove (1971). In fact, $x_{it} = 0.1t + 0.5x_{i,t-1} + z_{it}$, where z_{it} is uniformly distributed over the interval $[0, 2]$. The initial values x_{i0} are chosen as $(5 + 10z_{i0})$. For the disturbances, $u_{it} = \mu_i + \varepsilon_{it}$, $\varepsilon_{it} = \lambda \sum_{j=1}^N w_{ij}\varepsilon_{it} + \nu_{it}$ with $\nu_{it} \sim IIN(0, \sigma_\nu^2)$ and $\mu_i \sim N(0, \sigma_{\mu i}^2)$ where,

$$\sigma_{\mu_i}^2 = \sigma_{\mu_i}^2(\bar{x}_{i.}) = \sigma_\mu^2(1 + \theta\bar{x}_{i.})^2,$$

or

$$\sigma_{\mu_i}^2 = \sigma_{\mu_i}^2(\bar{x}_{i.}) = \sigma_\mu^2 \exp(\theta\bar{x}_{i.})^2, \tag{4.1}$$

and $\bar{x}_{i.}$ denoting the individual mean of x_{it}. Denoting the expected variance of μ_i by $\bar{\sigma}_{\mu_i}^2$ and following Roy (2002), we fix $\bar{\sigma}_{\mu_i}^2 + \sigma_\nu^2 = 20$. We let σ_ν^2 take the values 4 and 16. For each fixed value

of σ_ν^2, θ is assigned values 0, 1, 2 and 3 with $\theta = 0$ denoting the homoskedastic individual specific error. For a fixed value of σ_ν^2, we obtain the values of $\bar{\sigma}_{\mu_i}^2$ and using (4.2), we get the values for σ_μ^2 for each θ value considered. Then we obtain the values of $\sigma_{\mu_i}^2$ for each σ_μ^2 under the four different θ values considered. The matrix W is a rook or queen type weight matrix, and the rows of this matrix are standardized so that they sum to one. The spatial autocorrelation factor λ is varied over the positive range from 0 to 0.9 by increments of 0.1. Two values for $N = 25$ and 49, and three values for $T = 5, 7$ and 12 are chosen. For each experiment, 1000 replications are performed. We chose small values of N and T to demonstrate that the size of these tests work well even in small samples. Of course for larger samples, the computational difficulty increases, but the asymptotics should even be better behaved.

Table 1 gives the empirical size of the joint LM and LR tests for the null hypothesis $H_0^a : \theta = \lambda = 0$ at the 5% significance level when $N = 25, 49$ and $T = 5, 7, 12$ for both the queen and rook weight matrices. Additionally, we ran a limited set of experiments for $N = 100$ and $T = 7$ with a Rook design to show that the power improves as N gets large. These results are not shown here to save space but are available upon request from the authors. Table 2 gives the empirical size of the conditional LM and LR tests for the null hypothesis $H_0^b : \lambda = 0$ (given $\theta \neq 0$) and Tables 3a, 3b give the empirical size of the conditional LM and LR tests for the null hypothesis $H_0^c : \theta = 0$ (given $\lambda \neq 0$). As we can see from these Tables, the empirical size is not significantly different from 5% for a Bernoulli with 1000 replications and probability of success of 0.05. Table 4 gives the empirical size of the marginal LM and LR tests for the null hypothesis $H_0^d : \lambda = 0$ (given $\theta = 0$) and Tables 5a, 5b, give the empirical size of the marginal LM and LR tests for the null hypothesis $H_0^e : \theta = 0$ (given $\lambda = 0$) . As we can see from the Tables, the empirical size of the LM and LR tests are severely undersized for $\sigma_\nu^2 = 16$, and high values of λ, and this empirical size is significantly different from 5% for a Bernoulli with 1000 replications and probability of success of 0.05.

The power of these tests for various experiments are not shown here to save space and are available as Tables A1-A60 upon request from the authors. Figure 1 plots the empirical power for the joint LM and LR test for the null hypothesis $H_0^a : \theta = \lambda = 0$, for $N = 25$ and 49, and $T = 12$. This is done for various values of λ, when σ_ν^2 takes the values 4 and 16 and $\theta = 0, 1, 2, 3$. This is done for a Rook weight matrix and an exponential form of heteroskedasticity. The power of the joint

Table 1 : Estimated size of joint LM and LR tests for testing $H_0^a : \lambda = \theta = 0$.

					Quadratic heteroskedasticity				Exponential heteroskedasticity			
					N=25		N=49		N=25		N=49	
T	W	σ_v^2	θ	λ	LM	LR	LM	LR	LM	LR	LM	LR
5	Rook	4	0	0	0.053	0.059	0.051	0.053	0.039	0.071	0.051	0.058
		16	0	0	0.047	0.050	0.050	0.041	0.042	0.054	0.051	0.045
	Queen	4	0	0	0.060	0.064	0.047	0.052	0.036	0.056	0.046	0.054
		16	0	0	0.041	0.043	0.049	0.055	0.041	0.051	0.047	0.047
7	Rook	4	0	0	0.042	0.061	0.056	0.064	0.049	0.059	0.042	0.051
		16	0	0	0.050	0.051	0.061	0.055	0.047	0.049	0.041	0.047
	Queen	4	0	0	0.053	0.065	0.048	0.055	0.051	0.060	0.048	0.060
		16	0	0	0.047	0.043	0.044	0.035	0.037	0.046	0.037	0.047
12	Rook	4	0	0	0.048	0.066	0.042	0.043	0.050	0.057	0.049	0.055
		16	0	0	0.048	0.040	0.051	0.043	0.042	0.048	0.050	0.055
	Queen	4	0	0	0.041	0.046	0.048	0.052	0.037	0.057	0.033	0.043
		16	0	0	0.052	0.059	0.059	0.052	0.036	0.052	0.041	0.042

LM and LR tests climbs to one quickly as soon as λ exceeds 0.3 for various values of θ. Also, this power increases with θ for $0 < \lambda < 0.3$. The power also increases with N. Figure 2 repeats these power plots for the case of quadratic heteroskedasticity. Again, the same phenomenon is observed for this alternative form of heteroskedascity. Figure 3 plots the empirical power for the marginal and conditional LM and LR tests for the null hypothesis $H_0^b : \lambda = 0$ (given $\theta \neq 0$), for $N = 25$ and 49, and $T = 12$. This is done for various values of λ, when σ_ν^2 take the value 16 and $\theta = 3$. This is done for a Rook weight matrix and a quadratic form of heteroskedasticity. As clear from Figure 3, the conditional LM and LR tests perform better than their marginal counterparts for $\theta \neq 0$. However, the power of all these tests is close to one for $\lambda > 0.3$. Figure 4 plots the empirical power for the marginal and conditional LM and LR tests for the null hypothesis $H_0^c : \theta = 0$ (given $\lambda \neq 0$), for $N = 25$ and 49, and $T = 12$. This is done for various values of θ, when σ_ν^2 take the value 16 and $\lambda = 0.9$. This is done for a Rook weight matrix and an exponential form of heteroskedasticity. As clear from Figure 4, the conditional LM and LR tests perform better than their marginal counterparts for $\lambda \neq 0$, and this power is increasing with θ. These figures give a flavour of the power performance of these tests for a subset of the experiments. Of course, more plots can be given, but we refrain from doing so because of space limitation.

Table 2 : Estimated size of conditional LM and LR tests for testing $H_0^b : \lambda = 0$ (given $\theta \neq 0$).

					Quadratic heteroskedasticity				Exponential heteroskedasticity			
					N=25		N=49		N=25		N=49	
T	W	σ_v^2	θ	λ	LM	LR	LM	LR	LM	LR	LM	LR
5	Rook	4	0	0	0.059	0.052	0.044	0.047	0.045	0.067	0.045	0.051
			1	0	0.049	0.051	0.041	0.045	0.049	0.044	0.047	0.045
			2	0	0.050	0.058	0.048	0.051	0.053	0.052	0.047	0.051
			3	0	0.052	0.053	0.043	0.043	0.044	0.052	0.048	0.054
		16	0	0	0.047	0.050	0.050	0.047	0.040	0.066	0.049	0.049
			1	0	0.050	0.070	0.049	0.045	0.040	0.061	0.055	0.053
			2	0	0.047	0.047	0.058	0.051	0.052	0.064	0.047	0.055
			3	0	0.040	0.060	0.050	0.043	0.054	0.068	0.048	0.066
5	Queen	4	0	0	0.060	0.034	0.047	0.059	0.048	0.043	0.061	0.059
			1	0	0.037	0.050	0.054	0.032	0.049	0.059	0.054	0.043
			2	0	0.058	0.050	0.053	0.061	0.047	0.043	0.044	0.044
			3	0	0.046	0.045	0.051	0.052	0.048	0.044	0.052	0.057
		16	0	0	0.048	0.062	0.049	0.066	0.047	0.054	0.043	0.046
			1	0	0.041	0.053	0.031	0.048	0.050	0.055	0.044	0.056
			2	0	0.048	0.049	0.057	0.050	0.056	0.067	0.044	0.055
			3	0	0.045	0.059	0.046	0.050	0.048	0.069	0.058	0.062
7	Rook	4	0	0	0.053	0.054	0.052	0.053	0.056	0.056	0.041	0.044
			1	0	0.048	0.050	0.042	0.044	0.055	0.053	0.044	0.047
			2	0	0.060	0.059	0.048	0.049	0.050	0.051	0.044	0.047
			3	0	0.046	0.046	0.042	0.046	0.049	0.053	0.053	0.054
		16	0	0	0.049	0.050	0.055	0.059	0.047	0.051	0.054	0.053
			1	0	0.061	0.065	0.059	0.058	0.046	0.048	0.057	0.058
			2	0	0.047	0.048	0.057	0.055	0.046	0.054	0.052	0.052
			3	0	0.055	0.058	0.050	0.049	0.053	0.054	0.038	0.042
7	Queen	4	0	0	0.060	0.064	0.044	0.054	0.049	0.045	0.046	0.048
			1	0	0.051	0.054	0.057	0.053	0.053	0.051	0.049	0.054
			2	0	0.039	0.047	0.050	0.050	0.055	0.062	0.052	0.051
			3	0	0.041	0.051	0.054	0.055	0.050	0.046	0.053	0.058
		16	0	0	0.053	0.055	0.036	0.039	0.044	0.050	0.048	0.056
			1	0	0.049	0.055	0.048	0.051	0.053	0.061	0.056	0.060
			2	0	0.063	0.067	0.050	0.051	0.041	0.051	0.043	0.040
			3	0	0.060	0.064	0.046	0.049	0.052	0.056	0.047	0.053
12	Rook	4	0	0	0.052	0.050	0.042	0.040	0.048	0.049	0.046	0.049
			1	0	0.040	0.042	0.051	0.051	0.065	0.067	0.058	0.059
			2	0	0.056	0.056	0.054	0.054	0.058	0.056	0.050	0.049
			3	0	0.057	0.056	0.047	0.048	0.040	0.038	0.055	0.053
		16	0	0	0.055	0.057	0.051	0.051	0.054	0.059	0.048	0.052
			1	0	0.045	0.053	0.048	0.050	0.046	0.044	0.054	0.054
			2	0	0.054	0.058	0.039	0.041	0.053	0.057	0.046	0.048
			3	0	0.051	0.052	0.053	0.055	0.056	0.057	0.054	0.053
12	Queen	4	0	0	0.051	0.052	0.043	0.047	0.041	0.044	0.039	0.036
			1	0	0.047	0.046	0.041	0.044	0.048	0.050	0.056	0.058
			2	0	0.050	0.049	0.047	0.052	0.058	0.056	0.055	0.056
			3	0	0.062	0.065	0.057	0.053	0.041	0.041	0.053	0.054
		16	0	0	0.062	0.061	0.057	0.055	0.053	0.056	0.048	0.048
			1	0	0.045	0.053	0.052	0.051	0.047	0.054	0.049	0.052
			2	0	0.063	0.073	0.056	0.055	0.049	0.052	0.056	0.051
			3	0	0.047	0.047	0.047	0.051	0.034	0.035	0.053	0.053

Table 3a : Estimated size of conditional LM and LR tests for testing $H_0^c : \theta = 0$ (given $\lambda \neq 0$) when weight matrix is ROOK.

					Quadratic heteroskedasticity				Exponential heteroskedasticity			
					N=25		N=49		N=25		N=49	
T	W	σ_v^2	θ	λ	LM	LR	LM	LR	LM	LR	LM	LR
5	Rook	4	0	0.0	0.038	0.067	0.049	0.058	0.033	0.067	0.046	0.060
			0	0.1	0.050	0.063	0.049	0.064	0.043	0.063	0.042	0.054
			0	0.2	0.045	0.064	0.047	0.068	0.047	0.059	0.050	0.055
			0	0.3	0.041	0.060	0.038	0.057	0.045	0.058	0.042	0.052
			0	0.4	0.042	0.054	0.037	0.057	0.042	0.053	0.040	0.060
			0	0.5	0.036	0.067	0.050	0.065	0.039	0.056	0.042	0.047
			0	0.6	0.041	0.069	0.044	0.059	0.038	0.075	0.053	0.057
			0	0.7	0.043	0.066	0.054	0.064	0.045	0.055	0.050	0.057
			0	0.8	0.048	0.052	0.041	0.068	0.049	0.044	0.050	0.054
			0	0.9	0.043	0.055	0.044	0.057	0.038	0.046	0.047	0.048
5	Rook	16	0	0.0	0.053	0.038	0.040	0.046	0.041	0.048	0.035	0.053
			0	0.1	0.042	0.038	0.052	0.055	0.037	0.055	0.045	0.055
			0	0.2	0.051	0.028	0.048	0.038	0.033	0.045	0.053	0.047
			0	0.3	0.046	0.039	0.046	0.052	0.040	0.043	0.040	0.054
			0	0.4	0.051	0.036	0.047	0.046	0.033	0.036	0.039	0.055
			0	0.5	0.035	0.025	0.036	0.037	0.042	0.054	0.044	0.052
			0	0.6	0.043	0.038	0.044	0.041	0.036	0.043	0.048	0.054
			0	0.7	0.041	0.033	0.047	0.050	0.039	0.042	0.040	0.051
			0	0.8	0.043	0.035	0.040	0.031	0.039	0.053	0.044	0.046
			0	0.9	0.048	0.033	0.044	0.028	0.038	0.063	0.050	0.039
7	Rook	4	0	0.0	0.042	0.065	0.046	0.059	0.047	0.054	0.047	0.055
			0	0.1	0.041	0.060	0.050	0.059	0.048	0.067	0.043	0.058
			0	0.2	0.052	0.064	0.043	0.061	0.049	0.069	0.036	0.058
			0	0.3	0.050	0.062	0.055	0.064	0.042	0.068	0.049	0.060
			0	0.4	0.046	0.055	0.043	0.058	0.046	0.065	0.059	0.065
			0	0.5	0.039	0.066	0.054	0.068	0.042	0.059	0.039	0.043
			0	0.6	0.032	0.048	0.050	0.054	0.053	0.074	0.048	0.047
			0	0.7	0.037	0.057	0.043	0.047	0.042	0.060	0.042	0.047
			0	0.8	0.036	0.047	0.042	0.046	0.048	0.062	0.040	0.048
			0	0.9	0.046	0.048	0.046	0.060	0.039	0.046	0.046	0.049
7	Rook	16	0	0.0	0.051	0.037	0.051	0.045	0.044	0.056	0.050	0.055
			0	0.1	0.047	0.038	0.035	0.033	0.042	0.046	0.045	0.046
			0	0.2	0.049	0.035	0.054	0.050	0.040	0.054	0.045	0.048
			0	0.3	0.043	0.035	0.048	0.043	0.040	0.047	0.046	0.053
			0	0.4	0.040	0.043	0.049	0.043	0.035	0.044	0.050	0.054
			0	0.5	0.036	0.035	0.045	0.047	0.030	0.032	0.044	0.041
			0	0.6	0.036	0.028	0.047	0.040	0.029	0.038	0.047	0.049
			0	0.7	0.047	0.039	0.042	0.037	0.034	0.039	0.042	0.047
			0	0.8	0.042	0.025	0.042	0.035	0.037	0.046	0.045	0.051
			0	0.9	0.037	0.033	0.039	0.037	0.048	0.054	0.042	0.043
12	Rook	4	0	0.0	0.040	0.061	0.039	0.054	0.041	0.064	0.048	0.056
			0	0.1	0.038	0.055	0.049	0.057	0.050	0.062	0.049	0.062
			0	0.2	0.032	0.044	0.048	0.049	0.054	0.062	0.054	0.068
			0	0.3	0.057	0.058	0.048	0.054	0.027	0.050	0.046	0.051
			0	0.4	0.035	0.044	0.044	0.060	0.040	0.058	0.062	0.079
			0	0.5	0.043	0.060	0.039	0.045	0.048	0.057	0.047	0.059
			0	0.6	0.047	0.049	0.055	0.060	0.040	0.071	0.049	0.063
			0	0.7	0.044	0.044	0.040	0.046	0.037	0.050	0.045	0.060
			0	0.8	0.047	0.039	0.047	0.047	0.048	0.060	0.051	0.062
			0	0.9	0.046	0.032	0.045	0.048	0.033	0.043	0.045	0.054
12	Rook	16	0	0.0	0.048	0.033	0.050	0.044	0.040	0.045	0.050	0.057
			0	0.1	0.043	0.034	0.044	0.040	0.043	0.053	0.053	0.060
			0	0.2	0.043	0.030	0.046	0.034	0.054	0.071	0.045	0.053
			0	0.3	0.043	0.031	0.049	0.042	0.037	0.051	0.047	0.055
			0	0.4	0.051	0.030	0.050	0.041	0.033	0.051	0.049	0.051
			0	0.5	0.039	0.037	0.041	0.033	0.035	0.032	0.050	0.054
			0	0.6	0.038	0.025	0.046	0.039	0.040	0.053	0.050	0.055
			0	0.7	0.046	0.029	0.050	0.035	0.047	0.053	0.050	0.049
			0	0.8	0.036	0.028	0.058	0.038	0.059	0.061	0.047	0.056
			0	0.9	0.038	0.027	0.043	0.032	0.047	0.058	0.040	0.040

Table 3b : Estimated size of conditional LM and LR tests for testing $H_0^c : \theta = 0$ (given $\lambda \neq 0$) when weight matrix is QUEEN.

					Quadratic heteroskedasticity				Exponential heteroskedasticity			
					N=25		N=49		N=25		N=49	
T	W	σ_v^2	θ	λ	LM	LR	LM	LR	LM	LR	LM	LR
5	Queen	4	0	0.0	0.040	0.068	0.042	0.054	0.045	0.076	0.041	0.056
			0	0.1	0.046	0.061	0.044	0.052	0.045	0.059	0.043	0.058
			0	0.2	0.038	0.063	0.054	0.060	0.046	0.068	0.048	0.042
			0	0.3	0.040	0.069	0.040	0.067	0.041	0.052	0.048	0.062
			0	0.4	0.040	0.064	0.045	0.053	0.043	0.044	0.047	0.063
			0	0.5	0.046	0.066	0.039	0.056	0.039	0.062	0.045	0.052
			0	0.6	0.042	0.063	0.049	0.074	0.037	0.056	0.047	0.057
			0	0.7	0.044	0.066	0.045	0.063	0.048	0.056	0.046	0.054
			0	0.8	0.048	0.060	0.049	0.060	0.051	0.057	0.041	0.046
			0	0.9	0.050	0.061	0.047	0.049	0.042	0.055	0.046	0.053
5	Queen	16	0	0.0	0.044	0.037	0.040	0.046	0.038	0.048	0.047	0.053
			0	0.1	0.055	0.031	0.052	0.055	0.040	0.055	0.045	0.055
			0	0.2	0.049	0.034	0.048	0.038	0.041	0.045	0.050	0.047
			0	0.3	0.045	0.040	0.046	0.052	0.038	0.043	0.050	0.054
			0	0.4	0.040	0.025	0.047	0.046	0.045	0.036	0.044	0.055
			0	0.5	0.041	0.043	0.036	0.037	0.039	0.054	0.049	0.052
			0	0.6	0.044	0.033	0.044	0.041	0.041	0.043	0.044	0.054
			0	0.7	0.042	0.032	0.047	0.050	0.036	0.042	0.049	0.051
			0	0.8	0.041	0.031	0.040	0.031	0.046	0.053	0.044	0.046
			0	0.9	0.039	0.027	0.044	0.028	0.036	0.063	0.043	0.039
7	Queen	4	0	0.0	0.049	0.076	0.046	0.072	0.048	0.062	0.047	0.058
			0	0.1	0.040	0.052	0.049	0.060	0.030	0.049	0.048	0.048
			0	0.2	0.042	0.079	0.044	0.059	0.046	0.066	0.052	0.066
			0	0.3	0.033	0.053	0.037	0.056	0.045	0.066	0.044	0.055
			0	0.4	0.035	0.053	0.040	0.048	0.040	0.065	0.035	0.048
			0	0.5	0.051	0.068	0.045	0.054	0.046	0.063	0.054	0.069
			0	0.6	0.039	0.062	0.048	0.060	0.041	0.072	0.056	0.063
			0	0.7	0.048	0.043	0.050	0.054	0.048	0.070	0.046	0.059
			0	0.8	0.051	0.070	0.045	0.044	0.034	0.039	0.044	0.045
			0	0.9	0.050	0.053	0.055	0.069	0.044	0.063	0.046	0.049
7	Queen	16	0	0.0	0.048	0.038	0.043	0.041	0.044	0.052	0.039	0.044
			0	0.1	0.029	0.029	0.048	0.041	0.046	0.057	0.035	0.037
			0	0.2	0.039	0.034	0.050	0.043	0.042	0.046	0.047	0.046
			0	0.3	0.039	0.039	0.049	0.043	0.034	0.045	0.048	0.052
			0	0.4	0.043	0.024	0.043	0.045	0.031	0.040	0.057	0.060
			0	0.5	0.032	0.025	0.050	0.059	0.040	0.048	0.038	0.038
			0	0.6	0.029	0.034	0.045	0.044	0.031	0.049	0.055	0.052
			0	0.7	0.052	0.040	0.044	0.040	0.030	0.042	0.045	0.046
			0	0.8	0.040	0.031	0.042	0.038	0.045	0.044	0.045	0.045
			0	0.9	0.040	0.028	0.044	0.042	0.050	0.050	0.045	0.044
12	Queen	4	0	0.0	0.035	0.048	0.049	0.056	0.040	0.061	0.039	0.044
			0	0.1	0.040	0.049	0.042	0.055	0.043	0.064	0.033	0.035
			0	0.2	0.035	0.044	0.042	0.058	0.051	0.063	0.052	0.062
			0	0.3	0.034	0.051	0.047	0.073	0.038	0.051	0.055	0.068
			0	0.4	0.034	0.055	0.039	0.053	0.046	0.059	0.047	0.053
			0	0.5	0.034	0.057	0.039	0.056	0.038	0.059	0.057	0.062
			0	0.6	0.048	0.060	0.040	0.048	0.048	0.069	0.042	0.050
			0	0.7	0.051	0.052	0.037	0.048	0.049	0.059	0.054	0.055
			0	0.8	0.036	0.052	0.033	0.036	0.044	0.066	0.043	0.054
			0	0.9	0.040	0.027	0.038	0.039	0.043	0.054	0.047	0.045
12	Queen	16	0	0.0	0.050	0.043	0.046	0.043	0.038	0.050	0.045	0.051
			0	0.1	0.043	0.040	0.044	0.039	0.038	0.048	0.037	0.043
			0	0.2	0.043	0.034	0.049	0.043	0.031	0.043	0.045	0.043
			0	0.3	0.032	0.025	0.039	0.040	0.037	0.042	0.031	0.038
			0	0.4	0.044	0.042	0.055	0.043	0.043	0.047	0.039	0.050
			0	0.5	0.047	0.036	0.051	0.041	0.036	0.048	0.051	0.063
			0	0.6	0.056	0.030	0.049	0.040	0.035	0.049	0.041	0.044
			0	0.7	0.033	0.024	0.051	0.041	0.046	0.053	0.044	0.045
			0	0.8	0.039	0.027	0.055	0.046	0.040	0.050	0.038	0.042
			0	0.9	0.048	0.033	0.037	0.035	0.045	0.051	0.036	0.040

Table 4 : Estimated size of marginal LM and LR tests for testing $H_0^d : \lambda = 0$ (assuming $\theta = 0$).

					Quadratic heteroskedasticity				Exponential heteroskedasticity			
					N=25		N=49		N=25		N=49	
T	W	σ_v^2	θ	λ	LM	LR	LM	LR	LM	LR	LM	LR
5	Rook	4	0	0	0.043	0.045	0.052	0.058	0.049	0.054	0.040	0.040
			1	0	0.051	0.060	0.043	0.047	0.050	0.054	0.052	0.058
			2	0	0.045	0.058	0.037	0.039	0.049	0.052	0.058	0.058
			3	0	0.056	0.061	0.052	0.053	0.058	0.061	0.055	0.058
		16	0	0	0.050	0.068	0.048	0.052	0.039	0.043	0.051	0.051
			1	0	0.041	0.040	0.058	0.061	0.044	0.045	0.045	0.050
			2	0	0.042	0.052	0.051	0.051	0.054	0.055	0.055	0.057
			3	0	0.050	0.057	0.049	0.051	0.049	0.055	0.041	0.044
5	Queen	4	0	0	0.047	0.053	0.056	0.052	0.037	0.043	0.044	0.045
			1	0	0.036	0.039	0.055	0.056	0.047	0.046	0.060	0.060
			2	0	0.041	0.048	0.044	0.045	0.038	0.045	0.053	0.058
			3	0	0.044	0.051	0.057	0.054	0.053	0.057	0.044	0.051
		16	0	0	0.046	0.059	0.052	0.056	0.040	0.052	0.041	0.043
			1	0	0.046	0.053	0.047	0.048	0.037	0.049	0.044	0.045
			2	0	0.051	0.049	0.035	0.043	0.052	0.060	0.049	0.053
			3	0	0.038	0.058	0.048	0.053	0.046	0.060	0.046	0.044
7	Rook	4	0	0	0.052	0.053	0.044	0.046	0.054	0.055	0.049	0.048
			1	0	0.040	0.050	0.050	0.048	0.052	0.057	0.039	0.038
			2	0	0.050	0.049	0.048	0.053	0.057	0.061	0.056	0.058
			3	0	0.051	0.055	0.051	0.052	0.054	0.050	0.038	0.039
		16	0	0	0.050	0.054	0.043	0.045	0.057	0.060	0.059	0.058
			1	0	0.042	0.044	0.056	0.056	0.041	0.047	0.057	0.062
			2	0	0.058	0.059	0.058	0.060	0.047	0.053	0.045	0.042
			3	0	0.048	0.051	0.049	0.050	0.039	0.048	0.052	0.053
7	Queen	4	0	0	0.046	0.045	0.047	0.045	0.045	0.048	0.044	0.044
			1	0	0.048	0.050	0.054	0.054	0.041	0.043	0.031	0.032
			2	0	0.053	0.060	0.053	0.059	0.052	0.058	0.042	0.052
			3	0	0.051	0.054	0.040	0.040	0.049	0.047	0.045	0.047
		16	0	0	0.043	0.051	0.040	0.045	0.037	0.045	0.045	0.053
			1	0	0.053	0.056	0.060	0.061	0.040	0.056	0.047	0.050
			2	0	0.047	0.049	0.043	0.045	0.039	0.046	0.049	0.050
			3	0	0.046	0.057	0.052	0.053	0.047	0.051	0.046	0.050
12	Rook	4	0	0	0.049	0.051	0.044	0.046	0.055	0.056	0.038	0.039
			1	0	0.061	0.062	0.052	0.051	0.041	0.041	0.058	0.059
			2	0	0.053	0.056	0.052	0.052	0.044	0.049	0.048	0.049
			3	0	0.058	0.059	0.043	0.043	0.042	0.042	0.051	0.056
		16	0	0	0.047	0.056	0.040	0.042	0.049	0.047	0.048	0.048
			1	0	0.057	0.057	0.053	0.057	0.053	0.052	0.045	0.045
			2	0	0.059	0.062	0.058	0.058	0.057	0.059	0.045	0.044
			3	0	0.037	0.039	0.045	0.047	0.054	0.054	0.040	0.042
12	Queen	4	0	0	0.049	0.051	0.044	0.046	0.055	0.056	0.038	0.039
			1	0	0.061	0.062	0.052	0.051	0.041	0.041	0.058	0.059
			2	0	0.053	0.056	0.052	0.052	0.044	0.049	0.048	0.049
			3	0	0.058	0.059	0.043	0.043	0.042	0.042	0.051	0.056
		16	0	0	0.047	0.056	0.040	0.042	0.049	0.047	0.048	0.048
			1	0	0.057	0.057	0.053	0.057	0.053	0.052	0.045	0.045
			2	0	0.059	0.062	0.058	0.058	0.057	0.059	0.045	0.044
			3	0	0.037	0.039	0.045	0.047	0.054	0.054	0.040	0.042

Table 5a : Estimated size of marginal LM and LR tests for testing $H_0^e : \theta = 0$ (assuming $\lambda = 0$) when weight matrix is ROOK.

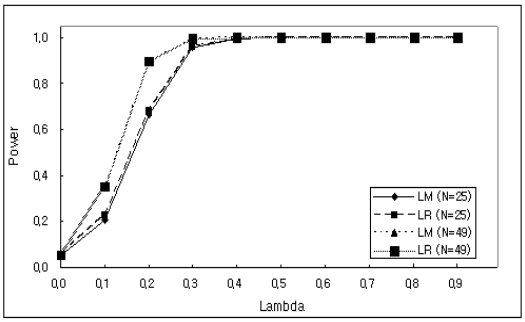
					Quadratic heteroskedasticity				Exponential heteroskedasticity			
					N=25		N=49		N=25		N=49	
T	W	σ_v^2	θ	λ	LM	LR	LM	LR	LM	LR	LM	LR
5	Rook	4	0	0.0	0.045	0.066	0.041	0.060	0.042	0.058	0.044	0.066
			0	0.1	0.035	0.064	0.061	0.082	0.041	0.058	0.049	0.051
			0	0.2	0.053	0.079	0.046	0.059	0.047	0.072	0.041	0.050
			0	0.3	0.038	0.061	0.043	0.049	0.037	0.056	0.040	0.046
			0	0.4	0.034	0.067	0.051	0.069	0.053	0.074	0.056	0.065
			0	0.5	0.040	0.059	0.050	0.079	0.041	0.055	0.040	0.047
			0	0.6	0.040	0.057	0.050	0.062	0.039	0.070	0.042	0.057
			0	0.7	0.043	0.059	0.050	0.057	0.046	0.062	0.033	0.048
			0	0.8	0.034	0.060	0.045	0.061	0.031	0.041	0.039	0.050
			0	0.9	0.027	0.041	0.045	0.047	0.044	0.047	0.050	0.061
5	Rook	16	0	0.0	0.043	0.039	0.046	0.047	0.039	0.049	0.039	0.044
			0	0.1	0.053	0.031	0.052	0.050	0.040	0.037	0.036	0.041
			0	0.2	0.037	0.026	0.040	0.037	0.037	0.046	0.041	0.043
			0	0.3	0.041	0.034	0.030	0.032	0.040	0.045	0.049	0.054
			0	0.4	0.047	0.040	0.050	0.047	0.037	0.037	0.040	0.041
			0	0.5	0.030	0.022	0.050	0.043	0.035	0.041	0.039	0.039
			0	0.6	0.035	0.018	0.039	0.028	0.034	0.037	0.035	0.039
			0	0.7	0.027	0.018	0.049	0.032	0.030	0.021	0.044	0.043
			0	0.8	0.027	0.009	0.039	0.016	0.032	0.022	0.026	0.022
			0	0.9	0.009	0.005	0.017	0.006	0.006	0.002	0.015	0.010
7	Rook	4	0	0.0	0.054	0.061	0.050	0.068	0.040	0.055	0.058	0.068
			0	0.1	0.042	0.059	0.043	0.056	0.042	0.068	0.044	0.055
			0	0.2	0.042	0.068	0.050	0.056	0.043	0.055	0.047	0.052
			0	0.3	0.043	0.061	0.042	0.055	0.031	0.055	0.045	0.048
			0	0.4	0.051	0.067	0.042	0.053	0.039	0.056	0.047	0.055
			0	0.5	0.029	0.054	0.042	0.049	0.036	0.058	0.048	0.055
			0	0.6	0.046	0.066	0.039	0.065	0.046	0.068	0.041	0.052
			0	0.7	0.054	0.056	0.043	0.059	0.035	0.052	0.052	0.061
			0	0.8	0.037	0.054	0.049	0.059	0.035	0.053	0.046	0.054
			0	0.9	0.038	0.053	0.044	0.050	0.042	0.055	0.046	0.054
7	Rook	16	0	0.0	0.036	0.030	0.068	0.061	0.046	0.054	0.041	0.045
			0	0.1	0.046	0.031	0.048	0.050	0.040	0.045	0.043	0.045
			0	0.2	0.041	0.029	0.037	0.045	0.044	0.060	0.054	0.058
			0	0.3	0.049	0.043	0.046	0.042	0.040	0.039	0.046	0.045
			0	0.4	0.047	0.041	0.033	0.027	0.037	0.042	0.047	0.054
			0	0.5	0.043	0.032	0.041	0.042	0.050	0.045	0.039	0.038
			0	0.6	0.046	0.036	0.047	0.037	0.034	0.030	0.043	0.039
			0	0.7	0.030	0.017	0.046	0.036	0.028	0.029	0.032	0.039
			0	0.8	0.037	0.011	0.036	0.024	0.022	0.021	0.049	0.037
			0	0.9	0.004	0.001	0.024	0.008	0.007	0.004	0.023	0.011
12	Rook	4	0	0.0	0.045	0.064	0.045	0.051	0.043	0.065	0.043	0.058
			0	0.1	0.044	0.060	0.071	0.074	0.046	0.056	0.054	0.057
			0	0.2	0.037	0.052	0.041	0.042	0.036	0.055	0.055	0.061
			0	0.3	0.041	0.050	0.037	0.046	0.037	0.061	0.042	0.056
			0	0.4	0.038	0.051	0.038	0.050	0.037	0.060	0.044	0.050
			0	0.5	0.045	0.044	0.040	0.045	0.048	0.077	0.046	0.048
			0	0.6	0.050	0.060	0.042	0.057	0.052	0.065	0.043	0.055
			0	0.7	0.033	0.038	0.038	0.051	0.057	0.076	0.047	0.055
			0	0.8	0.040	0.042	0.033	0.045	0.048	0.062	0.041	0.046
			0	0.9	0.059	0.058	0.049	0.054	0.042	0.052	0.051	0.063
12	Rook	16	0	0.0	0.042	0.039	0.053	0.047	0.042	0.058	0.036	0.042
			0	0.1	0.038	0.043	0.050	0.046	0.038	0.058	0.043	0.047
			0	0.2	0.035	0.035	0.041	0.041	0.037	0.045	0.043	0.048
			0	0.3	0.042	0.048	0.051	0.038	0.038	0.052	0.052	0.057
			0	0.4	0.033	0.029	0.052	0.035	0.050	0.059	0.051	0.053
			0	0.5	0.047	0.040	0.047	0.039	0.054	0.055	0.053	0.050
			0	0.6	0.034	0.024	0.038	0.037	0.042	0.053	0.039	0.043
			0	0.7	0.036	0.032	0.050	0.031	0.032	0.039	0.043	0.047
			0	0.8	0.037	0.029	0.040	0.033	0.032	0.034	0.033	0.041
			0	0.9	0.018	0.001	0.031	0.011	0.018	0.004	0.033	0.020

Table 5b : Estimated size of marginal LM and LR tests for testing $H_0^e : \theta = 0$ (assuming $\lambda = 0$) when weight matrix is QUEEN.

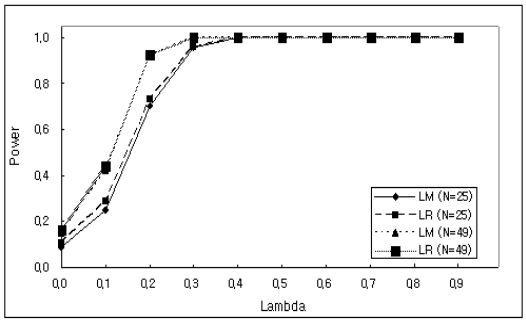
					Quadratic heteroskedasticity				Exponential heteroskedasticity			
					N=25		N=49		N=25		N=49	
T	W	σ_v^2	θ	λ	LM	LR	LM	LR	LM	LR	LM	LR
5	Queen	4	0	0.0	0.039	0.062	0.048	0.071	0.036	0.056	0.050	0.054
			0	0.1	0.048	0.069	0.049	0.062	0.047	0.055	0.053	0.069
			0	0.2	0.039	0.057	0.052	0.064	0.060	0.074	0.047	0.046
			0	0.3	0.045	0.066	0.055	0.060	0.031	0.058	0.041	0.052
			0	0.4	0.047	0.074	0.045	0.068	0.043	0.063	0.055	0.070
			0	0.5	0.042	0.055	0.037	0.054	0.034	0.043	0.034	0.043
			0	0.6	0.027	0.055	0.039	0.062	0.047	0.067	0.045	0.059
			0	0.7	0.053	0.065	0.044	0.055	0.044	0.061	0.033	0.051
			0	0.8	0.039	0.063	0.052	0.059	0.052	0.065	0.050	0.056
			0	0.9	0.043	0.046	0.038	0.047	0.031	0.040	0.041	0.041
5	Queen	16	0	0.0	0.038	0.032	0.046	0.045	0.038	0.032	0.039	0.044
			0	0.1	0.045	0.041	0.039	0.036	0.045	0.041	0.048	0.050
			0	0.2	0.049	0.038	0.049	0.046	0.049	0.038	0.047	0.054
			0	0.3	0.045	0.032	0.036	0.044	0.045	0.032	0.056	0.054
			0	0.4	0.042	0.024	0.035	0.036	0.042	0.024	0.043	0.037
			0	0.5	0.040	0.038	0.048	0.047	0.040	0.038	0.049	0.050
			0	0.6	0.036	0.017	0.040	0.033	0.036	0.017	0.049	0.047
			0	0.7	0.030	0.016	0.031	0.028	0.030	0.016	0.036	0.032
			0	0.8	0.028	0.009	0.040	0.020	0.028	0.009	0.034	0.029
			0	0.9	0.001	0.001	0.022	0.010	0.001	0.001	0.025	0.012
7	Queen	4	0	0.0	0.043	0.057	0.040	0.046	0.043	0.063	0.038	0.046
			0	0.1	0.043	0.059	0.042	0.064	0.047	0.065	0.045	0.059
			0	0.2	0.046	0.061	0.066	0.076	0.046	0.063	0.048	0.056
			0	0.3	0.043	0.055	0.059	0.076	0.049	0.072	0.045	0.053
			0	0.4	0.046	0.058	0.040	0.057	0.047	0.061	0.047	0.056
			0	0.5	0.041	0.063	0.035	0.054	0.049	0.066	0.031	0.037
			0	0.6	0.041	0.056	0.044	0.061	0.029	0.043	0.056	0.070
			0	0.7	0.050	0.064	0.052	0.077	0.046	0.056	0.048	0.052
			0	0.8	0.045	0.067	0.049	0.054	0.029	0.045	0.034	0.049
			0	0.9	0.048	0.052	0.039	0.060	0.049	0.065	0.053	0.060
7	Queen	16	0	0.0	0.037	0.032	0.049	0.055	0.046	0.050	0.046	0.056
			0	0.1	0.051	0.037	0.053	0.047	0.044	0.056	0.031	0.036
			0	0.2	0.036	0.033	0.058	0.050	0.042	0.047	0.052	0.058
			0	0.3	0.042	0.031	0.045	0.045	0.046	0.048	0.052	0.050
			0	0.4	0.038	0.034	0.054	0.056	0.046	0.053	0.045	0.056
			0	0.5	0.038	0.031	0.053	0.041	0.038	0.039	0.039	0.041
			0	0.6	0.041	0.024	0.046	0.039	0.039	0.041	0.053	0.049
			0	0.7	0.036	0.022	0.048	0.033	0.038	0.038	0.041	0.040
			0	0.8	0.032	0.009	0.047	0.021	0.023	0.012	0.038	0.039
			0	0.9	0.012	0.002	0.028	0.012	0.007	0.005	0.020	0.015
12	Queen	4	0	0.0	0.032	0.048	0.026	0.042	0.054	0.077	0.045	0.064
			0	0.1	0.041	0.046	0.053	0.061	0.041	0.064	0.054	0.057
			0	0.2	0.037	0.059	0.059	0.060	0.036	0.061	0.045	0.058
			0	0.3	0.053	0.062	0.051	0.057	0.048	0.074	0.042	0.051
			0	0.4	0.038	0.067	0.045	0.057	0.045	0.066	0.045	0.056
			0	0.5	0.049	0.055	0.056	0.060	0.042	0.057	0.054	0.057
			0	0.6	0.038	0.048	0.044	0.046	0.043	0.064	0.038	0.048
			0	0.7	0.048	0.057	0.056	0.074	0.034	0.052	0.044	0.056
			0	0.8	0.041	0.045	0.050	0.062	0.048	0.066	0.052	0.056
			0	0.9	0.042	0.035	0.035	0.044	0.038	0.050	0.048	0.048
12	Queen	16	0	0.0	0.053	0.046	0.047	0.043	0.049	0.059	0.051	0.056
			0	0.1	0.039	0.034	0.045	0.044	0.043	0.057	0.034	0.042
			0	0.2	0.042	0.029	0.044	0.037	0.032	0.043	0.045	0.054
			0	0.3	0.045	0.040	0.052	0.039	0.046	0.051	0.038	0.047
			0	0.4	0.039	0.027	0.044	0.033	0.044	0.058	0.043	0.055
			0	0.5	0.032	0.031	0.041	0.040	0.040	0.053	0.052	0.061
			0	0.6	0.040	0.029	0.046	0.031	0.043	0.045	0.049	0.054
			0	0.7	0.033	0.021	0.058	0.049	0.037	0.042	0.038	0.044
			0	0.8	0.041	0.023	0.041	0.025	0.038	0.027	0.061	0.059
			0	0.9	0.021	0.002	0.032	0.014	0.017	0.007	0.033	0.016

Figure 1. Frequency of rejections for $H_0^a : \lambda = 0$ and $\theta = 0$ N=25, 49 T=12, Exponential heteroskedasticity, weight matrix is ROOK.

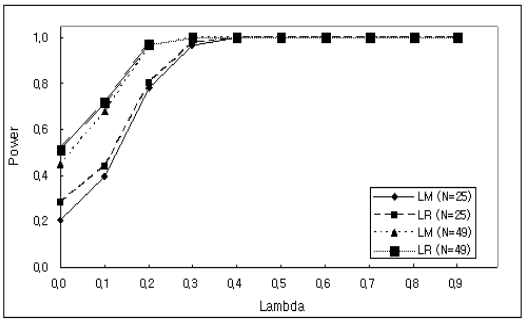
$\sigma_v^2 = 4,\ \theta = 0$



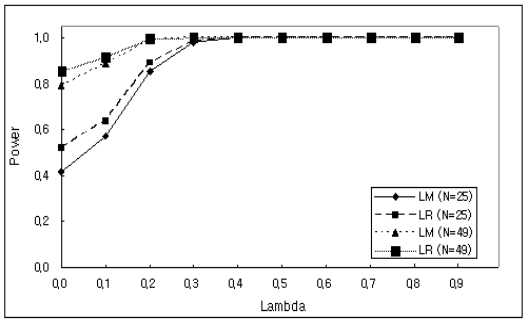
$\sigma_v^2 = 4,\ \theta = 1$



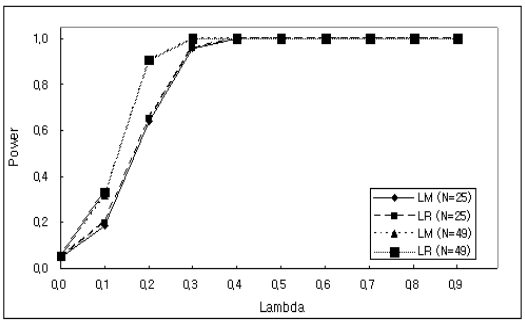
$\sigma_v^2 = 4,\ \theta = 2$



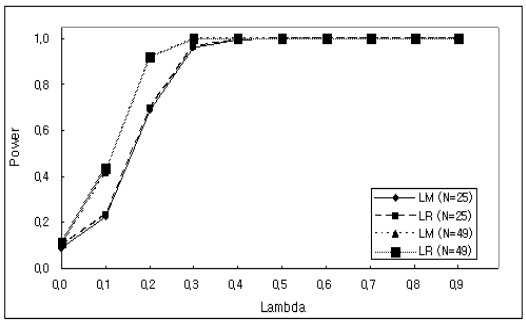
$\sigma_v^2 = 4,\ \theta = 3$



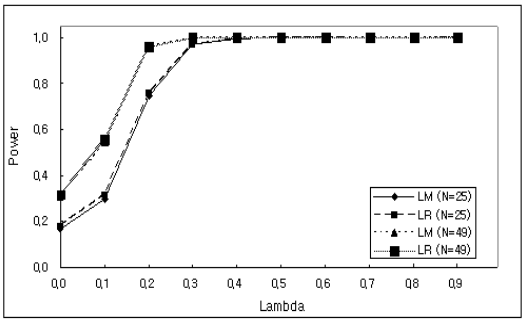
$\sigma_v^2 = 16,\ \theta = 0$



$\sigma_v^2 = 16,\ \theta = 1$



$\sigma_v^2 = 16,\ \theta = 2$



$\sigma_v^2 = 16,\ \theta = 3$

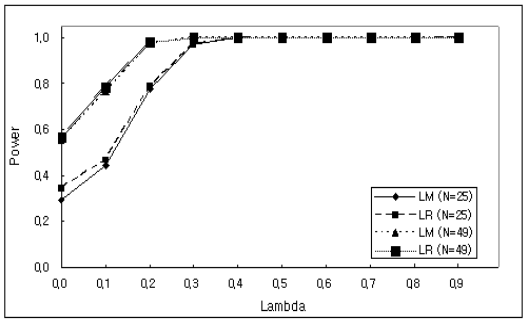
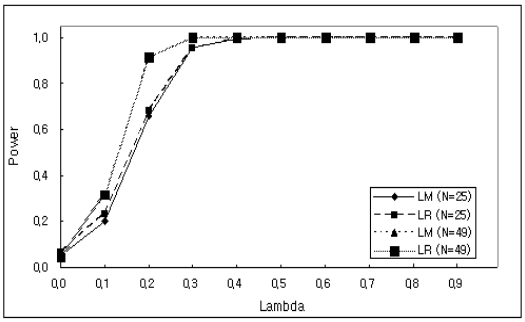
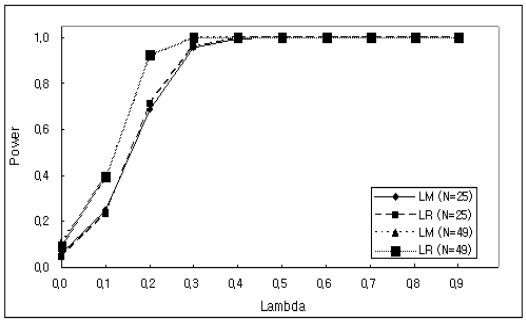


Figure 2. Frequency of rejections for $H_0^a : \lambda = 0$ and $\theta = 0$ N=25, 49 T=12, Quadratic heteroskedasticity, weight matrix is ROOK.

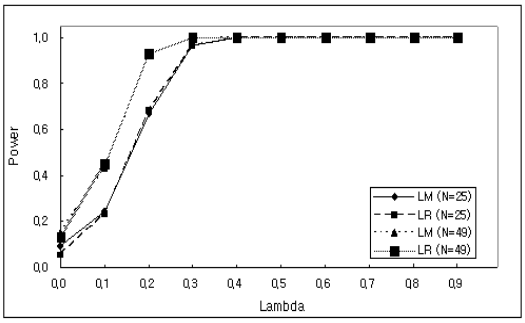
$\sigma_v^2 = 4,\ \theta = 0$



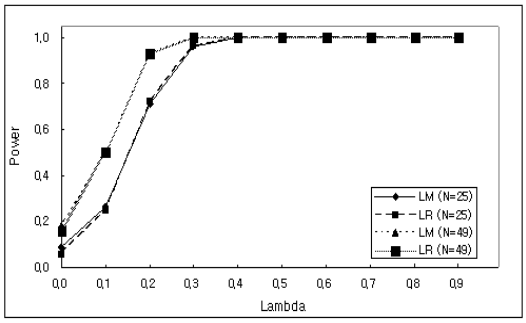
$\sigma_v^2 = 4,\ \theta = 1$



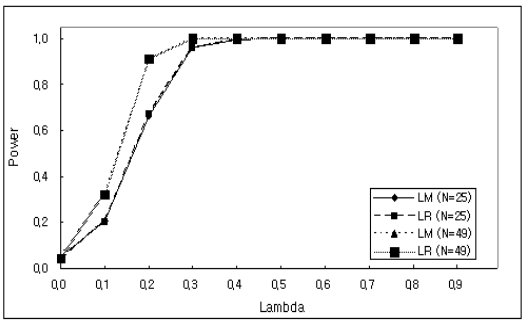
$\sigma_v^2 = 4,\ \theta = 2$



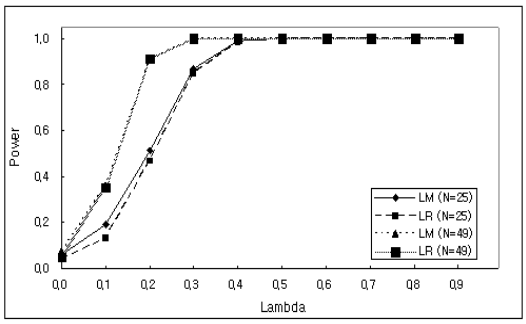
$\sigma_v^2 = 4,\ \theta = 3$



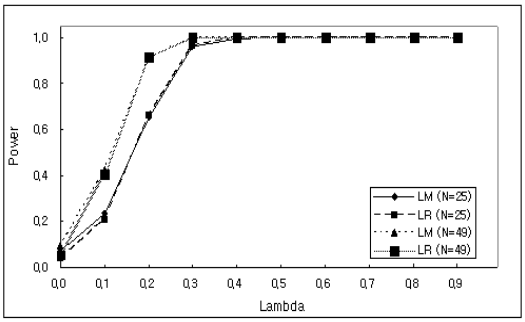
$\sigma_v^2 = 16,\ \theta = 0$



$\sigma_v^2 = 16,\ \theta = 1$



$\sigma_v^2 = 16,\ \theta = 2$



$\sigma_v^2 = 16,\ \theta = 3$

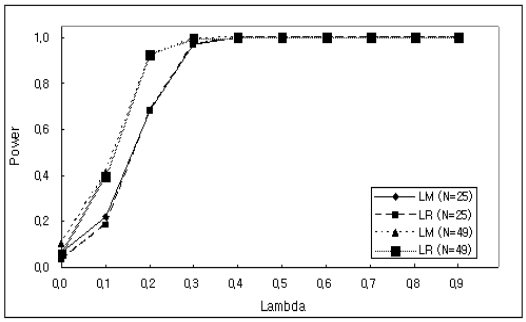
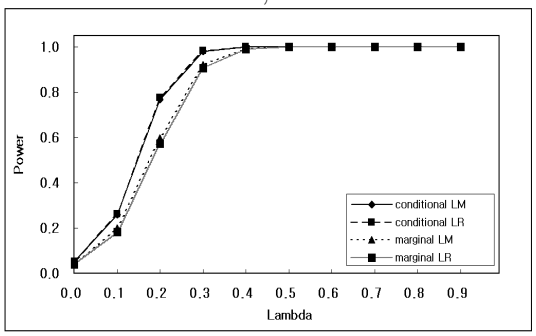


Figure 3. Frequency of rejections for $H_0^b : \lambda = 0$ (assuming $\theta \neq 0$) and for $H_0^d : \lambda = 0$ (assuming $\theta = 0$) T=12, $\sigma_v^2 = 16$, Quadratic heteroskedasticity, weight matrix is ROOK.

N=25, $\theta = 3$



N=49, $\theta = 3$

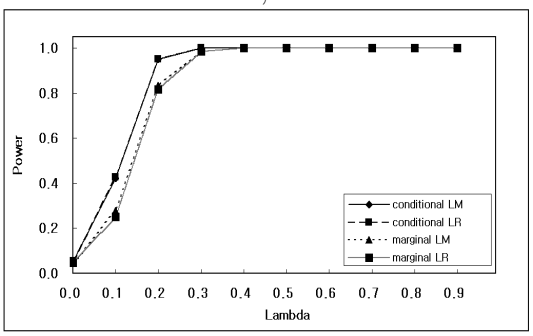
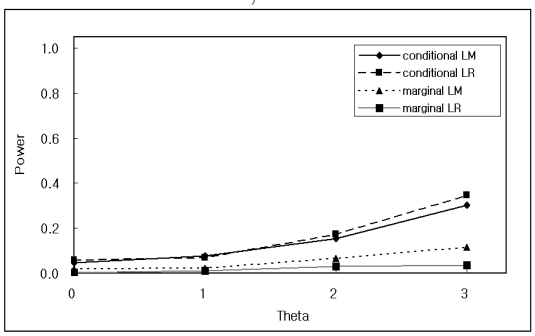
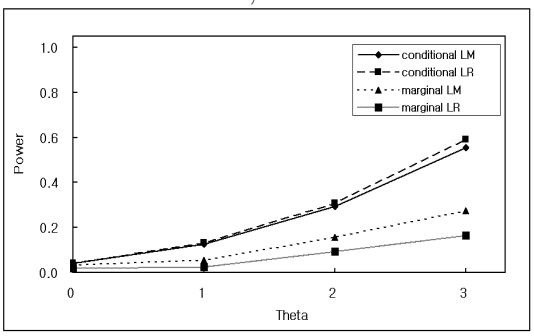


Figure 4. Frequency of rejections for $H_0^c : \theta = 0$ (assuming $\lambda \neq 0$) and for $H_0^e : \theta = 0$ (assuming $\lambda = 0$) T=12, $\sigma_v^2 = 16$, Exponential heteroskedasticity, weight matrix is ROOK.

N=25, $\lambda = 0.9$



N=49, $\lambda = 0.9$



5 Conclusion

This paper considered a panel data regression model with heteroskedasticity in the individual effects and spatial correlation in the remainder error. Testing for heteroskedasticity ignoring the spatial correlation as well as testing for spatial correlation ignoring heteroskedasticity is shown to lead to misleading results. The paper derived joint and conditional LM tests for heteroskedasticity and spatial correlation and studied their performance using Monte Carlo experiments. This paper generalized the Holly and Gardiol (2000) paper by allowing for spatial correlation in the remainder disturbances. The paper does not consider alternative forms of spatial lag dependence and this should be the subject of future research. In addition, it is important to point out that the asymptotic

distribution of our test statistics were not explicitly derived in the paper but that they are likely to hold under a similar set of primitive assumptions developed by Kelejian and Prucha (2001). The results in the paper should be tempered by the fact that the $N = 25, 49$ used in our Monte Carlo experiments may be small for a typical micro panel. Larger $N = 100$ did improve the performance of these tests whose critical values were based on their large sample distributions. Although the computational difficulty increases with N, one can use computational tricks given by Barry and Pace (1999) and Smirnov (2005).

Acknowledgement

This work was supported by **KOSEF**(R01-2006-000-10563-0). We would like to thank the guest editor Manfred M. Fischer and two anonymous referees for helpful comments and suggestions. We dedicate this paper in memory of our colleague and co-author Seuck Heun Song who passed away March, 2008.

Appendices

Appendix 1

This appendix derives the joint LM test for testing for no heteroskedasticity and no spatial correlation in a random effects panel data model, i.e., $H_o^a : \ \theta_1 = \cdots = \theta_p = 0$ and $\lambda = 0 | \ \sigma_\mu^2 > 0, \ \sigma_v^2 > 0$. The variance-covariance matrix of the disturbances in the unrestricted model is given by (2.8) and can be written as

$$\Omega = \sigma_\mu^2 \left(J_T \otimes diag \left(h \left(F\theta \right) \right) \right) + \sigma_v^2 (I_T \otimes \left(B'B \right)^{-1}), \tag{A.1}$$

where J_T is a matrix of ones of dimension T and F is $(N \times p)$ matrix of regressors that determine the heteroskedasticity. $diag(h(F\theta))$ denotes a diagonal $(N \times N)$ matrix with its ith diagonal element being the ith element of the $(N \times 1)$ vector $h(F\theta)$. The log-likelihood function under normality of the disturbances is given by

$$L(\beta, \varphi) = constant - \frac{1}{2} \log |\Omega| - \frac{1}{2} u' \Omega^{-1} u, \tag{A.2}$$

where $\varphi' = \left(\sigma_v^2, \sigma_\mu^2, \lambda, \theta_1, \ldots, \theta_p\right)$. The information matrix is block-diagonal between β and φ. Since H_0^a involves only φ, the part of the information due to β is ignored, see Breusch and Pagan (1980). In order to obtain the joint LM statistic, we need $D(\theta) = (\partial L/\partial\theta)$ and the information matrix $J(\theta) = E[-\partial^2 L/\partial\theta\partial\theta']$ evaluated at the restricted ML estimator $\widetilde{\varphi}$. Under the null hypothesis H_0^a, the variance-covariance matrix reduces to $\Omega_0 = \sigma_\mu^2(I_N \otimes J_T) + \sigma_\epsilon^2(I_N \otimes I_T)$. It is the familiar form of the random effects error component model with no spatial correlation or heteroskedasticity, see Baltagi (2005). Its inverse is given by

$$\Omega_0^{-1} = \frac{1}{\sigma_1^2}\left(\bar{J}_T \otimes I_N\right) + \frac{1}{\sigma_v^2}(E_T \otimes I_N), \tag{A.3}$$

where $\sigma_1^2 = \left(T\sigma_\mu^2 + \sigma_v^2\right)$, $E_T = I_T - \bar{J}_T$ and $\bar{J} = J_T/T$. Hartley and Rao (1967) or Hemmerle and Hartley (1973) give a general useful formula to derive the scores:

$$\frac{\partial L}{\partial \varphi_r} = -\frac{1}{2}tr\left[\Omega^{-1}\frac{\partial\Omega}{\partial\varphi_r}\right] + \frac{1}{2}\left[u'\Omega^{-1}\frac{\partial\Omega}{\partial\varphi_r}\Omega^{-1}u\right]. \tag{A.4}$$

Under the null hypothesis H_0^a, we get

$$\begin{aligned}
\frac{\partial\Omega}{\partial\theta_k}\Big|_{H_0^a} &= T\sigma_\mu^2 h^{(1)}(0)\left(\bar{J}_T \otimes diag(F_k)\right), \\
\frac{\partial\Omega}{\partial\lambda}\Big|_{H_0^a} &= \sigma_v^2\left(I_T \otimes \left(W' + W\right)\right), \\
\frac{\partial\Omega}{\partial\sigma_v^2}\Big|_{H_0^a} &= \left(I_T \otimes I_N\right) \\
\frac{\partial\Omega}{\partial\sigma_\mu^2}\Big|_{H_0^a} &= \left(J_T \otimes I_N\right),
\end{aligned} \tag{A.5}$$

where $h^{(1)}(0)$ denote the first derivative of h evaluated at zero, and F_k is the kth column of F. This makes use of the fact that $\partial\left(B'B\right)^{-1}/\partial\lambda = \left(B'B\right)^{-1}\left(W'B + B'W\right)\left(B'B\right)^{-1}$, see Anselin (1988, p.164). Substituting (A.5) into (A.4), we get

$$\begin{aligned}
\frac{\partial L}{\partial\sigma_v^2}\Big|_{H_0^a} &= -\frac{1}{2}tr\left[\left(\frac{1}{\sigma_1^2}\left(\bar{J}_T \otimes I_N\right) + \frac{1}{\sigma_v^2}(E_T \otimes I_N)\right)(I_T \otimes I_N))\right] \\
&\quad + \frac{1}{2}u'\left[\left(\frac{1}{\sigma_1^2}\left(\bar{J}_T \otimes I_N\right) + \frac{1}{\sigma_v^2}(E_T \otimes I_N)\right)(I_T \otimes I_N)\left(\frac{1}{\sigma_1^2}\left(\bar{J}_T \otimes I_N\right) + \frac{1}{\sigma_v^2}(E_T \otimes I_N)\right)\right]u \\
&= -\frac{1}{2}tr\left[\left(\frac{1}{\sigma_1^2}\left(\bar{J}_T \otimes I_N\right) + \frac{1}{\sigma_v^2}(E_T \otimes I_N)\right)\right] + \frac{1}{2}u'\left[\left(\frac{1}{\sigma_1^4}\left(\bar{J}_T \otimes I_N\right) + \frac{1}{\sigma_v^4}(E_T \otimes I_N)\right)\right]u
\end{aligned}$$

$$= -\frac{1}{2}\left(\frac{N}{\sigma_1^2}+\frac{N(T-1)}{\sigma_v^2}\right)+\frac{1}{2\sigma_1^4}u'(\bar{J}_T\otimes I_N)u+\frac{1}{2\sigma_v^4}u'(E_T\otimes I_N)u=0,$$

$$\begin{aligned}
\frac{\partial L}{\partial \sigma_\mu^2}\Big|_{H_0^a} &= -\frac{1}{2}tr\left[\left(\frac{1}{\sigma_1^2}\left(\bar{J}_T\otimes I_N\right)+\frac{1}{\sigma_v^2}(E_T\otimes I_N)\right)(J_T\otimes I_N)\right] \\
&\quad +\frac{1}{2}u'\left[\left(\frac{1}{\sigma_1^2}\left(\bar{J}_T\otimes I_N\right)+\frac{1}{\sigma_v^2}(E_T\otimes I_N)\right)(J_T\otimes I_N)\left(\frac{1}{\sigma_1^2}\left(\bar{J}_T\otimes I_N\right)+\frac{1}{\sigma_v^2}(E_T\otimes I_N)\right)\right]u \\
&= -\frac{T}{2\sigma_1^2}tr(\bar{J}_T\otimes I_N)+\frac{T}{2\sigma_1^4}u'(\bar{J}_T\otimes I_N)u \\
&= -\frac{TN}{2\sigma_1^2}+\frac{T}{2\sigma_1^4}u'(\bar{J}_T\otimes I_N)u=0,
\end{aligned}$$

$$\begin{aligned}
\frac{\partial L}{\partial \lambda}\Big|_{H_0^a} &= D\left(\lambda\right)=-\frac{1}{2}tr\left[\left(\frac{1}{\sigma_1^2}\left(\bar{J}_T\otimes I_N\right)+\frac{1}{\sigma_v^2}(E_T\otimes I_N)\right)\sigma_v^2(I_T\otimes(W'+W))\right] \\
&\quad +\frac{1}{2}u'\left[\left(\frac{1}{\sigma_1^2}\left(\bar{J}_T\otimes I_N\right)+\frac{1}{\sigma_v^2}(E_T\otimes I_N)\right)\sigma_v^2(I_T\otimes(W'+W))\left(\frac{1}{\sigma_1^2}\left(\bar{J}_T\otimes I_N\right)+\frac{1}{\sigma_v^2}(E_T\otimes I_N)\right)\right]u \\
&= -\frac{1}{2}tr\left[\frac{\sigma_v^2}{\sigma_1^2}\left(\bar{J}_T\otimes(W'+W)\right)+(E_T\otimes(W'+W))\right] \\
&\quad +\frac{1}{2\sigma_v^2}\left[u'(E_T\otimes(W'+W))u\right]+\frac{\sigma_v^2}{2\sigma_1^4}\left[u'(\bar{J}_T\otimes(W'+W))u\right] \\
&= \frac{1}{2\sigma_v^2}\left[u'(E_T\otimes(W'+W))u\right]+\frac{\sigma_v^2}{2\sigma_1^4}\left[u'(\bar{J}_T\otimes(W'+W))u\right],
\end{aligned}$$

$$\begin{aligned}
\frac{\partial L}{\partial \theta_k}\Big|_{H_0^a} &= -\frac{1}{2}tr\left[\left(\frac{1}{\sigma_1^2}\left(\bar{J}_T\otimes I_N\right)+\frac{1}{\sigma_v^2}(E_T\otimes I_N)\right)T\sigma_\mu^2h^{(1)}\left(0\right)\left(\bar{J}_T\otimes diag(F_k)\right)\right] \\
&\quad +\frac{1}{2}u'\begin{bmatrix}\left(\frac{1}{\sigma_1^2}\left(\bar{J}_T\otimes I_N\right)+\frac{1}{\sigma_v^2}(E_T\otimes I_N)\right)T\sigma_\mu^2h^{(1)}\left(0\right)\left(\bar{J}_T\otimes diag(F_k)\right) \\ \left(\frac{1}{\sigma_1^2}\left(\bar{J}_T\otimes I_N\right)+\frac{1}{\sigma_v^2}(E_T\otimes I_N)\right)\end{bmatrix}u \\
&= -\frac{T\sigma_\mu^2h^{(1)}\left(0\right)}{2\sigma_1^2}tr\left[\bar{J}_T\otimes diag(F_k)\right]+\frac{T\sigma_\mu^2h^{(1)}\left(0\right)}{2\sigma_1^4}u'\left[\bar{J}_T\otimes diag(F_k)\right]u \\
&= -\frac{T\sigma_\mu^2h^{(1)}\left(0\right)}{2\sigma_1^2}\iota_N'F_k+\frac{T\sigma_\mu^2h^{(1)}\left(0\right)}{2\sigma_1^4}S'F_k \quad \text{for } k=1,..,p,
\end{aligned}$$

$$\frac{\partial L}{\partial \theta'}\Big|_{H_0^a} = D\left(\widetilde{\theta}\right)=\frac{T\widetilde{\sigma}_\mu^2h^{(1)}\left(0\right)}{2\widetilde{\sigma}_1^4}S'F-\frac{T\widetilde{\sigma}_\mu^2h^{(1)}\left(0\right)}{2\widetilde{\sigma}_1^2}\iota_N'F$$

$$
\begin{aligned}
&= \frac{T\widetilde{\sigma}_{\mu}^{2}h^{(1)}(0)}{2\widetilde{\sigma}_{1}^{4}}S'F - \frac{T\widetilde{\sigma}_{\mu}^{2}h^{(1)}(0)}{2\widetilde{\sigma}_{1}^{2}\widetilde{\sigma}_{1}^{2}N}S'\iota_N\iota_N'F \\
&= \frac{T\widetilde{\sigma}_{\mu}^{2}h^{(1)}(0)}{2\widetilde{\sigma}_{1}^{4}}S'F - \frac{T\widetilde{\sigma}_{\mu}^{2}h^{(1)}(0)}{2\widetilde{\sigma}_{1}^{4}}S'\bar{J}_N F \\
&= \frac{T\widetilde{\sigma}_{\mu}^{2}h^{(1)}(0)}{2\widetilde{\sigma}_{1}^{4}}S'(I_N - \bar{J}_N)F = \frac{T\widetilde{\sigma}_{\mu}^{2}h^{(1)}(0)}{2\widetilde{\sigma}_{1}^{4}}S'\underline{F}, \qquad \text{(A.6)}
\end{aligned}
$$

This uses the fact that $tr(W) = tr(W') = 0$. From the second score equation, one gets that $\widetilde{\sigma}_1^2 = \frac{\widetilde{u}'(\bar{J}_T \otimes I_N)\widetilde{u}}{N}$,where $\tilde{u} = y - X\tilde{\beta}_{MLE}$ denote the restricted maximum likelihood residuals under H_0^a, i.e., under a random effects panel data model with no spatial correlation or heteroskedasticity. Substituting this in the first score equation, one gets $\widetilde{\sigma}_v^2 = \frac{\tilde{u}'(E_T \otimes I_N)\tilde{u}}{N(T-1)}$. Note that $\widetilde{\sigma}_1^2$ is used to simplify the last score equation. S is an $(N \times 1)$ vector with typical element $S_i = \tilde{u}_i'\bar{J}_T\tilde{u}_i$ and $\underline{F} = (I_N - \bar{J}_N)F$. Here, $\tilde{u}_i' = (\widetilde{u}_{i1}, .., \widetilde{u}_{iT})$, i.e., the residuals are re-ordered according to time for each individual. Therefore, the score with respect to each element of φ, evaluated at the resticted MLE is given by

$$
\widetilde{D}_{\varphi} = \begin{bmatrix} 0 \\ 0 \\ D\left(\widetilde{\lambda}\right) \\ D\left(\widetilde{\theta}\right) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{2\sigma_v^2}\left[\widetilde{u}'(E_T \otimes (W' + W))\widetilde{u}\right] + \frac{\sigma_v^2}{2\sigma_1^4}\left[\widetilde{u}'(\bar{J}_T \otimes (W' + W))\widetilde{u}\right] \\ \frac{T\widetilde{\sigma}_{\mu}^{2}h^{(1)}(0)}{2\widetilde{\sigma}_1^4}S'\underline{F} \end{bmatrix}.
$$

For the information matrix, it is useful to use the formula given by Harville(1977):

$$
J_{rs} = E\left[-\partial^2 L/\partial\varphi_r\partial\varphi_s\right] = \frac{1}{2}tr\left[\Omega^{-1}\frac{\partial\Omega}{\partial\varphi_r}\Omega^{-1}\frac{\partial\Omega}{\partial\varphi_s}\right],
$$

for r,s=1,2,..,p+3. Under the null hypothesis H_0^a,we get:

$$
\begin{aligned}
E\left[-\frac{\partial^2 L}{\partial^2\sigma_v^2}\right]\Big|_{H_0^a} &= \frac{1}{2}tr\begin{bmatrix} \left(\frac{1}{\sigma_1^2}\left(\bar{J}_T \otimes I_N\right) + \frac{1}{\sigma_v^2}(E_T \otimes I_N)\right)(I_T \otimes I_N) \\ \left(\frac{1}{\sigma_1^2}\left(\bar{J}_T \otimes I_N\right) + \frac{1}{\sigma_v^2}(E_T \otimes I_N)\right)(I_T \otimes I_N) \end{bmatrix} \\
&= \frac{1}{2}tr\left[\frac{1}{\sigma_1^4}\left(\bar{J}_T \otimes I_N\right) + \frac{1}{\sigma_v^4}(E_T \otimes I_N)\right] \\
&= \frac{N}{2}\left(\frac{1}{\sigma_1^4} + \frac{(T-1)}{\sigma_v^4}\right),
\end{aligned}
$$

$$
\begin{aligned}
E\left[-\frac{\partial^2 L}{\partial^2 \sigma_\mu^2}\right]\Big|_{H_0^a} &= \frac{1}{2} tr \begin{bmatrix} \left(\frac{1}{\sigma_1^2}\left(\bar{J}_T \otimes I_N\right) + \frac{1}{\sigma_v^2}(E_T \otimes I_N)\right)(J_T \otimes I_N) \\ \left(\frac{1}{\sigma_1^2}\left(\bar{J}_T \otimes I_N\right) + \frac{1}{\sigma_v^2}(E_T \otimes I_N)\right)(J_T \otimes I_N) \end{bmatrix} \\
&= \frac{T^2}{2\sigma_1^4} tr\left[(\bar{J}_T \otimes I_N)\right] = \frac{NT^2}{2\sigma_1^4},
\end{aligned}
$$

$$
\begin{aligned}
E\left[-\frac{\partial^2 L}{\partial^2 \lambda}\right]\Big|_{H_0^a} &= \frac{1}{2} tr \begin{bmatrix} \left(\frac{1}{\sigma_1^2}\left(\bar{J}_T \otimes I_N\right) + \frac{1}{\sigma_v^2}(E_T \otimes I_N)\right)\sigma_v^2(I_T \otimes (W' + W)) \\ \left(\frac{1}{\sigma_1^2}\left(\bar{J}_T \otimes I_N\right) + \frac{1}{\sigma_v^2}(E_T \otimes I_N)\right)\sigma_v^2(I_T \otimes (W' + W)) \end{bmatrix} \\
&= \frac{1}{2} tr\left[\frac{\sigma_v^4}{\sigma_1^4}\left(\bar{J}_T \otimes (W' + W)^2\right) + (E_T \otimes (W' + W)^2)\right] \\
&= \left(\frac{\sigma_v^4}{\sigma_1^4} + (T-1)\right) tr\left(W^2 + W'W\right),
\end{aligned}
$$

$$
\begin{aligned}
E\left[-\frac{\partial^2 L}{\partial \theta_k \partial \theta_l}\right]\Big|_{H_0^a} &= \frac{1}{2} tr \begin{bmatrix} \left(\frac{1}{\sigma_1^2}\left(\bar{J}_T \otimes I_N\right) + \frac{1}{\sigma_v^2}(E_T \otimes I_N)\right) T\sigma_\mu^2 h^{(1)}(0)\left(\bar{J}_T \otimes diag(F_k)\right) \\ \left(\frac{1}{\sigma_1^2}\left(\bar{J}_T \otimes I_N\right) + \frac{1}{\sigma_v^2}(E_T \otimes I_N)\right) T\sigma_\mu^2 h^{(1)}(0)\left(\bar{J}_T \otimes diag(F_l)\right) \end{bmatrix} \\
&= \frac{\left(T\sigma_\mu^2 h^{(1)}(0)\right)^2}{2\sigma_1^4} tr(\bar{J}_T \otimes diag(F_k) diag(F_l)) \\
&= \frac{\left(T\sigma_\mu^2 h^{(1)}(0)\right)^2}{2\sigma_1^4} tr\left[diag(F_k) diag(F_l)\right] = \frac{\left(T\sigma_\mu^2 h^{(1)}(0)\right)^2}{2\sigma_1^4} F_k' F_l,
\end{aligned}
$$

$$
\begin{aligned}
E\left[-\frac{\partial^2 L}{\partial \theta \partial \theta'}\right]\Big|_{H_0^a} &= \frac{\left(T\sigma_\mu^2 h^{(1)}(0)\right)^2}{2\sigma_1^4} F'F, \\
E\left[-\frac{\partial^2 L}{\partial \sigma_v^2 \partial \sigma_\mu^2}\right]\Big|_{H_0^a} &= \frac{1}{2} tr \begin{bmatrix} \left(\frac{1}{\sigma_1^2}\left(\bar{J}_T \otimes I_N\right) + \frac{1}{\sigma_v^2}(E_T \otimes I_N)\right)(I_T \otimes I_N) \\ \left(\frac{1}{\sigma_1^2}\left(\bar{J}_T \otimes I_N\right) + \frac{1}{\sigma_v^2}(E_T \otimes I_N)\right)(J_T \otimes I_N) \end{bmatrix} = \frac{NT}{2\sigma_1^4},
\end{aligned}
$$

$$
E\left[-\frac{\partial^2 L}{\partial \sigma_\mu^2 \partial \lambda}\right]\Big|_{H_0^a} = \frac{1}{2} tr \begin{bmatrix} \left(\frac{1}{\sigma_1^2}\left(\bar{J}_T \otimes I_N\right) + \frac{1}{\sigma_v^2}(E_T \otimes I_N)\right)(J_T \otimes I_N) \\ \left(\frac{1}{\sigma_1^2}\left(\bar{J}_T \otimes I_N\right) + \frac{1}{\sigma_v^2}(E_T \otimes I_N)\right)\sigma_v^2(I_T \otimes (W' + W)) \end{bmatrix} = 0,
$$

$$
\begin{aligned}
E\left[-\frac{\partial^2 L}{\partial\sigma_\mu^2\partial\theta_k}\right]\Big|_{H_0^a} &= \frac{1}{2}tr\left[\begin{array}{l}\left(\frac{1}{\sigma_1^2}\left(\bar{J}_T\otimes I_N\right)+\frac{1}{\sigma_v^2}(E_T\otimes I_N)\right)(J_T\otimes I_N)\\ \left(\frac{1}{\sigma_1^2}\left(\bar{J}_T\otimes I_N\right)+\frac{1}{\sigma_v^2}(E_T\otimes I_N)\right)T\sigma_\mu^2 h^{(1)}(0)\left(\bar{J}_T\otimes diag(F_k)\right)\end{array}\right]\\
&= \frac{T^2\sigma_\mu^2 h^{(1)}(0)}{2\sigma_1^4}i_N' F_k,\\
E\left[-\frac{\partial^2 L}{\partial\sigma_\mu^2\partial\theta'}\right]\Big|_{H_0^a} &= \frac{T^2\sigma_\mu^2 h^{(1)}(0)}{2\sigma_1^4}i_N' F,
\end{aligned}
$$

$$
\begin{aligned}
E\left[-\frac{\partial^2 L}{\partial\sigma_v^2\partial\lambda}\right]\Big|_{H_0^a} &= \frac{1}{2}tr\left[\begin{array}{l}\left(\frac{1}{\sigma_1^2}\left(\bar{J}_T\otimes I_N\right)+\frac{1}{\sigma_v^2}(E_T\otimes I_N)\right)(I_T\otimes I_N)\\ \left(\frac{1}{\sigma_1^2}\left(\bar{J}_T\otimes I_N\right)+\frac{1}{\sigma_v^2}(E_T\otimes I_N)\right)\sigma_v^2(I_T\otimes(W'+W))\end{array}\right]=0,\\
E\left[-\frac{\partial^2 L}{\partial\sigma_v^2\partial\theta_k}\right]\Big|_{H_0^a} &= \frac{1}{2}tr\left[\begin{array}{l}\left(\frac{1}{\sigma_1^2}\left(\bar{J}_T\otimes I_N\right)+\frac{1}{\sigma_v^2}(E_T\otimes I_N)\right)(I_T\otimes I_N)\\ \left(\frac{1}{\sigma_1^2}\left(\bar{J}_T\otimes I_N\right)+\frac{1}{\sigma_v^2}(E_T\otimes I_N)\right)T\sigma_\mu^2 h^{(1)}(0)\left(\bar{J}_T\otimes diag(F_k)\right)\end{array}\right]\\
&= \frac{T\sigma_\mu^2 h^{(1)}(0)}{2\sigma_1^4}i_N' F_k,\\
E\left[-\frac{\partial^2 L}{\partial\sigma_v^2\partial\theta'}\right]\Big|_{H_0^a} &= \frac{T\sigma_\mu^2 h^{(1)}(0)}{2\sigma_1^4}i_N' F,
\end{aligned}
$$

$$
\begin{aligned}
E\left[-\frac{\partial^2 L}{\partial\lambda\partial\theta_k}\right]\Big|_{H_0^a} &= \frac{1}{2}tr\left[\begin{array}{l}\left(\frac{1}{\sigma_1^2}\left(\bar{J}_T\otimes I_N\right)+\frac{1}{\sigma_v^2}(E_T\otimes I_N)\right)\sigma_v^2(I_T\otimes(W'+W))\\ \left(\frac{1}{\sigma_1^2}\left(\bar{J}_T\otimes I_N\right)+\frac{1}{\sigma_v^2}(E_T\otimes I_N)\right)T\sigma_\mu^2 h^{(1)}(0)\left(\bar{J}_T\otimes diag(F_k)\right)\end{array}\right]\\
&= 0. \qquad \text{(A.7)}
\end{aligned}
$$

Therefore, the information matrix under the null hypothesis H_0^a is given by

$$
\widetilde{J}_a(\varphi)=\left[\begin{array}{cccc}
\frac{N}{2}\left(\frac{1}{\sigma_1^4}+\frac{(T-1)}{\sigma_v^4}\right) & \frac{NT}{2\sigma_1^4} & 0 & \frac{T\sigma_\mu^2 h^{(1)}(0)}{2\sigma_1^4}i_N' F\\
\frac{NT}{2\sigma_1^4} & \frac{NT^2}{2\sigma_1^4} & 0 & \frac{T^2\sigma_\mu^2 h^{(1)}(0)}{2\sigma_1^4}i_N' F\\
0 & 0 & \left(\frac{\sigma_v^4}{\sigma_1^4}+(T-1)\right)b & 0\\
\frac{T\sigma_\mu^2 h^{(1)}(0)}{2\sigma_1^4}F' i_N & \frac{T^2\sigma_\mu^2 h^{(1)}(0)}{2\sigma_1^4}F' i_N & 0 & \frac{\left(T\sigma_\mu^2 h^{(1)}(0)\right)^2}{2\sigma_1^4}F' F
\end{array}\right],
$$

where $b = tr(W^2 + W'W)$. Since $\widetilde{D}'_\varphi = (0,\ 0,\ D(\widetilde{\lambda}),\ D(\widetilde{\theta}))$, and $\widetilde{J}_a(\varphi)$ is a block diagonal matrix with respect to $\varphi_1 = \left(\sigma_v^2,\ \sigma_\mu^2,\ \theta\right)$ and λ, we only need the lower $(p+1)\times(p+1)$ block of the inverse of $\widetilde{J}_a(\varphi)$. Let

$$\widetilde{J}_a(\varphi) = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix},$$

where

$$B_{22} = \begin{bmatrix} \left(\frac{\sigma_v^4}{\sigma_1^4} + (T-1)\right) b & 0 \\ 0 & \frac{\left(T\sigma_\mu^2 h^{(1)}(0)\right)^2}{2\sigma_1^4} F'F \end{bmatrix}.$$

And using the partitioned inverse formula, we get

$$\begin{aligned}
\left[\widetilde{J}_a(\varphi)\right]^{-1}_{(\lambda,\theta)} &= \left(B_{22} - B_{21}B_{11}^{-1}B_{12}\right)^{-1}, \text{ with} \\
B_{12} &= \begin{bmatrix} 0 & \frac{T\sigma_\mu^2 h^{(1)}(0)}{2\sigma_1^4} i'_N F \\ 0 & \frac{T^2\sigma_\mu^2 h^{(1)}(0)}{2\sigma_1^4} i'_N F \end{bmatrix}, \\
B_{21} &= \begin{bmatrix} 0 & 0 \\ \frac{T\sigma_\mu^2 h^{(1)}(0)}{2\sigma_1^4} F' i_N & \frac{T^2\sigma_\mu^2 h^{(1)}(0)}{2\sigma_1^4} F' i_N \end{bmatrix}, \\
B_{11}^{-1} &= \frac{1}{\frac{N^2T^2}{4\sigma_1^4}\left(\frac{T-1}{\sigma_v^4}\right)} \begin{bmatrix} \frac{NT^2}{2\sigma_1^4} & -\frac{NT}{2\sigma_1^4} \\ -\frac{NT}{2\sigma_1^4} & \frac{N}{2}\left(\frac{1}{\sigma_1^4} + \frac{(T-1)}{\sigma_v^4}\right) \end{bmatrix}, \\
B_{21}B_{11}^{-1} &= \frac{1}{N} \begin{bmatrix} 0 & 0 \\ 0 & \sigma_\mu^2 h^{(1)}(0) F' i_N \end{bmatrix}, \\
B_{21}B_{11}^{-1}B_{12} &= \begin{bmatrix} 0 & 0 \\ 0 & \frac{\left(T\sigma_\mu^2 h^{(1)}(0)\right)^2}{2\sigma_1^4} F'\bar{J}_N F \end{bmatrix}, \text{ and} \\
B_{22} - B_{21}B_{11}^{-1}B_{12} &= \begin{bmatrix} \left(\frac{\sigma_v^4}{\sigma_1^4} + (T-1)\right) b & 0 \\ 0 & \frac{\left(T\sigma_\mu^2 h^{(1)}(0)\right)^2}{2\sigma_1^4} \underline{F}'\underline{F} \end{bmatrix},
\end{aligned}$$

where $\underline{F} = (I_N - \bar{J}_N)F$. The resulting LM statistic for H_0^a is given by

$$LM_{\lambda\theta} = \widetilde{D}'_\varphi \widetilde{J}_a^{-1}(\varphi)\, \widetilde{D}_\varphi$$

$$
\begin{aligned}
&= \left(D\left(\widetilde{\lambda}\right), \frac{T\widetilde{\sigma}_{\mu}^{2} h^{(1)}(0)}{2\widetilde{\sigma}_{1}^{4}} S'\underline{F} \right) \begin{bmatrix} \left(\frac{\sigma_{\mu}^{4}}{\sigma_{1}^{4}} + (T-1)\right) b & 0 \\ 0 & \frac{\left(T\sigma_{\mu}^{2} h^{(1)}(0)\right)^{2}}{2\sigma_{1}^{4}} \underline{F}'\underline{F} \end{bmatrix}^{-1} \begin{bmatrix} D\left(\widetilde{\lambda}\right) \\ \frac{T\widetilde{\sigma}_{\mu}^{2} h^{(1)}(0)}{2\widetilde{\sigma}_{1}^{4}} \underline{F}'S \end{bmatrix} \\
&= \frac{D(\widetilde{\lambda})_{1}^{2}\widetilde{\sigma}_{1}^{4}}{\left(\widetilde{\sigma}_{v}^{4} + \widetilde{\sigma}_{1}^{4}(T-1)\right) b} + \frac{1}{2\widetilde{\sigma}_{1}^{4}} S'\underline{F}\left(\underline{F}'\underline{F}\right)^{-1}\underline{F}'S. \qquad \text{(A.8)}
\end{aligned}
$$

Under the null hypothesis, $LM_{\lambda\theta}$ should be asymptotically distributed as χ^{2}_{p+1}.

Appendix 2

This appendix derives the conditional LM test for testing no spatial correlation assuming there might be heteroskedasticity, i.e., $H_0^b : \lambda = 0$ (assuming some elements of θ may not be zero). Under the null hypothesis H_0^b , the variance-covariance matrix in (A.1) reduces to

$$
\Omega_0 = \sigma_{\mu}^{2}(J_T \otimes diag\left(h\left(F\theta\right)\right) + \sigma_{v}^{2}\left(I_T \otimes I_N\right). \qquad \text{(A.9)}
$$

Replacing J_T by $T\bar{J}_T$ and I_T by $(E_T + \bar{J}_T)$, where $E_T = I_T - \bar{J}_T$, and $\bar{J}_T = J_T/T$, see Wansbeek and Kapteyn (1982), one gets:

$$
\Omega_0 = \left[\left(\bar{J}_T \otimes diag\left(g_i\right)\right) + \sigma_{v}^{2}\left(E_T \otimes I_N\right)\right], \qquad \text{(A.10)}
$$

where $g_i = T\sigma_{\mu}^{2} h(f_i'\theta) + \sigma_{v}^{2}$. This also implies that

$$
\Omega_0^{-1} = \left(\bar{J}_T \otimes diag\left(\frac{1}{g_i}\right)\right) + \frac{1}{\sigma_{v}^{2}}\left(E_T \otimes I_N\right). \qquad \text{(A.11)}
$$

In this case:

$$
\begin{aligned}
\frac{\partial\Omega}{\partial\sigma_{\mu}^{2}}\Big|_{H_0^b} &= \left(J_T \otimes diag\left(h\left(F\theta\right)\right)\right), \\
\frac{\partial\Omega}{\partial\sigma_{v}^{2}}\Big|_{H_0^b} &= \left(I_T \otimes I_N\right), \\
\frac{\partial\Omega}{\partial\theta_k}\Big|_{H_0^b} &= T\sigma_{\mu}^{2}\left(\bar{J}_T \otimes diag\left(h^{(1)}\left(F\theta\right)\right) diag\left(F_k\right)\right), \text{ and} \\
\frac{\partial\Omega}{\partial\lambda}\Big|_{H_0^b} &= \sigma_{v}^{2}\left[I_T \otimes \left(W' + W\right)\right]. \qquad \text{(A.12)}
\end{aligned}
$$

Using (A.4), we get

$$
\frac{\partial L}{\partial\sigma_{v}^{2}}\Big|_{H_0^b} = -\frac{1}{2} tr\left[\left[\left(\bar{J}_T \otimes diag\left(\frac{1}{g_i}\right)\right) + \frac{1}{\sigma_{v}^{2}}\left(E_T \otimes I_N\right)\right]\left(I_T \otimes I_N\right)\right]
$$

$$
\begin{aligned}
&+\frac{1}{2}u'\left[[\left(\bar{J}_T\otimes diag\left(\frac{1}{g_i}\right)\right)+\frac{1}{\sigma_v^2}(E_T\otimes I_N)](I_T\otimes I_N)[\left(\bar{J}_T\otimes diag\left(\frac{1}{g_i}\right)\right)+\frac{1}{\sigma_v^2}(E_T\otimes I_N)]\right]u \\
= \quad & -\frac{1}{2}\left[\sum_{i=1}^{N}\left(\frac{1}{g_i}\right)+\frac{N(T-1)}{\sigma_v^2}\right]+\frac{1}{2}u'\left[[\left(\bar{J}_T\otimes diag\left(\frac{1}{g_i^2}\right)\right)+\frac{1}{\sigma_v^4}(E_T\otimes I_N)]\right]u=0,
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial L}{\partial\sigma_\mu^2}\Big|_{H_0^b} \quad = \quad & -\frac{1}{2}tr\left[[\left(\bar{J}_T\otimes diag\left(\frac{1}{g_i}\right)\right)+\frac{1}{\sigma_v^2}(E_T\otimes I_N)]\left(J_T\otimes diag\left(h\left(F\theta\right)\right)\right)\right] \\
& +\frac{1}{2}u'\begin{bmatrix} [\left(\bar{J}_T\otimes diag\left(\frac{1}{g_i}\right)\right)+\frac{1}{\sigma_v^2}(E_T\otimes I_N)]\left(J_T\otimes diag\left(h\left(F\theta\right)\right)\right) \\ [\left(\bar{J}_T\otimes diag\left(\frac{1}{g_i}\right)\right)+\frac{1}{\sigma_v^2}(E_T\otimes I_N)] \end{bmatrix}u \\
= \quad & -\frac{T}{2}\sum_{i=1}^{N}\left(\frac{h(f_i'\theta)}{g_i}\right)+\frac{T}{2}u'\left(\bar{J}_T\otimes diag\left(\frac{h(f_i'\theta)}{g_i^2}\right)\right)u=0,
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial L}{\partial\lambda}\Big|_{H_0^b} \quad = \quad & D\left(\lambda\right)=-\frac{1}{2}tr\left[[\left(\bar{J}_T\otimes diag\left(\frac{1}{g_i}\right)\right)+\frac{1}{\sigma_v^2}(E_T\otimes I_N)]\sigma_v^2(I_T\otimes(W'+W))\right] \\
& +\frac{1}{2}u'\begin{bmatrix} [\left(\bar{J}_T\otimes diag\left(\frac{1}{g_i}\right)\right)+\frac{1}{\sigma_v^2}(E_T\otimes I_N)] \\ \sigma_v^2(I_T\otimes(W'+W))[\left(\bar{J}_T\otimes diag\left(\frac{1}{g_i}\right)\right)+\frac{1}{\sigma_v^2}(E_T\otimes I_N)] \end{bmatrix}u \\
= \quad & -\frac{1}{2}tr\left[\sigma_v^2\left(\bar{J}_T\otimes diag\left(\frac{1}{g_i}\right)(W'+W)\right)+(E_T\otimes(W'+W))\right] \\
& +\frac{1}{2\sigma_v^2}\left[u'(E_T\otimes(W'+W))u\right]+\frac{\sigma_v^2}{2}\left[u'(\bar{J}_T\otimes diag\left(\frac{1}{g_i}\right)(W'+W)diag\left(\frac{1}{g_i}\right))u\right] \\
= \quad & \frac{1}{2\sigma_v^2}\left[u'(E_T\otimes(W'+W))u\right]+\frac{\sigma_v^2}{2}\left[u'(\bar{J}_T\otimes diag\left(\frac{1}{g_i}\right)(W'+W)diag\left(\frac{1}{g_i}\right))u\right],
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial L}{\partial\theta_k}\Big|_{H_0^b} \quad = \quad & -\frac{1}{2}tr\left[[\left(\bar{J}_T\otimes diag\left(\frac{1}{g_i}\right)\right)+\frac{1}{\sigma_v^2}(E_T\otimes I_N)]T\sigma_\mu^2\left(\bar{J}_T\otimes diag\left(h^{(1)}\left(F\theta\right)\right)diag\left(F_k\right)\right)\right] \\
& +\frac{1}{2}u'\begin{bmatrix} [\left(\bar{J}_T\otimes diag\left(\frac{1}{g_i}\right)\right)+\frac{1}{\sigma_v^2}(E_T\otimes I_N)]T\sigma_\mu^2\left(\bar{J}_T\otimes diag\left(h^{(1)}\left(F\theta\right)\right)diag\left(F_k\right)\right) \\ [\left(\bar{J}_T\otimes diag\left(\frac{1}{g_i}\right)\right)+\frac{1}{\sigma_v^2}(E_T\otimes I_N)] \end{bmatrix}u \\
= \quad & -\frac{T\sigma_\mu^2}{2}tr\left[(\bar{J}_T\otimes diag\left(\frac{1}{g_i}\right)diag\left(h^{(1)}\left(F\theta\right)\right)diag(F_k)\right] \\
& +\frac{T\sigma_\mu^2}{2}u'\left[\bar{J}_T\otimes diag\left(\frac{1}{g_i}\right)diag\left(h^{(1)}\left(F\theta\right)\right)diag(F_k)diag\left(\frac{1}{g_i}\right)\right]u \\
= \quad & 0 \quad \text{for } k=1,..,p. \qquad \text{(A.13)}
\end{aligned}
$$

Therefore, the score with respect to each element of φ, evaluated at the resticted MLE under H_0^b is given by

$$\widetilde{D}_\varphi = \begin{bmatrix} 0 \\ 0 \\ D\left(\widetilde{\lambda}\right) \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{2\widetilde{\sigma}_v^2}\left[\widetilde{u}'(E_T \otimes (W'+W))\widetilde{u}\right] + \frac{\widetilde{\sigma}_v^2}{2}\left[\widetilde{u}'(\bar{J}_T \otimes diag\left(\frac{1}{\widetilde{g}_i}\right)(W'+W)diag\left(\frac{1}{\widetilde{g}_i}\right))\widetilde{u}\right] \\ 0 \end{bmatrix},$$

where $\tilde{u} = y - X\tilde{\beta}_{MLE}$ denote the restricted maximum likelihood residuals under H_0^b, i.e., under a random effects panel data model with no spatial correlation but possible heteroskedasticity, $\widetilde{g}_i = (T\widetilde{\sigma}_\mu^2 h(f_i'\widetilde{\theta}) + \widetilde{\sigma}_v^2)$, where $\widetilde{\theta}$ and $\widetilde{\sigma}_v^2$ are the restricted MLE of θ and σ_v^2, under H_0^b.

Also, the elements of the information matrix for this model are given by

$$\begin{aligned} E\left[-\frac{\partial^2 L}{\partial^2 \sigma_v^2}\right]\Big|_{H_0^b} &= \frac{1}{2}tr\begin{bmatrix} [\left(\bar{J}_T \otimes diag\left(\frac{1}{g_i}\right)\right) + \frac{1}{\sigma_v^2}(E_T \otimes I_N)](I_T \otimes I_N) \\ [\left(\bar{J}_T \otimes diag\left(\frac{1}{g_i}\right)\right) + \frac{1}{\sigma_v^2}(E_T \otimes I_N)](I_T \otimes I_N) \end{bmatrix} \\ &= \frac{1}{2}tr\left[\left(\bar{J}_T \otimes diag\left(\frac{1}{g_i^2}\right)\right) + \frac{1}{\sigma_v^4}(E_T \otimes I_N)\right] \\ &= \frac{1}{2}\sum_{i=1}^{N}\left(\frac{1}{g_i^2}\right) + \frac{N(T-1)}{2\sigma_v^4}, \end{aligned}$$

$$\begin{aligned} E\left[-\frac{\partial^2 L}{\partial^2 \sigma_\mu^2}\right]\Big|_{H_0^b} &= \frac{1}{2}tr\begin{bmatrix} [\left(\bar{J}_T \otimes diag\left(\frac{1}{g_i}\right)\right) + \frac{1}{\sigma_v^2}(E_T \otimes I_N)]\left(J_T \otimes diag\left(h\left(F\theta\right)\right)\right) \\ [\left(\bar{J}_T \otimes diag\left(\frac{1}{g_i}\right)\right) + \frac{1}{\sigma_v^2}(E_T \otimes I_N)]\left(J_T \otimes diag\left(h\left(F\theta\right)\right)\right) \end{bmatrix} \\ &= \frac{1}{2}T^2 tr\left[\bar{J}_T \otimes diag\left(\frac{1}{g_i}\right) diag\left(h\left(F\theta\right)\right) diag\left(\frac{1}{g_i}\right) diag\left(h\left(F\theta\right)\right)\right] \\ &= \frac{T^2}{2}\sum_{i=1}^{N}\left(\frac{h(f_i'\theta)}{g_i}\right)^2, \end{aligned}$$

$$\begin{aligned} E\left[-\frac{\partial^2 L}{\partial^2 \lambda}\right]\Big|_{H_0^b} &= \frac{1}{2}tr\begin{bmatrix} [\left(\bar{J}_T \otimes diag\left(\frac{1}{g_i}\right)\right) + \frac{1}{\sigma_v^2}(E_T \otimes I_N)]\sigma_v^2\left[I_T \otimes (W'+W)\right] \\ [\left(\bar{J}_T \otimes diag\left(\frac{1}{g_i}\right)\right) + \frac{1}{\sigma_v^2}(E_T \otimes I_N)]\sigma_v^2\left[I_T \otimes (W'+W)\right] \end{bmatrix} \\ &= \frac{1}{2}tr\begin{bmatrix} \sigma_v^4\left(\bar{J}_T \otimes diag\left(\frac{1}{g_i}\right)(W'+W)\, diag\left(\frac{1}{g_i}\right)(W'+W)\right) \\ + \left(E_T \otimes \left[(W'+W)\right]^2\right) \end{bmatrix} \end{aligned}$$

$$= \frac{\sigma_v^4}{2} tr\left[diag\left(\frac{1}{g_i}\right)(W'+W)\, diag\left(\frac{1}{g_i}\right)(W'+W)\right] + (T-1)b,$$

$$\begin{aligned}
E\left[-\frac{\partial^2 L}{\partial\theta_k\partial\theta_l}\right]\Big|_{H_0^b} &= \frac{1}{2} tr\begin{bmatrix} [\left(\bar{J}_T \otimes diag\left(\frac{1}{g_i}\right)\right) + \frac{1}{\sigma_v^2}(E_T\otimes I_N)]T\sigma_\mu^2\left(\bar{J}_T\otimes diag\left(h^{(1)}(F\theta)\right) diag\left(F_k\right)\right) \\ [\left(\bar{J}_T \otimes diag\left(\frac{1}{g_i}\right)\right) + \frac{1}{\sigma_v^2}(E_T\otimes I_N)]T\sigma_\mu^2\left(\bar{J}_T\otimes diag\left(h^{(1)}(F\theta)\right) diag\left(F_l\right)\right) \end{bmatrix} \\
&= \frac{\left(T\sigma_\mu^2\right)^2}{2} tr\left[\begin{pmatrix} \bar{J}_T\otimes diag\left(\frac{1}{g_i}\right) diag\left(h^{(1)}(F\theta)\right) diag\left(F_k\right) \\ diag\left(\frac{1}{g_i}\right) diag\left(h^{(1)}(F\theta)\right) diag\left(F_l\right)\end{pmatrix}\right] \\
&= \frac{\left(T\sigma_\mu^2\right)^2}{2}\sum_{i=1}^N\left(\frac{h^{(1)}(f_i'\theta)}{g_i}\right)^2 f_{ki}f_{li} \\
&= \frac{\left(T\sigma_\mu^2\right)^2}{2} f_k' diag\left(r_i^2\right) f_l,
\end{aligned}$$

$$\begin{aligned}
E\left[-\frac{\partial^2 L}{\partial\theta'\partial\theta}\right]\Big|_{H_0^b} &= \frac{\left(T\sigma_\mu^2\right)^2}{2} F' diag\left(r_i^2\right) F, \\
E\left[-\frac{\partial^2 L}{\partial\sigma_\mu^2\partial\sigma_v^2}\right]\Big|_{H_0^b} &= \frac{1}{2} tr\begin{bmatrix} [\left(\bar{J}_T \otimes diag\left(\frac{1}{g_i}\right)\right) + \frac{1}{\sigma_v^2}(E_T\otimes I_N)]T(\bar{J}_T\otimes diag\left(h\left(F\theta\right)\right)) \\ [\left(\bar{J}_T \otimes diag\left(\frac{1}{g_i}\right)\right) + \frac{1}{\sigma_v^2}(E_T\otimes I_N)]\left(I_T\otimes I_N\right) \end{bmatrix} \\
&= \frac{T}{2} tr\left[\left(\bar{J}_T\otimes diag\left(\frac{1}{g_i}\right) diag\left(h\left(F\theta\right)\right) diag\left(\frac{1}{g_i}\right)\right)\right] \\
&= \frac{T}{2} tr\left[\left(diag\left(\frac{1}{g_i}\right) diag\left(h\left(F\theta\right)\right) diag\left(\frac{1}{g_i}\right)\right)\right] \\
&= \frac{T}{2}\sum_{i=1}^N\left(\frac{h(f_i'\theta)}{g_i^2}\right),
\end{aligned}$$

$$\begin{aligned}
E\left[-\frac{\partial^2 L}{\partial\sigma_v^2\partial\lambda}\right]\Big|_{H_0^b} &= \frac{1}{2} tr\begin{bmatrix} [\left(\bar{J}_T \otimes diag\left(\frac{1}{g_i}\right)\right) + \frac{1}{\sigma_v^2}(E_T\otimes I_N)]\left(I_T\otimes I_N\right) \\ [\left(\bar{J}_T \otimes diag\left(\frac{1}{g_i}\right)\right) + \frac{1}{\sigma_v^2}(E_T\otimes I_N)]\sigma_v^2\left[I_T\otimes(W'+W)\right] \end{bmatrix} \\
&= \frac{\sigma_v^2}{2} tr\left[\left(\bar{J}_T\otimes diag\left(\frac{1}{g_i}\right)^2(W'+W)\right) + \frac{1}{\sigma_v^4}(E_T\otimes(W'+W))\right] = 0, \\
E\left[-\frac{\partial^2 L}{\partial\sigma_v^2\partial\theta_i}\right]\Big|_{H_0^b} &= \frac{1}{2} tr\begin{bmatrix} \left[\left(\bar{J}_T \otimes diag\left(\frac{1}{g_i}\right)\right) + \frac{1}{\sigma_v^2}(E_T\otimes I_N)\right]\left(I_T\otimes I_N\right) \\ \left[\left(\bar{J}_T \otimes diag\left(\frac{1}{g_i}\right)\right) + \frac{1}{\sigma_v^2}(E_T\otimes I_N)\right] T\sigma_\mu^2\left(\bar{J}_T\otimes diag\left(h^{(1)}(F\theta)\right) diag\left(F_i\right)\right) \end{bmatrix}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{T\sigma_\mu^2}{2} tr\left[\bar{J}_T \otimes diag\left(\frac{1}{g_i^2}\right) diag\left(h^{(1)}\left(F\theta\right)\right) diag\left(F_i\right)\right] \\
&= \frac{T\sigma_\mu^2}{2} tr\left[diag\left(\frac{1}{g_i^2}\right) diag\left(h^{(1)}\left(F\theta\right)\right) diag\left(F_i\right)\right] \\
&= \frac{T\sigma_\mu^2}{2} \sum_{i=1}^{N}\left(\frac{h^{(1)}(f_i'\theta)}{g_i^2}\right) f_{ik} \\
&= \frac{T\sigma_\mu^2}{2} \mathbf{z}' F_k, \\
E\left[-\frac{\partial^2 L}{\partial\sigma_v^2 \partial\theta}\right]\Big|_{H_0^b} &= \frac{T\sigma_\mu^2}{2} \mathbf{z}' F, \\
E\left[-\frac{\partial^2 L}{\partial\sigma_\mu^2 \partial\lambda}\right]\Big|_{H_0^b} &= \frac{1}{2} tr\left[\begin{array}{l} \left[\left(\bar{J}_T \otimes diag\left(\frac{1}{g_i}\right)\right) + \frac{1}{\sigma_v^2}(E_T \otimes I_N)\right] T(\bar{J}_T \otimes diag\left(h\left(F\theta\right)\right) \\ \left[\left(\bar{J}_T \otimes diag\left(\frac{1}{g_i}\right)\right) + \frac{1}{\sigma_v^2}(E_T \otimes I_N)\right] \sigma_v^2 \left[I_T \otimes (W' + W)\right] \end{array}\right] \\
&= \frac{T\sigma_v^2}{2} tr\left[\left(\bar{J}_T \otimes diag\left(\frac{1}{g_i}\right) diag\left(h\left(F\theta\right)\right) diag\left(\frac{1}{g_i}\right)(W' + W)\right)\right] = 0, \\
E\left[-\frac{\partial^2 L}{\partial\sigma_\mu^2 \partial\theta_i}\right]\Big|_{H_0^b} &= \frac{1}{2} tr\left[\begin{array}{l} \left[\left(\bar{J}_T \otimes diag\left(\frac{1}{g_i}\right)\right) + \sigma_v^2 \left(E_T \otimes I_N\right)\right] T(\bar{J}_T \otimes diag\left(h\left(F\theta\right)\right) \\ \left[\left(\bar{J}_T \otimes diag\left(\frac{1}{g_i}\right)\right) + \sigma_v^2 \left(E_T \otimes I_N\right)\right] T\sigma_\mu^2 \left(\bar{J}_T \otimes diag\left(h^{(1)}\left(F\theta\right)\right) diag\left(F_k\right)\right) \end{array}\right] \\
&= \frac{T^2\sigma_\mu^2}{2} tr\left[\left(\bar{J}_T \otimes diag\left(\frac{1}{g_i}\right) diag\left(h\left(F\theta\right)\right) diag\left(\frac{1}{g_i}\right) diag\left(h^{(1)}\left(F\theta\right)\right) diag\left(F_k\right)\right)\right] \\
&= \frac{T^2\sigma_\mu^2}{2} tr\left[\left(diag\left(\frac{1}{g_i}\right) diag\left(h\left(F\theta\right)\right) diag\left(\frac{1}{g_i}\right) diag\left(h^{(1)}\left(F\theta\right)\right) diag\left(F_k\right)\right)\right] \\
&= \frac{T^2\sigma_\mu^2}{2} \sum_{i=1}^{N}\left(\frac{h(f_i'\theta)h^{(1)}\left(f_i'\theta\right)}{g_i^2}\right) f_{ki} \\
&= \frac{T^2\sigma_\mu^2}{2} \mathbf{s}' F_k, \\
E\left[-\frac{\partial^2 L}{\partial\sigma_\mu^2 \partial\theta}\right]\Big|_{H_0^b} &= \frac{T^2\sigma_\mu^2}{2} \mathbf{s}' F, \\
E\left[-\frac{\partial^2 L}{\partial\lambda \partial\theta_i}\right]\Big|_{H_0^b} &= \frac{1}{2} tr\left[\begin{array}{l} \left[\left(\bar{J}_T \otimes diag\left(\frac{1}{g_i}\right)\right) + \sigma_v^2 \left(E_T \otimes I_N\right)\right] \sigma_v^2 \left[I_T \otimes (W' + W)\right] \\ \left[\left(\bar{J}_T \otimes diag\left(\frac{1}{g_i}\right)\right) + \sigma_v^2 \left(E_T \otimes I_N\right)\right] T\sigma_\mu^2 \left(\bar{J}_T \otimes diag\left(h^{(1)}\left(F\theta\right)\right) diag\left(F_i\right)\right) \end{array}\right] \\
&= \frac{T\sigma_v^2\sigma_\mu^2}{2} tr\left[\bar{J}_T \otimes diag\left(\frac{1}{g_i}\right)(W' + W) diag\left(\frac{1}{g_i}\right) diag\left(h^{(1)}\left(F\theta\right)\right) diag\left(F_i\right)\right] \\
&= \frac{T\sigma_v^2\sigma_\mu^2}{2} tr\left[diag\left(\frac{1}{g_i}\right)(W' + W) diag\left(\frac{1}{g_i}\right) diag\left(h^{(1)}\left(F\theta\right)\right) diag\left(F_i\right)\right] = 0, \qquad \text{(A.14)}
\end{aligned}
$$

where $\mathbf{r}$,$\mathbf{z}$ and $\mathbf{s}$ are $(N \times 1)$ vectors with typical elements: $r_i = h^{(1)}(f_i'\theta)/g_i$, $z_i = h^{(1)}(f_i'\theta)/g_i^2$ and $s_i = h(f_i'\theta)h^{(1)}\left(f_i'\theta\right)/g_i^2$, respectively. Therefore, the information matrix under the null hypothesis

H_0^b is given by:

$$\widetilde{J}_b(\varphi) = \begin{bmatrix} \frac{1}{2}\sum_{i=1}^{N}\left(\frac{1}{g_i^2}\right) + \frac{NT-1)}{2\sigma_v^4} & \frac{T}{2}\sum_{i=1}^{N}\left(\frac{h(f_i'\theta)}{g_i^2}\right) & 0 & \frac{T\sigma_\mu^2}{2}\mathbf{z}'F \\ \frac{T}{2}\sum_{i=1}^{N}\left(\frac{h(f_i'\theta)}{g_i^2}\right) & \frac{T^2}{2}\sum_{i=1}^{N}\left(\frac{h(f_i'\theta)}{g_i}\right)^2 & 0 & \frac{T^2\sigma_\mu^2}{2}\mathbf{s}'F \\ 0 & 0 & \frac{\sigma_v^4}{2}d + (T-1)b & 0 \\ \frac{T\sigma_\mu^2}{2}F'\mathbf{z} & \frac{T^2\sigma_\mu^2}{2}s'F & 0 & \frac{\left(T\sigma_\mu^2\right)^2}{2}F'diag\left(r_i^2\right)F \end{bmatrix},$$

where $d = tr\left[diag\left(\frac{1}{g_i}\right)(W'+W)\,diag\left(\frac{1}{g_i}\right)(W'+W)\right]$. Since $\widetilde{D}_\varphi' = (0,\ 0,\ D(\widetilde{\lambda}),\ 0)$, and $\widetilde{J}_b(\varphi)$ is a block diagonal matrix with respect to $\varphi_1 = \left(\sigma_v^2,\ \sigma_\mu^2,\ \theta\right)$ and λ, the resulting conditional LM statistic for H_0^b is given by

$$LM_{\lambda|\theta} = \widetilde{D}_\varphi'\widetilde{J_b}^{-1}\widetilde{D}_\varphi = \frac{2D(\widetilde{\lambda})^2}{\sigma_v^4 d + 2(T-1)b}.$$

Under the null hypothesis H_0^b, $LM_{\lambda|\theta}$ should be asymptotically distributed as χ_1^2.

Appendix 3

This appendix derives the conditional LM test for testing for homoskedasticity assuming there might be spatial correlation, i.e $H_0^c : \theta = 0$ (assuming λ may not be zero). Under the null hypothesis H_0^c , the variance-covariance matrix in (A.1) reduces to

$$\Omega_0 = \sigma_\mu^2\left(J_T \otimes I_N\right) + \sigma_v^2(I_T \otimes \left(B'B\right)^{-1}). \tag{A.15}$$

Replacing J_T by $T\bar{J}_T$ and I_T by $(E_T + \bar{J}_T)$, where $E_T = I_T - \bar{J}_T$, and $\bar{J}_T = J_T/T$, see Wansbeek and Kapteyn (1982), one gets:

$$\Omega_0 = \sigma_v^2\left[\bar{J}_T \otimes \left(T\phi I_N + \left(B'B\right)^{-1}\right) + E_T \otimes \left(B'B\right)^{-1}\right], \tag{A.16}$$

where $\phi = \sigma_\mu^2/\sigma_v^2$. This also implies that

$$\Omega_0^{-1} = \frac{1}{\sigma_v^2}\left[\bar{J}_T \otimes Z + E_T \otimes (B'B)\right], \tag{A.17}$$

where $Z = \left(T\phi I_N + (B'B)^{-1}\right)^{-1}$. In this case:

$$\begin{aligned}
\frac{\partial\Omega}{\partial\sigma_\mu^2}\Big|_{H_0^c} &= T\left(\bar{J}_T \otimes I_N\right), \\
\frac{\partial\Omega}{\partial\sigma_v^2}\Big|_{H_0^c} &= (I_T \otimes (B'B)^{-1}), \\
\frac{\partial\Omega}{\partial\theta_k}\Big|_{H_0^c} &= T\sigma_\mu^2 h^{(1)}(0)\left(\bar{J}_T \otimes diag\left(F_k\right)\right), \text{ and} \\
\frac{\partial\Omega}{\partial\lambda}\Big|_{H_0^c} &= \sigma_v^2[I_T \otimes (B'B)^{-1}\left(W'B + B'W\right)(B'B)^{-1}].
\end{aligned} \tag{A.18}$$

Using (A.4), we get

$$\begin{aligned}
\frac{\partial L}{\partial\sigma_\mu^2}\Big|_{H_0^c} &= -\frac{1}{2}tr\left[\frac{1}{\sigma_v^2}\left[\bar{J}_T\otimes Z + E_T \otimes (B'B)\right]T\left(\bar{J}_T \otimes I_N\right)\right], \\
&\quad +\frac{1}{2}u'\begin{bmatrix} \frac{1}{\sigma_v^2}\left[\bar{J}_T\otimes Z + E_T \otimes (B'B)\right]T\left(\bar{J}_T \otimes I_N\right) \\ \frac{1}{\sigma_v^2}\left[\bar{J}_T\otimes Z + E_T \otimes (B'B)\right] \end{bmatrix} u \\
&= -\frac{T}{2\sigma_v^2}tr\left[Z\right] + \frac{T}{2\sigma_v^4}u'\left[\bar{J}_T \otimes Z^2\right]u = 0, \\
\frac{\partial L}{\partial\sigma_v^2}\Big|_{H_0^c} &= -\frac{1}{2}tr\left[\frac{1}{\sigma_v^2}\left[\bar{J}_T\otimes Z + E_T \otimes (B'B)\right](I_T \otimes (B'B)^{-1})\right] \\
&\quad +\frac{1}{2}u'\begin{bmatrix} \frac{1}{\sigma_v^2}\left[\bar{J}_T\otimes Z + E_T \otimes (B'B)\right](I_T \otimes (B'B)^{-1}) \\ \frac{1}{\sigma_v^2}\left[\bar{J}_T\otimes Z + E_T \otimes (B'B)\right] \end{bmatrix} u \\
&= -\frac{1}{2\sigma_v^2}tr\left[\bar{J}_T\otimes Z\left(B'B\right)^{-1} + E_T \otimes I_N\right] \\
&\quad +\frac{1}{2\sigma_v^4}u'\left[\bar{J}_T\otimes Z\left(B'B\right)^{-1}Z + E_T \otimes (B'B)\right]u = 0, \\
\frac{\partial L}{\partial\lambda}\Big|_{H_0^c} &= -\frac{1}{2}tr\left[\frac{1}{\sigma_v^2}[\bar{J}_T\otimes Z + E_T \otimes (B'B)]\,\sigma_v^2(I_T \otimes (B'B)^{-1}\left(W'B + B'W\right)(B'B)^{-1}\right] \\
&\quad +\frac{1}{2}u'\begin{bmatrix} \frac{1}{\sigma_v^2}\left[\bar{J}_T\otimes Z + E_T \otimes (B'B)\right]\sigma_v^2(I_T \otimes (B'B)^{-1}\left(W'B + B'W\right)(B'B)^{-1} \\ \frac{1}{\sigma_v^2}\left[\bar{J}_T\otimes Z + E_T \otimes (B'B)\right] \end{bmatrix} u \\
&= -\frac{1}{2}tr\left[\bar{J}_T \otimes Z(B'B)^{-1}\left(W'B + B'W\right)(B'B)^{-1} + E_T \otimes \left(W'B + B'W\right)(B'B)^{-1}\right] \\
&\quad +\frac{1}{2\sigma_v^2}u'\left[\bar{J}_T \otimes Z(B'B)^{-1}\left(W'B + B'W\right)(B'B)^{-1}Z + E_T \otimes \left(W'B + B'W\right)\right]u = 0, \\
\frac{\partial L}{\partial\theta_k}\Big|_{H_0^c} &= D\left(\theta_k\right) = -\frac{1}{2}tr\left[\frac{1}{\sigma_v^2}\left[\bar{J}_T\otimes Z + E_T \otimes (B'B)\right]T\sigma_\mu^2 h^{(1)}(0)\left(\bar{J}_T \otimes diag\left(F_k\right)\right)\right] \\
&\quad +\frac{1}{2}u'\begin{bmatrix} \frac{1}{\sigma_v^2}\left[\bar{J}_T\otimes Z + E_T \otimes (B'B)\right]T\sigma_\mu^2 h^{(1)}(0)\left(\bar{J}_T \otimes diag\left(F_k\right)\right) \\ \frac{1}{\sigma_v^2}\left[\bar{J}_T\otimes Z + E_T \otimes (B'B)\right] \end{bmatrix} u
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{T\sigma_\mu^2 h^{(1)}(0)}{2\sigma_v^2} tr\left[\bar{J}_T \otimes Z diag\left(F_k\right)\right] + \frac{T\sigma_\mu^2 h^{(1)}(0)}{2\sigma_v^4} u'\left[\bar{J}_T \otimes Z diag\left(F_k\right) Z\right] u \\
&= -\frac{T\sigma_\mu^2 h^{(1)}(0)}{2\sigma_v^2} d_1' F_k + \frac{T\sigma_\mu^2 h^{(1)}(0)}{2\sigma_v^4} u' D_k u \quad \text{for } k = 1, .., p,
\end{aligned}
$$

where d_1 is an $(N \times 1)$ vector with its typical element being the diagonal element of the matrix Z. $D_k = \left[\bar{J}_T \otimes Z diag\left(F_k\right) Z\right]$.

Therefore, the score with respect to each element of φ, evaluated at the resticted MLE under H_0^c is given by

$$
\widetilde{D}_\varphi = \begin{bmatrix} 0 \\ 0 \\ 0 \\ D\left(\widetilde{\theta}_1\right) \\ : \\ D\left(\widetilde{\theta}_p\right) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -\frac{T\sigma_\mu^2 h^{(1)}(0)}{2\sigma_v^2} d_1' F_1 + \frac{T\sigma_\mu^2 h^{(1)}(0)}{2\sigma_v^4} u' D_1 u \\ : \\ -\frac{T\sigma_\mu^2 h^{(1)}(0)}{2\sigma_v^2} d_1' F_p + \frac{T\sigma_\mu^2 h^{(1)}(0)}{2\sigma_v^4} u' D_p u \end{bmatrix},
$$

where $\widetilde{u} = y - X\widetilde{\beta}_{MLE}$ denote the restricted maximum likelihood residuals under H_0^c, i.e., under a random effects panel data model with no heteroskedasticity but possible spatial correlation.

Also, the elements of the information matrix for this model using (A.3) are given by

$$
\begin{aligned}
E\left[-\frac{\partial^2 L}{\partial^2 \sigma_v^2}\right]\Big|_{H_0^c} &= \frac{1}{2} tr \begin{bmatrix} \frac{1}{\sigma_v^2}\left[\bar{J}_T \otimes Z + E_T \otimes (B'B)\right]\left(I_T \otimes (B'B)^{-1}\right) \\ \frac{1}{\sigma_v^2}\left[\bar{J}_T \otimes Z + E_T \otimes (B'B)\right]\left(I_T \otimes (B'B)^{-1}\right) \end{bmatrix} \\
&= \frac{1}{2\sigma_v^4} tr\left[\bar{J}_T \otimes Z\left(B'B\right)^{-1} Z\left(B'B\right)^{-1} + E_T \otimes I_N\right] \\
&= \frac{1}{2\sigma_v^4} tr\left[Z\left(B'B\right)^{-1} Z\left(B'B\right)^{-1}\right] + \frac{N(T-1)}{2\sigma_v^4}, \\
E\left[-\frac{\partial^2 L}{\partial^2 \sigma_\mu^2}\right]\Big|_{H_0^c} &= \frac{1}{2} tr \begin{bmatrix} \frac{1}{\sigma_v^2}\left[\bar{J}_T \otimes Z + E_T \otimes (B'B)\right] T\left(\bar{J}_T \otimes I_N\right) \\ \frac{1}{\sigma_v^2}\left[\bar{J}_T \otimes Z + E_T \otimes (B'B)\right] T\left(\bar{J}_T \otimes I_N\right) \end{bmatrix} \\
&= \frac{T^2}{2\sigma_v^4} tr\left[\bar{J}_T \otimes Z^2\right] = \frac{T^2}{2\sigma_v^4} tr\left[Z^2\right], \\
E\left[-\frac{\partial^2 L}{\partial^2 \lambda}\right]\Big|_{H_0^c} &= \frac{1}{2} tr \begin{bmatrix} \frac{1}{\sigma_v^2}\left[\bar{J}_T \otimes Z + E_T \otimes (B'B)\right] \sigma_v^2 (I_T \otimes (B'B)^{-1}\left(W'B + B'W\right)\left(B'B\right)^{-1}) \\ \frac{1}{\sigma_v^2}\left[\bar{J}_T \otimes Z + E_T \otimes (B'B)\right] \sigma_v^2 (I_T \otimes (B'B)^{-1}\left(W'B + B'W\right)\left(B'B\right)^{-1}) \end{bmatrix} \\
&= \frac{1}{2} tr\left[Z(B'B)^{-1}\left(W'B + B'W\right)\left(B'B\right)^{-1}\right]^2
\end{aligned}
$$

$$
\begin{aligned}
& & & +\frac{(T-1)}{2} tr\left[\left(W'B+B'W\right)\left(B'B\right)^{-1}\right]^2, \\
E\left[-\frac{\partial^2 L}{\partial\theta_k\partial\theta_l}\right]\Big|_{H_0^c} & = & & \frac{1}{2} tr\begin{bmatrix} \frac{1}{\sigma_v^2}\left[\bar{J}_T\otimes Z+E_T\otimes\left(B'B\right)\right]T\sigma_\mu^2 h^{(1)}\left(0\right)\left(\bar{J}_T\otimes diag\left(F_k\right)\right) \\ \frac{1}{\sigma_v^2}\left[\bar{J}_T\otimes Z+E_T\otimes\left(B'B\right)\right]T\sigma_\mu^2 h^{(1)}\left(0\right)\left(\bar{J}_T\otimes diag\left(F_l\right)\right) \end{bmatrix} \\
& = & & \frac{\left(T\sigma_\mu^2 h^{(1)}\left(0\right)\right)^2}{2\sigma_v^4} tr\left[\bar{J}_T\otimes Z diag\left(F_k\right) Z diag\left(F_l\right)\right] \\
& = & & \frac{\left(T\sigma_\mu^2 h^{(1)}\left(0\right)\right)^2}{2\sigma_v^4} tr\left[Z diag\left(F_k\right) Z diag\left(F_l\right)\right] \\
& = & & \frac{\left(T\sigma_\mu^2 h^{(1)}\left(0\right)\right)^2}{2\sigma_v^4} F_k\left(Z\odot Z'\right)F_l, \\
E\left[-\frac{\partial^2 L}{\partial\theta\partial\theta'}\right]\Big|_{H_0^c} & = & & \frac{\left(T\sigma_\mu^2 h^{(1)}\left(0\right)\right)^2}{2\sigma_v^4} F'\left(Z\odot Z'\right)F, \\
E\left[-\frac{\partial^2 L}{\partial\sigma_\mu^2\partial\sigma_v^2}\right]\Big|_{H_0^c} & = & & \frac{1}{2} tr\begin{bmatrix} \frac{1}{\sigma_v^2}\left[\bar{J}_T\otimes Z+E_T\otimes\left(B'B\right)\right]T\left(\bar{J}_T\otimes I_N\right) \\ \frac{1}{\sigma_v^2}\left[\bar{J}_T\otimes Z+E_T\otimes\left(B'B\right)\right]\left(I_T\otimes\left(B'B\right)^{-1}\right) \end{bmatrix} \\
& = & & \frac{T}{2\sigma_v^4} tr\left[\bar{J}_T\otimes Z^2\left(B'B\right)^{-1}\right]=\frac{T}{2\sigma_v^4} tr\left[Z^2\left(B'B\right)^{-1}\right], \\
E\left[-\frac{\partial^2 L}{\partial\sigma_v^2\partial\lambda}\right]\Big|_{H_0^c} & = & & \frac{1}{2} tr\begin{bmatrix} \frac{1}{\sigma_v^2}\left[\bar{J}_T\otimes Z+E_T\otimes\left(B'B\right)\right]\left(I_T\otimes\left(B'B\right)^{-1}\right) \\ \frac{1}{\sigma_v^2}\left[\bar{J}_T\otimes Z+E_T\otimes\left(B'B\right)\right]\sigma_v^2(I_T\otimes\left(B'B\right)^{-1}\left(W'B+B'W\right)\left(B'B\right)^{-1} \end{bmatrix} \\
& = & & \frac{1}{2\sigma_v^2} tr\left[Z\left(B'B\right)^{-1}Z(B'B)^{-1}\left(W'B+B'W\right)\left(B'B\right)^{-1}\right] \\
& & & +\frac{(T-1)}{2\sigma_v^2} tr\left[\left(W'B+B'W\right)\left(B'B\right)^{-1}\right], \\
E\left[-\frac{\partial^2 L}{\partial\sigma_v^2\partial\theta_k}\right]\Big|_{H_0^c} & = & & \frac{1}{2} tr\begin{bmatrix} \frac{1}{\sigma_v^2}\left[\bar{J}_T\otimes Z+E_T\otimes\left(B'B\right)\right]\left(I_T\otimes\left(B'B\right)^{-1}\right) \\ \frac{1}{\sigma_v^2}\left[\bar{J}_T\otimes Z+E_T\otimes\left(B'B\right)\right]T\sigma_\mu^2 h^{(1)}\left(0\right)\left(\bar{J}_T\otimes diag\left(F_k\right)\right) \end{bmatrix} \\
& = & & \frac{T\sigma_\mu^2 h^{(1)}\left(0\right)}{2\sigma_v^4} tr\left[\bar{J}_T\otimes Z\left(B'B\right)^{-1}Z\left(diag\left(F_k\right)\right)\right] \\
& = & & \frac{T\sigma_\mu^2 h^{(1)}\left(0\right)}{2\sigma_v^4} tr\left[Z\left(B'B\right)^{-1}Z\left(diag\left(F_k\right)\right)\right] \\
& = & & \frac{T\sigma_\mu^2 h^{(1)}\left(0\right)}{2\sigma_v^4} d_2' F_k, \\
E\left[-\frac{\partial^2 L}{\partial\sigma_v^2\partial\theta'}\right]\Big|_{H_0^c} & = & & \frac{T\sigma_\mu^2 h^{(1)}\left(0\right)}{2\sigma_v^4} d_2' F,
\end{aligned}
$$

$$
\begin{aligned}
E\left[-\frac{\partial^2 L}{\partial\sigma_\mu^2\partial\lambda}\right]\Big|_{H_0^c} &= \frac{1}{2}tr\left[\begin{array}{l}\frac{1}{\sigma_v^2}\left[\bar{J}_T\otimes Z+E_T\otimes(B'B)\right]T\left(\bar{J}_T\otimes I_N\right)\\ \frac{1}{\sigma_v^2}\left[\bar{J}_T\otimes Z+E_T\otimes(B'B)\right]\sigma_v^2(I_T\otimes(B'B)^{-1}\left(W'B+B'W\right)(B'B)^{-1})\end{array}\right]\\
&= \frac{T}{2\sigma_v^2}tr\left[\bar{J}_T\otimes Z^2(B'B)^{-1}\left(W'B+B'W\right)(B'B)^{-1}\right]\\
&= \frac{T}{2\sigma_v^2}tr\left[Z^2(B'B)^{-1}\left(W'B+B'W\right)(B'B)^{-1}\right],\\
E\left[-\frac{\partial^2 L}{\partial\sigma_\mu^2\partial\theta_k}\right]\Big|_{H_0^c} &= \frac{1}{2}tr\left[\begin{array}{l}\frac{1}{\sigma_v^2}\left[\bar{J}_T\otimes Z+E_T\otimes(B'B)\right]T\left(\bar{J}_T\otimes I_N\right)\\ \frac{1}{\sigma_v^2}\left[\bar{J}_T\otimes Z+E_T\otimes(B'B)\right]T\sigma_\mu^2h^{(1)}(0)\left(\bar{J}_T\otimes diag\left(F_k\right)\right)\end{array}\right]\\
&= \frac{T^2\sigma_\mu^2h^{(1)}(0)}{2\sigma_v^4}tr\left[\bar{J}_T\otimes Z^2diag\left(F_k\right)\right]\\
&= \frac{T^2\sigma_\mu^2h^{(1)}(0)}{2\sigma_v^4}tr\left[Z^2diag\left(F_k\right)\right]=\frac{T^2\sigma_\mu^2h^{(1)}(0)}{2\sigma_v^4}d_3'F_k,\\
E\left[-\frac{\partial^2 L}{\partial\sigma_\mu^2\partial\theta'}\right]\Big|_{H_0^c} &= \frac{T^2\sigma_\mu^2h^{(1)}(0)}{2\sigma_v^4}d_3'F,\\
E\left[-\frac{\partial^2 L}{\partial\lambda\partial\theta_k}\right]\Big|_{H_0^c} &= \frac{1}{2}tr\left[\begin{array}{l}\frac{1}{\sigma_v^2}\left[\bar{J}_T\otimes Z+E_T\otimes(B'B)\right]\sigma_v^2(I_T\otimes(B'B)^{-1}\left(W'B+B'W\right)(B'B)^{-1})\\ \frac{1}{\sigma_v^2}\left[\bar{J}_T\otimes Z+E_T\otimes(B'B)\right]T\sigma_\mu^2h^{(1)}(0)\left(\bar{J}_T\otimes diag\left(F_k\right)\right)\end{array}\right]\\
&= \frac{T\sigma_\mu^2h^{(1)}(0)}{2\sigma_v^2}tr\left[Z(B'B)^{-1}\left(W'B+B'W\right)(B'B)^{-1}Zdiag\left(F_k\right)\right]\\
&= \frac{T\sigma_\mu^2h^{(1)}(0)}{2\sigma_v^2}d_4'F_k,\\
E\left[-\frac{\partial^2 L}{\partial\lambda\partial\theta'}\right]\Big|_{H_0^c} &= \frac{T\sigma_\mu^2h^{(1)}(0)}{2\sigma_v^2}d_4'F,
\end{aligned}
\tag{A.19}
$$

where $\odot$ stands for the Hadamard product, i.e., an element-by-element multiplication and d_2, d_3, d_4 are $(N\times 1)$ vectors corresponding to the diagonal elements of matrices: $Z(B'B)^{-1}Z$, Z^2 and $Z(B'B)^{-1}\left(W'B+B'W\right)(B'B)^{-1}Z$ respectively. Therefore, the information matrix under the null hypothesis H_0^c is given by:

$$\widetilde{J}_c(\varphi) = \begin{bmatrix} \frac{b_1}{2\sigma_v^4} + \frac{N(T-1)}{2\sigma_v^4} & \frac{Tb_2}{2\sigma_v^4} & \frac{1}{2\sigma_v^2}(b_3 + (T-1)b_4) & \frac{T\sigma_\mu^2 h^{(1)}(0)}{2\sigma_v^4} d_2' F \\ \frac{Tb_2}{2\sigma_v^4} & \frac{T^2 b_5}{2\sigma_v^4} & \frac{Tb_6}{2\sigma_v^2} & \frac{T^2\sigma_\mu^2 h^{(1)}(0)}{2\sigma_v^4} d_3' F \\ \frac{1}{2\sigma_v^2}(b_3 + (T-1)b_4) & \frac{Tb_6}{2\sigma_v^2} & \frac{1}{2}(b_7 + (T-1)b_8) & \frac{T\sigma_\mu^2 h^{(1)}(0)}{2\sigma_v^2} d_4' F \\ \frac{T\sigma_\mu^2 h^{(1)}(0)}{2\sigma_v^4} F' d_2 & \frac{T^2\sigma_\mu^2 h^{(1)}(0)}{2\sigma_v^4} F' d_3 & \frac{T\sigma_\mu^2 h^{(1)}(0)}{2\sigma_v^2} F' d_4 & \frac{\left(T\sigma_\mu^2 h^{(1)}(0)\right)^2}{2\sigma_v^4} F' \left(Z \odot Z'\right) F \end{bmatrix}$$

with

$$\begin{aligned} b_1 &= tr\left[Z\left(B'B\right)^{-1}\right]^2, \\ b_2 &= tr\left[Z^2\left(B'B\right)^{-1}\right], \\ b_3 &= tr\left[Z\left(B'B\right)^{-1} Z(B'B)^{-1}\left(W'B + B'W\right)\left(B'B\right)^{-1}\right], \\ b_4 &= tr\left[\left(W'B + B'W\right)\left(B'B\right)^{-1}\right], \\ b_5 &= tr\left[Z^2\right], \\ b_6 &= tr\left[Z^2(B'B)^{-1}\left(W'B + B'W\right)\left(B'B\right)^{-1}\right], \\ b_7 &= tr\left[Z(B'B)^{-1}\left(W'B + B'W\right)\left(B'B\right)^{-1}\right]^2, \text{ and} \\ b_8 &= tr\left[\left(W'B + B'W\right)\left(B'B\right)^{-1}\right]^2. \end{aligned} \tag{A.20}$$

The resulting conditional LM statistic for H_0^c is given by

$$LM_{\theta|\lambda} = \widetilde{D}_\varphi' \widetilde{J}_c^{-1}(\varphi)\, \widetilde{D}_\varphi.$$

Under the null hypothesis, the LM statistic should be asymptotically distributed as χ_p^2.

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