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Abstract

We model the classical transmission of a massless scalar field from a source to a detector on a background causal set. The predictions do not differ significantly from those of the continuum. Thus, introducing an intrinsic inexactitude to lengths and durations – or more specifically, replacing the Lorentzian manifold with an underlying discrete structure – need not disrupt the usual dynamics of propagation.

1 Introduction

Despite the variety of approaches the the problem of quantum gravity, it is hoped by some that agreement can be forged on suitably generic consequences of the as yet unknown theory. This would offer hope of deriving generic predictions of quantum gravity in a more or less heuristic fashion. However, deriving such predictions often turns out to demand a greater level of specificity, whereby disagreements return. For instance, a majority of workers would probably agree that the differentiable manifold structure of spacetime will break down near the Planck scale to be replaced by something of a more discrete, "quantised" or foamy nature. When potentially observable consequences of these general ideas are sought, however, the consensus evaporates. For example, disagreement arises over whether the expected break-down of General Relativity at Planck scales would give rise to modified dispersion relations or other Lorentz symmetry violating phenomena. Spacetime discreteness is often cited as motivation to consider Lorentz symmetry violation (see *e.g.* [1]), perhaps due to consideration of altered dispersion in simple lattice models. There are also quantum gravity inspired models that draw the same conclusion [2, 3]. Meanwhile, there are models of discrete spacetime which respect Lorentz invariance [4, 5, 6] and so do not result in Lorentz symmetry violating phenomenology. Another example of controversy is the arguments in references [7, 8, 9] which dispute the generic claim of reference [10] that any quantum gravitational "fuzzing" of the metric would disturb the coherence of light from distant sources by an amount greater than that allowed by the observations.

The lesson is that it is important to test expectations of phenomenological effects of quantum gravity against different concrete models in order to determine if they are generic. One might indeed be tempted to claim that any "fuzzing" of spacetime properties – be it quantum uncertainty in time and distance measurements, lack of definition of short distances due to discreteness, or from some other source – would naturally lead to some loss of coherence of light from distant sources (for a model inspired by similar considerations,

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see [11]). One purpose of this paper is to test this expectation in a definite model. More specifically, the model examines this question in the discrete context of causal set theory. We present a model in which spacetime metric relations do indeed have an approximate character that breaks down at around the Planck scale, but in which no significant loss of phase information results.

One crucial point is that, whereas other models introduce Lorentz violating assumptions, the causal set model allows for Lorentz invariance in the approximating continuum. That the transmission of light in this context must be consistent with Lorentz symmetry follows already from the results of [5, 6], and in agreement with this conclusion, the model we study below will exhibit the usual, strict proportionality of frequency to wavelength. However, we know of no equally general reasons that would ensure the coherence of light from distant sources; and so one might wonder whether the underlying discreteness would necessarily disrupt the coherence of propagated waves, or wrinkle the wave fronts or alter the wavelength, and hence the frequency of the signal.

A second purpose of this paper is to learn something about other possible effects of an underlying atomicity that one might expect to encounter in analogy with more familiar examples like the propagation of light through air or other material media. Assuming this analogy is valid, effects like scattering and extinction will be present at some level, and the question becomes whether one can expect them to rise to the level of observability with current technology.

In this paper we formulate a rudimentary model[1] with whose aid one can study some of these putative effects, assuming that the microscopic structure which replaces the continuum at the Planck scale is the causal set. The specific process we will analyze is the transmission of a signal from a source to a detector by a (classical) massless scalar field in Minkowski spacetime $\mathbb{M}^4$. Ideally, one would analyze this process with respect to a complete theory of electromagnetism formulated within the context of a complete theory of quantum gravity. In practise, however, one can hope that a simple model incorporating the elements of discreteness and wave-propagation can illuminate the range of possibilities to be expected. Here we study the simplest such model we could devise, based on a classical scalar field ϕ of zero mass. We do not formulate an independent dynamics for ϕ. Rather we describe its transmission from source to detector in terms of the discrete analog for the causal set of the retarded Green's function (a delta function on the future light cone) that yields the Lienard-Wiechert potentials in the continuum. In the future, we hope to compare the predictions of this model with those of a more complete theory of wave propagation in a causal set (for progress in this direction, see [13, 14, 15, 16]).

2 A continuum model

As illustrated in figure 1, we will imagine an idealised setup consisting — in a continuum description — of an oscillating point source of scalar charge q, together with a detector of rectangular shape, facing the source and at rest with respect to it in an ambient flat spacetime[2] . In a system of Cartesian coordinates in which the detector is at rest, let the source oscillate about the origin, tracing out the trajectory

$$x^1 = a \cos \omega x^0 \ , \ x^2 = x^3 = 0 \ , \tag{1}$$

[1] Some features of the model were previously reported in the proceedings of a conference [12].

[2] For a slightly more realistic model (at least for the comparison to propagation of electromagnetic radiation) the source should be neutral. To that end we will later be supplementing this point-charge with a static companion of opposite sign; this cancels the constant, "DC" component of the far field but leaves the AC component unaffected. We could also include more worldlines to build up an extended source, say in the dipole approximation. On the other hand, we could (since scalar charges need not be conserved) simplify our setup even further by keeping the charge at rest but letting its strength q vary with time. This would take us farther from the astrophysically relevant, electromagnetic case, however.

where a is a negligible fraction of the distance R to the detector. (On average, the source is thus at rest with respect to the detector, another simplification which one could easily relax.) Assuming that the detector has a resolution time of T, a two-dimensional area A, and a thickness d, this gives us in spacetime a *detection region* $\mathcal{D}$ of volume TAd, which we can take to be the rectangular domain $[R, R+T] \times [R, R+d] \times [0, \sqrt{A}] \times [0, \sqrt{A}]$. (In this paper we adopt Planckian units in which $c = \hbar = 8\pi G = 1$.) Let us assume further that R greatly exceeds $\lambda \sim \omega^{-1}$, the wavelength of the emitted radiation, which in turn is much greater than any detector dimension; and also that the detector's response rate is rapid compared to the period of the source oscillations. We have then

$$d, \sqrt{A} \ll \lambda \ll R, \quad T \ll \lambda, \quad \text{and } a \ll R \, .$$

Up to small corrections, the spatial distance between emitter and receiver is thus R. We remark that, in order to observe coherence effects, the detector actually must be operational for a much longer time than T, namely a time comparable to ω^{-1}. Finally, let us take the "output" F of our detector to be simply the integral of the field, ϕ, over the detection region $\mathcal{D}$. [3].

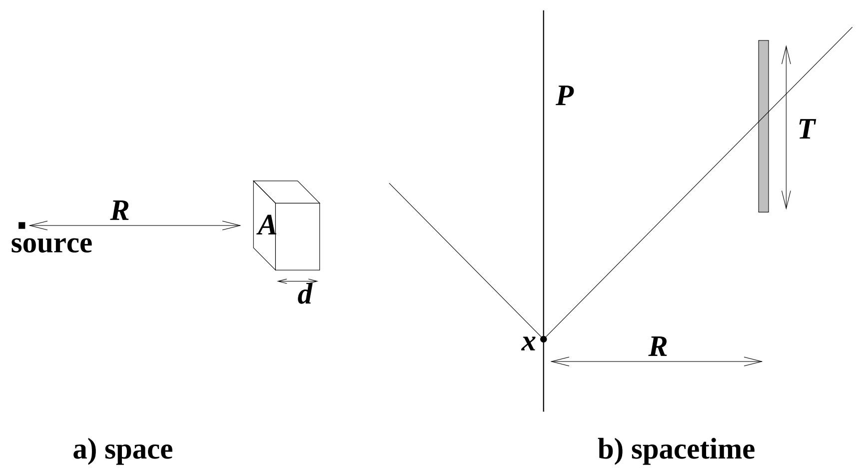

Figure 1: The Source and Detector. In (a) the spatial layout is shown, while (b) is a spacetime diagram (with only one spatial dimension represented) showing the source's worldline P, the light-cone of a point x on that worldline, and the detector region in grey.

Before proceeding to our causal set model of the same setup, let us first review the treatment of this situation in the continuum. The retarded Green's function for a massless scalar field in $(3+1)$-dimensions is a delta-function on the forward light-cone:

$$G(y, x) = \begin{cases} \frac{1}{2\pi}\delta(|y-x|^2) & \text{if } y \text{ is in the causal future of } x \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

$$= \frac{1}{4\pi r}\delta(y^0 - x^0 - r), \tag{3}$$

where r is the spatial distance from x to y. In terms of G, the field produced by our source is given by

$$\phi(y) = \int_P G(y, x(s))\, q\, ds \ , \tag{4}$$

[3]This model could be regarded as fairly realistic for radio wave reception. For optical frequencies, the time-resolution of realistic detectors is not better than the period of the wave, and the received signal is usually compared to another oscillator in order to diminish the effective value of T. In such cases our parameter T must be understood as an effective resolution-time characterizing the detector.

where P is the worldline of the source, q is its charge, and s is proper time along P.[4] According to the ansatz we have made, the output of the detector is proportional to the integral $\int_{\mathcal{D}} d^4y\, \phi(y)$, and thus can be expressed directly in terms of the source and the Green's function:

$$F = q \int_P ds \int_{\mathcal{D}} d^4y\, G(y, x(s)) \tag{5}$$

The two integrals can be done in either order. Integrating first with respect to s yields for $\phi(y)$

$$\frac{q}{4\pi R} \frac{ds/dt}{1 - dx/dt}\ , \tag{6}$$

where ds/dt and dx/dt are to be evaluated at the retarded time $t - R$ corresponding to the detection time t of interest. (Strictly speaking, this expression is correct only at a single point within the detector close to $(R, 0, 0)$, but we can ignore this caveat, since we have assumed the detector to be small compared to R and T small compared to the period of oscillation.)[5] In (6), the oscillating signal resides in the time-dependent factor multiplying $1/R$, which can be understood as just the Doppler effect in disguise. The y-integration is now trivial and furnishes for the detector output

$$F \approx \sqrt{\frac{1+\nu}{1-\nu}}\, \frac{q}{4\pi R}\, TAd\ , \tag{7}$$

where $\nu = dx/dt$ is the component of the source velocity toward the detector at the relevant retarded time. In a slow velocity approximation, $\sqrt{1+\nu/1-\nu}$ reduces to $1+\nu$, and the varying, "AC" part of the signal is a multiple of $\sin(\omega t)/4\pi R$.

In order explicitly to subtract off the DC part of the signal and create a more realistic model of electromagnetic radiation, we can add a static negative charge, $-q$, at the origin of spatial coordinates. Then, renaming F above as F_+ and calling the detector response to the negative charge, F_-, we have the total detector response

$$F_{\mathrm{total}} := F_+ + F_- = \sqrt{\frac{1+\nu}{1-\nu}}\, \frac{q}{4\pi R} T\,A\,d - \frac{q}{4\pi R} T\,A\,d \tag{8}$$

which, for low velocities is

$$\begin{aligned} F_{\mathrm{total}} &= \nu \frac{q}{4\pi R} T\,A\,d \qquad (9) \\ &= \frac{q}{4\pi R} T\,A\,d\,a\,\omega \sin(\omega t)\,. \qquad (10) \end{aligned}$$

The detector response is coherent in space as well as in time, so our model captures the coherence that spacetime uncertainty might be expected to disrupt.

Note that the $1/R$ dependence of the signal agrees with the usual falloff of far field electromagnetic radiation from a dipole source. Here the signal is proportional to the velocity of the source however, whereas the signal in the electromagnetic case is proportional to the acceleration. We note that we could make the model more realistic by pretending that the "field" $\phi(y)$ is the electrostatic potential A_0 of an electromagnetic field. The relevant spatial gradient of $A_0 = \phi(y)$ would arise from differential time delay (detectors at different spatial positions see the source at different times along its world line) and could give rise to an electric field proportional to the acceleration of the source.

[4] We have normalized G so that $\Box\, G = -\delta$, where our metric signature is $(-+++)$. Notice that only an infinitesimal portion of P contributes to $\phi(y)$.

[5] The fact that different points in the detector correspond to different values of s could not have been ignored if we had integrated first over y, however.

3 The discrete model

Now let us consider a causal set model for the same situation. Before turning to the model *per se* let us recall some kinematical results from causal set theory. (For a more detailed introduction see [4, 17, 13].) A causal set $\mathcal{C}$, or causet for short, is a locally finite, partially ordered set. When a causal set has an approximation by a continuum spacetime, the order-relation of $\mathcal{C}$ corresponds to the causal order of spacetime, while the number of elements in a subset of $\mathcal{C}$ equals (up to fluctuations) the volume of the corresponding spacetime region in fundamental units. It is conjectured that this information is all that is needed to recover the metric, differential structure, dimension and topology of the approximating Lorentzian manifold (at scales large compared to the fundamental scale).

With a nonzero discreteness scale, the correspondence of any Lorentzian manifold to a causet can only be approximate, and it seems to be necessary to define it stochastically. Although the causet is taken to be fundamental and the manifold is merely an effective description of it on large scales, some means must be found of determining whether a given spacetime approximates to a certain causal set. To this end we consider a method called *sprinkling*, which produces a random causet from a given Lorentzian manifold by means of a Poisson process [18]. One samples, at random, points from the manifold with a density of 1 in some fundamental units. (One might term these fundamental units "causet units". They are the analog of lattice units in lattice gauge theory, and one would naturally expect them to agree approximately with Planck units as defined above; for the purposes of this paper we will take the agreement to be exact. To determine the true factor relating the two sets of units would be to determine the fundamental discreteness scale and would be a key step in the development of the theory.) A causet can then be constructed using these "sprinkled" points as elements, the order-relation being that induced by the causal order of the Lorentzian manifold. A manifold is defined to approximate a causal set when the causal set is a typical result of the sprinkling process on that manifold (see *e.g.* []) .

Often, instead of finding an approximating spacetime for a given causal set, we are given the approximating spacetime, and wish to find a causal set to which it approximates. In that case, we can reverse the argument and construct possible underlying causal sets using the sprinkling process. If, in the sprinkling process, a property of the resulting causal set holds with probability 1, we say that the property holds for a typical sprinkling. Similarly, properties that hold with probabilities sufficiently near to 1 can be considered typical. Some care is needed to properly define this notion in all cases. However, without fully entering into this discussion, if the only quantity of interest is a function $x(\mathcal{C})$ of the causal set, the value for a typical sprinkling will be only a small number of standard deviations away from the mean over sprinklings $\langle x \rangle$. One can imagine cases in which x is an "ensemble average" of many variables pertaining to different regions.

In a Poisson process, the probability to sprinkle n elements into a spacetime region of volume V is given (for any measurable subset) by the Poisson distribution,

$$P(n) = \frac{V^n e^{-V}}{n!} \ . \tag{11}$$

Because a Poisson process depends only on the spacetime volume-element, it is Lorentz invariant. This invariance is exact for d-dimensional Minkowski spacetime, where the theorems on the existence and uniqueness of the Poisson process readily establish its invariance under all volume preserving linear maps, and in particular under arbitrary Poincaré transformations (see *e.g.* reference [18]). For a proof of invariance that holds even for individual realizations, see [6]. Thus, if the causal set is well-approximated by Minkowski space at all, then the discreteness does nothing to pick out a preferred frame in it. In a curved spacetime, Lorentz invariance is to be understood in the same, local and approximate sense that holds in General Relativity.

For future reference we define the terms "interval", "chain", "path" and "link". Given elements p and q in $\mathcal{C}$, the *order-interval* between them is $I(p,q) = \{r \in \mathcal{C} \,|\, p \prec r \prec q\}$, where $\prec$ is the fundamental precedence relation defining $\mathcal{C}$. A *chain* is a linearly ordered subset of $\mathcal{C}$ and a *path* is a saturated chain, i.e. one which is maximal within the order-interval between any two of its elements. A *link* is a causal relation $p \prec q$ between two elements, p and q, that is not implied via transitivity by other relations, i.e. there exists no third element r such that $p \prec r \prec q$.

We now turn to the causet description of the source-detector system. We will describe the discrete counterparts of the spacetime, the source and detector, and the model of propagation used above, and determine the detector output in this model. In place of 4-dimensional Minkowski space $\mathbb{M}^4$, we will have a typical sprinkling of $\mathbb{M}^4$, and in place of the detector region, the subset, $\widetilde{\mathcal{D}}$, comprising all the elements that were sprinkled into the region of spacetime $\mathcal{D}$. The field ϕ will be a function from the elements of the causet to $\mathbb{R}$.

We will ignore, for now, the static negative charge source and study only the discrete analogue of the oscillating positive charge. In analogy with the continuum model of the source as a point-charge, we may in the causet identify the source with a *path* $\widetilde{P}$ that approximately follows the spacetime worldline P of eqn. (1). Such a path will be "locally geodesic" in the sense of approximating a longest chain between any two sufficiently nearby elements of $\widetilde{P}$. We will say that an element of $\widetilde{P}$ is associated to a point x of P if it lies at a larger spacelike distance from x than any other point in P. The sprinkled points making up such a $\widetilde{P}$ will be distributed along the corresponding continuum curve with an approximate spacing in proper time along the curve that is near to one Planck unit [19, 20]. (The ratio of chain-length in the causet to proper length in the continuum does not seem to be known exactly in 4-dimensions. Simulations suggest a value somewhat greater than 1.2. This gives the average number of elements per length of P and this, along with the independence of the sprinkling process in different regions, turns out to be all that is needed for our calculations.) Such a locally geodesic path will exist, and the spatial distance in the sprinkling from the curve P to the points corresponding to elements of $\widetilde{P}$ will typically be a small number in Planck units.

Both of these definitions make use of regions in the spacetime $\mathbb{M}^4$ that we are to sprinkle, rather than being defined "intrinsically" by relations and/or fields on the causal set itself. Defining them by such means would be more satisfying physically but would complicate matters considerably, and if it led only to a change in the position of the two regions on or around the order of the Planck scale, that would not affect the results, as will become obvious during the calculation.

In the continuum, the relevant dynamics of the field were completely described by a retarded Green's function $G(x, y)$. To define an analogous "propagator" in the causal set, we must discover a discrete replacement for $G(x, y)$ that approximates it well on large scales, but relies only on the structure of the causal set, without appealing to any extra information from the continuum. This idea has already been explored in [21] and [22] (which are concerned primarily with the 2D situation).

The task of finding a discrete propagator is made easier by the simple form of equation (2), as we now describe, using the link concept defined above. In a sprinkling of $\mathbb{M}^4$, the future links from any given sprinkled point e_i are unlikely to stretch over long proper times. Nevertheless, thanks to the Lorentz invariance of the Poisson process, links from e_i will connect it with an infinite number of other sprinkled points ("future nearest neighbors") spaced out along, and just inside of, the future light-cone. This assertion can be proved as follows.

The mean number of links joining an element at point $x \in \mathbb{M}^4$ with elements in a region $\mathcal{R}$ contained within its future is

$$\langle n_x \rangle = \int_{\mathcal{R}} d^4y \, e^{-|I(x,y)|} \ , \qquad (12)$$

where $|I(x, y)|$ is the spacetime volume of $I(x, y) = J^+(x) \cap J^-1(y)$, the causal order-interval (or "Alexandrov set") between x and y. This expression can be understood as the sum over infinitesimal volumes d^4y in $\mathcal{R}$ of the probability that an element is sprinkled into d^4y times the probability that this element is linked to the element at x, *i.e.* the probability that no point was sprinkled into the interval $I(x,y)$. For a Poisson process, the former probability is d^4y itself and the latter is $e^{-|I(x,y)|}$, from which (12) follows. Now in four dimensions,

$$|I(x,y)| = \frac{\pi}{24}\tau^4(x,y)\ , \tag{13}$$

where $\tau(x,y)$ is the proper time between x and y. From this one sees that an element that was sprinkled a large proper time from x is highly unlikely to be linked to it; but by the same token, elements hugging the future light-cone of x which are close to it in proper time are quite likely to be linked to it.

Indeed, consider an element e_0 sprinkled at the point $x \in \mathbb{M}^4$. Choose coordinates $\xi, \eta, \theta, \varphi$ ($\eta \geq 0$), in which x is at the origin, $\xi = 0$, and the metric is

$$ds^2 = -d\xi^2 + \xi^2 d\eta^2 + \xi^2 \sinh^2 \eta d\Omega_2^2 \tag{14}$$

so that ξ is geodesic proper time from the origin. The expected number of future links from e_0 terminating in the region between the light-cone of x and the hyperboloid of points at a fixed proper time τ from x is given by (12) with $\mathcal{R}$ defined by $0 \leq \xi \leq \tau$, that is by

$$4\pi \left[\int_0^\tau d\xi\, \xi^3 e^{-\frac{\pi}{24}\xi^4}\right] \int_0^\infty d\eta \sinh\eta\ . \tag{15}$$

Since the integral over η diverges, the expected number of links is infinite, no matter how small τ is.

We introduce the causet function

$$L(e',e) = \begin{cases} \kappa & \text{whenever } e \prec e' \text{ and } \{e,e'\} \text{ is a link,} \\ 0 & \text{otherwise,} \end{cases} \tag{16}$$

where $e, e' \in \mathcal{C}$ are causet elements and κ is a normalising constant of order 1, to be decided later. In the limit of infinitely dense sprinkling, this function becomes a δ-function on the forward light-cone of e. This means that $L(e',e)$ is a Lorentz invariant discretisation of the continuum Green's function, and can be used to define the propagation of the scalar field on the causet.

3.1 Mean Detector Output

We will calculate the average and variance of the detector output over sprinklings of $\mathbb{M}^4$. Replacing $G(x,y)$ with $L(e',e)$, the calculation of the detector output is very similar to that in the continuum, although now it is a random variable. The double integral of eqn. (5) for the output of the detector $\widetilde{F}$ becomes a double sum over all elements in the source and detector:

$$\widetilde{F} = q \sum_{e\in\widetilde{P}} \sum_{e'\in\widetilde{\mathcal{D}}} L(e',e), \tag{17}$$

In other words $\widetilde{F}$ is proportional to the number of links from elements in the source region to elements in the detector region. The causal set is randomly generated by the sprinkling process in $\mathbb{M}^4$, with the source and detector regions as described above. Using the definition of $L(e',e)$ in (16), $\widetilde{F}$ can be rewritten as

$$\widetilde{F} = q\kappa \int_P \int_{\mathcal{D}\, x(s)\prec y} \sigma(ds)\chi(d^4y)\zeta(x(s),y), \tag{18}$$

where s and $x(s)$ are as in the continuum calculation, and $\sigma(ds)$, $\chi(d^4x)$ $\zeta(x,y)$ are random variables in the sprinkling process: the variable $\sigma(ds) = 1$ if any point of P with proper time in ds has a sprinkled element of $\widetilde{P}$ associated to it and is 0 otherwise, $\chi(d^4y) = 1$ if the volume element d^4y contains a sprinkled point and 0 otherwise, and $\zeta(x, y) = 1$ if the causal interval between x and y (not including the points x and y) is empty of sprinkled points and 0 otherwise. From this, we can find the mean value:

$$\langle \widetilde{F} \rangle = q\kappa \int_P \int_{\mathcal{D}\, x(s) \prec y} \langle \sigma(ds)\chi(d^4y)\zeta(x(s), y)\rangle, \tag{19}$$

$$= q\kappa \int_P \int_{\mathcal{D}\, x(s) \prec y} \langle \sigma(ds)\rangle \langle \chi(d^4y)\rangle \langle \zeta(x(s), y)\rangle, \tag{20}$$

$$= q\kappa \int_P \int_{\mathcal{D}\, x(s) \prec y} ds\, d^4y\, e^{-|I(x(s),y)|}. \tag{21}$$

As explained above, the average number of points in $\widetilde{P}$ associated to P per unit length, $\langle \sigma(ds)\rangle/ds$, is some number of order unity, which we absorb into κ. The calculation is unaffected by the Planck scale deviation of the spatial positions of elements in $\widetilde{P}$ from P. The second line above says that the three random variables are always uncorrelated: firstly $x(s)$ and y range over disjoint regions, so $\sigma(ds)$ and $\chi(d^4y)$ are uncorrelated, and secondly points sprinkled at $x(s)$ and y have no effect on $\zeta(x(s), y)$. Let us first calculate

$$\langle \widetilde{\phi}(y)\rangle = q\kappa \int_{P\, x(s) \prec y} ds\, e^{-|I(x(s),y)|}, \tag{22}$$

for $y \in D$.

(Despite the name this quantity is different to the discrete field, which is a function of the causal set elements). It is convenient to define $r = y^1 - x^1(s)$ and $t = y^0 - x^0(s)$, and use the co-ordinates $u = t - r$, $v = t + r$ to calculate the integral:

$$\langle \widetilde{\phi}(y)\rangle \approx q\kappa \int_{u(s)=\infty}^{u(s)=0} ds\, e^{-\frac{\pi}{24}(uv)^2}, \tag{23}$$

where we have used (13) and used $\sqrt{A} \ll R$ to approximate the proper time between $x(s)$ and y. Let us define $R_y = r|_{u=0}$. Using this,

$$v = \alpha u + 2R_y\,, \tag{24}$$

where

$$\alpha = \frac{1+\nu}{1-\nu}\,. \tag{25}$$

We also have

$$ds = \sqrt{1-\nu^2}\, dx^0 = -\sqrt{\alpha}\, du. \tag{26}$$

Inserting these into (23) yields

$$\langle \widetilde{\phi}(y)\rangle \approx q\kappa\sqrt{\alpha} \int_0^\infty du\, \exp\bigl[-\frac{\pi}{24}(\alpha u^2 + 2R_y u)^2\bigr] \tag{27}$$

$$\approx \frac{\sqrt{6}}{2}\, q\kappa\, \sqrt{\alpha}\frac{1}{R_y}, \tag{28}$$

since the integral is dominated by the region in which $u \ll R$, where the integral is approximately Gaussian. Note that R_y is approximately R, because the amplitude of the source $a \ll R$ and $d \ll R$. Inserting this result into eqn.(21) we obtain our result for the mean output,

$$\langle \widetilde{F} \rangle \approx \sqrt{\frac{1+\nu}{1-\nu}}\frac{q}{4\pi R} T A\, d \approx (1+\nu)\frac{q}{4\pi R} TAd, \tag{29}$$

in the low velocity approximation, where we have set $\kappa = 1/(2\sqrt{6}\pi)$. We see that the mean output in the discrete model matches the continuum result (7) given above. We have approximated the spatial distance from source points to detector points as R in all cases when finding $\widetilde{\phi}(y)$, but this is basically the same approximation we made in the continuum case.

It is of interest to find the size of the correction to the continuum result. The main extra approximation used in the discrete case was the approximation of the integral in equation (27) as Gaussian. Instead of throwing away all but the Gaussian factor, we can include the next most significant factor:

$$\langle\widetilde{\phi}(y)\rangle \approx q\kappa\sqrt{\alpha}\int_0^\infty du\,\exp\bigl[-\frac{\pi}{6}(R_y\alpha u^3 + R_y^2 u^2)\bigr]. \tag{30}$$

Solving this integral by symbolic computation, and expanding in powers of $1/R$, shows a first term equal to the continuum result, and a second term proportionally smaller by a factor of order $1/R^2$. This R is a large distance expressed in Planck units, and so the extra approximation introduced in the discrete case is insignificant.

Just as in the continuum case, we cancel the DC component of the response by adding a static, negative charge, modelled on the causal set as a path close to the origin. The mean of the total response is the sum of the means of the responses to the two sources and is

$$\langle\widetilde{F}_{\mathrm{total}}\rangle \approx \nu\frac{q}{4\pi R}TA\,d\,. \tag{31}$$

3.2 Fluctuations in the detector output

We have shown above how, in our model of propagation, the average value (over sprinklings) of the detector signal does not significantly differ from the signal in the standard case. However, this result must be strengthened. We want to make sure that the result does not differ significantly not just for the average, but for a typical sprinkling (moreover we want to make sure that the model predicts the correct result for many repeats of the experiment). To this end we calculate the variance of the signal. We start by calculating the variance when the source consists of a single oscillating positive charge.

As stated above, the detector output $\widetilde{F}$ is proportional to the number of links from the source to the detector. If the existence of any link was uncorrelated to the existence of any other link, we would expect a variance of $q\kappa\langle\widetilde{F}\rangle$ which is small. Correlations do exist, however, intuitively they will not be large. The result of this intuition – that the fluctuations, as a proportion of the signal, are small for reasonable values of the parameters – is correct, as we will now show.

Let us return to the expression (18) for the output as a random variable in the sprinkling process on $\mathbb{M}^4$. We have already calculated $\langle\widetilde{F}\rangle$ on this basis. There, the three random variables in the expression for $\widetilde{F}$ are uncorrelated. Now we seek to calculate $var(\widetilde{F}) = \langle\widetilde{F}^2\rangle - \langle\widetilde{F}\rangle^2$. The term $\langle\widetilde{F}^2\rangle$ can be written

$$\langle\widetilde{F}^2\rangle = q^2\kappa^2\int_P\int_{\mathcal{D}}\int_P\int_{\mathcal{D}\,(a)}\langle\sigma(ds_1)\chi(d^4y_1)\sigma(ds_2)\chi(d^4y_2)\zeta(x(s_1),y_1)\zeta(x(s_2),y_2)\rangle. \tag{32}$$

where s_1 and s_2 are both proper time along P, y_1 and y_2 range over the detector D, and we have used the symbol (a) as shorthand for the conditions $x(s_1) \prec y_1$, $x(s_2) \prec y_2$. In this case, not all of the random variables are uncorrelated. Firstly, $\chi(d^4y_2)$ is correlated to $\zeta(x(s_1),y_1)$ as y_2 can fall inside the causal interval $I(x(s_1),y_1)$. In this case if a point is sprinkled in d^4y_2, $\zeta(x(s_1),y_1) = 0$. Therefore there is no contribution to the integral from the range in which $y_2 \in I(x(s_1),y_1)$. Using similar reasoning we can put 4 restrictions on the region of integration without affecting its value:

$$x(s_1) \notin I_2, \tag{33}$$
$$y_1 \notin I_2, \tag{34}$$
$$x(s_2) \notin I_1, \tag{35}$$
$$y_2 \notin I_1, \tag{36}$$

where we introduce the notation $I_1 := I(x(s_1), y_1)$ and $I_2 := I(x(s_2), y_2)$. We will use the symbol (b) as shorthand for these restrictions. Inside the remaining region of integration, all χ and σ variables are uncorrelated with ζ variables.

Secondly, the χ and σ variables can be correlated with each other, as they do not all range over disjoint regions. When y_1 is in d^4y_2 there is a correlation. Taking this correlation into account is crucial to obtain the correct result (the situation is similar to accounting for the self-correlations in a discrete set of variables). Given a sprinkled point at y_1, when $y_2 \neq y_1$, the probability of finding a sprinkled point in d^4y_2 is infinitesimal. But when $y_1 = y_2$ the probability is 1, explaining the importance of this correlation. We deal with this by splitting the integral into a sum of 4 terms:

$$\langle \widetilde{F}^2 \rangle = J_1 + J_2 + J_3 + J_4\,. \tag{37}$$

These terms take the form

$$J_i = q^2\kappa^2 \int_P \int_D \int_P \int_{D\,(a)\,(b)\,(j_i)} \langle \sigma(ds_1)\chi(d^4y_1)\sigma(ds_2)\chi(d^4y_2)\zeta(x(s_1), y_1)\zeta(x(s_2), y_2)\rangle\,, \tag{38}$$

with the additional (b) bounds from eqns. (33-36) above, and also some new bounds (j_i) for each term:

$$(j_1)\ :\ s_1 = s_2,\ y_1 = y_2 \tag{39}$$
$$(j_2)\ :\ s_1 \neq s_2,\ y_1 = y_2 \tag{40}$$
$$(j_3)\ :\ s_1 = s_2,\ y_1 \neq y_2 \tag{41}$$
$$(j_4)\ :\ s_1 \neq s_2,\ y_1 \neq y_2\,. \tag{42}$$

In each term, once the obvious identifications have been made there are no remaining correlations between χ and σ variables. Finally there are correlations between the two ζ variables, which will be dealt with in due course. We will also define

$$K_4 = J_4 - \langle \widetilde{F} \rangle^2, \tag{43}$$

so that

$$var(\widetilde{F}) = J_1 + J_2 + J_3 + K_4. \tag{44}$$

We now proceed to calculate these four terms.

Let us start with the J_1 term. Due to the (j_1) bound, which can be used to eliminate s_2 and y_2, this term can be written

$$J_1 = q^2\kappa^2 \int_P \int_{\mathcal{D}\,x\prec y} \langle \sigma(ds)\chi(d^4y)\zeta(x, y)\rangle, \tag{45}$$
$$= q\kappa\langle \widetilde{F} \rangle. \tag{46}$$

Consider now the term J_2. Calling $y_1 = y_2 = y$ we have

$$J_2 = q^2\kappa^2 \int_P \int_P \int_{\mathcal{D}\,(a)\,(b)} \langle \sigma(ds_1)\sigma(ds_2)\chi(d^4y)\zeta(x(s_1), y)\zeta(x(s_2), y)\rangle \tag{47}$$

where the bounds should be interpreted with $y_1 = y_2 = y$. These bounds are incompatible. Bound (a) implies that $x(s_1) \prec y$ and $x(s_2) \prec y$. But all points on P are causally related, including $x(s_1)$ and $x(s_1)$, so $x(s_2) \prec x(s_1) \prec y$ or $x(s_1) \prec x(s_2) \prec y$. This contradicts either (33) or (35) of the (b) bounds. We therefore have

$$J_2 = 0. \tag{48}$$

The term K_4 suffers a similar fate:

$$J_4 = q^2\kappa^2 \int_P \int_{\mathcal{D}} \int_P \int_{\mathcal{D}\,(a)\,(b)} \langle \sigma(ds_1)\chi(d^4y_1)\sigma(ds_2)\chi(d^4y_2)\zeta(x(s_1),y_1)\zeta(x(s_2),y_2)\rangle \tag{49}$$

$$= q^2\kappa^2 \int_P \int_{\mathcal{D}} \int_P \int_{\mathcal{D}\,(a)\,(b)} ds_1\, d^4y_1\, ds_2\, d^4y_2 \langle \zeta(x(s_1),y_1)\zeta(x(s_2),y_2)\rangle \tag{50}$$

$$= q^2\kappa^2 \int_P \int_{\mathcal{D}} \int_P \int_{\mathcal{D}\,(a)\,(b)} ds_1\, d^4y_1\, ds_2\, d^4y_2 \exp(-|I_1 \cup I_2|), \tag{51}$$

where I_i is the causal interval between $x(s_i)$ and y_i as before, and $|.|$ indicates the volume of a region. The last line comes from seeing that the product $\zeta(x(s_1),y_1)\zeta(x(s_2),y_2)$ is only 1 when both of these intervals are empty of points, $\exp(-|I_1 \cup I_2|)$ being the probability for this to happen. Subtracting $\langle \widetilde{F}\rangle^2$,

$$\langle \widetilde{F}\rangle^2 = q^2\kappa^2 \int_P \int_{\mathcal{D}} \int_P \int_{\mathcal{D}\,(a)} ds_1\, d^4y_1\, ds_2\, d^4y_2 \exp(-|I_1| - |I_2|) \tag{52}$$

gives

$$K_4 \le q^2\kappa^2 \int_P \int_{\mathcal{D}} \int_P \int_{\mathcal{D}\,(a)\,(b)} ds_1\, d^4y_1\, ds_2\, d^4y_2 \Big[\exp(-|I_1 \cup I_2|) - \exp(-|I_1| - |I_2|)\Big], \tag{53}$$

where we have added the (b) bound to the $\langle \widetilde{F}\rangle^2$ integral, which can only increase the value of this upper bound on K_4. The integrand here is zero unless $I_1 \cap I_2 \neq \emptyset$. It can be seen that there is no region inside the bounds satisfying this condition. As in the J_2 case, we use the assertion that $x(s_1) \prec x(s_2)$ or $x(s_2) \prec x(s_1)$. Let us assume $x(s_2) \prec x(s_1)$, without loss of generality, as the conditions are symmetric between s_1 and s_2. The condition $I_1 \cap I_2 \neq \emptyset$ implies $x(s_1) \prec y_2$. Together with $x(s_2) \prec x(s_1)$ this gives $x(s_1) \in I_2$, contradicting (39) in the (a) bounds. Hence,

$$K_4 \le 0. \tag{54}$$

The only remaining term is J_3:

$$J_3 = q^2\kappa^2 \int_P \int_{\mathcal{D}} \int_{\mathcal{D}\,(a)\,(b)} ds\, d^4y_1\, d^4y_2 \exp(-|I_1 \cup I_2|), \tag{55}$$

where now, because of the identification $s_1 = s_2 = s$, we have $I_1 = J^+(x(s)) \cap J^-(y_1)$ and similarly for I_2. Noting that $|I_1 \cup I_2| \ge (|I_1| + |I_2|)/2$, we can write

$$J_3 \le q^2\kappa^2 \int_P \int_{\mathcal{D}\,(a)} ds\, d^4y_1 \exp(-\frac{1}{2}|I_1|) \int_{\mathcal{D}\,(a)\,(b)} d^4y_2 \exp(-\frac{1}{2}|I_2|). \tag{56}$$

Let us define $r_i = y_i^1 - x^1(s)$ and $t_i = y_i^0 - x^0(s)$, and $u_i = t_i - r_i$ and $v_i = t_i + r_i$ (note that in this case we will be holding the x variable constant while integrating over y, conversely to the method by which the mean was calculated). As a step to bounding J_3, consider

$$X(x(s), y_1) = \int_{\mathcal{D}\,(a)\,(b)} d^4y_2 \exp(-\frac{1}{2}|I_2|). \tag{57}$$

This range of integration for y_2 is much larger than necessary. Let us assume that the light-cone of $x(s)$ does not intersect the initial or final spatial boundaries of the detector region (for the points in P at which this does not hold, the region of $\mathcal{D}$ integrated over is smaller, so this results in an upper bound on $X(x(s), y_1)$ for all points in P). In this case, the bounds on the integral are $R - x^1(s) < r_2 < R - x^1(s) + d$, $0 < y_2^2 < \sqrt{A}$ and $0 < y_2^3 < \sqrt{A}$. As before we will use $\tau(x(s), y_2)^4 \approx (u_2 v_2)^2$. The bound (a) gives $u_2 > 0$ in this case, and so we have

$$X(x(s), y_1) \lesssim \frac{A}{2} \int_0^\infty du_2 \int_{2(R-x^1)+u_2}^{2(R-x^1+d)+u_2} dv_2 \, \exp\bigl(-\frac{\pi}{48}(u_2 v_2)^2\bigr). \tag{58}$$

As before we have used $\tau(s, y_2)^4 \approx (u_2 v_2)^2$. We have

$$X(x(s), y_1) \lesssim Ad \int_0^\infty du_2 \, \exp\bigl(-\frac{\pi}{48}(u_2(2(R - x^1) + u_2))^2\bigr) \tag{59}$$

$$\lesssim \sqrt{3} A d \frac{1}{R}, \tag{60}$$

since d and x^1 are negligible compared to R and as in the calculation of (27) we have neglected insignificant terms in the exponential. Inserting this back into (56) we have

$$J_3 \lesssim \sqrt{3} q^2 \kappa^2 A d \frac{1}{R} \int_P \int_{\mathcal{D}\,(a)} ds \, d^4 y_1 \exp(-\frac{1}{2}|I_1|). \tag{61}$$

This integral is similar to the one we have already calculated to find $\langle \widetilde{F} \rangle$ in eqns. (21–29). We find

$$J_3 \lesssim \sqrt{6}\, q \kappa A d \frac{1}{R} \langle \widetilde{F} \rangle \,. \tag{62}$$

Finally, summing up the remaining terms in eqns.(46) and (62) to find $var(\widetilde{F})$ from eqn.(44):

$$var(\widetilde{F}) \leq q \langle \widetilde{F} \rangle (1 + \sqrt{6} \kappa A d \frac{1}{R}). \tag{63}$$

Now, $var(\widetilde{F})$ represents the variance of the signal from only our oscillating positive charge. We are not actually interested in $var(\widetilde{F})$ but rather in the variance of the *total* detector response to the oscillating positive charge and the static negative charge together, $var(\widetilde{F}_{\text{total}}) = var(\widetilde{F}_+ + \widetilde{F}_-)$, where we have renamed $\widetilde{F}$ above as $\widetilde{F}_+$ as before. This total variance is

$$var(\widetilde{F}_{\text{total}}) = var(\widetilde{F}_+) + var(\widetilde{F}_-) + 2\, Cov(\widetilde{F}_+, \widetilde{F}_-), \tag{64}$$

where Cov is the covariance function,

$$Cov(\widetilde{F}_+, \widetilde{F}_-) = \langle (\widetilde{F}_+ - \langle \widetilde{F}_+ \rangle)(\widetilde{F}_- - \langle \widetilde{F}_- \rangle) \rangle. \tag{65}$$

It is a standard corollary of the Cauchy-Schwarz inequality that

$$Cov(\widetilde{F}_+, \widetilde{F}_-)^2 \leq var(\widetilde{F}_+) var(\widetilde{F}_-), \tag{66}$$

and so we have

$$var(\widetilde{F}_{\text{total}}) \leq var(\widetilde{F}_+) + var(\widetilde{F}_-) + 2\sqrt{var(\widetilde{F}_+) var(\widetilde{F}_-)} \tag{67}$$

$$< 4 var(\widetilde{F}_+), \tag{68}$$

where the result follows from (63) and (29) : $var(\widetilde{F}_-)$ is the same function as $var(\widetilde{F}_+)$ with the source velocity ν set to zero so $var(\widetilde{F}_-) \approx var(\widetilde{F}_+)$ for low velocities.

The standard deviation of the total signal as a proportion of the output is

$$\frac{\sqrt{var(\widetilde{F}_{\text{total}})}}{\langle \widetilde{F}_{\text{total}} \rangle} \sim \frac{\sqrt{4 var(\widetilde{F}_+)}}{\langle \widetilde{F}_{\text{total}} \rangle}. \tag{69}$$

Using our result (63) for $var(\widetilde{F}_+)$ (and the value set earlier for κ) gives

$$\frac{var(\widetilde{F}_{\rm total})}{\langle \widetilde{F}_{\rm total}\rangle^2} \lesssim 16\left[\frac{\pi R}{\nu^2 T A d} + \frac{1}{2\nu^2 T}\right] . \tag{70}$$

We can now input some values for the parameters to estimate the fluctuations for a model source and a model detector. We will obtain an upper bound on the fluctuations by making unrealistically restrictive estimates in some cases. For a description of a relevant source, we look to discussion of coherence of electromagnetic radiation from extragalactic sources in the literature. One coherent source that has been cited [10] is the Active Galactic Nucleus PKS 1413+135 [23]. This source is at a distance of order one Gigaparsec, and coherence was detected in radiation of wavelength $1.6\,\mu m$. Therefore R is set to be of order 10^{60} Planck lengths, and in order that the time resolution of detection be much better than the period of the radiation (one of the unrealistically restrictive estimates), we set T to be 10^{27} Planck units. The radiation from AGNs is thought to be synchrotronic, meaning that the source electrons have velocities close to 1. This doesn't fit with our low velocity assumption but we choose ν 0.1 as a compromise value for which our expansions are still approximately valid [6] . As for the spatial size of the detector, $\sqrt{A}$ and d can safely be set to be of order 1 nanometer, or 10^{26} Planck units. With these values of the parameters the first term inside the brackets in equation (70) is of order 10^{-43} and the second is of order 10^{-25}. With these values the standard deviation of the total signal is about one part in 10^{12}. This order of fluctuation is well below that which is possible to detect even in more realistic set-ups than this nano-size and femto-duration detection.

This shows that, with these values for the parameters of our model, the fluctuations are a small part of the total signal. Thus for a typical sprinkling, it is justified to claim that the results of the discrete model match the continuum model sufficiently well for the difference to be undetectable. The conclusion is that the spatial and temporal coherence of waves from distant sources is preserved in this model.

It is interesting to ask if models of this type can give good results for all types of source. In this connection, it should be noted that certain approximations and assumptions, mostly unfavourable to the eventual outcome, have been made to achieve the result given above for the distant AGN. The bound on fluctuations coming from equation (70) is well above that which could reasonably be achieved in such models, for three main reasons. Firstly, the values for the quantities involved are all set to give a higher result for the fluctuations than is realistic, to give an upper bound; the only one that might conceivably be less favourable for other sources is ν. Secondly the bound on $var(\widetilde{F}_+)$ given in (63) is fairly crude: not only could a smaller upper bound be found for J_3, but the K_4 term could be less than zero. Thirdly, making the model more realistic, by considering detection lasting for multiple oscillation periods, and/or spatially extended sources[7], would be expected to greatly reduce the relative size of the fluctuations. Thus it

[6] To fully satisfy scepticism on this point, we would have to repeat our calculations without this approximation. This would necessitate dropping the further assumption that the duration of the detection is much shorter than the period of oscillation of the source: when the maximum source velocity is close to the speed of light, the field from the postive charge would vary from much greater than its value for $\nu = 0$ to much smaller, and so it is possible that the fluctuations as a proportion of the dectector output would vary greatly over one oscillation. Such a large value for the relative fluctuations at only one time in the oscillation would not in practice be measureable, however; it is more realistic to consider a longer detection for the cited source in any case. Considering this, there is strong reason to think that this more complicated treatment would not lead to a different conclusion. When ν is positive, the ratio of fluctuations to detector output become even smaller as ν increases towards the relativistic regime, and when $vel \approx -1$ the contribution to the total detector output (and fluctuations) will be small.

[7] An extended source could be built by adding many positive and negative particles, in which case a bound on the total variance similar to (67) would still hold. The resulting bound would be proportional the square of the number of particles, and $\widetilde{F}_{\rm total}$ would of course be proportional to the number of particles, leaving (70) unchanged. However, in this case the result analogous to (67) would be a very much looser bound, as signals generated from significantly separated source particles would not be highly correlated. The relative fluctuations in such a model would arguably be greatly reduced as a result.

is reasonable to conjecture that models of this type could be constructed to model many realistic electromagnetic sources[8]. In any case, the main aim here was to show that the model is consistent with the coherence of waves from distant sources.

4 Conclusion

We have reviewed a simple model of the propagation of a scalar field from source to detector in a Minkowski spacetime, and demonstrated a model of the same physical system in which the continuous spacetime is replaced with a causal set. The main result is that the output of the detector is not significantly affected by the introduction of discreteness. This being so, the phase of the source as it oscillates is faithfully recorded at all astrophysical and cosmological distances: there is no loss of (classical) coherence. Nor is there any change in the dispersion relations. The discreteness *will* eventually show up when the distance, R, between source and detector becomes large enough that the mean number of links from source to detector region is of order one but the signature of the discreteness will be a "cutting out" of the signal altogether rather than any loss of coherence. Unfortunately, for this to happen for any reasonably sized detector, R would have to be a super-horizon distance and in any case the signal would by then be undetectably tiny.

The relevance for quantum gravity phenomenology of a model premised on a fixed background structure may be questioned. After all, the continuum seems likely to result from a "quantum superposition" of causal sets, whatever that turns out to mean. However, the purpose of this article was to show that discreteness *per se* does not pose any problem for the coherent propagation of waves over long distances. Claims about "generic" phenomenological predictions from quantum gravity will need to take account of this. For example, it cannot now be claimed that a fuzziness in spacetime metric relations necessarily disrupts Huygen's principle nor that modified dispersion relations always result from the existence of a minimum length scale. We emphasise, again, that the maintenance of coherence and the usual dispersion relations hold for any fixed causal set that arises with relatively high probability from the sprinkling process and is not the result of any averaging over causets.

The crucial aspect of causal set discreteness that distinguishes it from other approaches is that it respects Lorentz invariance. This is the key to preserving the usual dispersion relation in the above model: even without a detailed model, we would have claimed that variance in the speed of light as in [10] could not be present in a causal set model unless it were introduced "by hand" somehow. The concrete model is particularly illuminating because it shows explicitly that the speed of photons from distant sources is not affected by causet discreteness due to the existence in the causet of structure – the links – which traces lightcones extremely accurately. It is hoped that the study of Gamma ray pulses from Blazars and Gamma ray bursts [1, 7] will either detect, or place stringent limits on, variations in c and it remains to be seen which discrete models of propagation will be consistent with these observations.

We have thus seen that, due to the tininess of the Planck scale and the Lorentz invariance of the approach, causal set models can give results that are experimentally indistinguishable from the continuum. This is simultaneously encouraging and disappointing. Encouraging because every such result provides more evidence for the "Hauptvermutung" (Central Conjecture) of causal set theory, that causal sets can indeed be the deep structure of Lorentzian spacetimes. The disappointment arises because providing experimental

[8] Indeed, the situation might reasonably be expected to be better for a similar discretisation of the electromagnetic Green's functions, if the calculations were otherwise similar to those above. In the continuum, for the scalar field, the DC and AC parts of the signal have the same order of magnitude when $\nu \sim 1$, whereas in the electromagnetic case the the AC part will be larger by a factor of R. This would suppress the total variation of the signal but not the total mean signal. This could be analysed by investigating the causet version of the continuum model suggested at the end of section 2 in which the field ϕ is treated as an electostatic potential.

evidence for causal set theory will require predictions that are at variance with those made in the continuum. Though the simple model presented here seems not to be able to produce such phenomenology, other models of matter propagating on causal sets do hold out the hope of producing observable signatures of discreteness. One promising class of models is based on the idea that if spacetime is a causal set then this should cause small, random, Lorentz invariant fluctuations in the motion of particles through spacetime. For massive particles this results in a diffusion in momentum [5, 24] and for massless particles, both a diffusion and drift in frequency [24]. These effects are potentially observable and astrophysical and cosmological data has been used to bound the parameters of the models [5, 25, 26, 24]. Beyond these models, the discovery of a discrete, Lorentz invariant, D'Alembertian operator [13, 14] means that a discrete scalar wave equation can be solved and the behaviour of waves and wave packets investigated, providing more realistic models of matter and further opportunities for predicting phenomena that could reveal a fundamental spacetime discreteness.

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