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To What Type of Logic Does the "Tetralemma" Belong?

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Abstract

Although the so called "tetralemma" might seem to be incompatible with any recognized scheme of logical inference, its four alternatives arise naturally within the "anhomomorphic" logics proposed recently in order to accommodate certain features of microscopic (i.e. quantum) physics. This suggests that non-classical logics of a similar type might have been known in ancient India.

Considered from the standpoint of classical logic, the fourfold structure of the so-called tetralemma (catuṣkoṭi) appears to be irrational, and modern commentators have often struggled to explain its peculiar combination of alternatives, which at first sight appear to take the form, "A", "not-A", "A and not-A", "neither A nor not-A". (See for example [1] [2] [3] [4].) Such a combination fits into no recognized scheme of logical inference, and some authors have accordingly concluded that the tetralemmas hail from an unknown logical system, which rejects one or more cherished principles like the "law of the excluded middle" or the "law of contradiction". Others commentators have abandoned any attempt to make logical sense of the tetralemma, resorting rather to the hypothesis that its purpose is merely to illustrate that the propositions A that occur in it are meaningless or ill-defined, like the proposition, "The unicorn has black eyes".

According to Ruegg [3] the catuṣkoṭi form occurs frequently in the early Buddhist literature, and he provides (in translation) several examples, one being the question whether the world is

finite, infinite, both finite and infinite, or neither finite nor infinite,

another being the question whether

> the world (of living beings) is eternal, not eternal, both eternal and not eternal, neither eternal nor not eternal.

Similarly, Shcherbatskoi [1] writes that, according to tradition, the founder of Buddhism refused to answer

> four questions regarding the beginning of the world, viz., there is a beginning, there is not, both, or neither.

Almost the same fourfold structure appears in another quotation from [3] (page 3):

> Entities of any kind are not ever found anywhere produced from themselves, from another, from both [themselves and another], and also from no cause.

In all these examples, the four offerings are evidently intended as an exhaustive set of alternatives, which, moreover, all derive from a single "event" A (except possibly in the final example concerning causation). In the second example, for illustration, A is the event of the world lasting forever, and if we abbreviate not-A as $\bar{A}$, then it seems as if we could express the four proffered alternatives as four propositions representable as: A, $\bar{A}$, A and $\bar{A}$, neither A nor $\bar{A}$. The difficulty of course is that the final two alternatives are self-contradictory when stated this way. (The tetralemma concerning causation might fall into a different category than the other three examples, depending on whether or not one assumes that every "entity" needs a cause. Nevertheless, it still clashes with classical logic in its denial that any one of the alternatives obtains. This case is discussed further in Appendix I.)

It seems clear that either the tetralemmas are unadorned nonsense or something other than classical logic is involved. In particular, one cannot explain away the evident contradictions merely by hypothesizing that questions about the beginning of the world or the infinity of existence are meant to be like questions about unicorns. Even if that were the case, why would it not have sufficed to state the two alternatives A and $\bar{A}$, after which one could either refuse to choose between them or deny both of them? Why did they need to be supplemented with two further alternatives, apparently trivial or lacking meaning altogether? With respect to classical logic, this would have added nothing. But could it be that the questioners were aware of broader types of logic, with respect to which the first two alternatives failed to cover all the relevant possibilities, and with respect to which the

two additional alternatives were not in fact nonsense? If so, what might these other logics have been?

A possible answer comes from quantum mechanics, where certain alternative logics have been proposed as a solution to the paradoxes that arise in the attempt to describe subatomic reality [5] [6] [7]. In the early proposals of this sort, known collectively as "quantum logic", the laws for combining propositions were modified in such a way that the distributive law no longer holds. More recently, though, a different type of logical structure has been put forward, in which the rules for combining propositions* are the classical ones, but what changes are the rules of inference [8] [9]. It is these "anhomomorphic" logics, I would suggest, that hold the key to understanding the catuṣkoṭi form.

In order to appreciate how anhomomorphic logics differ from classical logic, one needs to distinguish carefully between what have been called "asserted" and "unasserted" propositions. † That is, one needs to avoid confusing a proposition-in-itself with the affirmation or denial of that proposition. Unaccompanied by either affirmation or denial, a proposition functions rather like a question or a predicate. It only indicates a possible event or state of affairs, without taking a position on whether that event or state of affairs actually obtains. For example, let A be the proposition "It rained all day yesterday". In itself, A tells us nothing about yesterday's weather; it only raises an implicit question. But if we then either deny or affirm A, we answer the implicit question, either negatively or positively, as the case may be.

In ordinary speech we rarely if ever make this distinction explicitly, though something rather like it is implicit in the grammar of conditional and counterfactual constructions. Perhaps for this reason, formal logic also seems to lack an agreed upon symbolism to portray the distinction cleanly. To remedy this lack, we can (following [8]) introduce a

* In relation to physics, the word "event" would be more evocative than "proposition", but the latter accords better with the usage traditional in the subject of logic.

† In [10] Russel discusses this distinction at some length, contrasting his own attitude toward it with that of Frege.

function ϕ that expresses affirmation or denial explicitly. Given a proposition A, we can write $\phi(A) = 1$ in order to affirm A, and $\phi(A) = 0$ in order to deny it.[♭]

The distinction between a proposition per se and its affirmation or denial is closely related to the distinction between what, using a different language, might have been called "positive" and "negative" propositions. Consider the ("unasserted") propositions, $A =$ "the electron is here" and $\bar{A} =$ "the electron is elsewhere". A statement like "I see the electron here" is in some sense positive, corresponding roughly to $\phi(A) = 1$ (i.e. to the affirmation of A). Similarly, "I see the electron there" is also positive, and corresponds roughly to $\phi(\bar{A}) = 1$. In contrast, a statement like "I don't see see the electron here" is in some sense negative, corresponding roughly to $\phi(A) = 0$, while "I don't see see the electron there" corresponds roughly to $\phi(\bar{A}) = 0$. In this way, the affirmation of A is decoupled from the denial of $\bar{A}$, and four distinct alternatives arise.

Physically, the motivation for this type of de-coupling stems from the phenomenon of *interference*, in which, for example, an electron can act as if it were "in two places at once" or "in neither place separately but both together" [11]. Anhomomorphic logics accommodate this type of behavior by admitting, for instance, the possibility that both of the propositions, "the electron is here" and "the electron is elsewhere" might be false. In other words, one can have $\phi(A) = \phi(\bar{A})$=0 in such logics, just as one can have $\phi(A) = \phi(\bar{A})$=1.[⋆] Thus, there is no necessary correlation in anhomomorphic logic between $\phi(A)$ and ϕ(not-A).

Consider now the sentence "The electron is not elsewhere" (or if you wish, "The world is not infinite"). With the possibility of an anhomomorphic logic in mind, we can see that its meaning is ambiguous. Perhaps it is simply uttering a proposition-in-itself, namely the above proposition $A =$ "the electron is here". On the other hand, perhaps it is trying to *deny* the complementary proposition $\bar{A}$, i.e. to express that $\phi(\bar{A}) = 0$. Or perhaps it actually means to *affirm* the proposition A, in which case one should render it as $\phi(A) = 1$. Accordingly, three different renderings of our sentence are possible: (i) A, (ii) $\phi(\bar{A}) = 0$,

[♭] In the nomenclature of [8], such a function is a "co-event". Its value $\phi(A)$ is commonly called the "truth-value" of A.

[⋆] In the "multiplicative scheme" of [8], only the first of these two possibilities can occur.

(*iii*) $\phi(A) = 1$. In classical logic, there is no good reason to distinguish these meanings, but anhomomorphically it is crucial to do so. I would claim that most of the confusion over the tetralemma could be traceable to a wrong resolution of unrecognized ambiguities of this kind.

Take for definiteness the question about whether the world is finite or not, letting A be the proposition "the world is finite", and $\bar{A}$ the proposition "the world is infinite", which is the logical negation of A.[†] The key question now is how to interpret the alternatives that constitute the tetralemma. If we interpret them as "propositions-per-se", we will immediately land back in the usual difficulties, since even within anhomomorphic logic, the propositions, "both A and $\bar{A}$" and "neither A nor $\bar{A}$" are still nonsensical. (More precisely, both of them reduce to the "zero proposition", and including them as alternatives adds nothing.) Moreover, we can gain no comfort from any ambiguity in the symbolic form of these propositions, since for example, the propositions, $\bar{A} \wedge \bar{\bar{A}}$ and $A \bar{\vee} \bar{A}$ are strictly equivalent within anhomomorphic logic (and both equal to zero).[♭] (For this reason the proposal in [4] to distinguish between these two forms of the fourth alternative, would get nowhere within anhomomorphic logic.) The symbolic notation thus exhibits the difficulties particularly clearly. At the same time, though, it conceals an implicit assumption that the alternatives are meant as propositions-in-themselves as opposed to affirmations or denials of propositions, for which the usual notation does not even provide.

However if — taking advantage of the ambiguity pointed out above — we read the four alternatives differently, namely as the four possible combinations of affirmation and denial of the two complementary propositions A (finite) and $\bar{A}$ (infinite), then the tetralemma makes perfect sense. In symbols, its four alternatives become then:

(1) $\phi(A) = 1$ and $\phi(\bar{A}) = 0$

(2) $\phi(A) = 0$ and $\phi(\bar{A}) = 1$

(3) $\phi(A) = 1$ and $\phi(\bar{A}) = 1$

[†] It is is not clear that an analysis in terms of complementary propositions A and $\bar{A}$ applies to the tetralemma about causation. See the appendices on this and related issues.

[♭] Here, I've used for brevity the common symbolism, $\vee$='or', $\wedge$='and'.

(4) $\phi(A) = 0$ and $\phi(\bar{A}) = 0$

On this exegesis, the tetralemma format is simply the one that you adopt if you are seeking maximum generality, once you have admitted that the denial of a proposition A need not entail the affirmation of its logical negation $\bar{A}$, and vice versa. To the extent that Indian thinkers in the time of Gotama were aware of this possibility, they would naturally have phrased their questions in "tetralemmatic" form.

But do we possess independent evidence that they were in fact aware of such more general forms of logic? Perhaps attention to the precise wording of the various catuṣkoṭi in their original languages would shed some light on this question, but we also have at least one indication which ought to be more independent of that kind of delicate textual analysis. In reference [3], Ruegg writes that Indian grammarians recognized a type of "absolute" or "pure" negation that did not entail "affirmation of the contrary". Could this type of "negation" be what we have been expressing as $\phi(A) = 0$ (and to which one should oppose the propositional negation that exchanges A with $\bar{A}$)? And if so, could its introduction have been their way to decouple the denial, $\phi(A) = 0$, from the affirmation, $\phi(\bar{A}) = 1$?

It would be very interesting to know what led ancient thinkers to recognize — if they did — the possibility of an anhomomorphic logic. They cannot have had access to the kind of technology that has led in modern times to quantum physics. Are there then other experiences that one could point to which were in fact available to them and to which anhomomorphic inference is more suited than homomorphic inference? If so, we might gain a better intuition for the microworld by ourselves paying more attention to those experiences.

Appendix I: A second-order application of anhomomorphic logic?

The analysis given in the body of the paper applies naturally to all but the last example cited at the outset (about causation), and it would explain why these questions were posed in fourfold form. But if that were the whole story, then the answer to any given tetralemma should be a selection of one of the four alternatives, (1)-(4), as the correct one. What then are we to make of the fact [3] that (unlike Gotama, who refused to answer at all) Nagarjuna in his writings rejected all four of the alternatives?

Perhaps, as is commonly suggested, Nagarjuna was simply trying to express a mystical rejection of analytical thought itself. However, it seems worth pointing out that anhomomorphic logic opens up another interpretation, perhaps consistent with the mystical one, but not really requiring it. Namely one can imagine that Nagarjuna's blanket denial represents a kind of "second order" application of anhomomorphic logic, one that reasons anhomomorphically, not just about "reality as such", but also about the logical processes employed to grasp that reality. When one is dealing with two propositions A and B (whether or not they are mutual negations so that $B = \bar{A}$), there will be four possible combinations of affirmation and denial of each, as explained above, i.e. four possible combinations of $\phi(A)$ and $\phi(B)$. When one is dealing with not two but four propositions, there will be 16 possible combinations, including the one that denies all four. Nagarjuna could be exercising this last option in respect of the four "second-order propositions" listed above as (1)-(4). It is natural to regard these as of second order since they are in some sense "propositions about propositions", speaking not just about "first order events", but about the values of ϕ on these "events". [Thus, for example, one might try to symbolize the denial of alternative (3) by writing $\phi(\,\phi(A) = \phi(\bar{A}) = 1\,) \;=\; 0$.]

Whether the tetralemma on causation also involves such a "denial at second order" seems uncertain. Perhaps it can be fit into the same mold as the others, if one treats "caused by self" and "caused by another" as mutual negations, and if one interprets its negative phrasing as the denial of all four alternatives. In that case, it would have the same character as the other Nargarjunian examples. However, such an exegesis would only be consistent if one assumed that every event (or "entity" in the quoted translation) must have a cause. Without that assumption, one has in this tetralemma merely an exhaustive set of alternatives, A_1-A_4, which happen to be four in number, not because they are formed from a single proposition A in the manner of the other tetralemmas, but because they represent all possible subsets of the two-element set {"caused by self", "caused by another"}. In that case, denying all the alternatives is only a "first order" activity, symbolizable as $\phi(A_1) = \phi(A_2) = \phi(A_3) = \phi(A_4) = 0$.

Appendix II: Contradiction and excluded middle

It is often felt that the tetralemma's phrasing conflicts with the so-called laws of contradiction and of the excluded middle. Is such a conflict still present on the anhomomorphic

interpretation? If not, this would furnish further evidence for the interpretation, inasmuch as — far from denying these two laws of logic — prominent schools of Buddhist logic seem to have embraced them, in particular the Madhyamika school [3].

What these laws mean in modern terms seems to be not exactly settled, but to the extent that they govern the formation of compound propositions from simpler ones, they hold automatically in anhomomorphic logic. For example not-not-$A = A$ holds for any proposition A.

To the extent, however, that they are understood as laws of inference, the possibility of conflict does arise. For present purposes let us take "excluded middle" to mean that 0 (false) and 1 (true) are the only values a proposition can assume, and let us take "non-contradiction" to mean that no proposition can take both of these values at once. Thus interpreted, both laws are honored in anhomomorphic logic, at least as it has developed so far. The former is obeyed because $\phi(A) = 0$ and $\phi(A) = 1$ are the only two values provided for, the latter simply because ϕ is assumed to be a function, as opposed to what used to be called a "multiply valued function".

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[11] The experimental grounds for such descriptions can be found in any number of textbooks. For a particularly vivid account see Section 1-5 in:
Richard P. Feynman, Robert B. Leighton and Matthew Sands, *The Feynman Lectures on Physics, vol. III: Quantum Mechanics* (Addison–Wesley, 1965)